1): ergodic: P_1, P_2, P_4

2): (a) $a=0$ $b\in[0,1]$ or $b=0$, $a\in[0,1]$

(b) $a\in(0,1]$, $b\in(0,1]$

3) (a) $E(X_2)=1$ (b) $E(X_2-3X_4)=-2$ (c) $E(A_5)=1$

(d) $E(A_5-X_1)=0$ (e) $Var(A_5)=\frac{1}{5}$ (f) $Stdl(A_5)=\frac{1}{\sqrt{5}}$

(g): $\mu=1$ $\sigma^2=\frac{1}{5}$

4) Cumulative distribution Function: $F(t)=P(Y<t)=\begin{cases}1 & \text{if } t\geq 2\\ (t-1)^3 & \text{if } t\in(1,2)\\ 0 & \text{if } t\leq 1.\end{cases}$

Density: $f(t)=F'(t)=\begin{cases}0 & \text{if } t\geq 2\\ 3(t-1)^2 & \text{if } t\in(1,2)\\ 0 & \text{if } t\leq 1.\end{cases}$

5). (a) $P(|X|>2)=2\cdot 0.1587=0.3174$

(b) $P(X<-2)=0.1587$

(c) $P(X>2)=0.1587$

(d) $P(e^X<e^4)=P(X<4)=0.9772$

(e) $E(X^2)=4$

6). (a): $P(X^2>1)=P(X>1)+P(X<-1)=0.5+0.0228=0.5228$

(b): $P(X<0)=0.1587$. (c) $P(X=0)=0$. (d) $P(X>0)=1-0.1587=0.8413$

(e): $E((X-1)^2)=1$

(7): $H_0: p_0 = 0.6$
$H_a: p > 0.6$

$$z = \frac{\frac{350}{500} - 0.6}{\sqrt{\frac{0.6(1-0.6)}{500}}} \approx 4.56$$

P-Value: $\approx P(z > 4.56) \approx 0 < 0.05$

So enough evidence.

(8): $\lambda_2 = 2\lambda_0, \quad \lambda_0 = \frac{2}{13} \quad \lambda_1 = \frac{4}{13}$.

(9): $\mu = 0 \quad \sigma^2 = \frac{13}{2}$

(10): steps: ① Solve $f_x = 0 \quad f_y = 0$

② Find Hessian: $D = \det \begin{pmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{pmatrix}$ $\quad D = 4(1-x^2) - 4y^2$

& plug in solutions from ①:

if $D \leq 0$, NOT max/min pt.

③ for $D > 0$ if $f_{xx} > 0$ $f_{yy} > 0$ local min pt

if $f_{xx} < 0$ $f_{yy} < 0$ local MAX

pts to consider: $(1, \sqrt{3})$ $(1, -\sqrt{3})$, ~~$(0, -2)$~~ $(-2, 0)$ $(0, 0)$

$D = 4(1-x^2) - 4y^2$

local min at $(0, 0)$.

11: (a) B, All of them.

(b): P_4 P_5 P_6

(c): P_1 P_2 P_3

(d): For $P_4 = \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{matrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & 0 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}$ S_1 and S_3 are Absorbing states

Time to Absorption: $\begin{matrix} S_4 \\ S_2 \end{matrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and { S_1 has time to Absorption $\emptyset$; S_3 has time to Absorption $\emptyset$ }

Prob to Absorption: $\begin{matrix} & S_3 & S_1 \\ S_4 & 0 & 1 \\ S_2 & \frac{1}{3} & \frac{2}{3} \end{matrix}$ $\begin{matrix} & S_1 & S_3 \\ S_1 & 1 & 0 \\ S_3 & 0 & 1 \end{matrix}$

For P_5 $\begin{matrix} S_1 \\ S_2 \end{matrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Abs. states: S_1 S_2

Time to Absorption: S_1 & S_2 — $\emptyset$.

prob. to be absorbed: $\begin{matrix} & S_1 & S_2 \\ S_1 & 1 & 0 \\ S_2 & 0 & 1 \end{matrix}$

For $P_6 = \begin{matrix} S_1 \\ S_2 \end{matrix}\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ Abs state: S_1

Time to Abs. S_2 $\begin{matrix} S_1 \\ (1) \end{matrix}$

S_1 time to Abs: $\emptyset$

{ prob. to be absorbed $\begin{matrix} S_1 \\ S_2\,(1) \end{matrix}$

S_1 — prob. tobe abs. 1

12. (a) State space = {A, B, Home, Bar} = {1, 2, 3, 4}

(b)
$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} Q & \quad R \end{array} \\ \left(\begin{array}{cc|cc} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \end{array}$$

(c): $P^2 = \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

distribution

$\left(\frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right) P^2 = \left(\frac{1}{16}, \frac{1}{16}, \frac{7}{16}, \frac{7}{16}\right)$

$P(X_2 = A) = \frac{1}{16}$, $P(X_2 = B) = \frac{1}{16}$

$P(X_2 = \text{Home}) = \frac{7}{16}$ $P(X_2 = \text{Bar}) = \frac{7}{16}$

(d): $\frac{1}{2}$ (e) $\frac{1}{2}$ (f) = 0 (g) = 0

Note: Absorption Prob = $\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} \text{Home} & \text{Bar.} \end{array} \\ \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \end{array}$ Time to Abs: $\begin{array}{c} \\ A \\ B \end{array} \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

13 (e)

14 True

15: (c)