

EJERCICIOS DELTAMATH

MATRICES

$$\textcircled{3} \quad \begin{bmatrix} 0 & -3 & -5 \\ -1 & 0 & -2 \\ 0 & -5 & -2 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & -1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 8 & -5 \\ 3 & 1 & -1 \\ -8 & 7 & -2 \end{bmatrix}$$

$$C_{11} = 0(-1) + (-3)(2) + (-5)(-1)$$

$$= -6 + 5$$

$$\boxed{-1}$$

$$C_{12} = 0(+1) + (-3)(-1) + (-5)(-1)$$

$$= 3 + 5 = \boxed{8}$$

$$C_{13} = 0 + (-5)(1) = \boxed{-5}$$

$$C_{21} = -1(-1) + 0(-2) + (-2)(-1)$$

$$\cancel{+ C} = 1 + 2 = \boxed{3}$$

$$C_{22} = (-1)(1) + (0)(-1) + (-2)(-1)$$

$$= -1 + 2 = 1$$

$$C_{23} = (-1)(-1) + (-2)(1)$$

$$= 1 - 2 = -1$$

$$C_{31} = 0(+1) + (-5)(2) + (-2)(-1)$$

$$= -10 + 2 = -8$$

$$C_{32} = 0(+1) + (-5)(-1) + (-2)(-1)$$

$$= +5 + 2 = \boxed{7}$$

$$C_{33} = 0(-1) + (-5)(0) + (-2)(1)$$

$$= -2$$

Find the product of the following two matrices

(2)

$$\begin{bmatrix} 3 & -5 & -2 \\ 3 & -3 & 0 \\ -1 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 3 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 2 \\ 6 & 3 & 0 \\ 8 & -1 & -7 \end{bmatrix},$$

$$C_{11} = 3(1) + (-5)(-1) + (-2)(3) \\ = 3 + 5 + (-6) \\ = 8 - 6 = 2,$$

$$C_{12} = 3,$$

$$C_{13} = 3(1) + (-5)(1) + (-2)(-2) \\ = 3 + (-5) + (4) \\ = -2 + 4 = 2,$$

$$C_{21} = 3(1) + (-3)(-1) + (0)(3) \\ = 3 + 3 = 6,$$

$$C_{22} = 3(1) = 3,$$

$$C_{23} = 3(1) + (-3)(1) + (0)(-2) \\ = 3 - 3 = 0,$$

$$C_{31} = (-1)(1) + (0)(-1) + (3)(3) \\ = -1 + 9 = 8,$$

$$C_{32} = (-1)(1) = -1,$$

$$C_{33} = (-1)(1) + (0)(1) + (3)(-2) \\ = -1 + (-6) \\ = -1 - 6 = -7,$$

Find the product of the following two matrices.

$$\textcircled{3} \quad \begin{bmatrix} 0 & 4 & 2 \\ 0 & -4 & 2 \\ -1 & 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} -6 & 4 & 8 \\ 2 & -4 & 8 \\ -1 & 2 & -7 \end{bmatrix}$$

$$C_{11} = (0)(1) + (4)(-1) + (2)(-1) \\ C_{11} = -4 - 2 = -6$$

$$C_{12} = (0)(0) + (4)(1) + (2)(0) \\ C_{12} = 4$$

$$C_{13} = (0)(-1) + (4)(0) + (2)(4) \\ C_{13} = 8$$

$$C_{21} = (0)(1) + (-4)(-1) + (2)(-1) \\ C_{21} = 4 - 2 = 2$$

$$C_{22} = (0)(0) + (-4)(1) + (2)(0) \\ C_{22} = -4$$

$$C_{23} = (0)(-1) + (-4)(0) + (2)(4) \\ C_{23} = 8$$

$$\textcircled{4} \quad \begin{bmatrix} 3 & -1 & -5 \\ 2 & 0 & -1 \\ -3 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -6 & -1 & -5 \\ -4 & 1 & 1 \\ 6 & -3 & -3 \end{bmatrix}$$

$$C_{11} = (3)(-2) = -6$$

$$C_{12} = 3(0) + (-1)(4) + (-5)(-1) \\ C_{12} = -4 + 5 = 1$$

$$C_{13} = 3(0) + (-1)(0) + (-5)(-1) \\ C_{13} = 5$$

$$C_{21} = 2(-2) = -4$$

$$C_{22} = 2(0) + 0(4) + (-1)(0) \\ C_{22} = 0$$

$$C_{23} = 2(0) + 0(0) + (-1)(-1) \\ C_{23} = 1$$

$$C_{31} = -3(-2) = 6$$

$$C_{32} = -3(0) + 0(4) + 3(0) \\ C_{32} = 0$$

$$C_{33} = -3(0) + 0(0) + 3(-1) \\ C_{33} = -3$$

$$\textcircled{5} \quad \begin{bmatrix} 3 & 0 & 2 \\ -4 & 0 & 0 \\ -2 & -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 5 & -14 \\ 0 & -4 & 4 \\ 0 & 4 & 0 \end{bmatrix}$$

