Formal Languages, Regular Expressions, Automata, Transducers

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Outline

- Formal Languages in the Chomsky Hierarchy
- Regular Expressions
- Finite State Automata
- Finite State Transducers
- Some Sample CL tasks using Regexps
- Concluding Remarks

Formal Language = Set of Strings of Symbols

- A Formal Language Can Model a Phenomenon, e.g., written English
- Examples
  - All Combinations of the letters A and B: ***ABAB, AABB, AAAB***, etc.
  - Any number of As, followed by any number of Bs: ***AB, AABB, AB, AAAAAAAAABBB***, etc.
  - Mathematical Equations: ***1 + 2 = 5, 2 + 3 = 4 + 1, 6 = 6***
  - All the sentences of a simplified version of written English, e.g., ***My pet wombat is invisible.***
  - A sequence of musical notation (e.g., the notes in Beethoven's 9th Symphony), e.g., ***A-sharp B-flat C G A-sharp***

What is a Formal Grammar for?

- A formal grammar
  - set of rules
  - matches **<u>all and only</u>** instances of **a formal language**
- A formal grammar defines a formal language
- In Computer Science, formal grammars are used to both **generate** and to **recognize** formal languages.
  - Parsing a string of a language involves:
    - Recognizing the string and
    - Recording the analysis showing it is part of the language
  - A compiler translates from language X to language Y, e.g.,
    - This may include parsing language X and generating language Y

A Formal Grammar Consists of:

- **N**: a Finite set of nonterminal symbols
- T: a Finite set of terminal symbols
- R: a set of rewrite rules, e.g., XYZ → abXzY
  - Replace the symbol sequence XYZ with abXzY
- S: A special nonterminal that is the start symbol

