Introduction to Some Signal Processing Models

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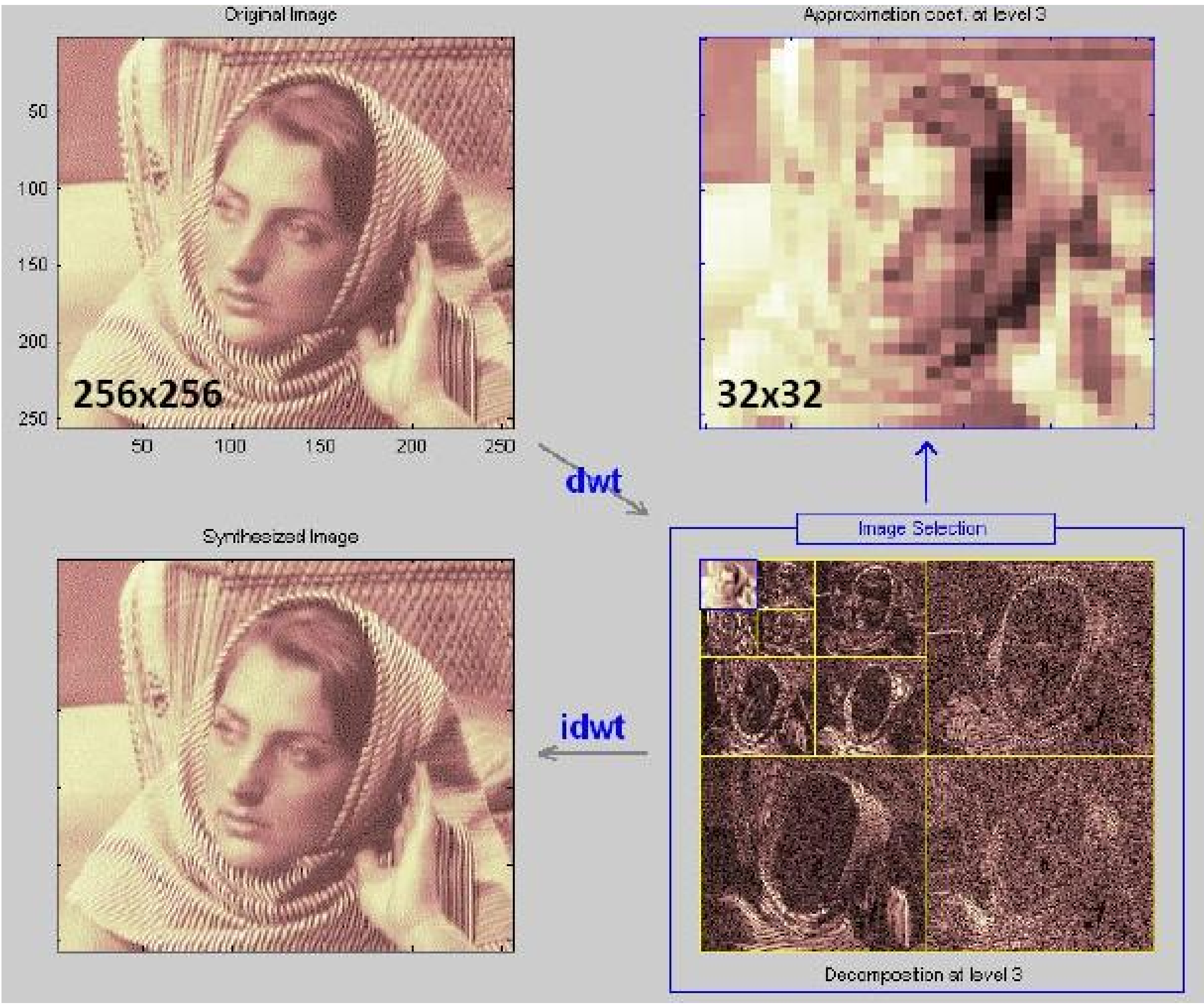
Models

Discrete Wavelet Transform (DWT)

DWT is widely used in signal processing. For example, the compression image formatting JPEG-2000 is based on DWT. Compared to Fourier transform, wavelet transform captures both frequency and location information. It can reveal the trends and periodicity in unstationary time series. Below is a result of a wavelet transform showing how it decomposes an image.



Principle Component Analysis (PCA)

PCA is the most popular algorithm among all the methods introduced in this article. It is used to identify the uncorrelated factors behind the signals. One can usually find practical justifications for low-order principle components.

