TAREA SEMANA 7

Resuelva las siguientes ecuaciones diferenciales por el metodo mas conveniente.

1. $\frac{dS}{dr} = kS$.

$\frac{ds}{s} = kdr$

$\int \frac{ds}{S} = \int kdr$

$\ln|S| = Kr + C$

$S = e^{kr+c}$

$S = c'e^{kr}$

$S(r) = Ce^{kr}$

2. $\frac{dQ}{dt} = k(Q-70)$

$\int \frac{dQ}{Q-70} = \int kdt$

$\ln|Q-70| = kt + C$

$|Q-70| = e^{kt+C}$

$Q-70 = C'e^{kt}$

$Q(t) = 70 + Ce^{kt}$

3. $\frac{dy}{dx} = \frac{x-2}{y^2}$ si $f(-3) = 2$

$y^2 \frac{dy}{dx} = xy^2 - 2$

$\frac{dy}{dx} = x - \frac{2}{y^2}$

$\frac{y^2}{y^2-2} dy = xdx$

$\int \frac{y^2}{y^2-2} dy = \int xdx$

$\frac{y^2}{y^2-2} = 1 + \frac{2}{x^2-2}$

$$\int\left(1+\frac{2}{y^2-2}\right)dy=\int x\,dx$$

$$\int 1\,dy+\int\frac{2}{y^2-2}\,dy=\int x\,dx$$

$$y+\int\frac{2}{y^2-2}\,dy=\frac{x^2}{2}+C$$

$$\int\frac{2}{y^2-2}\,dy=\sqrt{2}\ln|y-\sqrt{2}|-\sqrt{2}\ln|y+\sqrt{2}|$$

$$=\sqrt{2}\ln\left|\frac{y-\sqrt{2}}{y-\sqrt{2}}\right|$$

$$=y+\sqrt{2}\ln\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right|=\frac{x^2}{2}+C$$

$$=2+\sqrt{2}\ln\left|\frac{2-\sqrt{2}}{2+\sqrt{2}}\right|=\frac{9}{2}+C$$

$$C=2+\sqrt{2}\ln\left|\frac{2-\sqrt{2}}{2+\sqrt{2}}\right|-\frac{9}{2}$$

$$y+\sqrt{2}\ln\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right|=\frac{x^2}{2}+2+\sqrt{2}\ln\left|\frac{2-\sqrt{2}}{2+\sqrt{2}}\right|-\frac{9}{2}\ //$$