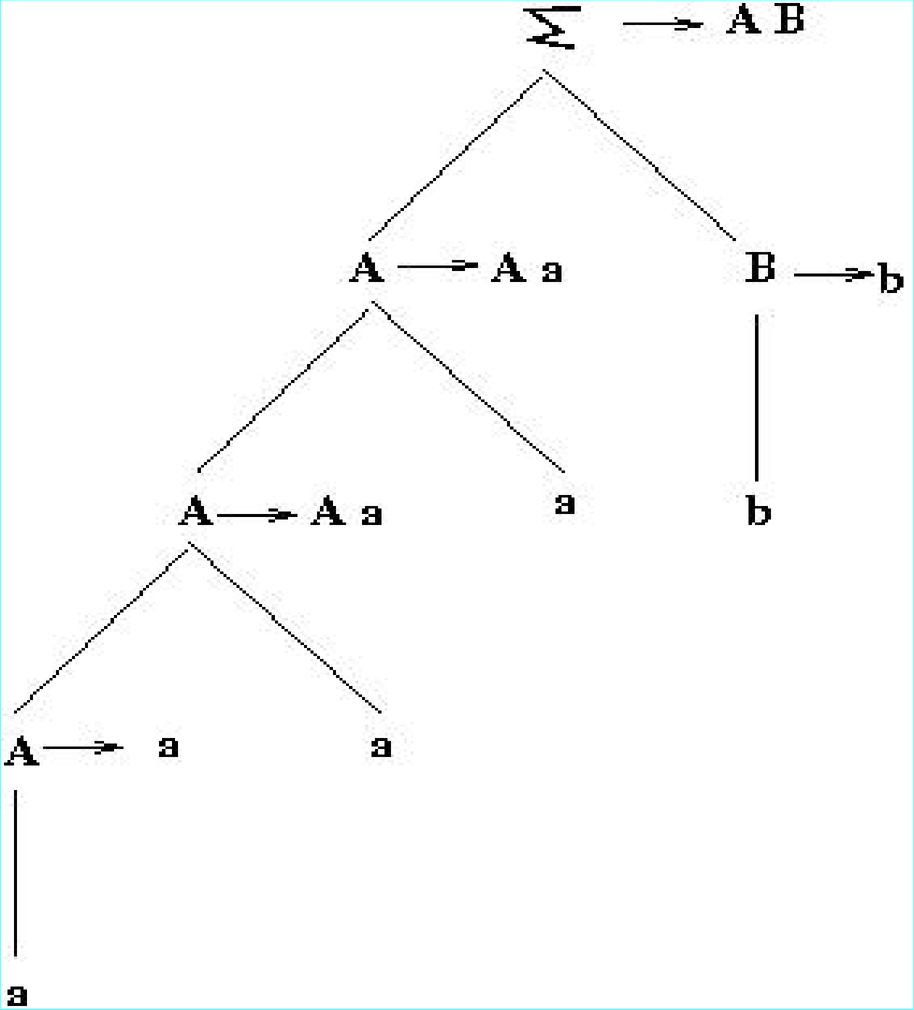
A Very Simple Formal Grammar

- **Language_AB = 1 or more a, followed by 1 or more b, e.g., ab, aab, abb, aaaaaaabb, etc.**
- **N = {A,B}**
- **T={a,b}**
- **S=Σ**
- **R={A→a, A→Aa, B→b B→Bb, Σ→AB}**

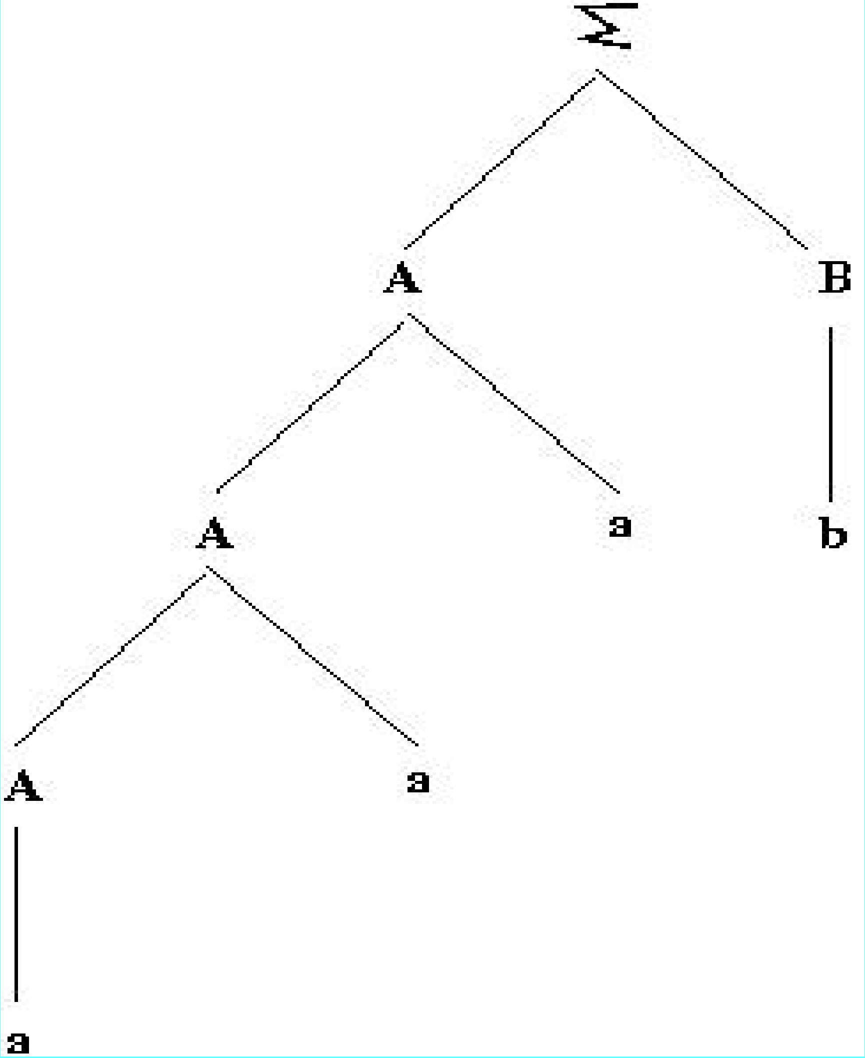
Generating a Sample String

- Start with **Σ**
- Apply **Σ→AB**, Generate A B
- Apply **A→Aa**, Generate A a B
- Apply **A→Aa**, Generate A a a B
- Apply **A→a**, Generate a a a B
- Apply **B→b**, Generate a a a b