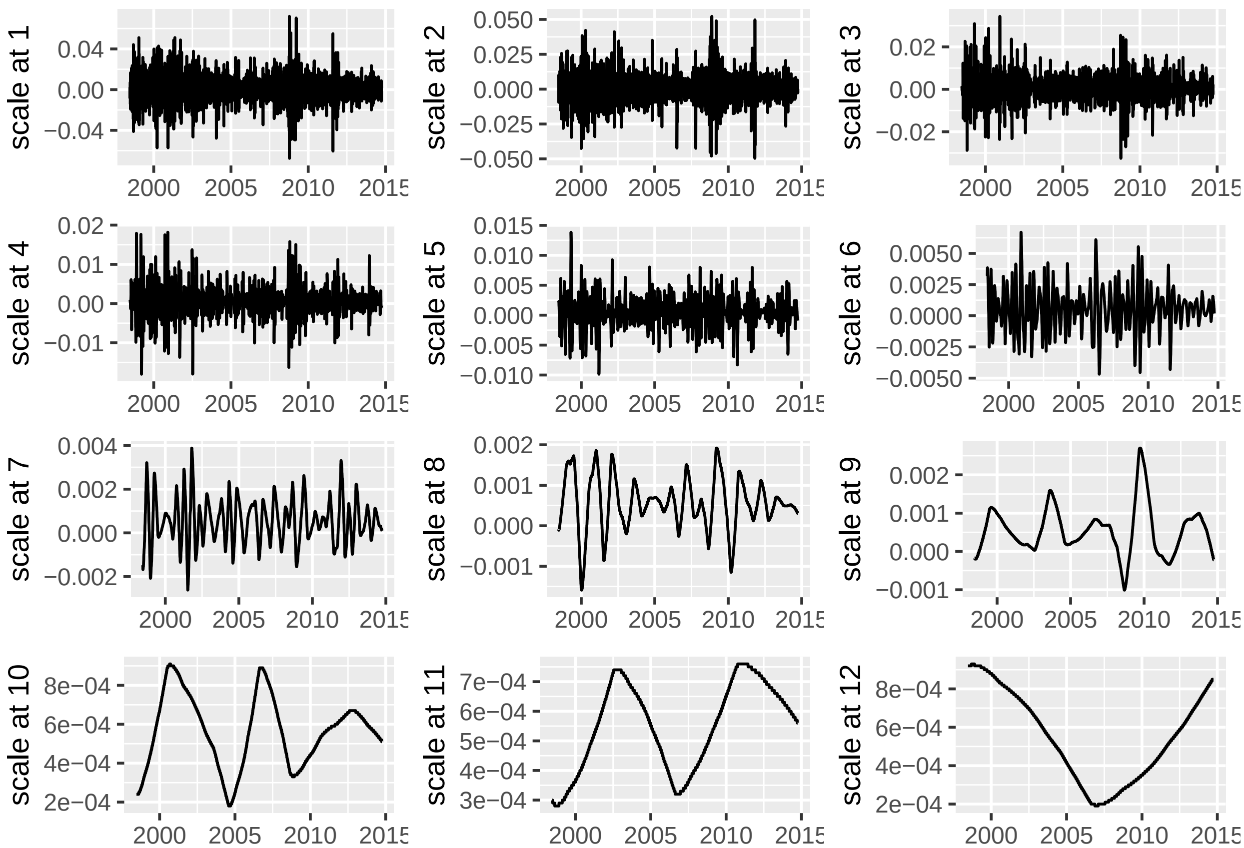
Kalman Filter

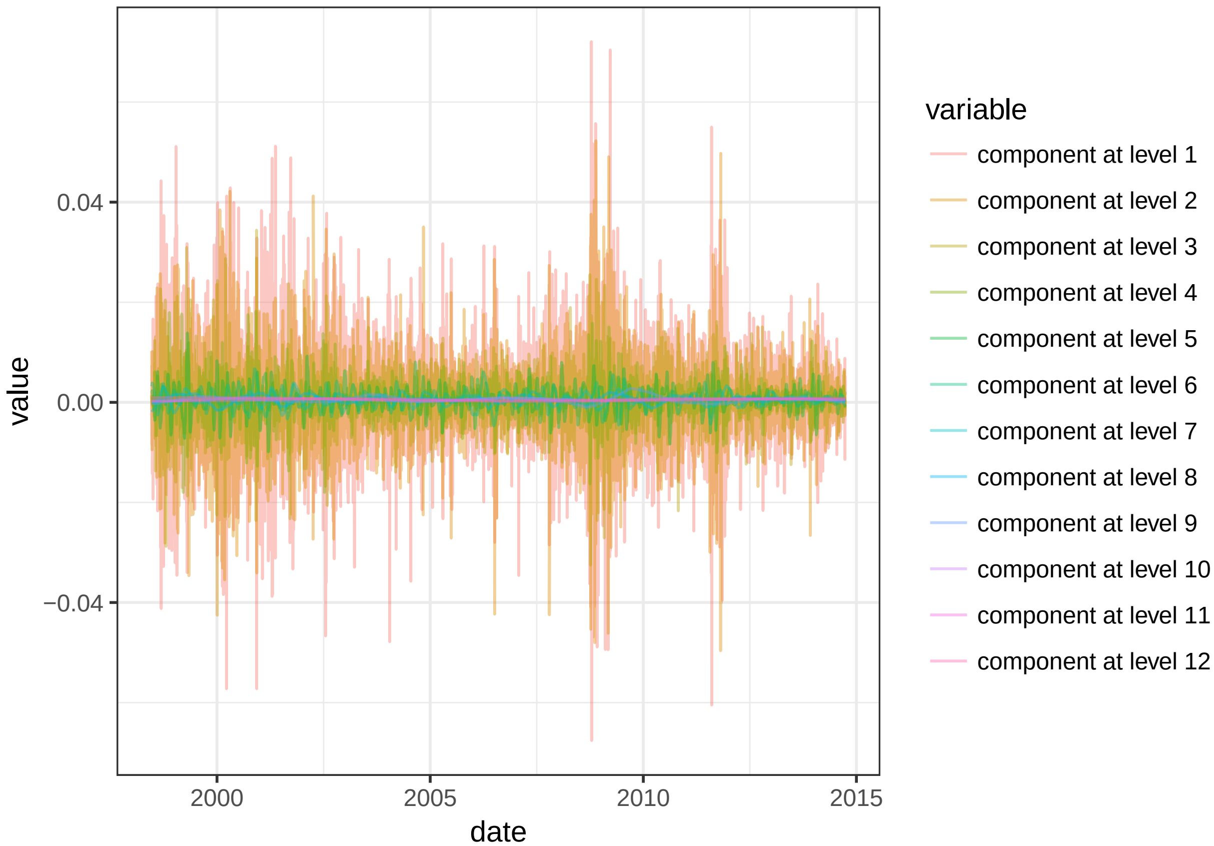
Kalman filter is widely used in signal processing such as signal prediction and smoothing given very little prior information. In our case, with more than 500 dimensions of every states, it is really hard to use ordinary regression because of the rank deficiency at beginning. The Kalman filter, however, can give estimates at very beginning and quickly converges to the BLUE estimator. There are many derivatives of Kalman filter, such as Extended KF and Unscented KF. Adaptive KF is used in this project. The methodology can be found in Improving Adaptive Kalman Estimation in GPSINS Int.pdf, Market Risk Beta Estimation using Adaptive Kalman Filter.pdf.

