

## Some terms

Square of L2  $||\theta||_2^2$

Sum of all squares

L2 norm  $||\theta||_2$

Square root of sum of all squares

Collaborative filtering - collaborative filtering is a method of making automatic predictions (filtering) about the interests of a user by collecting preferences or taste information from many users

Assumes that one person has same opinion as the other person

Inductive learning Assumption - after seeing many training examples result in high accuracy on unobserved examples

Hypothesis approximates well

average hypothesis only depends on  $H$ , does not always lie in  $H$

$$H_2 \subset H_{10}$$

Target function  $y = f(x)$

Unbiased learner cannot generalise  $\rightarrow$  inductive bias

$h$  is +ve when  $h(x) = 1$

$\geq_g$  more general than or equal to

$$h_j \geq_g h_k \equiv \forall x \in X (h_k(x) = 1) \rightarrow (h_j(x) = 1)$$

Not a total order as there may have the same specificity but different hypothesis (maximally specific  $h$ )

Consistency:  $h$  is consistent iff  $h(x) = f(x)$

$$h(\langle 0, 1 \rangle) = 1 \neq f(\langle 0, 1 \rangle) = 0.$$

### Proposition 1

$h$  is consistent with  $S$  iff every +ve training instance satisfies  $h$  and every -ve training instance does not satisfy  $h$ .

### Proposition 2

If  $h$  belongs to  $H$ , then it is consistent

## Naive Bayes

### Bayes Rule

There can be relatively lesser occurrences of one class

prior odds / likelihood = posterior odds

Bayesian thinking

How likely is something not going to work?

MATH: BUSINESS	
Prior odds ratio	1 : 10
Likelihood ratio	75 : 15
Posterior odds ratio	1 : 2

Hold your beliefs, shift your beliefs as u encounter with the world

$$P(y|x) = \frac{P(x|y) \times P(y)}{P(x)}$$

$$P(X_1 = 0, X_2 = -1) = P(X_1 = 0 | C_1) \times P(X_2 = -1 | C_1) \times P(C_1) + P(X_1 = 0 | C_2) \times P(X_2 = -1 | C_2) \times P(C_2)$$

**a priori** - equally probable

Naive - Assuming that features are conditionally independent of each other

Insensitive to the number of training examples

Calculate which is more likely

$$1. P(Outcome|x) = P(X_1|Outcome) * P(X_2|Outcome) * P(Outcome)$$

$$2. P(Not Outcome|x) = P(X_1|Not Outcome) * P(X_2|Not Outcome) * P(Not Outcome)$$

Calculate probability outcome occurs

$$\#1 / (\#1 + \#2)$$

### Perceptron classifier

$$x_0\theta_0 + x_1\theta_1 + x_2\theta_2$$

of the previous weight vector

first instance will compute based on initial vector

For -1, first point checks == -1, != 1 for the points after

Update weight = - (x × learning\_rate) for the misclassified instance

## Supervised learning

Identifying objects

Output would be a continuous value

Given many emails, you want to **determine** if they are Spam or Non-Spam emails.

Given historical weather records, **predict** if tomorrow's weather will be sunny or rainy.

## Unsupervised learning

No labels (no output) Kmeans

Input: Given a set of data points  $P$ , number of clusters  $k$

1. Randomly pick  $k$  points from  $P$  as centers  $c_j = 1..k$
  2. Iterate (until max-iterations or  $c_j$  no longer changes up to some threshold)
- Assign each point to nearest center:  $y_i = \arg \min \| p_i - c_j \|^2$
  - $c_j$  = Re-estimate each center as mean of points assigned to it

Given a set of news articles from many different news websites, find out what are the main topics covered.

From the user usage patterns on a website, figure out what different groups of users exist.

## Motivation

Importance of models, learning a probability distribution

Can learn an arbitrary distribution, not all distribution can be described by the same model

Restricted so small number of parameters to be learned in a reasonable amount of time

Can tell water drops and rain

Shaking the toy causes the rattling noise

Get a good approximation on the cost and structure using limited amount of resources

Data is generated by a physical process

Only wants to capture the signal

Intrinsicity - How many independent dimension are there in each layer

### **PCA**

$\hat{z}$  is the reconstruction and  $v$  is the principal component

Chooses orthonormal bases

Ellipse is defined by the eigenvectors and values

### **SVD**

$V$  - principal components

$\Sigma$  - eigenvalues

Top left corner will have the largest eigenvalue

Bottom right corner have the least significant eigenvalue

0 on off-diagonal

$U$  - projection of data

### **Clustering**

k means++ default initialization by sklearn

Generalized EM when the approximation is intractable

Incremental EM estimates on small number of points, like difference between batch gd and sgd

