

Let:

- $A = \{ \text{natural numbers from 1 to 1000 divisible by 3} \}$
- $B = \{ \text{natural numbers from 1 to 1000 divisible by 5} \}$
- $C = \{ \text{natural numbers from 1 to 1000 divisible by 9} \}$

We are finding:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Where sample space is:

$$\Omega = \{1 \leq n \leq 1000 \mid n \text{ divisible by 3, 5, or 9}\}$$

**Step 1: Compute cardinalities**

$$A = \{3, 6, 9, \dots, 999\} \rightarrow |A| = \left\lfloor \frac{1000}{3} \right\rfloor = 333 \Rightarrow P(A) = \frac{333}{1000}$$

$$B = \{5, 10, 15, \dots, 1000\} \rightarrow |B| = \left\lfloor \frac{1000}{5} \right\rfloor = 200 \Rightarrow P(B) = \frac{200}{1000}$$

$$C = \{9, 18, 27, \dots, 999\} \rightarrow |C| = \left\lfloor \frac{1000}{9} \right\rfloor = 111 \Rightarrow P(C) = \frac{111}{1000}$$

**Step 2: Compute intersections**

$$P(A \cap B) = P(\text{divisible by 15}) = \frac{\left\lfloor \frac{1000}{15} \right\rfloor}{1000} = \frac{1}{15}$$

$$P(A \cap C) = P(\text{divisible by 9}) = \frac{\left\lfloor \frac{1000}{9} \right\rfloor}{1000} = \frac{1}{9}$$

$$P(B \cap C) = P(\text{divisible by 45}) = \frac{\left\lfloor \frac{1000}{45} \right\rfloor}{1000} = \frac{1}{45}$$

$$P(A \cap B \cap C) = P(\text{divisible by LCM(3,5,9)}) = \frac{\left\lfloor \frac{1000}{45} \right\rfloor}{1000} = \frac{1}{45}$$

**Step 3: Plug in the inclusion-exclusion formula**

$$\begin{aligned} P(A \cup B \cup C) &= \frac{333}{1000} + \frac{200}{1000} + \frac{111}{1000} - \left( \frac{66}{1000} + \frac{111}{1000} + \frac{22}{1000} \right) + \frac{22}{1000} \\ &= \frac{333 + 200 + 111 - (66 + 111 + 22) + 22}{1000} = \frac{467}{1000} = 0.467 \end{aligned}$$