20.4 Multilevel Power - Simplified

Set number of groups and number within groups.

$$\mathrm{SE}(\bar{y}) = \sqrt{\frac{\sigma_y^2}{n} + \frac{\sigma_\alpha^2}{J}}$$

Recall split plot design and corCompSymm:
$\rho = \frac{\sigma_\alpha^2}{\sigma_y^2+\sigma_\alpha^2}$ so let $n_i = m$ and
$\mathrm{SE}(\bar{y}) = \sqrt{\frac{\sigma_{total}^2}{Jm}[1+(m-1)\rho]}$.
NYC example they pick $\rho = 0.15$. Reasoning?

20.5 Simulation of Power

Continuous response

$y_{jt} \sim N(\alpha_j + \beta_j t, \sigma_y^2)$ is the sqrt CD4% of children w/out zinc supplement. Using lmer we get
$\widehat{\sigma}_y = 0.7$, $\widehat{\sigma}_\alpha = 1.3$, $\widehat{\sigma}_\beta = 0.7$, $\widehat{\rho} = 0.1$, with means for intercepts and slopes of 4.8 and -0.5, respectively.
Step 1: Does the fitted model generate data which looks like the original data? Fig 20.5.
Step 2: Find sample size to detect a shift in mean for slope of $\underline{+0.5}$ with 80% power at $\alpha = .05$. $n_j = 7$ for each kid.
Assume MVN for $\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix}$ with correlation ρ and shift γ_1^β under treatment.

Quick & Dirty

β_j's are data points – one per kid – with SD $= .7$. To detect a shift of Δ, we need

$$J \geq \left(\frac{2\cdot 2.8\sigma}{\Delta}\right)^2 = \left(\frac{2\cdot 2.8}{\Delta}\right)^2 (\sigma_\beta^2 + \frac{\sigma_y^2}{SSX}) =$$

$$\left(\frac{2\cdot 2.8}{\Delta}\right)^2 (0.7^2 + .7\times 1.13)^2 = 150$$

R function to simulate fake data

Build function taking J, K $= 7$ as inputs returns data matrix of fake data.
Generate 1000 datasets, test each to see if H_0 gets rejected. Keep track of rejection rate $=$ power. power we need 130 kids.