

1) ergodic: P_1, P_2, P_4

2) (a) $a=0, b \in [0,1)$ or $b=0, a \in [0,1)$

(b) $a \in (0,1], b \in (0,1]$

3) (a) $E(x_2) = 1$ (b) $E(x_2 - 3x_4) = -2$ (c) $E(A_5) = 1$

(d) $E(A_5 - x_1) = 0$ (e) $\text{Var}(A_5) = \frac{1}{5}$ (f) $\text{Std}(A_5) = \frac{1}{\sqrt{5}}$

(g): $\mu = 1, \sigma^2 = \frac{1}{5}$

4) Cumulative ~~distribution~~ Function: $F(t) = P(Y < t) = \begin{cases} 1 & \text{if } t \geq 2 \\ (t-1)^2 & \text{if } t \in (1,2) \\ 0 & \text{if } t \leq 1. \end{cases}$

Density: $f(t) = F'(t) = \begin{cases} 0 & \text{if } t \geq 2 \\ 2(t-1) & \text{if } t \in (1,2) \\ 0 & \text{if } t \leq 1. \end{cases}$

5) (a) $P(|x| > 2) = 2 \cdot 0.1587 = 0.3174$

(b) $P(x < -2) = 0.1587$

(c) $P(x > 2) = 0.1587$

(d) $P(e^x < e^4) = P(x < 4) = 0.9772$

(e) $E(x^2) = 4$

6) (a): $P(x^2 > 1) = P(x > 1) + P(x < -1) = 0.5 + 0.0228 = 0.5228$

(b): $P(x < 0) = 0.1587$ (c) $P(x=0) = 0$ (d) $P(x > 0) = 1 - 0.1587 = 0.8413$

(e): $E((x-1)^2) = 1$

$$(7): \begin{array}{l} H_0: P_0 = 0.6 \\ H_a: P > 0.6 \end{array} \quad \left\{ \begin{array}{l} z = \frac{\frac{350}{500} - 0.6}{\sqrt{\frac{0.6(1-0.6)}{500}}} \approx 4.56 \end{array} \right.$$

$$\text{P-Value: } \approx P(Z > 4.56) \approx 0 < 0.05.$$

So enough evidence.

$$(8): \lambda_1 = 2\lambda_0, \quad \lambda_0 = \frac{2}{13}, \quad \lambda_1 = \frac{4}{13}.$$

$$(9): \mu = 0, \quad \sigma^2 = \frac{13}{2}$$

$$(10): \text{steps} \quad \textcircled{1} \text{ solve } f_x = 0 \quad f_y = 0$$

$$\textcircled{2} \text{ Find Hessian: } D = \det \begin{pmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{pmatrix}$$

$$D = 4(1-x^2) - 4y^2$$

& plug in solutions from $\textcircled{1}$:

if $D \leq 0$, NOT Max/Min pt.

$\textcircled{3}$ for $D > 0$ if $f_{xx} > 0$ $f_{yy} > 0$ local min pt

if $f_{xx} < 0$ $f_{yy} < 0$ local MAX

pts to consider: $(1, \sqrt{3})$ $(1, -\sqrt{3})$, $(0, 2)$ $(0, 0)$

$$D = 4(1-x^2) - 4y^2$$

local min at $(0, 0)$.

11: (a) ~~0~~, All of them.

(b): P_4 P_5 P_6

(c): P_1 P_2 P_3

(d): For $P_4 = \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & 0 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}$ S_1 and S_3 are Absorbing states.

Time to Absorption: $S_4 \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\left\{ \begin{array}{l} S_1 \text{ has time to Absorption } \emptyset \\ S_3 \text{ has time to Absorption } \emptyset \end{array} \right.$

Prob to Absorption: $\begin{matrix} S_3 & S_1 \\ S_4 & \begin{pmatrix} 0 & 1 \\ 1/3 & 2/3 \end{pmatrix} \end{matrix}$ $\begin{matrix} S_1 & S_3 \\ S_1 & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \\ S_3 & \end{matrix}$

For $P_5 \begin{matrix} S_1 \\ S_2 \end{matrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Abs. states: S_1 S_2

Time to Absorption: S_1 & $S_2 - \emptyset$.

prob. to be absorbed: $\begin{matrix} S_1 & S_2 \\ S_1 & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \\ S_2 & \end{matrix}$

For $P_6 = \begin{matrix} S_1 \\ S_2 \end{matrix} \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ Abs state: S_1

Time to Abs. $S_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\left\{ \begin{array}{l} \text{prob. to be absorbed} \\ S_1 \text{ time to Abs: } \emptyset \\ S_2 (1) \\ S_1 - \text{prob. to be abs. } 1 \end{array} \right.$

12: (a) State space = $\{ \begin{matrix} A & B & \text{Home} & \text{Bar} \\ 1 & 2 & 3 & 4 \end{matrix} \}$

(b)
$$P = \begin{matrix} & \begin{matrix} Q & R \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

(c):
$$P^2 = \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

distribution

$(\frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}) P^2 = (\frac{1}{16}, \frac{1}{16}, \frac{7}{16}, \frac{7}{16})$

$P(X_2 = A) = \frac{1}{16}, \quad P(X_2 = B) = \frac{1}{16}$

$P(X_2 = \text{Home}) = \frac{7}{16}, \quad P(X_2 = \text{Bar}) = \frac{7}{16}$

(d): $\frac{1}{2}$

(e) $\frac{1}{2}$

(f) = 0

(g) = 0

Note: Absorption Prob =

$$\begin{matrix} & \begin{matrix} \text{Home} & \text{Bar} \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \end{matrix}$$

Time to Abs: $\begin{matrix} A(2) \\ B(2) \end{matrix}$

13: (e)

14: True

15: (c)