Derivation of a a a b



Phrase Structure Tree for a a a b



The Chomsky Hierarchy: Type 0 and 1

- Type 0: No restrictions on rules
  - Equivalent to Turing Machine
    - General System capable of Simulating any Algorithm
- Type 1: Context-sensitive rules
  - $\alpha A \beta \rightarrow \alpha \gamma \beta$
    - Greek chars = 0 or more nonterms/terms
    - A = nonterminal
    - γ = 1 or more nonterms/terms
  - For example,
    - DUCK DUCK DUCK→ DUCK DUCK GOOSE
    - Means convert DUCK to a GOOSE, if preceded by 2 DUCKS

Chomsky Hierarchy Type 2

- Context-free rules
- A → αγβ
- Like context-sensitive, except left-hand side can only contain exactly one nonterminal
- Example Rule from linguistics:
  - NP → POSSP n PP
  - NP → Det n
  - NP → n
  - POSSP → NP 's
  - PP → p NP
  - [NP [POSSP [NP [Det ***The***] [*n* ***group***]] 's]

    [n ***discussion***]

    [PP [p ***about***][NP [n ***food***]]]]
    - ***The group's discussion about food***

Chomsky Hierarchy Type 3

- Regular (finite state) grammars
  - $A \rightarrow \beta a$ or $A \rightarrow \epsilon$ (left regular)
  - $A \rightarrow a\beta$, or $A \rightarrow \epsilon$ (right regular)
- Like Type 2, except
  - non-terminals can either precede (left) or follow (right) terminals, but not both
  - null string is allowed
- Example Rule from linguistics:
  - NP → POSSP n
  - NP → n
  - NP → det n
  - POSSP → NP 's
- [NP [POSSP [NP [det ***The***] [n ***group***]] 's]

  [n ***discussion***]]
  - ***The group's discussion***

Chomsky Hierarchy

- $Type0 \supseteq Type1 \supseteq Type2 \supseteq Type3$
- Type 3 grammars
  - Least expressive, Most efficient processors
- Processors for Type 0 grammars
  - Most expressive, Least efficient processors
- Complexity of recognizer for languages:
  - Type 0 = exponential; Type 1 = polynomial; Type 2 = $O(n^3)$; Type 3 = $O(n \log n)$

CL mainly features Type 2 & 3 Grammars

- Type 3 grammars
  - Include regular expressions and finite state automata (aka, finite state machines)
  - The focal point of the rest of this talk
  - Also see Nooj CL tools: www.nooj4nlp.net/
- Type 2 grammars
  - Commonly used for natural language parsers
  - Used to model syntactic structure in many linguistic theories (often supplemented by other mechanisms)
  - Will play a key roll in the next talk on parsing

Regular Expressions

- The language of ***regular expressions*** (regexps)
  - A standardized way of representing search strings
  - Kleene (1956)
- Computer Languages with regexp facilities:
  - Python, JAVA, Perl, Ruby, most scripting languages, …
  - If not officially supported, a library still may exist
- Many UNIX (linux, Apple, etc.) utilities
  - grep (grep -E regexp file), emacs, vi, ex, ...
- Other
  - Mysql, Microsoft Office, Open Office, ...

My T-Shirt

- My T-Shirt says: **/(BB|[^B]{2})/**
  - The “/”, “(“ and “)” can be ignored for now
  - B represents the string “B”
  - “|” represents the operator 'inclusive or'
  - “^” represents the negative operator
  - [] represents a single character
  - {N}, where N is a number represents N repetitions of the preceding item
- What famous quote could this represent?
- What details are different from the quote?