$$C_{11} = 3(0) = 0$$

$$C_{12} = 3(1) + 0(1) + 2(-1) \\ C_{12} = 3 - 2 = 1$$

$$C_{13} = 3(-1) + 0(1) + 2(4) \\ C_{13} = -3 + 8 = 5$$

$$C_{21} = -4(0) = 0$$

$$C_{22} = -4(1) + 0(1) + 0(-1) \\ C_{22} = -4$$

$$C_{23} = -4(-1) + 0(1) + 0(4) \\ C_{23} = 4$$

$$C_{31} = -2(0) = 0$$

$$C_{32} = -2(1) + 0(1) + 0(-1) \\ C_{32} = -2$$

$$C_{33} = -2(-1) + 0(1) + 0(4) \\ C_{33} = 2$$

SISTEMAS DE ECUACIONES 3x3.

$$\begin{aligned} 4x + 9y - 9z &= 6 \quad (1) \\ 4x + 4y - 3z &= -10 \quad (2) \\ x - 2y + z &= -1 \quad (3) \end{aligned}$$

(2) y (3)

$$-3(4x + 4y - 3z = -10) = 12x - 12y + 9z = 30$$

$$9(x - 2y + z = -1) = 9x - 18y + 9z = -9$$

nueva ecuación

$$\begin{array}{l} 4x + 9y - 9z = 6 \quad (1) \\ -12x + 12y + 9z = 30 \quad (2) \\ 9x - 18y + 9z = -9 \quad (3) \end{array}$$

(1) y (2)

$$\begin{array}{r} 4x + 9y - 9z = 6 \\ -12x + 12y + 9z = 30 \\ \hline -8x - 3y = 36 \end{array}$$

(1) y (3)

$$\begin{array}{r} 4x + 9y - 9z = 6 \\ + 9x - 18y + 9z = -9 \\ \hline 13x - 9y = -3 \end{array}$$

$$\begin{array}{l} (-3) - 8x - 3y = 36 \rightarrow 24x + 9y = -108 \\ 13x - 9y = -3 \\ \hline 37x = -111 \end{array}$$

(y)

$$\begin{array}{l} 37x = -111 \\ \frac{37x}{37} = \frac{-111}{37} \\ x = -3 \end{array}$$

$$\begin{array}{l} 13(-3) - 9y = -3 \\ -39 - 9y = -3 \\ -9y = -3 + 39 \\ -9y = 36 \\ \frac{-9y}{-9} = \frac{36}{-9} \\ y = -4 \end{array}$$

$$\boxed{y = -4}$$

(z)

$$\begin{array}{l} 4x + 9y - 9z = 6 \\ 4(-3) + 9(-4) - 9z = 6 \\ -12 - 36 - 9z = 6 \\ -48 - 9z = 6 \\ -9z = 6 + 48 \\ -9z = 54 \\ \frac{-9z}{-9} = \frac{54}{-9} \\ z = -6 \end{array}$$

$$-48 - 9z = 6$$

$$\cancel{-48} \cancel{-9z} \cancel{6}$$

$$-9z = 6 + 48$$

$$-9z = 54$$

$$z = \frac{54}{-9}$$

$$\boxed{z = -6}$$

$$\begin{aligned} -8x + 7y - 8z &= -9 \\ -8x + 7y - 10z &= -7 \\ x - 2y + z &= 9 \end{aligned}$$

$$\begin{aligned} ① y \quad ③ & \quad 8x + 7y - 8z = -9 \\ -8x + 7y - 8z &= -9 \\ 8x - 16y + 8z &= 72 \\ -9y &= 63 \\ y &= \frac{63}{-9} \end{aligned}$$

$$y = -7$$

$$\begin{aligned} -8x + 7y - 8z &= -9 \\ -8x + 7(-7) - 8(-1) &= -9 \\ -8x - 49 + 8 &= -9 \\ -8x &= -9 + 49 - 8 \\ -8x &= -17 + 49 \\ -8x &= 32 \\ x &= \frac{32}{-8} \\ x &= -4 \end{aligned}$$

