#### Some terms

Model-free: not learning the transition and reward, only need to get observations from the real world or simulator Prediction to control - prediction into a control policy

#### Monte Carlo

$$\pi_{UCT}(n) = argmax_a(Q(n, A) + c\sqrt{\frac{log_2(N(n))}{N(n, a)}})$$

$$V(A) = \frac{\#A|A=1}{\#A+\#B}$$

## Generalized policy iteration

Take argmax get the policy Run the policy and estimate a new Q function Don't have to evaluate the policy fully Do one step and update one data (Q function) Update greedily wrt the Q function By being greedy (get a new policy) and improve it

Monte Carlo

Take many trajectories and average the trajectories
Temporal difference learning
Immediately update our prediction just after one step
But there is no return from the long trajectory
Current return of u
U is the estimate
Monte Carlo error: considers all the previous steps

TD error: how difference are my difference at this time step Do gradient descent and minimize squared loss

Gradient is the direction of maximum increase

So we need to find the opposite to find the min which is why it is called gradient descent

### n step TD

better tradeoff between bias and variance When n is  $\infty$ , it becomes Monte Carlo Average can somewhat reduce variance (need less data to get good estimate, learn faster as it converges faster)

 $\mathrm{TD}(\lambda)$ n step is weighted by  $\frac{\lambda}{n-1}$ 0 forget the future take 1 step 1 close to 1 important for a long time We are averaging the estimates (move expensive)

#### ADP

Run vi or pi not trying to estimate the model only the values need to have simulator of the real world to go on with RL Build a model using function approximation What is the transition function to use? What reward to use

Successful has been parameterised Q function, supervised learning etc. Model-free are easier to work with

We usually mix MC with TD in practice, less estimate and not much variance

## TD

Assumption is it is always markovian in real RL tasks in the environment TD learning applies assumption to estimate Initializer can be 0

A, 0, B, 0

Markovian assumption, if it sees A and B then it gives the reward of 0. Does not give the right values of the states because it makes an assumption that is not true of the environment

If it is true, then it will converge to correct value functions

Update equation

$$V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(R(s) + \gamma V^{\pi}(s') - V^{\pi}(s))$$

 $1-\alpha$  can be obtain by extracting -  $V^{\pi}(s)$  out and simplifying 1 -  $\alpha$  can be obtain by extracting -  $V^{\pi}(s)$  out and simplifying

# **SARSA**

TD control

Uses GPI, on policy (improve the same policy)

 $target = R(s) + \gamma Q(s', a')$ 

SARSA will converge to optimal policy on tabular cases

MC on the windy gridworld

Converges slower but eventually reaches optimal

Have to use old policy until end of the trajectory

SARSA changes policy at every time step, may wander around at the start, but will continue learning

 $\epsilon$  greedy: Take the action suggested by the Q function or random exploration

Improving the current policy with  $\epsilon$  greedy, when present does not converging to the optimal policy

trying to improve the policy

environment might change, world isn't quite a MDP, it can be robust, model may change over time

Might still fall off the cliff

Predictive problem learning (passive TD) has some restrictions

Update only at next state

## Q-learning

Only have the q function (acts as policy on its own)

Not in SARSA:  $\gamma$  max a

best action at the next action of Q function

E.g currently have a deployed policy, at wild change the policy with SARSA, train the target policy

Behavior policy can be some other policy that is running, generate data to train the policy

If stop learning then Q-learning will outperform Sarsa

Similar concept to  $\epsilon$  greedy Let's go to part of the space that is exploit less Issues when state space is large

Update policy then take  $\operatorname{argmax}$  - given the current policy from the Q function, estimate a new Q function and construct a new policy by taking  $\operatorname{argmax}$  of Q function

Q learning can learn off policy (doesn't follow the policy given, uses different policy)

Take the max over the next state

Minimizing the difference between the target and the current estimate If the count is small give bonus, if we try many times reduce bonus

Why don't u update all the Q at each state?

