ERGODIC AND REGULAR MARKOV CHAINS

ERGODIC AND REGULAR MARKOV CHAINS

- **ergodic chain** : it is possible to go from every state to every state (not necessarily in one move).

- **regular chain :** some power of the transition matrix has **only** positive elements.

- **absorbing chain:**
  1) it has at least one absorbing state,
  2) from every state it is possible to go to an absorbing state (not necessarily in one step).

- Relationships:

- {set of **absorbing** chains} $\cap$ {set of **ergodic chains)** = $\varnothing$

- **regular implies ergodic** , **ergodic does not imply regular.**

EXAMPLE

EHRENFEST MODEL

- We have two urns that, between them, contain four balls. At each step, one of the four balls is chosen at random and moved from the urn that it is in into the other urn.
- We choose, as states, the number of balls in the first urn.
- What is the transition matrix?
- Absorbing Markov? Ergodic Markov? Regular?
- You may find this example “Ehrenfest Model” in Grinstead.pdf
  Example 11.8 Page 410
  Example 11.17 Page 433
  Example 11.23 Page 441
  etc.

EXAMPLE

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- We choose, as states, the number of balls in the first urn.

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1/4 & 0 & 3/4 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 3/4 & 0 & 1/4 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

MORE MARKOV CHAINS

DISCRETE-TIME MARKOV CHAINS

COMPUTER REPAIR EXAMPLE

- **Summarize: the state space = {at the beginning of each day the status of working computers} and the one-step transition matrix P**
- Two aging computers are used for word processing. When both are working in morning, there is a 30% chance that one will fail by the evening and a 10% chance that both will fail.
- If only one computer is working at the beginning of the day, there is a 20% chance that it will fail by the close of business.
- If neither is working in the morning, the office sends all work to a typing service.
- Computers that fail during the day are picked up the following morning, repaired, and then returned the next morning.
- The system is observed after the repaired computers have been returned and before any new failures occur.

DISCRETE-TIME MARKOV CHAINS

COMPUTER REPAIR EXAMPLE

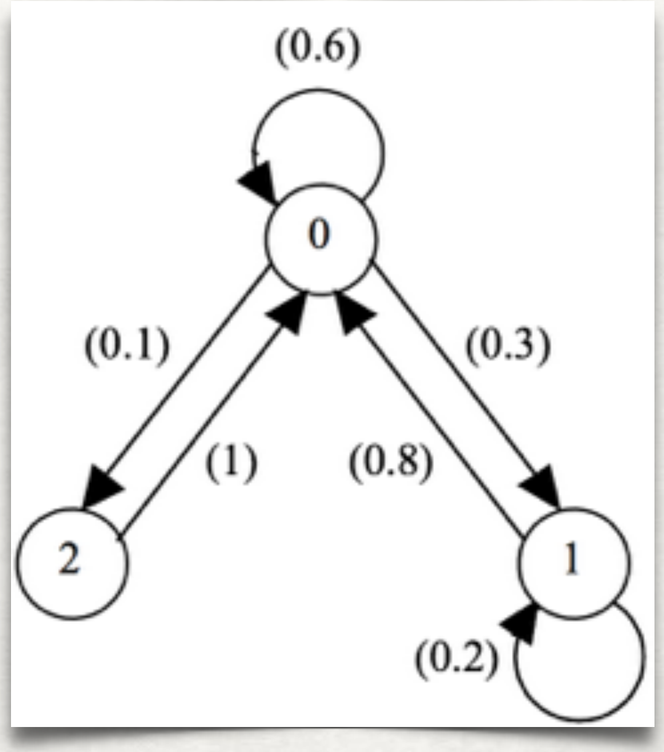
- <u>**State Space, State-Transition Matrix and Network**</u>

- State space = {0,1,2}
  0 = no computer failed; 1 = one computer failed; 2 = both failed

- The events associated with a Markov chain can be described by the 3 × 3 matrix: P = (pij).

$$\mathbf{P} = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.8 & 0.2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- State-Transition Network：
  Node for each state
  Arc from node i to node j if pij > 0.



DISCRETE-TIME MARKOV CHAINS

COMPUTER REPAIR EXAMPLE

- **State Space, State-Transition Matrix and Network**

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- Questions:
  1)Find 2-step transition matrix for the above Markov chain
  2)Find 2-step distribution for the above Markov chain
  3)Absorbing? Ergodic? Regular?

DISCRETE-TIME MARKOV CHAINS

COMPUTER REPAIR EXAMPLE

- **State Space, State-Transition Matrix and Network**

- The events associated with a Markov chain can be described by the 3 × 3 matrix: P = (pij).

$$\mathbf{P} = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.8 & 0.2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P^2 = \begin{pmatrix} 0.7 & 0.24 & 0.06 \\ 0.64 & 0.28 & 0.08 \\ 0.6 & 0.3 & 0.1 \end{pmatrix}$$

- Questions:
  1)Find 2-step transition matrix for the above Markov chain
  2)Find 2-step distribution for the above Markov chain
  3)Absorbing? Ergodic? Regular?
  4) stationary matrix

DISCRETE-TIME MARKOV CHAINS

TRANSFORM A PROCESS TO A MARKOV CHAIN

- Pr{ rain tomorrow | rain last 2 days (RR) } = 0.7
- Pr{ rain tomorrow | rain today but not yesterday (NR) } = 0.5
- Pr{ rain tomorrow | rain yesterday but not today (RN) } = 0.4
- Pr{ rain tomorrow | no rain in last 2 days (NN) } = 0.2

- Does the Markovian Property Hold ?
- For this question, we cannot get a Markov chain by setting state space as: {R,N}.
- But we can set state space as: {RR,NR,RN,NN}

DISCRETE-TIME MARKOV CHAINS

TRANSFORM A PROCESS TO A MARKOV CHAIN

- **state space as: {0 = RR,1 = NR, 2=RN, 3=NN}**

- **Note: here you consider 3 consecutive days: and transition matrix P as the following**

	0(RR)	1(NR)	2(RN)	3(NN)
0 (RR)	0.7	0	0.3	0
1 (NR)	0.5	0	0.5	0
2 (RN)	0	0.4	0	0.6
3 (NN)	0	0.2	0	0.8

DISCRETE-TIME MARKOV CHAINS

TRANSFORM A PROCESS TO A MARKOV CHAIN

- **state space as: {0 = RR,1 = NR, 2=RN, 3=NN}**

- **Note: here you consider 3 consecutive days: and transition matrix P as the following**

	0(RR)	1(NR)	2(RN)	3(NN)
0 (RR)	0.7	0	0.3	0
1 (NR)	0.5	0	0.5	0
2 (RN)	0	0.4	0	0.6
3 (NN)	0	0.2	0	0.8

- **eg: P02 = P{ day2=R, day3=N |day1=R, day2=R}**