Applications

Discrete Wavelet Transform (DWT)

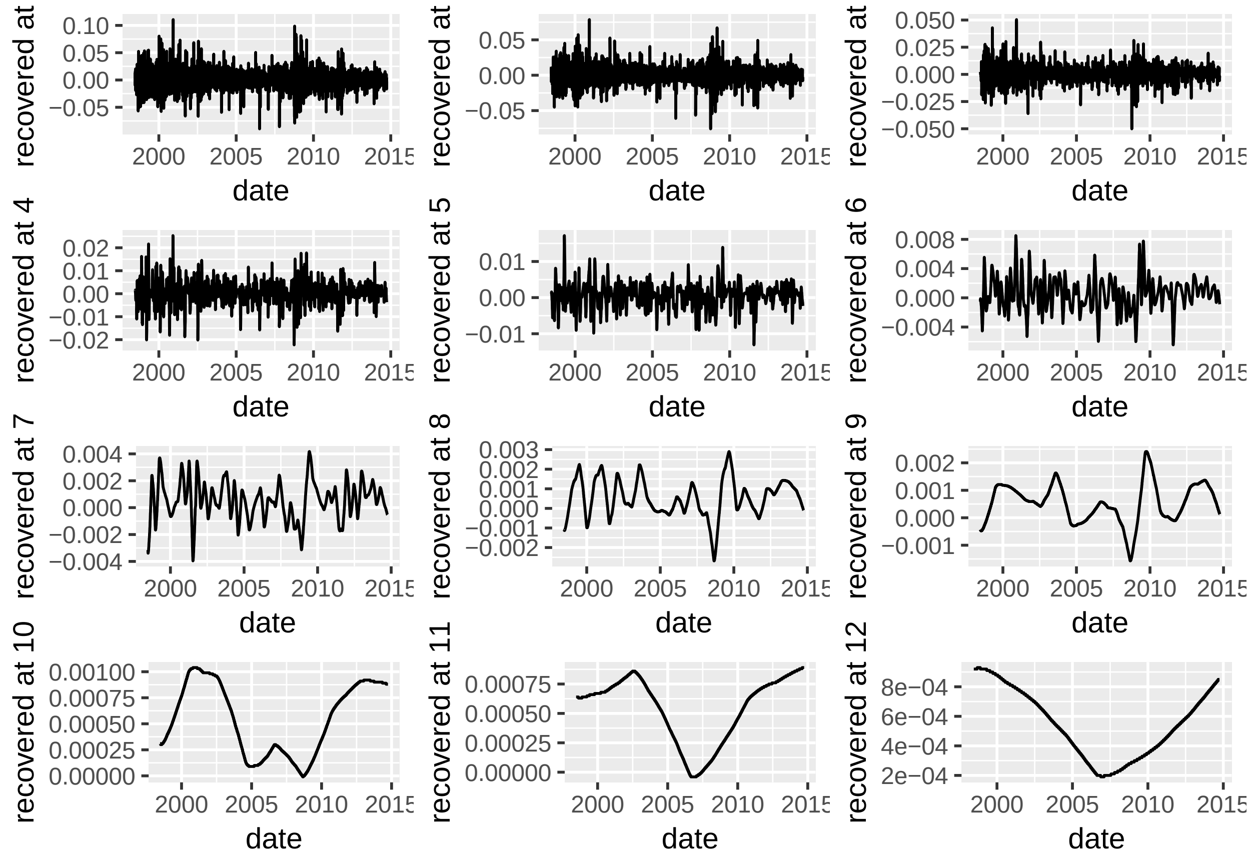
Wavelet transform can be a good tool to analyse the seasonal features in our case. It decomposes the return signal into different scale levels.





value
0.04
0.00
−0.04
2000
2005
2010
2015
date
variable
component at level 1
component at level 2
component at level 3
component at level 4
component at level 5
component at level 6
component at level 7
component at level 8
component at level 9
component at level 10
component at level 11
component at level 12

