Stat 446 Homework #9 Due: Friday, November 13, 2015.

1. In a water pollution study, a sample of lakes is selected from the 100 lakes in a study region by the following procedure. A rectangle of length L and width W was drawn around the study region on a map with the southwest corner being the origin $(0, 0)$. Random numbers l and w are generated in the intervals $(0, L)$ and $(0, W)$, respectively. The point (l, w) is located on the map, and if the point lies in a lake then the lake is selected. The process continues until $n = 8$ lakes are sampled. One lake was sampled three times, another lake two times, while the remaining three lakes were sampled once. The pollutant concentrations (in parts per million) were were 1.5 ppm for the lake sampled thrice (Lake 1), 2 ppm for the lake sampled twice (Lake 1) and 4 ppm, 5 ppm and 10 ppm for the other three lakes (Lakes 3, 4, and 5). The respective sizes of the lakes in km^2 were 2.5 (Lake 1), 2.0 (Lake 2), 1.5 (Lake 3), 1.0 (Lake 4), and 0.5 (Lake 5). In the study region, lakes covered a total of 200 km^2.

   (a) (2pt) Calculate the Hansen-Hurwitz estimate of the mean pollution concentration per lake in the population.

   (b) (3pt) Calculate the standard error of the Hansen-Hurwitz in (a).

2. A population consists of 6 units having y-values $y_1 = 4$, $y_2 = 4$, $y_3 = 5$, $y_4 = 8$, $y_5 = 10$, and $y_6 = 8$. A sample of size $n = 4$ is to be taken. There are 15 possible samples of size 4. These samples and their probability of being selected are given below.

Units	1,2,3,4	1,2,3,5	1,2,3,6	1,2,4,5	1,2,4,6	1,2,5,6	1,3,4,5	1,3,4,6
Prob.	.02	.02	.02	.04	.04	.04	.08	.08

Units	1,3,5,6	1,4,5,6	2,3,4,5	2,3,4,6	2,3,5,6	2,4,5,6	3,4,5,6
Prob.	.08	.10	.10	.10	.10	.08	.10

   (a) (1.5pt) What are the first order inclusion probabilities π_i for $i = 1, 2, 3, 4, 5, 6$?

   (b) (1pt) Suppose the sample with units $1, 2, 5, 6$ was taken. What is the Horvitz-Thompson estimate of the population total t?

   (c) (2.5pt) What are the second order inclusion probabilities π_{ij} for the fifteen i, j combinations?

   (d) (3pt) Suppose the $1, 2, 4, 6$ sample of units was taken. Calculate the estimated variance of the Horvitz-Thompson estimator of t (using equation (72) on page 142 of the course notes).