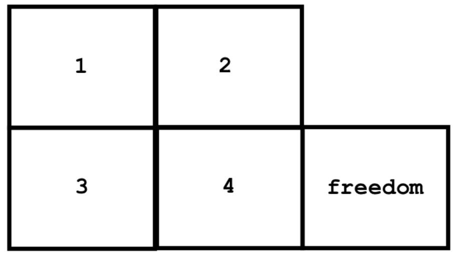

Written Homework 13 · MATH331

Markov Chain (chapter 11 in Grinstead)

Due on Thursday Apr 26 11:30AM

1. Markov Chain (Continue from Homework 12) Consider a rat in a maze with 4 cells, indexed as 1,2,3,4, and the outside (freedom), indexed by 5 (that can only be reached via cell 4). The rat starts initially in a given cell and then takes a move to another cell, continuing to do so until finally reaching freedom. We assume that at each move (transition) the rat, independent of the past, is equally likely to choose from among the neighboring cells (sharing an edge). eg. if it is in cell 4, then it is equal likely that the rat moves to cell 3, 2, or freedom.

   Please pay attention to my state labeling here $\{s_1, s_2, s_3, s_4, s_5\}$ where s_i corresponds to cell $i, i = 1, 2, 3, 4$ and s_5 corresponds to freedom

   Then corresponding transition matrix P is a 5 by 5 matrix:

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Find the none zero matrix S, which stabilizes P, that is

$$SP = S$$

   (b) Find the probability that the rat eventually ends up with freedom

   (c) How many steps, on average, does it take for the rat to reach freedom if it starts from cell 3?

   (d) How many steps, on average, does it take for the rat to reach freedom if it starts from cell 1?

2. Consider a matrix

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

What conditions you need to enforce on p_{ij}'s so that P becomes a transition matrix for an absorbing Markov Chain with states $= 1, 2, 3$?

3.

$$P_1 = \begin{pmatrix} 2/3 & 1/3 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) Determine if the above matrices are transition matrices for some absorbing Markov Chain.

(b) If so, find their stationary matrices

(c) If so, find the probability that each transient state to be absorbed and number of steps, on average, it takes them to be absorbed.

4. Grinstead's Chapter 11.2, Exercise 15