Regexp = formula specifying set of strings

- Regexp = ∅
  - The empty set
- Regexp = ε
  - The empty string
- Regexp = a sequence of one or more characters from the set of characters
  - *X*
  - *Y*
  - *This sentence contains characters like &T^**%P*
- Disjunctions, concatenation, and repetition of regexps yield new regexps

Concatenation, Disjunction, Repetition

- Concatenation
  - If X is a regexp and Y is a regexp, then XY is a regexp
  - Examples
    - If ***ABC*** and ***DEF*** are regexps, then ***ABCDEF*** is a regexp
    - If ***AB**** and ***BC**** are regexps, then ***AB*BC**** is a regexp
      - Note: Kleene * is explained below
- Disjunction
  - If X is a regexp and Y is a regexp, then X | Y is a regexp
  - Example: ***ABC|DEF*** will match either ***ABC*** or ***DEF***
- Repetition
  - If X is a regexp than a repetition of X will also be a regexp
    - The Kleene Star: ***A**** means 0 or more instances of ***A***
    - Regexp{number}: ***A{2}*** means exactly 2 instances of ***A***

Regexp Notation Slide 2

- Disjunction of characters
  - ***[ABC]*** – means the same thing as ***A* | *B* | *C***
  - ***[a-zA-Z0-9]*** – ranges of characters equivalent to listing characters, e.g., a|b|c|...|A|B|...|0|1|...|9|
  - ^ inside of bracket means complement of disjunction, e.g., ***[^a-z]*** means a character that is neither ***a*** nor ***b*** nor ***c*** … nor ***z***
- Parentheses
  - Disambiguate scope of operators
    - ***A(BC)|(DEF)*** means ABC or ADEF
    - Otherwise defaults apply, e.g., ***ABC|D*** means ***ABC*** or ***ABD***
- ***?*** signifies optionality
  - ***ABC?*** is equivalent to ***(ABC)|(AB)***
- **+** indiates 1 or more
  - ***A(BC)**** is equivalent to A|(A(BC)+)

Regexp Notation Slide 3

- Special Symbols:
  - ***A.*B*** – matches A and B and any characters between (period = any character)
  - ^***ABC*** – matches ABC at beginning of line (^ represents beginning of line)
  - ***[\.?!]$*** – matches sentence final punctuation (***$*** represents end of line)
- Python's Regexp Module
  - Searching
    - Groups and Group Numbers
  - Compiling
  - Substitution
- Similar Modules for: Java, Perl, etc.

Regexp in NLTK's Chatbot

- Running eliza
  - import nltk
  - from nltk.chat.eliza import *
  - eliza_chat()
- NLTK's chatbots:
  - /usr/local/lib/python2.6/site-packages/nltk/chat or
  - /usr/lib/pymodules/python2.7/nltk/chat
  - See util.py and eliza.py
- How it works
  - It creates a Chat object (defined in util.py) that includes a substitute method
  - The settings for this chat object are in eliza.py
  - For each pair in pairs, the 1st item is matched against the input string, to produce an answer listed as the 2nd item. The use of %1 indicates repeated parts of the strings.
  - In util.py – note that the matching pattern for the 1st item is created with ***re.compile***, a method that turns a regular expression into a match-able pattern, although in the current examples (.*), a very simple (and general) regexp.

Regexps in Python (2 and 3)

- import re imports regexp package
- Example re functions
  - re.search(regexp,input_string) creates a search object
  - re.sub (regexp,repl,string)
- search_object methods
  - start() and end() -- respectively output start and end position in the string
  - group(0) – outputs whole match
  - group(N) – outputs the nth group (item in parentheses)
- Patterns can be compiled
  - Pattern1 = re.compile(r'[Aa]Bc')
  - Efficient, can take re functions as methods
  - Methods takes additional parameters (e.g., starting position)
    - Pattern1.search('ABcaBc',2)
      - starts search at position 2

