Sampling: HW9

Leslie Gains-Germain

1. (a) The Hansen Hurwitz estimate of the mean pollution concentration per lake in the population is 7.867 ppm. See my work below.

```
N <- 100
n <- 8
y <- c(rep(1.5, 3), rep(2, 2), 4, 5, 10)
p <- c(rep(2.5/200, 3), rep(2/200, 2), 1.5/200, 1/200, 0.5/200)
ybarhh.fun <- function(N, n, y.vec, p.vec) {
  1/(N*n)*sum(y.vec/p.vec)
}

ybarhh <- ybarhh.fun(N, n, y, p)
```

   (b) The standard error of the Hansen-Hurwitz estimate in part (a) is 4.716. See my work below.

```
thh <- ybarhh*100
var.ybarhh.fun <- function(N, n, y.vec, p.vec) {
  1/(N^2*n*(n-1))*sum((y.vec/p.vec-thh)^2)
}

se.ybarhh <- sqrt(var.ybarhh.fun(N, n, y, p))
```

2. (a) The first order inclusion probabilities are shown in the table below. I included the code for this part in the appendix.

	pi	pi.vals
1	pi_1	0.52
2	pi_2	0.56
3	pi_3	0.7
4	pi_4	0.74
5	pi_5	0.74
6	pi_6	0.74

   (b) The Horvitz-Thompson estimate for the population total is 39.16. My work is shown below.

```
y <- c(4,4,10,8)
pi <- c(pi.1, pi.2, pi.5, pi.6)
t.ht <- sum(y/pi)
```

(c) The second order inclusion probabilities for the fifteen i, j combinations are shown in the table below. I included the code for this part in the appendix.

	pi	pi.vals
1	pi_12	0.18
2	pi_13	0.3
3	pi_14	0.36
4	pi_15	0.36
5	pi_16	0.36
6	pi_23	0.36
7	pi_24	0.38
8	pi_25	0.38
9	pi_26	0.38
10	pi_34	0.48
11	pi_35	0.48
12	pi_36	0.48
13	pi_45	0.5
14	pi_46	0.5
15	pi_56	0.5

(d) The variance of the Horvitz-Thompson estimator of t is estimated to be -29.40 using Equation (72) on page 142 of the notes. See my work below. Clearly the variance estimate is not accurate, because a variance cannot be negative. In this case, it would be more appropriate to use the Brewer and Hanif variance estimate.

```
y <- c(4, 4, 8, 8)
pi <- c(pi.1, pi.2, pi.4, pi.6)
pi.order2 <- c(pi.12, pi.14, pi.16, pi.24, pi.26)

var.tht <- (1/(pi.1^2) - 1/pi.1)*y[1]^2 +
  (1/(pi.2^2) - 1/pi.2)*y[2]^2 +
  (1/(pi.4^2) - 1/pi.4)*y[3]^2 +
  (1/pi.6^2 - 1/pi.6)*y[4]^2 +
  2*((1/pi.1*1/pi.2-1/pi.12)*y[1]*y[2]+
  (1/pi.1*1/pi.4-1/pi.14)*y[1]*y[3] +
    (1/pi.1*1/pi.6 - 1/pi.16)*y[1]*y[4] +
    (1/pi.2*1/pi.4-1/pi.24)*y[2]*y[3] +
    (1/pi.2*1/pi.6-1/pi.26)*y[2]*y[4] +
  (1/pi.4*1/pi.6 - 1/pi.46)*y[3]*y[4])
```

R code appendix

```
pi.1 <- 0.02*3+0.04*3+0.08*3+0.10
pi.2 <- 0.02*3 + 0.04*3 + 0.10*3 + 0.08
pi.3 <- 0.02*3 + 0.08*3 + 0.10*4
pi.4 <- 0.02 + 0.04*2 + 0.08*2 + 0.10*3 + 0.08 + 0.10
pi.5 <- 0.02 + 0.04*2 + 0.08*3 + 0.10*4
pi.6 <- 0.02 + 0.04*2+ 0.08*3 + 0.10*4

pi <- c("pi_1", "pi_2", "pi_3", "pi_4", "pi_5", "pi_6")
pi.vals <- c(pi.1, pi.2, pi.3, pi.4, pi.5, pi.6)

require(xtable)
xtable(cbind(pi, pi.vals))
```

```
pi.12 <- 0.02*3 + 0.04*3
pi.13 <- 0.02*3 + 0.08*3
pi.14 <- 0.02 + 0.04*2 + 0.08*2 + 0.1
pi.15 <- 0.02 + 0.04*2 + 0.08*2 + 0.1
pi.16 <- 0.02 + 0.04*2 + 0.08*2 + 0.1
pi.23 <- 0.02*3 + 0.1*3
pi.24 <- 0.02 + 0.04*2 + 0.1*2 + 0.08
pi.25 <- 0.02 + 0.04*2 + 0.1*2 + 0.08
pi.26 <- 0.02 + 0.04*2 + 0.1*2 + 0.08
pi.34 <- 0.02 + 0.08*2 + 0.1*3
pi.35 <- 0.02 + 0.08*2 + 0.1*3
pi.36 <- 0.02 + 0.08*2 + 0.1*3
pi.45 <- 0.04 + 0.08 + 0.1*3 + 0.08
pi.46 <- 0.04 + 0.08 + 0.1*3 + 0.08
pi.56 <- 0.04 + 0.08*2 + 0.1*3

pi <- c("pi_12", "pi_13", "pi_14", "pi_15", "pi_16", "pi_23", "pi_24", "pi_25", "pi_26", "pi_34", "pi_35", "pi
pi.vals <- c(pi.12, pi.13, pi.14, pi.15, pi.16, pi.23, pi.24, pi.25, pi.26,
             pi.34, pi.35, pi.36, pi.45, pi.46, pi.56)

require(xtable)
xtable(cbind(pi, pi.vals))
```