4. $\frac{dy}{dx}=\frac{x+2}{y}$; halle $y=f(x)$ sabiendo que $f(6)=-6$

$$y\left(\frac{dy}{dx}\right)=x+2$$

$$y\,dy=(x+2)\,dx$$

$$\int y\,dy=\int(x+2)\,dx$$

$$\frac{y^2}{2}=\frac{x^2}{2}+2x+C$$

$$y^2=x^2+4x+C$$

$$6^2=6^2+4(6)+C$$

$$36=36+24+C$$

$$C=36-60=-24$$

$$\boxed{y^2=x^2+4x-24}$$

$$y=\pm\sqrt{x^2+4x-24}$$

$$6=\pm\sqrt{6^2+4(6)-24}$$

$$6=\pm\sqrt{36+24-24}$$

$$6=\pm\sqrt{36+24-24}$$

$$6=\pm\sqrt{36}$$

$$6=\pm 6$$

$$y=\sqrt{x^2+4x-24}\ //$$

5. $\frac{dy}{dx} = \left(\frac{2y+3}{4x+5}\right)^2$

$$\frac{dy}{(2y+3)^2} = \frac{dx}{(4x+5)^2}$$

$$\int \frac{du}{2u^2} = \frac{1}{2}\int y^{-2}\,du$$

$$= \frac{1}{2}\left(-\frac{1}{u}\right) = -\frac{1}{2(2y+3)}$$

$$\int \frac{dv}{4v^2} = \frac{1}{4}\int v^{-2}\,dv$$

$$= \frac{1}{4}\left(-\frac{1}{v}\right) = -\frac{1}{4(4x+5)}$$

$$-\frac{1}{2(2y+3)} = -\frac{1}{4(4x+5)} + C$$

$$\frac{1}{2(2y+3)} = \frac{1}{4(4x+5)} - C$$

$$2(2y+3) = \frac{4(4x+5)}{1-4C}$$

$$4y+6 = \frac{16x+20}{1-4C}$$

$$4y = \frac{16x+20}{1-4C} - 6$$

$$y = \frac{16x+20}{4(1-4C)} - \frac{6}{4}$$

$$y = \frac{16x+20}{4(1-4C)} - \frac{3}{2}$$

6. $\dfrac{dP}{dt} = P - P^2$

$$\frac{dP}{dt} = P(1-P)$$

$$\frac{dP}{P(1-P)} = dt$$

$$\frac{1}{P(1-P)} = \frac{A}{P} + \frac{B}{1-P}$$

$$1 = A(1-P) + BP$$

$$1 = A - AP + BP$$

$$1 = A + (B-A)P$$

$$\frac{1}{P(1-P)} = \frac{1}{P} + \frac{1}{1-P}$$

$$\int \left(\frac{1}{P} + \frac{1}{1-P} \right) dP = \int dt$$

$$\ln|P| - \ln|1-P| = t + C$$

$$\ln\left|\frac{P}{1-P}\right| = t + C$$

$$\frac{P}{1-P} = e^{t+C}$$

$$\frac{P}{1-P} = C'e^{t}$$

$$P = C'e^{t}(1-P)$$

$$P + C'e^{t}P = C'e^{t}$$

$$P(1 + C'e^{t}) = C'e^{t}$$

$$P = \frac{C'e^{t}}{1 + C'e^{t}}$$

$$P_0 = \frac{C'}{1 + C'}$$

$$C' = \frac{P_0}{1 - P_0}$$

$$P = \frac{P_0 / 1 - P_0 \, e^{t}}{1 + \frac{P_0}{1 - P_0} e^{t}}$$

$$P = \frac{P_0 e^{t}}{(1-P_0) + P_0 e^{t}}$$

$$P(t) = \frac{C'e^{t}}{1 + C'e^{t}}$$

$$P(t) = \frac{P_0 e^{t}}{(1-P_0) + P_0 e^{t}} \; //$$

7. $\frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y-x-3}$

$\frac{dy}{dx} = \frac{y(x+2) - (x+2)}{y(x-3) - (x+3}$

$\frac{dy}{dx} = \frac{(x+2)(y-1)}{(x+3)(y-1)} = \frac{dy}{dx} = \frac{(y-1)(x+2)}{(y-1)(x+3)}$

$\frac{dy}{dx} = \frac{x+2}{x+3} = dy = \frac{x+2}{x+3}dx$

$dy = \left(1 - \frac{1}{x+3}\right)dx$

$dy = dx - \frac{dx}{x+3}$

$\int dy = \int dx - \int \frac{dx}{x+3}$

$y = x - \ln|x+3| + C$

$y = x - \ln|x+3| + C$

8. $\frac{dy}{dy} = \frac{3x^2+2}{y}$; halle $y = f(x)$ sabiendo que $f(-2) = -6$

$y\,dy = (3x^2+2)dx$

$\int y\,dy = \int (3x^2+2)\,dx$

$\frac{y^2}{2} = \int 3x^2 dx + \int 2dx$

$\frac{y^2}{2} = x^3 + 2x + C$

$y^2 = 2x^3 + 4x + C$

$6^2 = 2(2)^3 + 4(2) + C$

$36 = 2(8) + 8 + C$

$36 = 16 + 8 + C$

$C = 36 - 24 = 12$

$y^2 = 2x^3 + 4x + 12$

$y = \pm\sqrt{2x^3+4x+12}$

$6 = \pm\sqrt{2(2)^3+4(2)+12}$

$6 = \pm\sqrt{16+8+12}$

$6 = \pm\sqrt{36}$

$6 = \pm 6$

$y = \sqrt{2x^3+4x+12}$

9. $xy^2\frac{dy}{dx} = y^3 - x^3 \; ; y(1) = 2$

$$\frac{dy}{dx} = \frac{y^3 - x^3}{xy^2}$$

$$\frac{dy}{dx} = \frac{y^3}{xy^2} - \frac{x^3}{xy^2}$$

$$\frac{dy}{dx} = \frac{y}{x} - \frac{x^2}{y^2}$$

$$\frac{dy}{\frac{y}{x} - \frac{x^2}{y^2}} = dx$$

10. $x^2\frac{dy}{dx} - 2xy = 3y^4 ; y(1) = \frac{1}{2}$

$$x^2 \frac{dy}{dx} = 2xy + 3y^4$$

$$\frac{dy}{dx} = \frac{2y}{x} + \frac{3y^4}{x^2}$$

$$v = y^{-3} \Rightarrow \frac{dv}{dx} = -3y^{-4}\frac{dy}{dx}$$

11. $\frac{dy}{dx} = \frac{1-x-y}{x+y}$

$$v = x + y$$

$$y = v - x$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - = \frac{1 - x - (v - x)}{v}$$