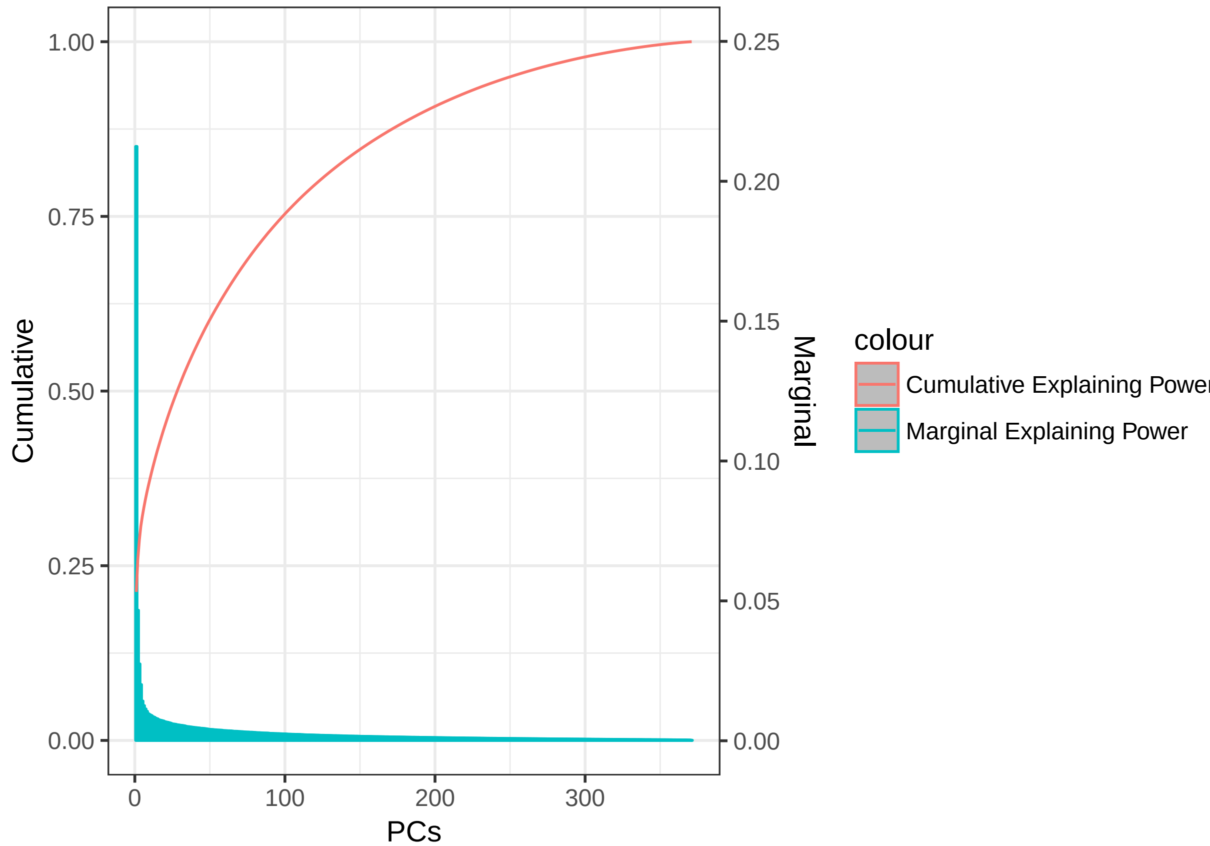
Through Wavelet transform, we can look into the signal recovered at different scales.



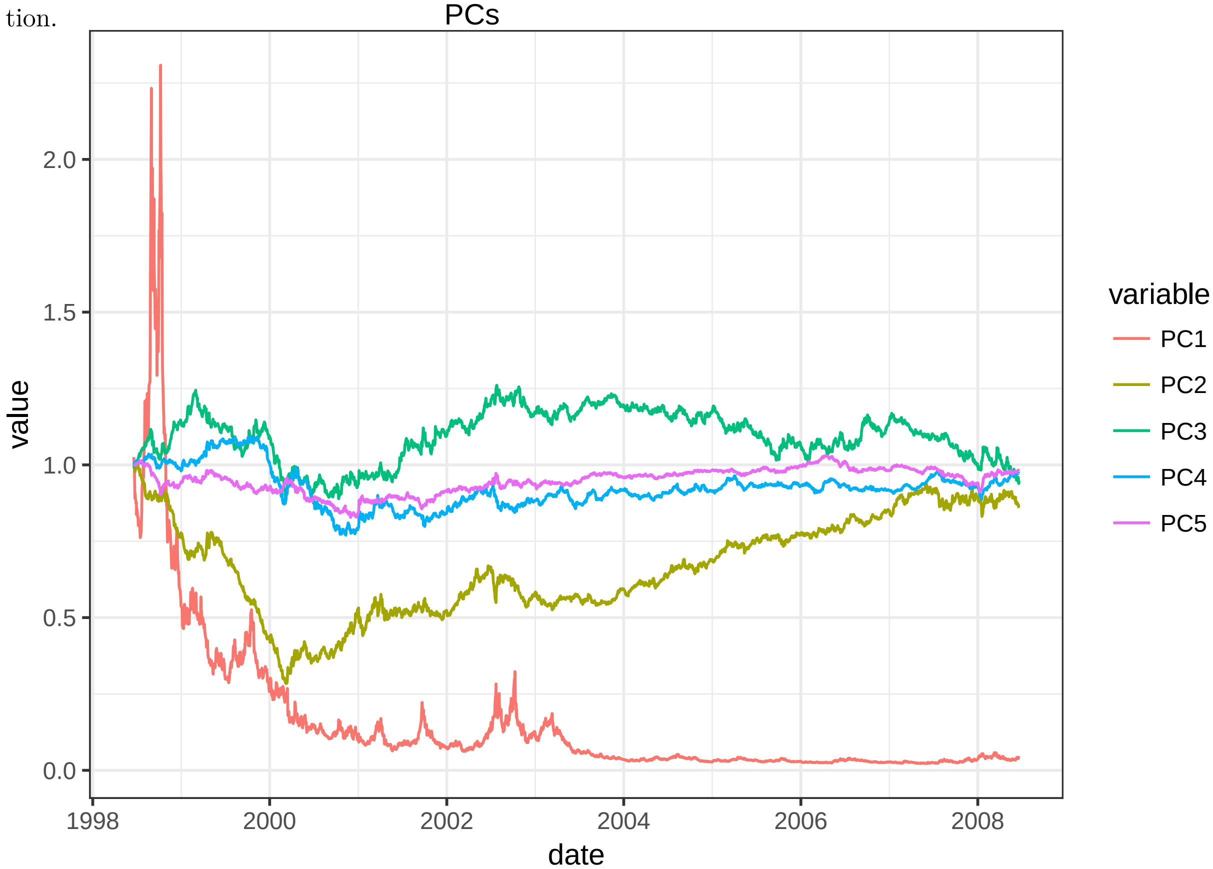
Although there seems to exist some seasonalities in the signal, the intensity of the seasonality is low. Most of the energy of the return signal is in high-frequency band. I think it might be better if we could use monthly data instead of daily data. I want to emphasis that Wavelet transform does not require the signal to be stationary, while it is required in ARIMA model. And in fact, the return signals mostly are not stationary in reality.