Update is only done for all the state-action

Not the final Q-table, it will interact with the environment and update the table

What if the transitions are non-deterministic?

probability to move to a state

Even it is probability, it will move to that state, no longer a probability so take Q value of that state, expected Q learning

# **Function approximation**

Number of states are huge, huge state space Approximate (estimate) Q value with function approximator

Figure out what are the important features and extract out to define each state E.g. for a chess, queen is worth 5 points, pawn worth, knight worth RL will learn the parameter vector  $\Theta$ 

Allows generalisation observations so that it can work on entire state space Small fraction of the state

Applications: Go, Deep neural network, OpenAI5

MCT Search on simulator, for online search value function and combine with MCTS

Do learning with function approximation

Runs in the form of ridge regression

#### Online learning

One example, learn an example and update the weight vector Update theta to improve estimate, step in the direction of the negative gradient

 $\hat{U}(s)$  function approximator, estimate from the function target  $u_j(s)$ At every time step it is the sum of the reward to the future, measured from the environment

It could be the q function Semi gradient - don't include in the computation gradient wrt the parameter  $\Theta$  we treat  $\gamma \hat{U}(s')$  as a fixed target When we move on-policy we lose most of our policy, have to do lots of tuning e.g. doesn't converge Deep Q-learning

Techniques to do better: Experience replay large buffer D, old sample does not mix with the recent one

 $(s_t, a_t, r_{t+1}, s_{t+1})$  max over a without the k learn and freeze it and use as target  $\theta^ \theta^-$  Q function from current - c steps

High profile achievement as one architecture can work across 50 atari games Set a new target and change every c steps

Extract with a linear function of features
E.g. position of the ball
Need two frames to determine the direction we are going
provably good - how long would it take to go to epsilon

#### Policy search

Mapping from state to action Drop the assumption that policy is of Q(s, a) Deep learning is continuous Max function is not differentiable everywhere max function isn't linear

Gradient descent can differentiate that Actions are discrete can't find gradient wrt a

#### POMDP

Approximate sometimes doesn't work but usually work well

P space hard (as hard as any problem that can be solved with polynomial space, consist of NP)

If there is P(b', a) and P(b), we have a MDP

Function from belief to action through an observation b - o - a

b input vector [b(1), b(2), b(3)],  $\alpha$  is the weights or coefficient of the linear function (alpha vectors) then it becomes a linear function  $b(1)\alpha(1) + ...$  $b(n)\alpha(n)$ 

 $\alpha$  (1) is the expected return at the state

Optimal policy to choose conditional plan

$$U(b) = max_p \ b \cdot \alpha_p$$

Do induction on the depth of the tree

wrt to state action, then if u take an action then u go to a state

Zero step plan only take into regard the state, so are initialised with rewards at the state

Each 2-step plan can draw a straight line

Each region has a corresponding value function

Converge into a convex function

Construct a depth d plan

$$\Gamma ransition = \sum_{s'P(s'|s,a)}$$

Transition =  $\sum_{s'P(s'|s,a)}$ Average over observations (with probability of receiving o) =  $\sum_{e} P(e|s')$ 

Probability to use alpha vector at  $s' = \alpha_{p,e}(s')$ 

$$\alpha_p(s) = R(s) + \gamma(\sum_s' P(s'|s,a)$$
 Possible subtrees that can put in position d - 1 action

- 1. Use initialised values to compute alpha vectors
- 2. Remove dominated vectors for depth 1 (Largest difference between belief to small enough)

3. Use existing alpha vectors to compute alpha vectors at depth 2 and so on

Know the belief instead of being perfect of knowing the state Be clever on which subset to pick

Intractable - no of state variables grow Also never reach same belief twice, have to enumerate through all the states, a lot of multiplications and integrations

Belief tracking is expensive, generating beliefs is continuous

Instead use history, sample some states for approximating beliefs Tree search until a certain depth, build the tree from simulation Particle filtering Have a simulator, so represent a set of states

