1. Problems

1.1. Verification part 1, (3P). Given the function

$$f(x) = \frac{1}{2}x - 3\,.$$

1. Approximate the value $\int_0^8 f(x)\,\mathrm{d}x$

   First, we note that f is continuous. Hence we can apply all quadrature rules.

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1) Aproximation
        a) using the midpoint rule            :  -8
        b) using the trapezoidal rule         :  -8
        c) using the composite trapezoidal rule:  -8
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2. Are the values exact? Justify why.

   YES, all results are exact up to rounding errors, since f is affine linear

1.2. Verification part 2, (5P). Given the function

$$f(x) = e^x - 1\,.$$

approximate $\int_0^3 f(x)\,\mathrm{d}x$.

1. Design a composite rule similar to (3) using the midpoint rule.

$$\int_a^b f(x)\,\mathrm{d}x = \sum_{i=1}^{n-1} \int_{x_i}^{x_{i+1}} f(x)\,\mathrm{d}x \approx \sum_{i=1}^{n-1} (x_{i+1} - x_i) \cdot f\left(\frac{x_{i+1} + x_i}{2}\right).$$

2. Apply this rule and approximate $\int_0^3 f(x)\,\mathrm{d}x$ on 5 subdivisions of the interval $[0, 3]$ into $n \in \{1, 2, 4, 8, 16\}$ subintervals.

   a) calculate (theoretically) the exact value.

$$\rightarrow \int_0^3 f(x)\,\mathrm{d}x = e^3 - 4 \approx 16.0855369231877$$

   b) for each $n \in \{1, 2, 4, 8, 16\}$, record the approximated values and fill the following table:

Again, we note that f is continuous. Hence we can apply the quadrature rule.

n	approximate value	relative error
1	10.44507	0.3506547366
2	14.40710	0.1043442411
4	15.64545	0.0273590958
8	15.97416	0.0069237563
16	16.05761	0.0017362609