Regexp with Unix tools

- grep -E '$[0-9\.,]+' all-OANC |less
- In the program less
  - /\$[0-9.,]
    - Highlights numeric instances
    - Note some of the problems with this regexp for characterizing money strings

RegExp to Search for Common Types of Numeric Strings

- An XML (or html) tag
  - `<[^>]+>`
- Money
  - $[0-9\.,]+
  - Would this match the string '$,,,,,,'?
    - Maybe that doesn't matter?
  - How might we handle cases like “$4 million”?
  - What might be a better regexp for money?
- Others
  - Dates, Roman Numerals, Social Security, Telephone Numbers, Zip Codes, Library Call Numbers, etc.
- Time of Day – Let's Do this one as a joint exercise

Time of Day

- Let's agree on the components of a time of day as printed
  - **** fill in here ****
- For 5 minutes, Everyone should attempt to write such an expression independently. You can test your regexp with Python or grep.
- Let's look at some of the proposed answers, test them and possibly combine aspects.

NLTK's Regexp Language for Chunking

- sentence = '''The big grey dog with three heads was on my lap'''
- tokens = nltk.word_tokenize(sentence)
- pos_tagged_items = nltk.pos_tag(tokens)
- chunk_grammar = nltk.RegexpParser(r"""

```
NG: {(<DT|JJ|NN|PRP\$>)*(<NN|NNS>)}
VG: {<MD|VB|VBD|VBN|VBZ|VBP|VBG>*<VB|VBD|VBN|VBZ|VBP|VBG><RP>?}
"""
```

- chunk_grammar.parse(pos_tagged_items)
- Structure:
  - 1 rule per line
  - Nonterminal: Regexp
  - Regexp = terminals, nonterminals & operators (*+?{}...)

Chunking Rules with NonTerminal on Right Hand Side

- chunks2 = r"""

```
DTP: {<PDT><DT|CD>}
NG: {(<DT|JJ|NN|DTP|PRP\$>)*(<NN|NNS>)}
VG:{<MD|VB|VBD|VBN|VBZ|VBP|VBG>*<VB|VBD|VBN|VBZ|VBP|VBG><RP>?}
PP:{<IN|TO><NG>}
VP: {<VG> <NG|PP>}
"""
```

