1 Kinematics

Definitions

1. Displacement refers to the change in particle's position

$$\Delta x = x_f - x_i$$

2. Distance is a scalar.
3. Average velocity is defined as the ratio of displacement Δx and the time interval Δy

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i}$$

4. Average speed is defined as the ratio of total distance travelled to the total time taken

$$\text{Av.Speed} = \frac{\text{Distance}}{\text{Time}}$$

5. Instantaneous velocity

$$v = \frac{dx}{dt}$$

6. Average acceleration is defined as the change in velocity divided by the time in which the change occured

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i}$$

7. Instantaneous acceleration

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Constant acceleration equations

1. Use when no displacement given

$$v = v_0 + at$$

2. Use when required to find displacement or when given displacement

$$s = s_0 + v_0 t + \frac{1}{2}at^2$$

3. Use when no time is given

$$v^2 = v_0^2 + 2a(s - s_0)$$

4. Use when no acceleration is given

$$s - s_0 = \frac{1}{2}(v + v_0)t$$

Free Fall

If air resistance can be neglected, the motion of a falling object is free fall. Free fall due to gravity is 9.81 m/s^2.

2 Two Dimensional Motion

The same formulas apply for motion in 2D.

1. Instantaneous Velocity - Differentiate the x and y components separately.
$$\vec{v} = \frac{d\vec{r}}{dt}$$
2. Average Velocity
$$\bar{v} = \frac{\Delta\vec{r}}{\Delta t}$$
3. Instantaneous Acceleration
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$
4. Average Acceleration
$$\bar{a} = \frac{\Delta v}{\Delta t}$$

The equations of motion can be split up into x and y components

$$v_x = v_{x0} + a_x t$$

$$v_y = v_{y0} + a_y t$$
$$\vec{v} = \vec{v_0} + \vec{a}t$$

Projectile Motion
The horizontal motion and the vertical motion are independent of each other.
Without air resistance, the horizontal motion will remain constant, and the vertical motion will be affected by gravity (-9.81m/s^2).
The range of the projectile motion is determined by the horizontal velocity, while the time of the the motion is determined by the vertical velocity.
Resolve the vectors to get separate the x and y components.

Basically, projectile motion is constant horizontal velocity and vertical free fall, without air resistance.
Horizontal Motion
$$v_x = v_{x0} = v_0 \cos\alpha_0$$
$$x = v_0 \cos\alpha_0 \cdot t$$

Vertical Motion
$$v_y = v_{y0} - gt = v_0 \sin\alpha_0 - gt$$
$$y = y_0 + v_0 \sin\alpha_0 \cdot t - \frac{1}{2}gt^2$$

Trajectory Formulas
$$y = (\tan\alpha_0)x - \left(\frac{g}{2v_0^2\cos^2\alpha_0}\right)x^2$$

Maximum Height – At maximum height, the vertical velocity v_y is zero.

$$h = y_{max} = \frac{v_0^2\sin^2\alpha_0}{2g}$$

Range of Motion – Horizontal Range. This is why maximum range occurs at 45 degrees.

$$R = x = \frac{v_0^2\sin 2\alpha_0}{g}$$

3 Circular Motion

Uniform Circular Motion

Uniform circular motion occurs when there is a change in direction of velocity, but the magnitude of velocity remains the same.

Acceleration occurs when there is a change in direction or magnitude of velocity or both. The radial centripetal acceleration during uniform circular motion always points towards the center of the circle.

$$T = \frac{2\pi R}{v}$$

$$v = \frac{2\pi R}{T}$$

$$|\vec{a}| = a_{rad} = \frac{v^2}{r} = \frac{4\pi^2 R}{T}$$

$$\omega = 2\pi f = \frac{2\pi}{T}\text{rads}^{-1}$$

$$\omega = \frac{v}{R}$$

$$v = r\omega$$

$$|\vec{a}| = a_{rad} = \omega^2 R$$

Non-uniform Circular Motion

Non-uniform circular motion occurs when the magnitude of velocity is not constant. For example, vertical circle.