### **RL**

Key usage: Planning problems, app placements product recommendations

### **Goal**

Low risk when performing actions

Goal of markov property: operate in ease

choices that maximise the outcome

Precision vs control

Decisions we make the optimize

Prediction allows us to evaluate how good a policy is for a state space  
Control  
Reward at any step is optimal, highest reward  
continuous MDP - common in robots  
No supervisor, only reward signal  
Reward is a scalar feedback  
whether one state vs the other is preferred

RL Challenge: Does not have a good idea if it is right to do, which one  
is feasible, figuring out rules, which gives the best reward  
Given action will give observation and reward  
Does not know the rules

Planning Challenge:  
A\* search - Going through the search space to figure the appropriate action  
what if i did this, what kind of consequences  
Rules are known  
Uses a search space which requires tree search

Action affects the subsequent states  
Action  $\rightarrow$  Observation (what state it is in)  $\rightarrow$  Reward  
time  $t$  will not take the final reward but  $t - 1$   
Sometimes actions affect the observations  
May contain irrelevant information of the state at time  $t$   
Although we have large field of vision, our neural cortex is trying to observe  
 $t$  changes in the environment  
Function of history is not visible to the environment and is specific to agent  
State - what information that we have in the brain is what we are going to  
need to predict the future

### **Fully observable**

The state can be represented different based on how we want the agent to  
see the sequence  
congruent between the agent and the environment

### **Partially observable**

Align/ localize itself based on the map e.g. game  
Represented state can influence the agent  
Depending on the agent, we can end up in a different state

E.g. if we want to turn the rotor around the corner, it may turn out not to behave that way

Recurrent neural networks

Using ReLu to construct each state in deep learning, easy to compute as no exponential operations

## **Policy**

Look through the mapping function

Deterministic  $s_1$  to  $a_1$

Stochastic Take a particular action with a probability

Value function - future reward

Sum of expected reward, reward farther in the future will have gamma to discount it

Model - probability that it will go to the next state, sum over all possible actions has to be 1

## **Agent taxonomy**

Value Based

Policy - in this state we take a particular action

Actor Critic - model based

## **Exploration vs Exploitation**

Going to the restaurant that i like

Exploration - discover more information about the environment

Exploitation - Exploits known information to maximise reward

## **State transition probability matrix**

Represented with the symbol P

some of the values are 0 in the transition matrix if it cannot move to a particular state

episodes are finite

there are always probability attach to the state, eventually it will reach the terminal state

## 0.1 Markov Reward process

Attach the reward and discount

Differentiating reward, create two different states with different reward

$E(r_{t+1} | \cdot)$  immediately receive the reward in next time step

Discount factor  $\gamma$

Near sighted - given where i am now where should I go

Far sighted - sum all the rewards add all of them, favor longer samples (should have more reward)

### Role of Discount

Infinitely sample the cycle, a lot of uncertainties on the future

## 0.2 Bellman equation

Reward based on executing the state

### Value iteration

Idea of value iteration

Instead of looking what policy to take, look back and determine what is the best action brought us to get the best value function

Emits the observation and reward

Recover information or  $t$  and  $r$

Value iteration is a simple form of policy iteration

Use optimal  $V^*$  to create back the policy

## Q learning

Use  $q$  value to engineer the policy (monodically increasing)

$r(s, a)$  reward at comes at next state

Higher value of  $q$  means we see it a lot of times, smooth it over time

## Neural Networks

$\Theta$  dimension =  $(d_{l-1} + 1) \times d_l$  where  $l$  is the layer before the activation function (denote by  $x$  or  $a$ )

$g$  function will be  $\text{relu}$  for deep learning and  $\tanh$  for NN

For the hard threshold, when it is close to 0 it becomes a linear line

$$a = g(s) = g\left(\sum_{j=0}^{d[l-1]} \Theta_{i,j}^{[l]} x_j^{[l-1]}\right)$$

1 represent correct classification

-1 to 1 step function

Going from -1 to 1  $h = [1; -1; -1]$  where 1 represents the output unit to be positive (for eg first from the top  $a_1^{[l]}$ )

Number of kernels  $N = 2$  for gray, 6 for RGB

$W_c$  = Number of weights of the Conv Layer.

$B_c$  = Number of biases of the Conv Layer.

