Monte Carlo Integration

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This Monte Carlo Integration method is used to approximate the summation below.

$$I = \sum_{k=0}^{\infty} \frac{\cos(\cos k)}{k!}$$

$$\sum_{k=0}^{\infty} \frac{\cos(\cos k)}{k!} \qquad f(x) = \frac{\cos(\cos k)}{k!}$$

$$\sum_{k=0}^{\infty} \frac{\cos(\cos k)}{k!} \qquad g(x) = \frac{e^{-\lambda}\lambda^k}{k!}$$

$$E\left[\frac{f(x)}{g(x)}\right] = \sum_{k=0}^{\infty} \frac{f(x)}{g(x)} \cdot g(x) = \sum_{k=0}^{\infty} \frac{\cos(\cos k)}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{\cos(\cos k)}{e^{-\lambda}\lambda^k} \cdot \frac{e^{-\lambda}\lambda^k}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{\cos(\cos k)}{e^{-\lambda}\lambda^k} \cdot P(K = k)$$

When $\lambda = 1$:

$$\frac{\cos(\cos k)}{e^{-1}} \cdot \frac{e^{-1}}{k!}$$

$$E\left[\frac{f(x)}{g(x)}\right] = e \sum_{k=0}^{\infty} \cos(\cos k) \cdot \frac{1}{k!\, e}$$

$$K \sim \text{Pois}(\lambda = 1)$$

$$I \approx \frac{e}{n} \sum_{i=1}^{\infty} \cos(\cos k_i) \quad \text{as } n \to \infty$$

Where $k_i = \frac{e^{-1}}{k!}$ when $k = 0, 1, 2, \ldots$