There will be one more component of acceleration, instead of just centripetal / radial acceleration, there will also be tangential acceleration.

$$\vec{a} = \vec{a}_t + \vec{a}_{rad}$$

$$a_t = \frac{d\vec{v}}{dt}$$

$$a_{rad} = \frac{v^2}{r}$$

The tangential acceleration is always perpendicular to the centripetal acceleration. The overall direction of acceleration $\vec{a}$ may not be towards the center.

To determine tangential acceleration, you may need to look at what forces are acting in the tangential direction.

As long as you are going in an arc of some sort, there will be centripetal acceleration.

4 Newton's Laws of Motion

Contact forces represent the result of physical contact between two objects, and is present only when two surfaces are in contact.

Field forces do not involve physical contact between 2 objects, but act through empty space.

At the atomic level forces classified as contact forces are actually repulsive electrical (field) forces.

Newton's 1st Law
A body acted on by no net force moves with constant velocity (which may be zero) and zero acceleration.

$$\sum \vec{F} = 0$$

$$\vec{v} = \text{constant}$$

$$\vec{a} = 0$$

Gives rise to inertia, the tendency of an object to resist any change in motion. Heavier object, more resistant to change in motion.

Newton's 2nd Law
If a net force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The net force vector is equal to the mass of the body times the acceleration.

$$\sum F = ma$$

Weight = mg, and acts downwards. Ratio of the weight of two objects is the same as ratio of the mass of the two objects.

Newton's 3rd Law
If body A exerts a force on body B, then body B exerts a force on body A. These forces have the same magnitude but are opposite in direction. These forces act on different bodies.

$$F_{ab} = -F_{ba}$$

Normal Contact Force Normal force is what balances out the motion

5 Applications of Newton's Laws

Basic Strategy

1. Draw a simple FBD of the object, showing all forces on the object (by something else)
2. Establish the x and y direction, and resolve accordingly to solve.

Common Forces

1. Normal Contact Force
2. Tension: Magnitude of the force a rope exerts on an object it is attached to.
3. Weight

Spring Balance

When looking at spring balance, look at the tension on the side of the spring balance with the hook. The spring balance can only show a reading if it is in equilibrium.

Apparent Weight in Lift

Draw the free body diagram. If the life is accelerating upwards, then the net force is upwards. Then you use $\sum F_y = n - w = ma_y$.

Friction Force

Frictional force always act opposite to the direction of motion.

1. Static Friction f_s
   The force that prevents the object from moving.
   Static friction is at a maximum when the object is on the verge of moving.
   $$f_s \leq \mu_s n$$
2. Kinetic Friction f_k
   The retarding force that acts on an object in motion. The kinetic friction is relatively constant, while static friction will increase as more force is applied.
   $$f_k = \mu_k n$$

Generally, $f_{s\text{max}} > f_k$. Once the object starts to move (accelerate), the resulting frictional force (f_k) becomes less than $f_{s\text{max}}$.

Also, the value of μ_k is usually less than μ_s. Coefficient of kinetic friction is less than coefficient of static friction.

Resistive Forces

A fluid medium exerts a resisitive force opposite to the direction of motion.

1. Small objects at low speed e.g. dust in air
   $$f = kv$$
   Resultant force
   $$mg - kv = ma$$
   $$a = g - \frac{k}{m}v$$
   Terminal speed occurs when the a = 0, in that case, the terminal velocity, v_t
   $$v_t = \frac{mg}{k}$$
2. Large objects at high speed e.g. skydiver
   $$f = Dv^2$$

Archimedes' Principle When a body is partially or fully submerged in a fluid, the fluid exerts an upwards force (buoyant force) on the body equal to the weight of fluid displaced by the body.

Buoyant Force The upward force a fluid exerts on any submerged object. Due to the difference in pressure of liquid. Buoyant force acts upwards.

