Inverse Transformation

leta199

January 2026

1 Introduction

This document demonstrates the inverse transformation method.

Let $r \sim \text{Uniform}[1, 4]$.

$$\text{Area} = \pi r^2$$

Probability Density Function

For a uniform distribution on $[a, b]$,

$$f_r(x) = \frac{1}{b - a}$$

Here, $a = 1$ and $b = 4$, so

$$f_r(x) = \begin{cases} 0, & x \leq 1 \\ \frac{1}{3}, & 1 < x < 4 \\ 0, & x \geq 4 \end{cases}$$

Cumulative Distribution Function

$$F_r(x) = \int_1^x \frac{1}{3}\, dx$$

$$F_r(x) = \frac{1}{3}(x - 1)$$

Inverse CDF Method

Let $U \sim \text{Uniform}(0, 1)$.

$$\frac{1}{3}(x - 1) = U$$

$$x - 1 = 3U$$

$$x = 3U + 1$$

Expected Value

$$\mathbb{E}[r] = \int_1^4 \frac{1}{3} x \, dx$$

$$= \frac{1}{6}(4^2 - 1^2) = \frac{15}{6} = \frac{5}{2}$$

Expected Area

$$\text{Area} = \pi r^2$$

$$\mathbb{E}[\text{Area}] = \pi \left(\mathbb{E}[r]\right)^2 = \pi \left(\frac{5}{2}\right)^2$$

$$= \frac{25\pi}{4} \approx 19.63$$