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# PyLops

**Feb 23, 2020**



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## Getting started:

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This Python library is inspired by the MATLAB [Spot – A Linear-Operator Toolbox](#) project.

Linear operators and inverse problems are at the core of many of the most used algorithms in signal processing, image processing, and remote sensing. When dealing with small-scale problems, the Python numerical scientific libraries `numpy` and `scipy` allow to perform most of the underlying matrix operations (e.g., computation of matrix-vector products and manipulation of matrices) in a simple and expressive way.

Many useful operators, however, do not lend themselves to an explicit matrix representation when used to solve large-scale problems. PyLops operators, on the other hand, still represent a matrix and can be treated in a similar way, but do not rely on the explicit creation of a dense (or sparse) matrix itself. Conversely, the forward and adjoint operators are represented by small pieces of codes that mimic the effect of the matrix on a vector or another matrix.

Luckily, many iterative methods (e.g. `cg`, `lsqr`) do not need to know the individual entries of a matrix to solve a linear system. Such solvers only require the computation of forward and adjoint matrix-vector products as done for any of the PyLops operators.

Here is a simple example showing how a dense first-order first derivative operator can be created, applied and inverted using `numpy/scipy` commands:

```
import numpy as np
from scipy.linalg import lstsq

nx = 7
x = np.arange(nx) - (nx-1)/2

D = np.diag(0.5*np.ones(nx-1), k=1) - \
    np.diag(0.5*np.ones(nx-1), k=-1)
D[0] = D[-1] = 0 # take away edge effects

# y = Dx
y = np.dot(D, x)
# x = D'y
xadj = np.dot(D.T, y)
# xinv = D^-1 y
xinv = lstsq(D, y)[0]
```

and similarly using PyLops commands:

```
from pylops import FirstDerivative

Dlop = FirstDerivative(nx, dtype='float64')

# y = Dx
y = Dlop*x
# x = D'y
xadj = Dlop.H*y
# xinv = D^-1 y
xinv = Dlop / y
```

Note how this second approach does not require creating a dense matrix, reducing both the memory load and the computational cost of applying a derivative to an input vector `x`. Moreover, the code becomes even more compact and expressive than in the previous case letting the user focus on the formulation of equations of the forward problem to be solved by inversion.



# CHAPTER 1

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## Terminology

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A common *terminology* is used within the entire documentation of PyLops. Every linear operator and its application to a model will be referred to as **forward model (or operation)**

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

while its application to a data is referred to as **adjoint modelling (or operation)**

$$\mathbf{x} = \mathbf{A}^H\mathbf{y}$$

where  $\mathbf{x}$  is called *model* and  $\mathbf{y}$  is called *data*. The *operator*  $\mathbf{A} : \mathbb{F}^m \rightarrow \mathbb{F}^n$  effectively maps a vector of size  $m$  in the *model space* to a vector of size  $n$  in the *data space*, conversely the *adjoint operator*  $\mathbf{A}^H : \mathbb{F}^n \rightarrow \mathbb{F}^m$  maps a vector of size  $n$  in the *data space* to a vector of size  $m$  in the *model space*. As linear operators mimics the effect a matrix on a vector we can also loosely refer to  $m$  as the number of *columns* and  $n$  as the number of *rows* of the operator.

Ultimately, solving an inverse problems accounts to removing the effect of  $\mathbf{A}$  from the data  $\mathbf{y}$  to retrieve the model  $\mathbf{x}$ .

For a more detailed description of the concepts of linear operators, adjoints and inverse problems in general, you can head over to one of Jon Claerbout's books such as [Basic Earth Imaging](#).



# CHAPTER 2

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## Implementation

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PyLops is build on top of the [scipy class `scipy.sparse.linalg.LinearOperator`](#).

This class allows in fact for the creation of objects (or interfaces) for matrix-vector and matrix-matrix products that can ultimately be used to solve any inverse problem of the form  $\mathbf{y} = \mathbf{A}\mathbf{x}$ .

As explained in the [scipy LinearOperator official documentation](#), to construct a `scipy.sparse.linalg.LinearOperator`, a user is required to pass appropriate callables to the constructor of this class, or subclass it. More specifically one of the methods `_matvec` and `_matmat` must be implemented for the *forward operator* and one of the methods `_rmatvec` or `_adjoint` may be implemented to apply the *Hermitian adjoint*. The attributes/properties `shape` (pair of integers) and `dtype` (may be `None`) must also be provided during `__init__` of this class.