6 Work and Energy

Energy exist in various forms, and can be transformed from one form to another.

Work Done

Work done by an agent exerting a constant force is the product of the component of the force in the direction of the displacement and the magnitude of the displacement of the force. Work done by a force = Force × Displacement SI Unit is Joules (J)

$$W = Fs$$

Work is not done if the object does not move in the direction of the force. i.e., work done by a force is zero if the force is perpendicular to the direction of motion.

Work done by a force is negative if the object moves in a direction opposite to the force.

Work done by a force is positive if the object moves in a direction of the force.

Kinetic Energy

Kinetic Energy is the energy associated with a body in motion.

$$K = \frac{1}{2}mv^2$$

$$W = K_f - K_i = \Delta K$$

The work done is positive if speed increases, and negative if speed decreases.

Power

Power is the rate of doing work

SI Unit is in Watts (W)

1 kWh = 1000W × 3600s = 3.6MJ

1 horsepower = 746W

1. Average Power

$$P_{av} = \frac{\Delta W}{\Delta t}$$

2. Instantaneous Power

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Potential Energy

Potential energy is the energy associated with the position (GPE) or configuration (EPE) of an object.

Considered as stored energy.

Gravitational Potential Energy Work done by gravitational force is unaffected by any horizontal motion, only the initial and final height matters.

$$U = mgy$$

$$W = \Delta U = U_f - U_i$$

Elastic Potential Energy For example, the work done by a spring. Work done by a spring is never negative. It is only zero if the spring is at its natural position.

$$F_x = kx$$

$$U = \frac{1}{2}kx^2$$

Conservation of Energy
When energy is changed from one form to another, the total energy in an isolated system remains the same.

Mechanical energy basically refers to KE, GPE, EPE (outside of the structure of objects)

Internal energy refers to the energy of molecules of a body.

Total mechanical energy $= E = KE + U$

7 Conservative Forces

A force is conservative if the work it does on a particle moving between any two points is independent of the path taken by the particle.

Basically, the work done by a conservative force is zero when moving through a closed loop. If there is work done by a force when moving through a closed loop, it is a non-conservative force.

Non-conservative forces
Non-conservative force includes electromagetic force, and dissipative forces such as kinetic friction and resistive forces. This is because resistive forces always act in a direction opposite to the direction of the motion, the overall work done is negative.

Conservation of Energy

$$KE_i + GPE_i + EPE_i + W_{other} = KE_f + GPE_f + EPE_f$$

W_{other} refers to the work done by non-conservative force.

If the non-conservative force is dissipative, the work done is negative. Otherwise, it is positive. E.g kinetic friction

$$K_i + GPE_i + EPE_i - f_k d = K_f + GPE_f + EPE_f$$

Note that it is the distance, and not displacement.

Internal Energy
Energy of molecules

$$\Delta U_{internal} = -W_{other}$$

Internal energy is gained (+ve) if the W_{other} is by dissipative forces.

Internal energy is lost (-ve) if the W_{other} is by propelling forces (e.g. explosion)

$$K_i + GPE_i + EPE_i + \Delta U_{internal} = K_f + GPE_f + EPE_f$$

Mass-Energy Equivalence
The total mass-energy in the universe is constant.

$$E = mc^2$$

Potential Energy Function
dx is the displacement, U_0 is the initial potential energy.

$$U(x) = \int_{x_0}^{x} F_x dx + U_0$$

Force is given by the negative gradient of the potential energy graph. Mainly used to derive formulas.

$$F = -\frac{du}{dx}$$

Equilibrium Types

1. Stable Equilibrium
   When U(x) has a minimum value, on the u-x graph. $\frac{du}{dx} = 0$
   Will return to the same position
2. Unstable Equilibrium
   When U(x) has a maximum value, on the u-x graph. $\frac{du}{dx} = 0$
   Will topple.
3. Neutral Equilibrium
   When U(x) = constant
   E.g. ball rolling on flat surface