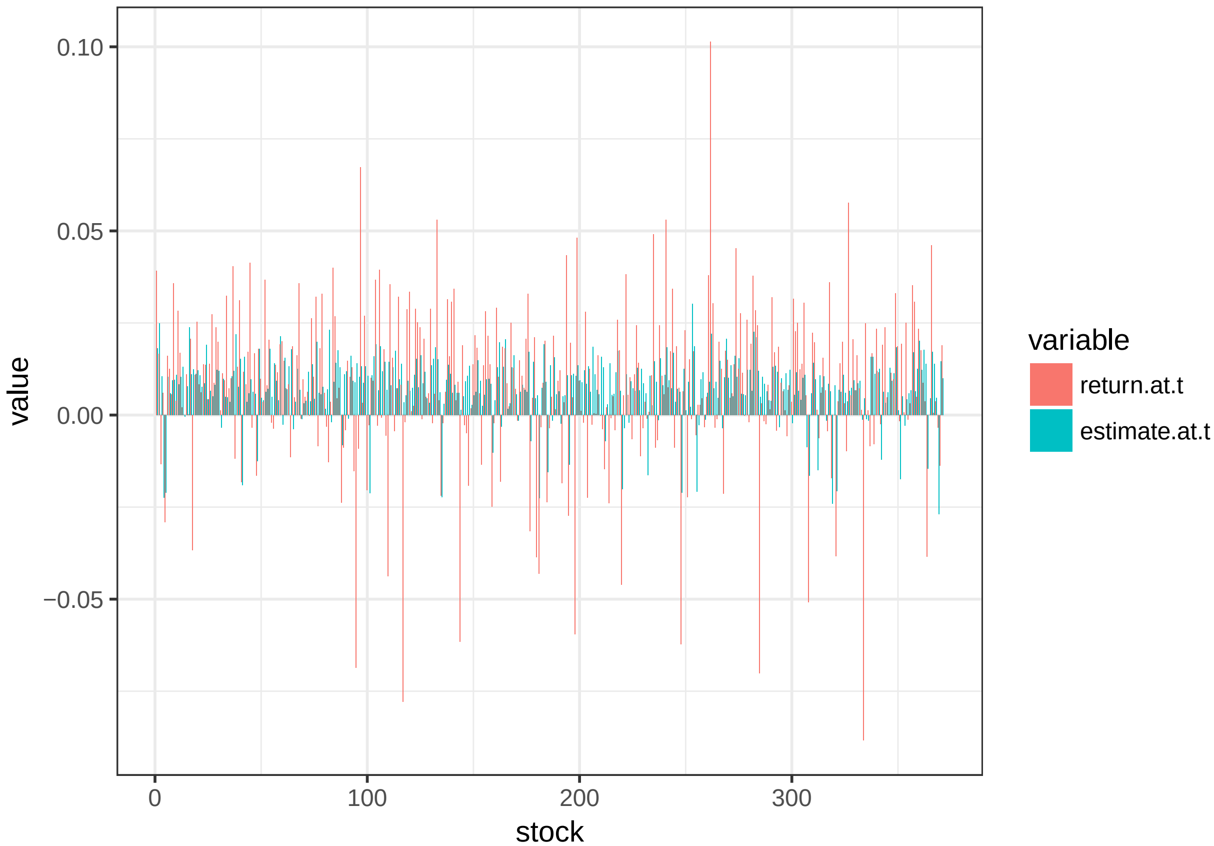
Principle Component Analysis (PCA)