$$\begin{aligned} ② y \quad ③ & \quad 8x + 7y - 10z = -7 \\ -8x + 7y - 10z &= -7 \\ 8x - 16y + 8z &= 72 \\ -9y - 2z &= 65 \end{aligned}$$

$$\begin{aligned} -9(-7) - 2z &= 65 \\ 63 - 2z &= 65 \\ -2z &= 65 - 63 \\ -2z &= 2 \\ z &= -1 \end{aligned}$$

$$\begin{aligned} 2x + 5y + 10z &= -4 \\ 2(8) + 5(-7) + 10(-1) &= -4 \\ -16 - 35 - 10 &= -4 \\ -61 &\neq -4 \end{aligned}$$

$$\begin{aligned} 2x + 5y + 10z &= -4 \\ 3x + 2y + 10z &= 10 \\ x + 5y + z &= -3 \end{aligned}$$

$$\begin{aligned} -1(3x + 2y + 10z = 10) & \rightarrow -3x - 2y - 10z = 10 \\ -10(x + 5y + z = -3) & \rightarrow -10x - 50y - 10z = 30 \end{aligned}$$

nueva ecuación

$$\begin{aligned} 2x + 5y + 10z &= -4 \quad ① \\ -3x - 2y - 10z &= -10 \quad ② \\ -10x - 50y - 10z &= 30 \quad ③ \end{aligned}$$

$$\begin{aligned} ④ y \quad ⑤ & \quad 2x + 5y + 10z = -4 \\ 2x + 5y + 10z &= -4 \\ -3x - 2y - 10z &= -10 \\ -x + 3y &= -14 \end{aligned}$$

$$15(-x + 3y = -14) \rightarrow -15x + 45y = -210$$

$$\begin{aligned} -15x + 45y &= -210 \\ -8x - 45y &= 26 \\ -23x &= -184 \end{aligned}$$

$$x = \frac{-184}{23} \quad x = 8$$

$$\begin{aligned} ④ y \quad ③ & \quad 2x + 5y + 10z = -4 \\ 2x + 5y + 10z &= -4 \\ -10x - 50y - 10z &= 30 \\ -8x - 45y &= 26 \end{aligned}$$

$$-8x - 45y = 26$$

$$\begin{aligned} ④ y \quad ③ & \quad 2x + 5y + 10z = -4 \\ 2(8) + 5(-7) + 10(-1) &= -4 \\ 16 - 35 - 10 &= -4 \\ 6 + 10z &= -4 \end{aligned}$$

$$\begin{aligned} 10z &= -4 - 6 \\ 10z &= -10 \end{aligned}$$

$$\begin{aligned} z &= \frac{-10}{10} \\ z &= -1 \end{aligned}$$

$$\begin{aligned} -6x + 8y + 3z &= 9 \quad (1) \\ -6x + y + 5z &= 2 \quad (2) \\ x + 2y - z &= 5 \quad (3) \end{aligned}$$

$$-(-6x + 8y + 3z = 9) \Rightarrow 6x - 8y - 3z = -9 \quad (4)$$

$$\begin{array}{l} 6x - 8y - 3z = -9 \quad (4) \\ -6x + y + 5z = 2 \quad (2) \\ 6x + 12y - 6z = 30 \quad (5) \end{array}$$

$$\begin{array}{l} (2) - y \quad (4) \\ -6x + y + 5z = 2 \\ 6x - 8y - 3z = -9 \\ \hline 13y - 1z = 32 \end{array}$$

$$\begin{array}{l} 26y - 2z = 64 \\ -7y + 2z = -7 \\ \hline 19y = 57 \\ y = \frac{57}{19} \\ y = 3 \end{array}$$

$$\begin{array}{l} (2) - 7y + 2z = -7 \\ 10 - 7(3) + 2z = -7 \\ 10 - 21 + 2z = -7 \\ 2z = -7 + 21 \\ 2z = 14 \\ z = \frac{14}{2} \\ z = 7 \end{array}$$

$$\begin{array}{l} 6x - 8y - 3z = -9 \\ 6x - 8(3) - 3(7) = -9 \\ 6x - 24 - 21 = -9 \\ 6x - 45 = -9 \\ 6x = -9 + 45 \\ x = \frac{36}{6} \\ x = 6 \end{array}$$

$$\begin{array}{l} 2x + 9y - 4z = -5 \\ -x + 2y - z = -3 \\ x - 4y + 2z = 4 \end{array}$$