$$\frac{dv}{dx} - 1 = \frac{1 - v}{v}$$

$$\frac{dv}{dx} = \frac{1 - v}{v} + 1$$

$$\frac{dv}{dx} = \frac{1 - v + v}{v} = \frac{1}{v}$$

$$v\,dv = dx$$

$$\int v\,dv = \int dx$$

$$\frac{v^2}{2} = x + C$$

$$(x+y)^2 = 2x + 2C$$

$$(x+y)^2 = 2x + 2C_{//}$$

12. $\frac{dy}{dx} = \cos(x+y);\ y(0) = \frac{\pi}{4}$

$$\frac{dy}{dx} = \cos(x+y)$$

$$y = v - x$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 = \cos(v)$$

$$\frac{dv}{dx} = \cos(v) + 1$$

$$\frac{dv}{\cos(v)+1} = dx$$

$$\cos(v) + 1 = 2\cos^2\left(\frac{v}{2}\right)$$

$$\int \frac{dv}{2\cos^2(v/2)} = \int dx$$

$$\int \frac{dv}{2\cos^2} = \int dx$$

$$\tan\left(\frac{v}{2}\right) = x + C$$

$$\tan\left(\frac{x+y}{2}\right) = x + C$$

$$\tan\left(\frac{0+\frac{\pi}{4}}{2}\right) = 0 + C$$

$$\tan\left(\frac{\pi}{8}\right) = C$$

$$\tan\left(\frac{x+y}{2}\right) = x + \tan\left(\frac{\pi}{8}\right)$$

13) $y' + 3x^2 y = x^2$

$u(x) = e^{\int p(x)\,dx}$

$u(x) = e^{\int 3x^2 dx} = e^{x^3}$

$e^{x^3} y' + 3x^2 e^{x^3} y = x^2 e^{x^3}$

$\frac{d}{dx}(e^{x^3} y) = e^{x^3} y' + 3x^2 e^{x^3} y$

$\frac{d}{dx}(e^{x^3} y) = x^2 e^{x^3}$

$\int \frac{dx}{dx}(e^{x^3} y)\,dx = \int x^2 e^{x^3} dx$

$\int x^2 e^{x^3} dx = \frac{1}{3}\int e^u du = \frac{1}{3} e^u = \frac{1}{3} e x^3$

$e^{x^3} y = \frac{1}{3} e^{x^3} + C$

$y = \frac{1}{3} + Ce^{-x^3}$ //.

14) $(1+x)\frac{dy}{dx} - xy = x + x^2$

$(1+x)\frac{dy}{dx} = x + x^2 + xy$

$\frac{dy}{dx} = \frac{x + x^2 + xy}{1+x}$

$\frac{dy}{dx} = \frac{x(1+x+y)}{1+x}$

$\frac{dy}{dx} = x + y$

$\frac{dy}{dx} - y = x$

$u(x) = e^{\int p(x)\,dx}$

$u(x) = e^{\int -1\,dx} = e^{-z}$

$e^{-x}\frac{dy}{dx} - e^{-x} y = x e^{-x}$

$\frac{d}{dx}(e^{-x} y) = x e^{-x}$

$\int \frac{d}{dx}(e^{-x} y)\,dv = \int x e^{-x} dx$

$\int u\,dv = uv - \int v\,du$

$\int x e^{-x} dx = -x e^{-z} + \int e^{-x} dx = -x e^{-z} - e^{-x}$

$e^{-x} y = -x e^{-x} - e^{-x} + C$

$y = -x - 1 + Ce^{x}$ //.