$P_c$  = Number of parameters of the Conv Layer.

$K$  = Size (width) of kernels used in the Conv Layer.

$N$  = Number of kernels.

$C$  = Number of channels of the input image.

$$W_c = K^2 \times C \times N$$

$$B_c = N$$

$$P_c = W_c + B_c$$

## Motivation

In traditional ml, it is difficult to handcraft features, neural networks are good in feature learning

NN generalise badly

As the number of parameters (weights and layers) increase in a neural network, it can easily overfit the data.

Backpropagation is a complicated and tedious process. Moreover, the surface plotted by the error of a neural network in non-convex and high-dimensional and can consist of many hills and valleys - making it hard to reach the optimal set of weights.

Do all the neurons in a layer always share the same bias value?

Yes, that is correct. The bias value is the same value ( $x_0 = 1$ ) for all units at all layers, because we are going to learn the particular weight for the bias for each individual unit,  $\theta_0$ , separately. It's just pictorially cleaner to think that each layer has a separate bias unit, but it doesn't matter, because the



value of the bias unit is always (by convention) set to unity, such that the individual weights  $\theta_0$  (or in the NN matrix notation  $\theta_0^{(l)}$ ) are the controls for the biases.

Random initialisation of weights for sign function  
the  $\text{sign}()$  function takes different constant values in different regions and therefore has gradient 0.  
the  $\text{sign}()$  function is discontinuous at  $x=0$ .

Stack in together in a linear unit  
Generalise things that are out of sample

Regularisation for NN  
Weight decay square of the weights are small so it is more toward linear regression (L2 regularisation)

Is the Soft Weight Elimination function a Cost Function?  
No, but it closely related. It is a regulariser, a separate, artificial part of the augmented loss function, as what we learned in Week 05. We denote regulariser functions as  $\Omega(\theta)$ .

Would it have been a good idea to stop the first time the validation error goes up?  
The validation error might fluctuate and therefore stopping it the first time it goes up will not give the optimal result. One can consider using a moving average or some other metric.

## Deep Learning

Capability to take in multiple input and give multiple outputs

Dropout - Generalise well rather than approximate well and overfit ReLU - Vanishing gradients  
Bias- Variance tradeoff - Not generalizing due to too many parameters, model representation fit the particularities. Not fitting the additional noise.

Higher budget will have a wide set of solution and encompasses  $\theta_{lin}$  (the ideal solution) such that no regularisation is used

$\theta_{reg}$  is proportional to the size of the circle, larger C causes smaller regularisation, smaller circle leads to larger gradient

GAN create adversary to help u train better

E.g. mock test paper to do better in exam

RNN time series inputs are correlated to each other

E.g. determiner followed by noun

Weights of output from one node to another model layer of another time step is the same

Model constraint input uses the same weights so that we can fewer parameters than 2

1 to Many: Image captioning

Many to 1: Sentiment Classification

Many to Many: NMachine translation Many to Many (time sensitive): Stock market prediction

## **RNN**

Tries to get the softmax chance higher

### **Truncated backprop**

Instead of going to the start, backprop only to the start of the partition block

## **CNN**

Finding context where locality is important or somehow related to the things nearby

Key idea: Make it invariant to position

E.g. Non-linear transformation to find a horizontal lines

AND and OR gate to form a high level representation

Image that maximises the strength of the corresponding units activation

Tied samples over all the subsamples of the image  
Sliding windows to build new feature maps  
Sub-sampling (Pooling layers)  
Stride - ignore intermediate squares Max pooling gives the maximum of the units based on the filter size

Layers at the beginning of the network has smaller activations as it is the problem of vanishing gradients (gradient is very small at end layers) stacking all of the layers together, when run on training (after pre-training)  
Layers get more positive or negative further down the network as compound-ing signals get more extreme, causing vanishing problem, doesn't give the same magnitude of gradient at different parts of the network  
- Learning rate has to compensate  
- Weight decay regularisation

Or when values are close to 1, then we have issues with learning  
ReLu loses representation

softmax cannot be used in nlp to predict next word as it can be computationally expensive  
hierarchical softmax can be computed in parallel For unsupervised pre-training, weights by supervised learning and fine tuning it  
Final softmax layer needs supervision so we can only pre-train the middle layers  
GPU allows operation to happen simultaneously  
each part of the GPU computing one feature map

performance will only get better for resnet with more layers as the residual will be learned by the next layer only if in the previous layer the residual is not captured.