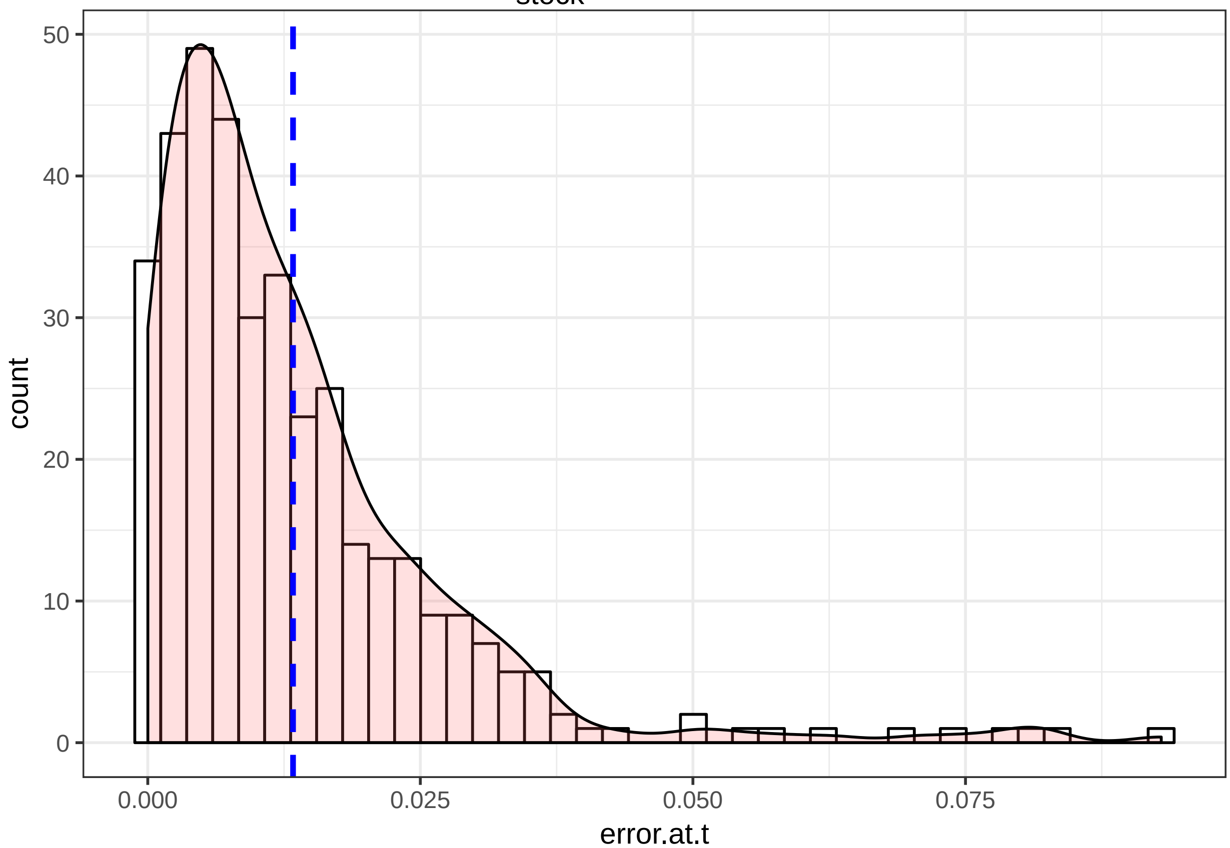
PCA is mostly used in finding underlying factors. In our case, it can also be used to deal with the high dimensional problem, namely the number of variables exceeding the number of obeservations, which leads to singular matrix in calculation. It is not perfect solution however, since it losses information when doing dimension reduc-


Cumulative
Marginal
PCs
colour
Cumulative Explaining Power
Marginal Explaining Power
1.00
0.75
0.50
0.25
0.00
0.25
0.20
0.15
0.10
0.05
0.00
0
100
200
300

tion.

