

# Union Find

- $N = V$  = number of nodes;  
 $M = E$  = number of edges
- weighted (use sz) + path compression:
- individual operation:  
Union(p,q):  $\lg V$   
## improved by path compression, attach smaller tree to the larger—> tree height won't increase  
Root(p):  $\lg V$   
## when call root, rewind  $\text{id}[p] = \text{id}[\text{id}[p]]$  to trim tree height  
ifconnected(p,q):  $\lg V$   
## this repeatedly calling root, so it trims the height on the way
- Whole operation:  
N node, for each edge need to call functions!  
initialization: V  
The whole algorithm:  $V + E \lg V$  (used E many union)

```
public class UF
{
    UF(int N) initialize union-find data structure with
               N objects (0 to N - 1)

    void union(int p, int q) add connection between p and q

    boolean connected(int p, int q) are p and q in the same component?

    int find(int p) component identifier for p (0 to N - 1)

    int count() number of components
}
```

# DFS — both dir + undir

applications:

1. (dir+undir) Find all vertices connected to a given source vertex (union find) — reachability
2. (dir+undir) Find ONE path between two vertices
3. Topological sort
4. Directed cycle detection

Connected or strongly connected components (dir+undir):

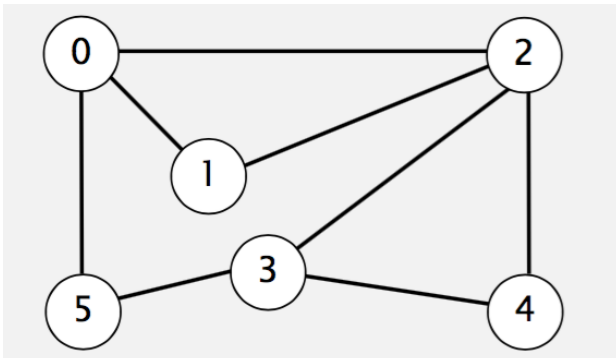
Time =  $O(\text{sum of (vertex degree)}) = O(2E)$

General Time:  $O(E+V)$

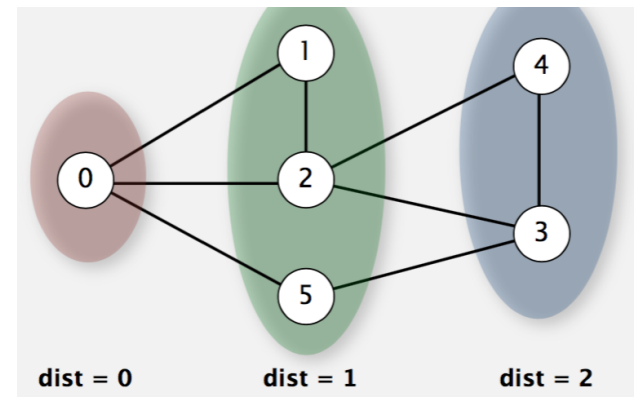
equivalent to Stack

# BFS — both dir+undir

- shortest paths: time =  $O(E + V)$
- when the actual problem could have a very long path, dfs may NOT work well. we use BFS



# BFS



- BFS :shortest paths: time =  $O(E + V)$
- eg LC 286: You are given a  $m \times n$  grid initialized with these three possible values.  
 1 - A wall or an obstacle; 0 - A gate; INF - empty room.  
 Fill each empty room with the distance to its nearest gate. If it is impossible, fill it with INF.
- Number of vertices:  $V = m \times n$   
 Each vertex connect at most 4 other vertices:  $E = 4 \times V$   
 So, BFS loops all vertices takes:  $O(E+V) = O(m \times n)$

# BFS vs DFS both (undir + dig)

	DFS	BFS
Find a path between a, b	auto shortest	just a path
put unvisited	on a queue (FIFO)	on a stack
(undir )Time	$O(E+V)$	$O(E+V)$
Topological sort+	Yes (build a cycle detection within the algo)	NO
dir cycle detect	Yes	NO
find connected component	Yes for undir	Yes for undir
strong component	find reverse graph top order; then find graph cmpnts	NO