$$-4(-x + 2y - z = -3) \Rightarrow 4x - 8y + 4z = 12 \quad 2(x - 4y + 2z = 4) \rightarrow 2x - 8y + 4z = 8$$

$$\begin{array}{l} 2x + 9y - 4z = -5 \\ 4x - 8y + 4z = 12 \\ 2x - 8y + 4z = 8 \end{array}$$

$$\begin{array}{l} (1) y \quad (2) \\ 2x + 9y - 4z = -5 \\ 4x - 8y + 4z = 12 \\ \hline 6x + y = 7 \end{array}$$

$$\begin{array}{l} (1) y \quad (3) \\ 2x + 9y - 4z = -5 \\ 2x - 8y + 4z = 8 \\ \hline 4x + y = 3 \end{array}$$

$$\begin{array}{l} -6x - y = -7 \\ 4x + y = 3 \\ -2x = -4 \\ x = 2 \end{array}$$

$$\begin{array}{l} 4x + y = 3 \\ 4(2) + y = 3 \\ 8 + y = 3 \\ y = \frac{3}{8} - 8 \\ y = -5 \end{array}$$

$$\begin{array}{l} 2x + 9y - 4z = -5 \\ 2(2) + 9(-5) - 4z = -5 \\ 4 - 45 - 4z = -5 \\ -41 - 4z = -5 \\ -4z = -5 + 41 \\ -4z = 36 \\ z = \frac{36}{-4} \\ z = -9 \end{array}$$

$$\begin{array}{l} p = 300 + (50 + x) \cdot 2 \\ p = 300 + 50 + 2x + 2x \\ p = 300 + 50 + 4x \\ p = 300 + 50 - 50 \\ p = 300 + 4x \\ 2p = 600 + 8x \\ 2p - 600 = 8x \\ 2(p - 300) = 8x \\ 2p - 600 = 8x \\ 2p = 8x + 600 \\ 2p = 8x + 600 \\ 2p = 8x + 600 \end{array}$$

DERIVATES OF POLYNOMIALS

(1)

$$y = 5x^5 - 8x^4 - 8x^3 + 4x^2 + 2x + 5$$

$$\frac{dy}{dx} = 25x^4 - 32x^3 - 24x^2 + 8x + 2 //$$

(2)