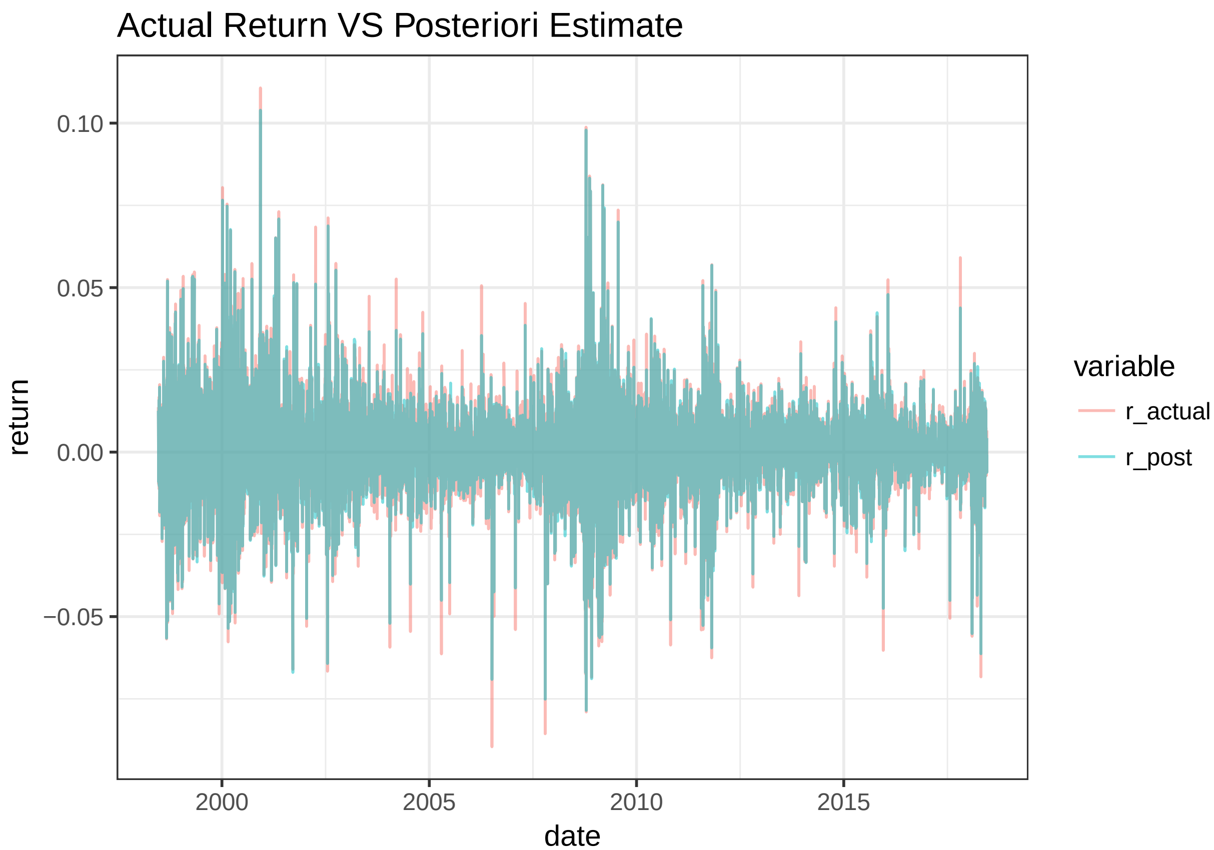

value
date
variable
PC1
PC2
PC3
PC4
PC5
2.0
1.5
1.0
0.5
0.0
1998
2000
2002
2004
2006
2008

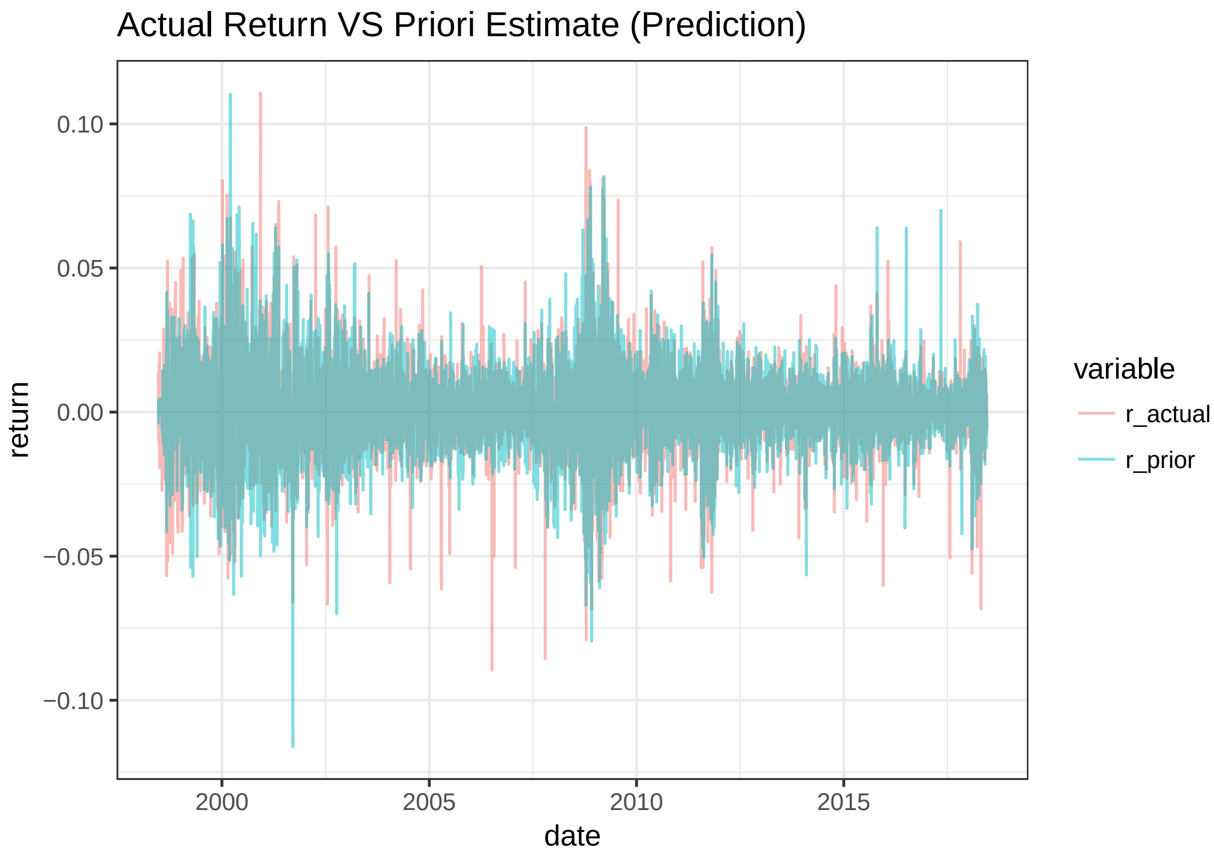

0.10
0.05
0.00
−0.05
value
0
100
200
300
stock
variable
return.at.t
estimate.at.t


