Random Walk Gambler's Ruin

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(a) Find $P(4600 < T < 6000)$.

Given:

$$P(G = 1) = \frac{20}{38}, \quad P(G = -1) = \frac{18}{38}$$

The expected value of G is

$$E(G) = 1\left(\frac{20}{38}\right) + (-1)\left(\frac{18}{38}\right) = \frac{2}{38} = \mu$$

The variance of G is

$$\mathrm{Var}(G) = E(G^2) - \mu^2 = 1 - \left(\frac{2}{38}\right)^2 = \frac{360}{361} \approx 0.9972$$

Now let

$$T = \sum_{i=1}^{100000} G_i$$

Then

$$E(T) = n\mu = 100000 \cdot \frac{2}{38} \approx 5263.16$$

$$\mathrm{Var}(T) = n\sigma^2 = 100000 \cdot \frac{360}{361} \approx 99{,}723$$

Thus the standard deviation is

$$\sigma_T = \sqrt{99{,}723} \approx 315.8$$

Using the normal approximation,

$$P(4600 < T < 6000) = P\left(\frac{4600 - 5263.16}{315.8} < Z < \frac{6000 - 5263.16}{315.8}\right)$$

$$= P(-2.11 < Z < 2.33)$$

$$= 0.9901 - 0.0179 = 0.9722$$