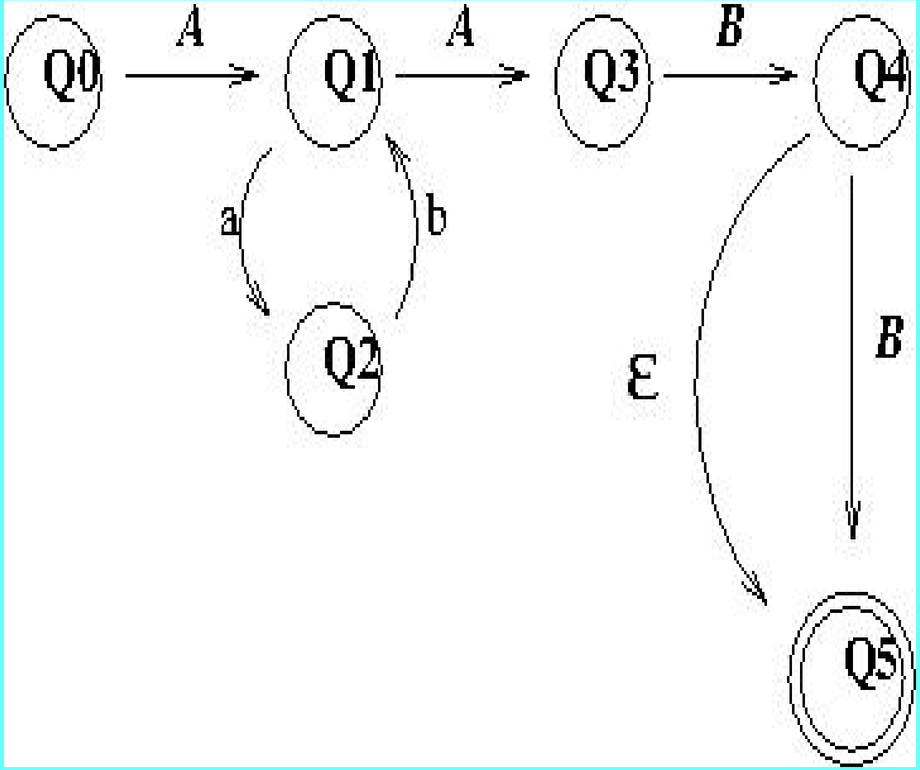
The Penn Treebank II POS tagset

- Verbs: VB, VBP, VBZ, VBD, VBG, VBN
  – base, present-non-3rd, present-3rd, past, -ing, -en
- Nouns: NNP, NNPS, NN, NNS
  – proper/common, singular/plural (singular includes mass + generic)
- Adjectives: JJ, JJR, JJS (base, comparative, superlative)
- Adverbs: RB, RBR, RBS, RP (base, comparative, superlative, particle)
- Pronouns: PRP, PP$ (personal, possessive)
- Interogatives: WP, WP$, WDT, WRB (compare to: PRP, PP$, DT, RB)
- Other Closed Class: CC, CD, DT, PDT, IN, MD
- Punctuation: **# $. , : ( ) “ ” '' ' `**
- Weird Cases: FW(***deja vu***), SYM (**@**), LS (***1, 2, a, b***), TO (***to***), POS(***'s, '***), UH (***no, OK, well***), EX (***it/there***)
- Newer tags: HYPH, PU

Finite State Automata

- Devices for recognizing finite state grammars (including regular expressions)
- Two types
  - Deterministic Finite State Automata (DFSA)
    - Rules are unambiguous
  - NonDeterministic FSA (NDFSA)
    - Rules are ambiguous
      - Sometimes more than one sequence of rules must be attempted to determine if a string matches the grammar
        - » Backtracking
        - » Parallel Processing
        - » Look Ahead
  - Any NDFSA can be mapped into an equivalent (but larger) DFSA

DFSA for Regexp: *A(ab)*ABB?*



State	Input				
	A	B	a	b	ε
Q0	Q1				
Q1	Q3		Q2		
Q2				Q1	
Q3		Q4			
Q4		Q5			Q5
Q5					

DFSA algorithm

- D-Recognize(tape, machine)

    pointer ← beginning of tape

    current state ← initial state Q0

    repeat until the end of the input is reached

        look up (current state,input symbol) in transition table

        if found: set current state as per table look up

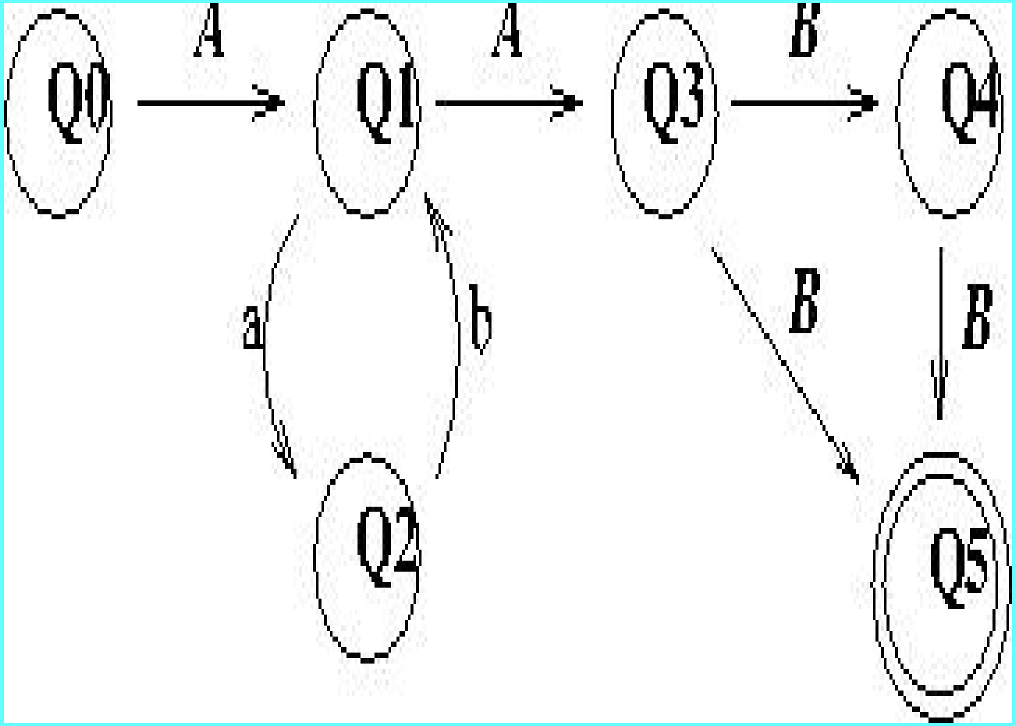
                advance pointer to next position on tape