Any linear operator developed within the PyLops library follows this philosophy. As explained more in details in [Implementing new operators](#) section, a linear operator is created by subclassing the `scipy.sparse.linalg.LinearOperator` class and `_matvec` and `_rmatvec` are implemented.



# CHAPTER 3

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## History

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PyLops was initially written and it is currently maintained by [Equinor](#). It is a flexible and scalable python library for large-scale optimization with linear operators that can be tailored to our needs, and as contribution to the free software community.

## 3.1 Installation

The PyLops project strives to create a library that is easy to install in any environment and has a very limited number of dependencies. However, since *Python2* will retire soon, we have decided to only focus on a *Python3* implementation. If you are still using *Python2*, hurry up!

For this reason you will need **Python 3.5 or greater** to get started.

### 3.1.1 Dependencies

Our mandatory dependencies are limited to:

- `numpy`
- `scipy`

We advise using the [Anaconda Python distribution](#) to ensure that these dependencies are installed via the Conda package manager. This is not just a pure stylistic choice but comes with some *hidden* advantages, such as the linking to Intel MKL library (i.e., a highly optimized BLAS library created by Intel).

If you simply want to use PyLops for teaching purposes or for small-scale examples, this should not really affect you. However, if you are interested in getting better code performance, read carefully the [Advanced installation](#) page.

### 3.1.2 Optional dependencies

PyLops's optional dependencies refer to those dependencies that we do not include in our `requirements.txt` and `environment.yml` files and thus are not strictly needed nor installed directly as part of a standard installation (see

below for details)

However, we sometimes implement additional back-ends (referred to as `engine` in the code) for some of our operators in order to improve their performance. To do so, we rely on third-party libraries. Those libraries are generally added to the list of our optional dependencies. If you are not after code performance, you may simply stick to the mandatory dependencies and `pylops` will ensure to always fallback to one of those for any linear operator.

If you are instead after code performance, take a look at the *Optional dependencies* section in the [Advanced installation](#) page.

### 3.1.3 Step-by-step installation for users

#### Python environment

Activate your Python environment, and simply type the following command in your terminal to install the PyPi distribution:

```
>> pip install pyllops
```

If using Conda, you can also install our conda-forge distribution via:

```
>> conda install -c conda-forge pyllops
```

Note that using the `conda-forge` distribution is recommended as all the dependencies (both mandatory and optional) will be correctly installed for you, while only mandatory dependencies are installed using the `pip` distribution.

Alternatively, to access the latest source from github:

```
>> pip install https://github.com/equinor/pylops/archive/master.zip
```

or just clone the repository

```
>> git clone https://github.com/equinor/pylops.git
```

or download the zip file from the repository (green button in the top right corner of the main github repo page) and install PyLops from terminal using the command:

```
>> make install
```

#### Docker

If you want to try PyLops but do not have Python in your local machine, you can use our `Docker` image instead.

After installing Docker in your computer, type the following command in your terminal (note that this will take some time the first time you type it as you will download and install the docker image):

```
>> docker run -it -v /path/to/local/folder:/home/jupyter/notebook -p 8888:8888  
→mrava87/pylops:notebook
```

This will give you an address that you can put in your browser and will open a jupyter-notebook environment with PyLops and other basic Python libraries installed. Here `/path/to/local/folder` is the absolute path of a local folder on your computer where you will create a notebook (or containing notebooks that you want to continue working on). Note that anything you do to the notebook(s) will be saved in your local folder.

A larger image with Conda distribution is also available. Simply use `conda_notebook` instead of `notebook` in the previous command.

### 3.1.4 Step-by-step installation for developers

Fork and clone the repository by executing the following in your terminal:

```
>> git clone https://github.com/your_name_here/pylops.git
```

The first time you clone the repository run the following command:

```
>> make dev-install
```

If you prefer to build a new Conda environment just for PyLops, run the following command:

```
>> make dev-install_conda
```

To ensure that everything has been setup correctly, run tests:

```
>> make tests
```

Make sure no tests fail, this guarantees that the installation has been successful.

If using Conda environment, always remember to activate the conda environment every time you open a new *bash* shell by typing:

```
>> source activate pylops
```

## 3.2 Advanced installation

In this section we discuss some *important details* regarding code performance when using PyLops.

To get the most out of PyLops operators in terms of speed you will need to follow these guidelines as much as possible or ensure that the Python libraries used by PyLops are efficiently installed (e.g., allow multithreading) in your system.

### 3.2.1 Dependencies

PyLops relies on the `numpy` and `scipy` libraries and being able to link these to the most performant `BLAS` will ensure optimal performance of PyLops when using only *required dependencies*.