15) $y\,dx = (ye^y - 2x)\,dy$

$$\frac{dx}{dy} = \frac{ye^y - 2x}{y}$$

$$dx = \frac{ye^y - 2x}{y}\,dy$$

$$y\,dx + 2x\,dy = ye^y\,dy$$

$$y\,dx + 2x\,dy = ye^y\,dy$$

$$\frac{dx}{dy} + \frac{2x}{y^2} = \frac{e^y}{y} - \frac{2x}{y^2}$$

16) $\frac{dP}{dt} + 2tP = P + 4t - 2$

$$\frac{dP}{dt} + 2tP = P + 4t - 2$$

$$\frac{dP}{dt} - P = 4t - 2 = \frac{dP}{dt} + p(t)P = q(t)$$

$p(t) = -1, \quad q(t) = 4t - 2$

$y(t) = e^{\int p(t)dt} = e^{\int -1dt} = e^{-t}$

$$e^{-t}\frac{dP}{dt} - e^{-t}P = (4t-2)e^{-t}$$

$$\frac{d}{dt}(e^{-t}P) = (4t-2)e^{-t}$$

$$\int d(e^{-t}P) = \int (4t-2)e^{-t}dt$$

$$\int u\,dv = uv - \int v\,du$$

$$\int (4t-2)e^{-t}dt = -(4t-2)e^{-t} + \int 4e^{-t}dt$$

$$\int 4e^{-t}dt = -4e^{-t} = \int (4t-2)e^{-t}dt = (4t-2)e^{-t} - 4e^{-t}$$

$$= -e^{-t}(4t-2+4) = -e^{-t}(4t+2) = e^{-t}P = -e^{-t}(4t+2) + C$$

$P = -(4t+2) + Ce^{t}$

$P = Ce^{t} - 4t - 2$ //

17) $(x^2-1)\frac{dy}{dx} + 2y = (y+1)^2$

$$(x^2-1)\frac{dy}{dx} + 2y = (y+1)^2$$

$$(x^2-1)\frac{dy}{dx} = (y+1)^2 - 2y$$

$$(y+1)^2 = y^2 + 2y + 1$$

$$(y+1)^2 - 2y = y^2 + 2y + 1 - 2y = y^2 + 1$$

$$(x^2-1)\frac{dy}{dx} = y^2 + 1$$

$$\frac{dy}{dx} = \frac{y^2+1}{x^2-1}$$

$$\frac{dy}{y^2+1} = \frac{dx}{x^2-1}$$

$$\int \frac{dy}{y^2+1} = \tan^{-1}(y)$$

$$\int \frac{dx}{(x-1)(x+1)} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$1 = A(x+1) + B(x-1)$$

$$1 = A(1+1) + B(1-1)$$

$$1 = 2A \Rightarrow A = \frac{1}{2}$$

$$1 = A(-1+1) + B(-1-1)$$

$$1 = -2B \Rightarrow B = -\frac{1}{2}$$

$$\frac{1}{(x-1)(x+1)} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1}$$

$$= \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|$$

$$y = \tan\left(\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C\right) //$$

VERNAZA

18) $(2x-1)\,dx + (3y+7)\,dy = 0$

$(2x-1)\,dx + (3y+7)\,dy = 0$

$M(x,y)\,dx + N(x,y)\,dy = 0$

$M(x,y) = 2x-1, \quad N(x,y) = 3y+7$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2x-1) = 0$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(3y+7) = 0$$

$$\frac{\partial F}{\partial x} = M(x,y) = 2x-1$$

$$F(x,y) = \int (2x-1)\,dx = x^2 - x + g(y)$$

$$\frac{\partial F}{\partial y} = N(x,y) = 3y+7$$

$$\frac{\partial}{\partial y}(x^2 - x + g(y)) = g'(y)$$

$$g'(y) = 3y+7$$

$$g(y) = \int (3y+7)\,dy = \frac{3}{2}y^2 + 7y + C$$

$$F(x,y) = x^2 - x + \frac{3}{2}y^2 + 7y + C$$

$F(x,y) = C$

$$x^2 - x + \frac{3}{2}y^2 + 7y = C$$

$$x^2 - x + \frac{3}{2}y^2 + 7y = C_{//}.$$

19) $(x^3+y^3)\,dx + 3xy^2\,dy = 0$

$M(x,y)dx + N(x,y)dy = 0$

$M(x,y) = x^3+y^3,\quad N(x,y) = 3xy^2$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^3+y^3) = 3y^2$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(3xy^2) = 3y^2 = \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial F}{\partial x} = M(x,y) = x^3+y^3$$

$$F(x,y) = \int (x^3+y^3)\,dy = \frac{x^4}{4} + y^3x + g(y)$$

$$\frac{\partial F}{\partial y} = N(x,y) = 3xy^2$$

$$\frac{\partial}{\partial y}\left(\frac{x^4}{4} + y^3x + g(y)\right) = 3xy^2 + g'(y)$$