## **Sparse Autoencoders**

Have to learn a smaller set of weights like a lossy compression algorithm  
L1 to encourage sparsity

## **Dropout**

Idea: Random forest feature projection  
Keep input fixed and only the output at other layers change

Rotate over multiple batches

Only certain neuron get the weight which can help in regularisation

### **Transfer learning**

Unsupervised feature extraction layers, or bottom and middle layers to use for new tasks

Hyperparameter to decide where to cut off softmax, FC, pooling

Not useful for

VGG net has learned color or natural images that are not part of the domain

### **Representation Learning**

Uses embedding in the compressed layers

### **SVM**

Another linear unit

## Find S

1. Initialise h to most specific h
2. If the positive instance differs, then replace with ?

## Version space

List all hypothesis, remove any hypothesis that is inconsistent with any training example  $h(x) \neq f(x)$

## Candidate Elimination

prefers positive examples over negatives as there can only be one maximally specific hypothesis in each iteration ? don't care

no value

Set  $S_0 = \langle \emptyset \emptyset \emptyset \emptyset \emptyset \emptyset \rangle$

$G_0, G_1 = \langle \text{?????} \rangle$

For the negative example,

Create h from the specific hypothesis when it differs based on previous instance

Remove those that are inconsistent from the previous general hypotheses

Keep S to the previous instance S

For the positive example,

replace with ? when it differs

## Cost functions

Logistic regression

Is binary classification

Need meta strategies for multi class and it chooses the model that has the highest confidence in predictions

Only gives values 0 and 1 so it is not a good result

$$J(\theta) = \frac{1}{m} \sum_{j=1}^m \ln(1 + \exp^{-y^j \theta^T x^j})$$

Linear regression

$$J(\theta) = \frac{1}{m} \sum_{j=1}^m (\theta^T x^j - y^j)^2$$

## Linear algebra

$$\text{trace}(\text{AB}) = \text{trace}(\text{BA}) = \text{trace}(\text{I}_{n+1}) = n + 1$$

*Idempotency*

$$H^2 = H$$

$$(I - H)^2 = I - 2H + H = I - H$$

## Pseudoinverse

$$(X^T X)^{-1} X^T$$

$$\theta = X^+ y$$

$$X = \begin{pmatrix} 1 & .. & .. \\ 1 & .. & .. \\ 1 & .. & .. \end{pmatrix}$$

## Maximizing likelihood

$$\text{Minimize cross entropy} = - \frac{1}{m} \sum_{j=1}^m \ln g(y^j \theta^T x^j)$$

## Ridge Regression

$$\frac{1}{m} \sum_{i=1}^m (y_i - w^T x_i)^2 + \lambda \|w\|^2$$

$$-2X^T(Y - X * \theta) + 2 * \alpha * \theta$$

## Noise

Not deterministic only deterministic for  $f(x) = 0$

$$y - f(x)$$

$$P(y|x)$$

$$\text{Noisy target} = E[y|x] + (y - f(x))$$

## Stochastic noise vs Deterministic

To reduce stochastic noise, the only way is to re-measure  $y$

To reduce deterministic noise, change  $H$

## Underfitting and Overfitting

Why do we regularize? We fit the data too much, we are fitting to the noise, balance fitting to the observed data. If we fit the data too well, the performance can be poor called overfitting. Overfitting can happen even if the data is not noisy.

**Overfit** - high level of noise, high complexity of  $f(x)$

**Underfit** - high bias and low variance

Reduce bias by increasing complexity of hypotheses

simple representation of more complex reality

## Non-linear transformation

Transform this non-linear equation to a linear one:  $y = ax^b$ ?

Under the condition  $y > 0$  and  $x > 0$

$$\ln(y) = \ln(ax^b)$$

$$\ln(y) = \ln(a) + \ln(x^b)$$

$$\ln(y) = \ln(a) + b\ln(x)$$

let  $y' = \ln(y)$ ,  $x' = \ln(x)$ ,  $a' = \ln(a)$ , the equation can be written as  $y' = a' + bx'$

## Decision trees

Diversifying dataset similar to validation set

Piecewise linear but ends up with a complex function

Decision tree uses the entire hypothesis space

Can overfit so we need pruning

Divide and conquer log  $n$  to find  $\theta$  for decision stump (one node of decision tree)

If it is in the wrong class, then the weight of the misclassified is increased, shrink those that are correct

Random forest (Feature subspaces)

Does not use all the data

To classify new data,  
Same as any other ensembling method  
Give the input to all the trees  
And take the vote

