Machine Learning - Homework Week 1

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1 Exercise 1

To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

Solution

Let A be an event that someone testing positive for disease.
Let B be an event that someone tested with new test.

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(A|B) = \frac{(0.05.0.98)}{(0.05.0.98) + (0.95.0.03)}$$

$$P(A|B) = \frac{98}{155}$$

The probability that someone testing positive for Hansen's disease under this new test actually has it is nearly 63.2%.

2 Exercise 2

Univariate Normal Distribution. The distribution is normalized if the area under the curve equals 1.

Assume

$$I = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp(\frac{-(x-\mu)^2}{2\sigma^2})\, dx$$

Let

$$z = \frac{x-\mu}{\sigma}$$

$$dx = \sigma du$$

Now integral I becomes:

$$I = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp(\frac{-z^2}{2})\, dz$$

Let

$$y^2 = \frac{z^2}{2}$$

$$y = \frac{z}{\sqrt{2}}$$

$$dz = \sqrt{2}\, dy$$

Then, I becomes:

$$I = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp(-y^2\,\sqrt{2})\, dy$$

$$I = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} exp(-y^2)\, dy$$

Apply Gaussian Integral:

$$\int_{-\infty}^{\infty} exp(-y^2)\, dy = \sqrt{\pi}$$

Hence,

$$I = \frac{1}{\sqrt{\pi}} * \sqrt{\pi} = 1$$

Mean of Univariate Normal Distribution

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} x\, exp(\frac{-(x-\mu)^2}{2\sigma^2})\, dx$$

Let

$$x = \sigma\, y + \mu$$

$$dx = \sigma\, dy$$

Then,

$$E(X) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} (\sigma\, y + \mu)\, exp(\frac{-((\sigma\, y + \mu) - \mu)^2}{2\sigma^2})\, dy$$

$$E(X) = \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y\, exp(-y^2/2)\, dy \,+\, \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} exp(-y^2/2)\, dy$$

$$E(X) = -\frac{\sigma}{\sqrt{2\pi}} \left[\frac{-y^2}{2}\right]_{-\infty}^{\infty} \,+\, \frac{\mu}{\sqrt{2\pi}} \sqrt{2\pi}$$

$$E(X) = 0 + \mu = \mu$$

Calculating $E(X^2)$

$$E(X^2) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} x^2 \, exp(\frac{-(x-\mu)^2}{2\sigma^2}) \, dx$$

Let

$$x^2 = \sigma^2 \, y^2 + 2\sigma \, y \, \mu + \mu^2$$

$$dx = \sigma \, dy$$

Then,

$$E(X^2) = \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y^2 \, exp(-y^2/2) \, dy + \frac{2\sigma\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y \, exp(-y^2/2) \, dy + \frac{\mu^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} exp(-y^2/2) \, dy$$

$$E(X^2) = \frac{\sigma^2}{\sqrt{2\pi}} \sqrt{2\pi} + \frac{2\sigma\mu}{\sqrt{2\pi}} * 0 + \frac{\mu^2}{\sqrt{2\pi}} \sqrt{2\pi}$$

$$E(X^2) = \sigma^2 + \mu^2$$

Standard deviation of Univariate Normal Distribution

$$std(X) = \sqrt{var(X} = \sqrt{E(X^2) - E(X)^2} = \sqrt{\sigma^2 + \mu^2 - \mu^2} = \sqrt{\sigma^2}$$