        else: reject string and exit function

    if current state is a final state: accept the string

    else: reject the string

NDFSA for Regexp: *A(ab)*ABB?*



State	Input			
	A	B	a	b
Q0	Q1			
Q1	Q3		Q2	
Q2				Q1
Q3		Q4 Q5		
Q4		Q5		

NDFSA algorithm

- ND-Recognize(tape, machine)

  agenda ← {(initial state, start of tape)}

  current state ← next(agenda)

  repeat until accept(current state) or agenda is empty

      agenda ← Union(agenda,look_up_in_table(current state,next_symbol))

      current state ← **next**(agenda)

  if accept(current state): return(True)

  else: false
- Accept if at the end of the tape and current state is a final state
- **Next** defined differently for different types of search
  - Choose most recently added state first (depth first)
  - Chose least recently added state first (breadth first)
  - Etc.

A Right Regular Grammar Equivalent to: *A(ab)*ABB?*

(Red = Terminal, Black = Nonterminal)

- Q→ARS
- R→ϵ
- R→abR
- S→ABB
- S→AB

Readings

- Jurafsky and Martin, Chapters 2 and 3
- NLTK Chapters 2 and 3

Homework # 2: Slide 1

- Create 2 Programs using regular expressions to identify the following in a corpus
  - Program 1 should identify dollar amounts
    - Cover as many cases as possible (including those with words like million or billion)
  - Program 2 should identify telephone numbers
    - Attempt to handle as many cases as possible: with and without area codes, different punctuation, etc.
- Design and test the programs using the OANC corpus from the class website and any other corpora that you choose.
  - http://cs.nyu.edu/courses/fall15/CSCI-UA.0480-006/all-OANC.txt
- Programming language: the program can be in any standard programming language
  - Even shell scripts that support regexp
    - sed -E 's/(19|20)[0-9][0-9]/[&]/g' all-OANC.txt > output_file
      - This would put brackets around years
      - It would overgenerate, e.g., number greater than 4 digits
      - It would undergenerate, e.g., years before 1900
- More Details On Next Slide

Homework # 2: Slide 2

- Output format: insert brackets around money expressions
  - The Picasso print costs [$5 billion dollars and 50 cents] on Ebay.
- The program should be self-contained and include instructions for running it, e.g., it could be run as follows:
  - Program INPUT_FILE OUTPUT_FILE
- Submit program via NYUClasses as a zip, tar, tar.gz or tgz file in the following format: YourName-HW2.extension, e.g., AdamMeyers-HW2.tgz
- Programs will be graded by how well they do on the OANC corpus according to 2 metrics:
  - Precision: Number of Correct Answers / Number of Answers
    - If there are many answers to grade, we may use sampling to estimate precision
  - Coverage: Number of Correct Answers

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Optional HW

- Read through the Bots that are part of NLTK and use their libraries to make your own
- The current bots mostly use the regexp (.*). Add bots that use more elaborate regexps