50
40
30
20
10
0
count
0.000
0.025
0.050
0.075
error.at.t

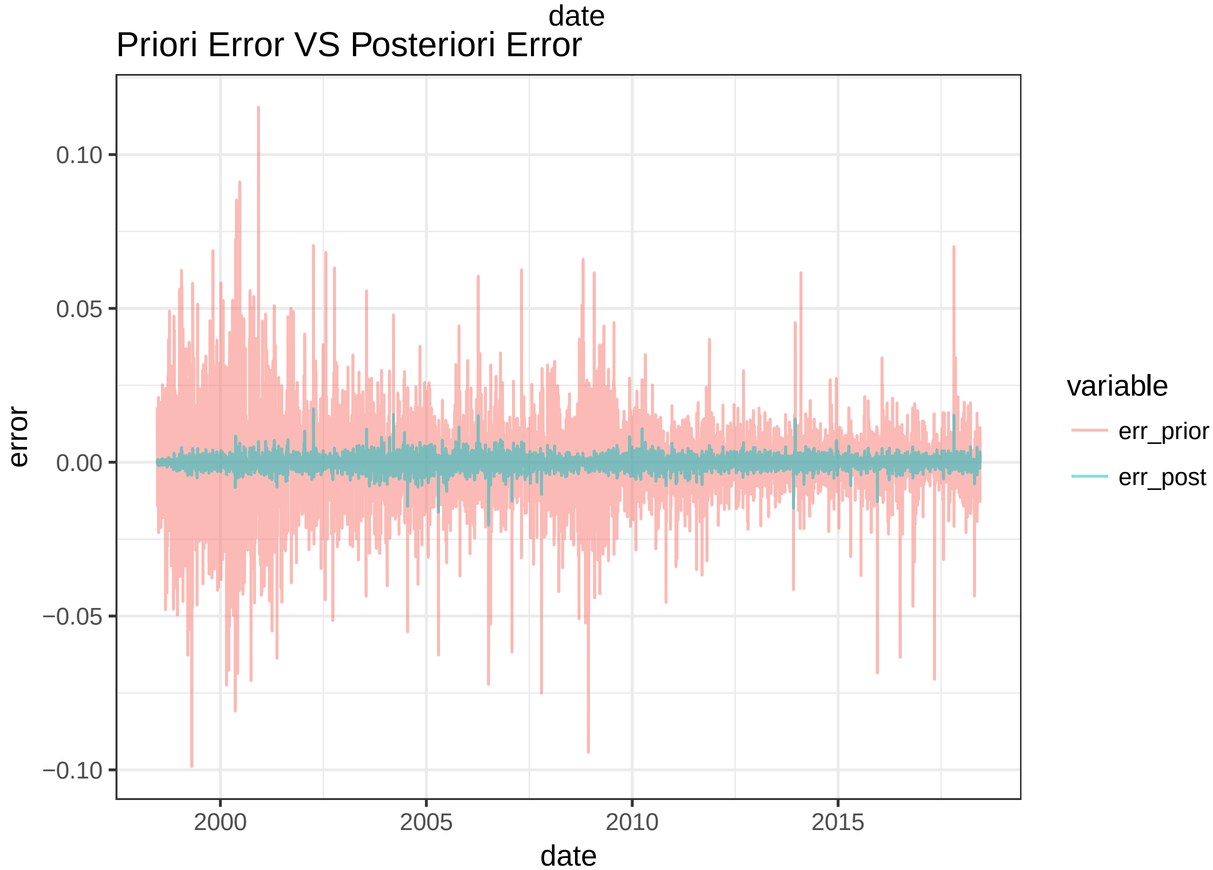
Kalman Filter

In this part, all stock data in the S&P 500 universe is used for estimate the stock return of MMM. As the result has shown below, it seems that Kalman filter can provide a very accurate posteriori estimate and good one step prediction. There is one concern that the model's accuracy to some degree relies on the initial values of Q, the covariance matrix of state transition noise, and R, the variance of estimate noise. Adaptive Kalman filter is designed to eliminate the influence of inaccurate initial values of Q and R.





Actual Return VS Priori Estimate (Prediction)
0.10
0.05
0.00
−0.05
−0.10
return
2000
2005
2010
2015
date
variable
r_actual
r_prior



Priori Error VS Posteriori Error
0.10
0.05
0.00
−0.05
−0.10
error
2000
2005
2010
2015
date
variable
err_prior
err_post

	Prior.MSE	Std.Dev.of.Priori.MSE	Posteri.MSE	Std.Dev.of.Posteri.MSE
Kalman Filter Estimates	1.68e-06	0.0145	5.88e-06	0.00225

Conculsion

These models are commonly used in signal processing. They can also be categorized into machine learning in a broad sense. Although some of them may not be used in building models, but all of them are very helpful in analyzing data and finding features. For example, Kalman filter and Wavelet transform are good tools to deal with nonstationary signals, which exist everywhere in financial world.