$3xy^2 + g'(y) = 3xy^2$

$g'(y) = 0 \Rightarrow g(y) = C$

$$F(x,y) = \frac{x^4}{4} + y^3x + C$$

$F(x,y) = C$

$$\frac{x^4}{4} + y^3x = C \;//$$

20) $(4t^3y - 15t^2 - y)\,dt + (t^4 + 3y^2 - t)\,dy = 0$

$M(t,y)\,dt + N(t,y)\,dy = 0$

$M(t,y) = 4t^3y - 15t^2 - y$, $N(t,y) = t^4 + 3y^2 - t$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(4t^3y - 15t^2 - y) = 4t^3 - 1$$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial t}(t^4 + 3y^2 - t) = 4t^3 - 1$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$ exacta

$$\frac{\partial F}{\partial t} = M(t,y) = 4t^3y - 15t^2 - y$$

$$F(t,y) = \int (4t^3y - 15t^2 - y)\,dt$$

$$F(t,y) = y\int 4t^3 dt - \int 15t^2 dt - \int y\,dt$$

$$F(t,y) = y(t^4) - 5t^3 - yt + g(y)$$

$$F(t,y) = t^4y - 5t^3 - yt + g(y)$$

$$\frac{\partial F}{\partial y} = N(t,y) = t^4 + 3y^2 - t$$

$$\frac{\partial}{\partial y}(t^4y - 5t^3 - yt + g(y)) = t^4 - t + g'(y)$$

$t^4 - t + g'(y) = t^4 + 3y^2 - t \quad = g'(y) = 3y^2$

$$g(y) = \int 3y^2\,dy = y^3 + C$$

$$F(t,y) = t^4y - 5t^3 - yt + y^3 + C$$

$F(t,y) = C \qquad t^4y - 5t^3 - yt + y^3 = C$ //

21) $(10-6y+e^{-3x})dx - 2dy = 0$

$M(x,y)dx + N(x,y)dy = 0$

$M(x,y) = 10-6y+e^{3x}, \quad N(x,y) = -2$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(10-6y+e^{3x}) = -6$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-2) = 0 \qquad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\mu(y) = e^{\int \frac{-6}{-2}dy} = e^{\int 3dy} = e^{3y}$$

$$e^{3y}(10-6y+e^{3x})dx - e^{3x}(2)dy = 0$$

$$(10e^{3y} - 6ye^{3y} + e^{3x+3y})dx - 2e^{3y}dy = 0$$

$$M(x,y) = 10e^{3y} - 6ye^{3y} + e^{3x+3y}, \quad N(x,y) = -2e^{3y}$$

$$\frac{\partial M}{\partial y} = 30e^{3y} - 18ye^{3y} - 6e^{3y} + 3e^{3x+3y}$$

$$\frac{\partial N^*}{\partial x} = 0 \,//$$

22) $(y^2+yx)dx - x^2dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(y^2+yx) = 2y+x$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-x^2) = -2x$$

$$\mu = \frac{1}{x^2}$$

$$\left(\frac{y^2}{x^2}+\frac{y}{x}\right)dx - dy = 0$$

$$\left(\frac{y^2}{x^2}+\frac{y}{x}\right)dx = dy$$

$$dy = \left(\frac{y^2}{x^2}+\frac{y}{x}\right)dx$$

$$\frac{dy}{y^2+xy} = \frac{dx}{x^2}$$

23) $(x^2+2y^2)\frac{dx}{dy}=xy$; $y(-1)=1$

$y(-1)=1$

$$\frac{dx}{dy}=\frac{xy}{x^2+2y^2}$$

$$v=\frac{x}{y} \Rightarrow x=vy$$

$$\frac{dx}{dy}=v+y\frac{dv}{dy}$$

$$((v^2y^2)+2y^2)\left(v+y\frac{dv}{dy}\right)=vy^2$$

$$y^2(v^2+2)\left(v+y\frac{dv}{dy}\right)=vy^2$$

$$(v^2+2)\left(v+y\frac{dv}{dy}\right)=v$$

$$(v^2+2)v+(v^2+2)y\frac{dv}{dy}=v$$

$$(v^3+2v)+(v^2+2)y\frac{dv}{dy}=v$$

$$(v^2+2)y\frac{dv}{dy}=v-v^3-2v$$

$$(v^2+2)y\frac{dv}{dy}=-v^3-v$$

$$(v^2+2)y\frac{dv}{dy}=-v(v^2+1)$$

$$\frac{(v^2+2)}{v(v^2+1)}dv=-\frac{dy}{y}$$