Bagging - Sample with replacement  
Diversity in features  
Some subset of features

Ensemble usage  
Continuously verify the presence of the user

Adaboost

1. Find a weak estimator
2. Update the weights, to classify incorrect inputs to be correct
3. Find another estimator

If we take the best hypothesis, error rate is larger than 0.5, then no estimator will give a better result

Based on probabilities, not all positives/ negatives

To lower variance:

DTL - pruning

Bootstrap - Average to lower variance

To have more variation:

Feature projection for Random forest

### **0.3 Ethics of ML**

Sensitive attributes: race, ethnicity, sex

Fine line between personalization and in building our bias in the system

Can look at correlations on search data

Sometimes created by people themselves on social media, what people like,  
or malicious users when the user is not at home

Quantifying the risk



Techniques that can preserve those properties, privacy-preserving, fair and explainable

Defense mechanisms - don't want to share the data, share compressed data so everyone can benefit from the data without leaking the information

Trustworthy ML - large capacity if u expect the model to do very well, vulnerable to data leakage or manipulation attacks.

When is trained to be more fair, it can craft more data in the final model compared to the model that is not fair, data poisoning

Robust to data poisoning, can i trust in a typical setting

Critic-ability, can it convince me? Is it trustworthy?

Fairness, privacy and robustness

Engineering bias in the machine learning algorithm

Hiding corrupt practices behind mathematics

Going through the data to double/ triple check

Triplet ratings, build a machine and scale it in a massive international scale

ML vs Mathematics - Predict people instead of markets

Algorithms are nothing more than opinions embedded in code

1. Data u train the algorithm on
2. Objective / Penalty at stake

Make more meals that are success in the past

There is an agenda that is imposed, person building the algorithm defines success, but may not be success for the stakeholders

Widespread - Mysterious - formula is not available, not aware they are being spotted. Constituency right to know what is right

Destructive - Ruin people's lives unknowingly, opportunities taken from them, destructive feedback loops

If u label a teacher as bad if the students' proficiency scores were bad

Value added teacher model - It is better if they score more than expected, dinged for the points lower than expected

Students should score higher in their finals, punished for previous teachers' teaching (elevated scores)

Teachers should be able to understand how the scoring work

Misusing personality tests, algorithms that sort through resumes

Looks at who are successful in the past, picking out past patterns and re-

peating them

Non violent crime are more prevalent and predictable

Pre-emptive punishing people for things they haven't done

Embedded in a risk score

Go back to prison due to high risk scores

Same algorithm, different use case can be very different

Can be anonymous but related to the person in a precise way

Build a model by construction is fair, alter the algorithm for fairness

Big data is not a silver bullet, is just a tool

How many miles for a self driving car for the teacher evaluation model

Fairness to auditing

Restrict people from using decision trees, if u have to explain why

When to disclose? when it involves human preference and taste, subject to vetting

Automatic bias, easy to use machine learning. But used only when there is no infrastructure.

Group attribution, generalised to the distribution

Implicit bias, not taking into fact of missing information

## **EM - Expectation Maximization**

Investigate one of the things (latent variables, membership) and hold the others constant (model)

# EM for GMMs

```

for  $k = 1$  to  $K$  do:           # initialize randomly
     $\mu_k \leftarrow$  some random location,  $\sigma_k^2 \leftarrow 1, \pi_k \leftarrow 1/K$ 

repeat until converged:
    for  $j = 1$  to  $m$  do:
        for  $k = 1$  to  $K$  do:           # E. Step. Compute Expected Assignment
             $z_k^{(j)} \leftarrow \pi_k [2\pi\sigma_k^2]^{-\frac{1}{2}} \exp\left[-\frac{1}{2\sigma_k^2} \|\mathbf{x}^{(j)} - \mu_k\|^2\right]$ 
             $z^{(j)} \leftarrow \frac{1}{\sum_k z_k^{(j)}} z^{(j)}$ 

        for  $k = 1$  to  $K$  do:           # M. Step. Re-Estimate the Model
             $\pi_k \leftarrow \frac{1}{m} \sum_j z_k^{(j)}$ 
             $\mu_k \leftarrow \frac{\sum_j z_k^{(j)} \mathbf{x}^{(j)}}{\sum_j z_k^{(j)}}, \sigma_k^2 \leftarrow \frac{\sum_j z_k^{(j)} \|\mathbf{x}^{(j)} - \mu_k\|^2}{\sum_j z_k^{(j)}}$ 

    return  $\mathbf{Z}$ 

```