PROJECT – QUADRATURE

1. PROJECT SPECIFIC RULES

- How to submit your project:
  Comunidad ITAM → Trabajos y Exámenes → Proyecto ?
- Last possible date of submission: some date and hour
- ¿What you should upload as part of your project? → see general rules.
- For each problem you need to write and hand in a *MatLab/Octave* script that reproduces the results in your document. It is recommended that the first line of each script reads:

```
clear all;   close all;   clc;
```

2. THEORY: SIMPLE QUADRATURE RULES

For simplicity, let $f\colon [a,b] \to \mathbb{R}$ by a continuous function.
We aim to approximate the value

$$\int_a^b f(x)\,\mathrm{d}x\,.$$

If the function f is almost an affine function (i.e., a polynomial of degree 1) then this can be done using one of the following quadrature rules:

$$\int_a^b f(x)\,dx \approx (b-a) f\left(\frac{a+b}{2}\right) \qquad \text{, the \textit{midpoint rule},} \tag{1}$$

$$\int_a^b f(x)\,dx \approx (b-a)\left(\frac{f(a)+f(b)}{2}\right) \qquad \text{, the \textit{trapezoidal rule}.} \tag{2}$$

If the function is more complicated we can split up the interval $[a,b]$ into, say n, subintervals and apply our favourite rule on each subinterval, *e.g.* the trapezoidal rule:

$$\int_a^b f(x)\,\mathrm{d}x = \sum_{i=1}^{n-1} \int_{x_i}^{x_{i+1}} f(x)\,\mathrm{d}x \approx \sum_{i=1}^{n-1} \frac{x_{i+1}-x_i}{2}\Big(f(x_i)+f(x_{i+1})\Big)\,. \tag{3}$$

3. Problems

Hint: The values below should be calculated by your implementations of the quadrature rules (1), (2), (3) and the one defined below.

3.1. Verification part 1, (5P). Given the function

$$f(x) = \frac{1}{2}x - 3\,.$$

1. Approximate the value $\int_0^8 f(x)\,\mathrm{d}x$
   a) using the midpoint rule (1)
   b) using the trapezoidal rule (2)
   c) using the *composite rule* given in (3) on $n = 10$ equidistant subintervals, *i.e.* of the shape $[a + ih, a + (i+1)h]$ where $h = (b-a)/n$ and $i = 0, \ldots, n-1$.
2. Are the values exact? Justify why.

3.2. Verification part 2, (5P). Given the function

$$f(x) = e^x - 1\,.$$

approximate $\int_0^3 f(x)\,\mathrm{d}x$.

1. Design a composite rule similar to (3) using the midpoint rule.
2. Apply this rule and approximate $\int_0^3 f(x)\,\mathrm{d}x$ on 5 subdivisions of the interval $[0, 3]$ into $n \in \{1, 2, 4, 8, 16\}$ subintervals.
   a) calculate (theoretically) the exact value.
   b) for each $n \in \{1, 2, 4, 8, 16\}$, record the approximated values and fill the following table:

n	approximate value	relative error
1	$\cdot$	$\cdot$
$\vdots$	$\cdot$	$\cdot$
16	$\cdot$	$\cdot$

   To fill the table, recall the definition

$$\text{relative error} = \frac{\text{absolute error}}{\text{exact value}}\,.$$

Congratulations. You have implemented and verified the quadrature rules described above. Now you can use your implementation to approximate integrals which are not known, *e.g.*

$$f(x) = e^{x^2}\,.$$