As already mentioned in the [Installation](#) page, we strongly encourage using the [Anaconda Python distribution](#) as `numpy` and `scipy` will be automatically linked to the `Intel MKL` library, which is per today the most performant library for basic linear algebra operations (if you don't believe it, take a read at this [blog post](#)).

The best way to understand which `BLAS` library is currently linked to your `numpy` and `scipy` libraries is to run the following commands in ipython:

```
import numpy as np
import scipy as sp
print(np.__config__.show())
print(sp.__config__.show())
```

You should be able to understand if your `numpy` and `scipy` are linked to `Intel MKL` or something else.

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**Note:** Unfortunately, PyLops is so far only shipped with [PyPI](#), meaning that if you have not already installed `numpy` and `scipy` in your environment they will be installed as part of the installation process of the `pylops` library, all of those using `pip`. This comes with the disadvantage that `numpy` and `scipy` are linked to `OpenBlas` instead of `Intel MKL`, leading to a loss of performance. To prevent this, we suggest the following strategy:

- create conda environment, e.g. `conda create -n envname python=3.6.4 numpy scipy`
  - install pyllops using `pip install pyllops`
- 

Finally, it is always important to make sure that your environment variable `OMP_NUM_THREADS` is correctly set to the maximum number of threads you would like to use in your code. If that is not the case `numpy` and `scipy` will underutilize your hardware even if linked to a performant BLAS library.

For example, first set `OMP_NUM_THREADS=1` (single-threaded) in your terminal:

```
>> export OMP_NUM_THREADS=1
```

and run the following code in python:

```
import os
import numpy as np
from timeit import timeit

size = 4096
A = np.random.random((size, size)),
B = np.random.random((size, size))
print('Time with %s threads: %f s' \
      %(os.environ.get('OMP_NUM_THREADS'),
        timeit(lambda: np.dot(A, B), number=4)))
```

Subsequently set `OMP_NUM_THREADS=2`, or any higher number of threads available in your hardware (multi-threaded):

```
>> export OMP_NUM_THREADS=2
```

and run the same python code. By both looking at your processes (e.g. using `top`) and at the python print statement you should see a speed-up in the second case.

Alternatively, you could set the `OMP_NUM_THREADS` variable directly inside your script using `os.environ['OMP_NUM_THREADS']=str(2)`. Moreover, note that when using Intel MKL you can alternatively set the `MKL_NUM_THREADS` instead of `OMP_NUM_THREADS`: this could be useful if your code runs other parallel processes which you can control independently from the Intel MKL ones using `OMP_NUM_THREADS`.

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**Note:** Always remember to set `OMP_NUM_THREADS` (or `MKL_NUM_THREADS`) in your environment when using PyLops

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### 3.2.2 Optional dependencies

To avoid increasing the number of *required* dependencies, which may lead to conflicts with other libraries that you have in your system, we have decided to build some of the additional features of PyLops in such a way that if an *optional* dependency is not present in your python environment, a safe fallback to one of the required dependencies will be enforced.

When available in your system, we recommend using the Conda package manager and install all the mandatory and optional dependencies of PyLops at once using the command:

```
>> conda install -c conda-forge pyllops
```

in this case all dependencies will be installed from their conda distributions.

Alternatively, from version 1.4.0 optional dependencies can also be installed as part of the pip installation via:

```
>> pip install pyllops[advanced]
```

Dependencies are however installed from their PyPI wheels.

## numba

Although we always strive to write code for forward and adjoint operators that takes advantage of the perks of numpy and scipy (e.g., broadcasting, ufunc), in some cases we may end up using for loops that may lead to poor performance. In those cases we may decide to implement alternative (optional) back-ends in `numba`.

In this case a user can simply switch from the native, always available implementation to the `numba` implementation by simply providing the following additional input parameter to the operator `engine='numba'`. This is for example the case in the `pylops.signalprocessing.Radon2D`.

If interested to use `numba` backend from conda, you will need to manually install it:

```
>> conda install numba
```

Finally, it is also advised to install the additional package `icc_rt`.

```
>> conda install -c numba icc_rt
```

or pip equivalent. Similarly to Intel MKL, you need to set the environment variable `NUMBA_NUM_THREADS` to tell `numba` how many threads to use. If this variable is not present in your environment, `numba` code will be compiled with `parallel=False`.

## fft routines

Two different *engines* are provided by the `pylops.signalprocessing.FFT` operator for `fft` and `ifft` routines in the forward and adjoint modes: `engine='numpy'` (default) and `engine='fftw'`.