If initial belief is bad, then it might affect the approximation Deterministic will take longer time to compute

# Partial Observability

Fog of war in DoTa single network that controls all 5 players each agent runs independently Break into hierarchical operation RNN to learn what is good to remember and propagate (hidden unit) to memory

Each player has own policy (POMDP)
For model based how is it well defined?
Model of the opponent becomes an environment
For a 2 player, will converge to an optimal structure
Value iteration for MDP, what u observe is different from what the enemy observe

Does not work with multi-agent, belief is no longer meaningful but policy still holds

Optimal collection of trees(policies), each agent to have a tree

Each agent to not do its own observation

Reward shaping might give incorrect reward, go around collecting rewards instead of what u want

It helps bring consequences that are in the distant future closer to the present, so it's easier for an agent to learn.

Where win and loss are very far

Not so much for chess, pawn and queen worth is defined using function approximator

Adding a constant does not change the optimal policy

# Game theory

Are they solutions (strategy profile where all players are maximizing expected utility?

equilibrium - no one wants to change their utility

No one can benefit from switching

Result in local optimum

q is the minimizer, E is the maximiser

minimax theorem - doesn't matter who goes first Can be formulated to a linear problem

p - probability vector

q - single vector

don't see the exact action that the player is going to take, only sees the strategy

What are everyone's strategy when it is stable? mixed strategy - doesn't matter who goes first, would end up with same expected value

#### Game theory

Equilibrium no agent can improve themselves by changing their strategy Pure strategy every agent plays an action

Pareto optimal - What is good for me may not be good for another agent, solutions that are not dominated

Dominated if the policy can be improved without making it worse for somebody else

If there is multiple equilibrium, can be selected that is good for everybody

Mixed strategy

What is good for one is bad for another

Symmetric, good for player A and bad for player B or bad for player A and good for player B

Is computationally difficult

Zero-sum game can computationally efficient

Minimax theorem

If row player picks mixed strategy x, and column players plays after knowing the strategy is the same as letting the column player go first, second player can play a pure strategy

 $\rightarrow$  does not matter who picks the strategy first

x and y depends on the game matrix

can be solved with linear program (polynomial time algorithms)

Minimize/ maximize subjected to linear constraints using interior points methods

If we know everybody was rational, the possible outcomes are only when everyone is at equilibrium

Lacking the compute power to be rational

A lot of modelling issues if someone is not rational

Once the row player have decided a probability vector, the second player can play a mixed strategy, the min is only with one of the pure strategy (min of one of the two endpoints of the linear, convex function)

#### Sequential game

Sequence of turns

Normal form: 1 move using matrix or table

Extensive form: Multi dimension table, trees

Belief does not make sense if the agents are optimising their strategy Observation history: what are the observation that the player has seen Nodes with same observation history are placed into an information set Perfect information- No ambiguity on which part of the tree that the node is on

Policy - Function of history

The player plays this strategy based on the history or distribution Chance player expands all non-zero possibilities, order of the observation matters

Subgame - requires that player 2 sees player 1 moves (information set including all nodes descended from it)

Cannot improve by changing strategy

## Prisoner's dilemna

Better off if two players cooperated

testify no matter what they play in the history, for a fixed number of rounds, both players will testify

In each round 1% chance they will never play again

If any player betrays, they will get punished forever (don't cooperate forever)

#### Perpetual punishment

Geometric series -1 / 1 - 0.99

If finite then will not cooperate, then we can reason from the last game and reason backwards, would always betray and not change the strategy If gamma is 0, then we have 1 game, then there would be no continuity. It should be a range of gammas, for some gammas it would be close to 0 reward > punishment \* 1/(1-r)

which equilibrium that u end up in is unclear, if u don't play equilibrium u can improve  $\rightarrow$  would play some equilibrium because u would improve by changing the strategy