$$y = \overbrace{5x^3 + 5x + 4}$$

$$\frac{dy}{dx} = 15x^2 + 5 //$$

$$(3) f(x) = -5x^3 + 8x - 4$$

$$f'(x) = -15x^2 + 8 //$$

$$(4) y = 7x^5 + 9x$$

$$\frac{dy}{dx} = 35x^4 + 9 //$$

$$(5) y = -4x^5 - 8x^3 + x^2 + 9x$$

$$\frac{dy}{dx} = -20x^4 - 24x^2 + 2x + 9$$

DERIVATES OF E

$$(1) y = e^{6x^5}$$

$$y' = e^{6x^5}$$

$$y' = 30x^4$$

$$y' = e^{6x^5} \cdot (30x^4)$$

$$y' = \underline{\underline{30x^4 e^{6x^5}}}$$

$$(3) y = e^{-9x^6}$$

$$y' = -54x^5$$

$$y' = e^{-9x^6} (-54x^5)$$

$$y' = -54x^5 e^{-9x^6} //$$

$$(2) y = e^{-2x^5}$$

$$y' = -10x^4$$

$$y' = e^{-2x^5} (-10x^4)$$

$$y' = -10x^4 e^{-2x^5} //$$

$$(4) y = e^{-x^5 - 4x^4}$$

$$y' = e^{(-5x^4 - 16x^3)}$$

$$y' = e^{-x^5 - 4x^4} (-5x^4 - 16x^3)$$

$$(5) y = e^{-6x^2 + 6x}$$

$$y' = e^{(-12x + 6)}$$

$$y' = e^{-6x^2 + 6x} (-12x + 6)$$

~~DERIVATIVE~~ DERIVATIVE OF IN

$$\textcircled{1} \quad y = \ln(2x^4 + x^3)$$

$$y' = 8x^3 + 3x^2$$

$$y' = \frac{1}{2x^4 + x^3} \cdot (8x^3 + 3x^2)$$

$$y = \frac{8x^3 + 3x^2}{x(x^3 + x^2)}$$

$$\textcircled{2} \quad y = \ln(8x^3)$$

$$y' = \frac{1}{8x^3} \cdot (24x^2)$$

y

$$\textcircled{3} \quad y = \ln(7x^4 + 4x^3)$$

$$y' = \frac{1}{7x^4 + 4x^3} \cdot (28x^3 + 12x^2)$$

$$y' = \frac{4(7x+3)}{x(7x+4)}$$

$$\textcircled{4} \quad y = \ln(9x^2 + 6x)$$

$$y' = \frac{1}{9x^2 + 6x} \cdot (18x + 6)$$

$$y' = \frac{2(3x+1)}{x(3x+2)}$$

$$\textcircled{5} \quad y = \ln(7x^6 - 6x^5)$$

$$y' = \frac{1}{7x^6 - 6x^5} \cdot (42x^5 - 30x^4)$$

$$y' = \frac{6(7x-5)}{x(7x-6)}$$

$$\textcircled{6} \quad y = \ln(5x^6)$$

$$y' = \frac{1}{5x^6} \cdot (30x^5)$$

$$y' = \left(\frac{6}{x^5} - 10x^4 \right)$$

Implicit Derivative at a Point (Level 1)

$y - y^2 + 2x^3 = 0$ then find $\frac{dy}{dx}$ at the point $(1, 2)$

$$\frac{d}{dx}(y - y^2 + 2x^3) = \frac{d}{dx}(0)$$

$$y' - 2yy' + 6x = 0$$

$$y' - 2(2)y' + 6(1) = 0$$

$$y' - 4y' + 6 = 0$$

$$-3y' + 6 = 0$$

$$-3y' = -6$$

$$\boxed{y' = 2}$$

If $-x^2 - y^3 = -1$ then find $\frac{dy}{dx}$ at the point $(3, -2)$

$$\frac{d}{dx}(-x^2 - y^3) = \frac{d}{dx}(-1)$$

$$-2x - 3y^2y' = 0$$

$$-2(3) - 3(-2)^2y' = 0$$

$$-6 - 12y' = 0$$

$$-12y' = -6$$

$$y' = \frac{-6}{-12} = \frac{1}{2}$$

$$y' = -\frac{1}{2}$$

if $-4 - 5x^3 - x = y^2 - 2x^2$ then find $\frac{dy}{dx}$ at the point $(-1, -2)$

$$\frac{d}{dx} (-4 - 5x^3 - x) = \frac{d}{dx} (y^2 - 2x^2)$$

$$-15x^2 - 1 = 2yy' - 4x$$

$$-15(-1)^2 - 1 = (2)(-2)y' - 4(-1)$$

$$-15 - 1 = -4y' + 4$$

$$-16 = -4y' + 4$$

$$4y' = 20$$

$$y' = \frac{20}{4}$$

$$y' = 5$$

if $x^2 + 4y^2 = -4y^3 + 4x$ then find $\frac{dy}{dx}$ at the point $(4, -1)$

$$2x + 8yy' = -12y^2y' + 4$$

$$2(4) + 8(-1)y' = -12(-1)^2y' + 4$$

$$8 - 8y' = -12y' + 4$$

$$4y' = -4$$

$$\boxed{y' = -1}$$

if $O = -2x^2 + y^2 - 2x^3$ then find $\frac{dy}{dx}$ at the point $(1, -2)$

$$O = -4x + 2yy' - 6x^2$$

$$O = -4(1) + 2(-2)y' - 6(1)^2$$

$$O = -4 - 4y' - 6$$

$$4y' = -10$$

$$\boxed{y' = -\frac{10}{4}}$$

If $-4 - y^2 - 3x^2 - y = 2x^3$ then find $\frac{dy}{dx}$ at the point $(-2, -1)$

$$-2yy' - 6x - y' = 6x^2$$

$$-2(-1)y' - 6(-2) - y' = 6(-2)^2$$

$$2y' + 12 - y' = 24$$

$$2y' - y' = 24 - 12$$

$$\boxed{y' = 12} //$$

If $2y - x + y^3 = x^2$ then find $\frac{dy}{dx}$ at the point $(3, 2)$

$$2y' - 1 + 3y^2y' = 2x$$

$$2y' - 1 + 3(2)^2y' = 2(3)$$

$$2y' - 1 + 12y' = 6$$

$$2y' + 12y' = 6 + 1$$

$$14y' = 7$$

$$y' = \frac{7}{14} //$$

$$\boxed{y' = 1/2} //$$

If $2x^2 - 2y^2 = 5 - y$ then find $\frac{dx}{dy}$ at the point $(-2, -1)$

$$4x - 4yy' = y'$$

$$4(-2) - 4(-1)y' = y'$$

$$-8 + 4y' = -y$$

$$4y' + y' = 8$$

$$5y' = 8$$

$$y' = \frac{8}{4} //$$