Monte Carlo Integration

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This document will document the mathematical intuition for Monte Carlo Integration of a definite integral

$$\int_0^\infty \sin(x)e^{-x^2}dx \qquad \frac{\sin(x)e^{-x^2}}{e^{-x}}$$

$$I = \int_0^\infty f(x)\,dx \qquad \text{Where}$$

$$I \Leftarrow E[f(X)] \approx f(x_1) + f(x_2) + \ldots + f(x_n)$$

$$\text{Where } X \sim \text{r.v with some support}$$

as $n \to \infty$
Therefore, for the function $\sin(x)e^{-x^2}$ we can have

$$f(x) = \sin(x)e^{-x^2}$$

$$X \sim \text{Exp}(1) \qquad he^{-hx} = (1)e^{-(x)} = e^{-x}$$

$$\int_0^\infty f(x)\,dx = \int_0^\infty \frac{f(x)}{g(x)}g(x) = \int_0^\infty h(x)\,g(x)\,dx$$

$$E[h(x)] = E\left[\frac{f(x)}{g(x)}\right]$$

$$E\left[\frac{f(x)}{g(x)}\right] = \left[\frac{f(x_1)}{g(x_1)} + \ldots + \frac{f(x_n)}{g(x_n)}\right] \cdot \frac{1}{n}$$

Where $\dfrac{f(x)}{g(x)} = \dfrac{\sin(x)e^{-x^2}}{e^{-x}} = \sin(x)e^{-x^2+x} = \sin(x)e^{x-x^2}$