The first engine comes as default as numpy is part of the dependencies of PyLops and automatically installed when PyLops is installed if not already available in your Python distribution.

The second engine implements the well-known FFTW via the python wrapper `pyfftw.FFTW`. This optimized fft tends to outperform the one from numpy in many cases, however it has not been inserted in the mandatory requirements of PyLops, meaning that when installing PyLops with pip, `pyfftw.FFTW` will *not* be installed automatically.

Again, if interested to use FFTW backend from conda, you will need to manually install it:

```
>> conda install -c conda-forge pyfftw
```

or pip equivalent.

## skfmm

This library is used to compute traveltimes tables with the fast-marching method in the initialization of the `pylops.waveeqprocessing.Demigration` operator when choosing `mode == 'eikonal'`.

As this may not be of interest for many users, this library has not been inserted in the mandatory requirements of PyLops. If interested to use `skfmm`, you will need to manually install it:

```
>> conda install -c conda-forge scikit-fmm
```

or pip equivalent.

### spgl1

This library is used to solve sparsity-promoting BP, BPDN, and LASSO problems in `pylops.optimization.sparsity.SPGL1` solver.

If interested to use `spgl1`, you can manually install it:

```
>> pip install spgl1
```

### pywt

This library is used to implement the Wavelet operators.

If interested to use `pywt`, you can manually install it:

```
>> conda install pywavelets
```

or pip equivalent.

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**Note:** If you are a developer, all the optional dependencies can also be installed automatically by cloning the repository and installing pyllops via `make dev-install` or `make dev-install_conda`.

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## 3.3 Extensions

PyLops brings to users the power of linear operators in a simple and easy to use programming interface.

While very powerful on its own, this library is further extended to take advantage of more advanced computational resources, either in terms of **multiple-node clusters** or **GPUs**.

Two spin-off projects aim at extending the capabilities of PyLops to such computational environments:

- [PyLops-GPU](#)
- [PyLops-Distributed](#)

## 3.4 Tutorials

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**Note:** Click [here](#) to download the full example code

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### 3.4.1 01. The LinearOperator

This first tutorial is aimed at easing the use of the PyLops library for both new users and developers.

Since PyLops heavily relies on the use of the `scipy.sparse.linalg.LinearOperator` class of `scipy`, we will start by looking at how to initialize a linear operator as well as different ways to apply the forward and adjoint operations. Finally we will investigate various *special methods*, also called *magic methods* (i.e., methods with the double underscores at the beginning and the end) that have been implemented for such a class and will allow summing, subtracting, chaining, etc. multiple operators in very easy and expressive way.

Let's start by defining a simple operator that applies element-wise multiplication of the model with a vector  $d$  in forward mode and element-wise multiplication of the data with the same vector  $d$  in adjoint mode. This operator is present in PyLops under the name of `pylops.Diagonal` and its implementation is discussed in more details in the [Implementing new operators](#) page.

```
import timeit
import matplotlib.pyplot as plt
import numpy as np
import pylops

n = 10
d = np.arange(n) + 1.
x = np.ones(n)
Dop = pylops.Diagonal(d)
```

First of all we apply the operator in the forward mode. This can be done in four different ways:

- `_matvec`: directly applies the method implemented for forward mode
- `matvec`: performs some checks before and after applying `_matvec`
- `*`: operator used to map the special method `__matmul__` which checks whether the input  $x$  is a vector or matrix and applies `_matvec` or `_matmul` accordingly.
- `@`: operator used to map the special method `__mul__` which performs like the `*` operator

We will time these 4 different executions and see how using `_matvec` (or `matvec`) will result in the faster computation. It is thus advised to use `*` (or `@`) in examples when expressivity has priority but prefer `_matvec` (or `matvec`) for efficient implementations.

```
# setup command
cmd_setup = """\
import numpy as np
import pylops
n = 10
d = np.arange(n) + 1.
x = np.ones(n)
Dop = pylops.Diagonal(d)
DopH = Dop.H
"""

# _matvec
cmd1 = 'Dop._matvec(x)'

# matvec
cmd2 = 'Dop.matvec(x)'

# @
cmd3 = 'Dop@x'

# *
cmd4 = 'Dop*x'

# timing
t1 = 1.e3 * np.array(timeit.repeat(cmd1, setup=cmd_setup,
                                    number=500, repeat=5))
t2 = 1.e3 * np.array(timeit.repeat(cmd2, setup=cmd_setup,
                                    number=500, repeat=5))
t3 = 1.e3 * np.array(timeit.repeat(cmd3, setup=cmd_setup,
```

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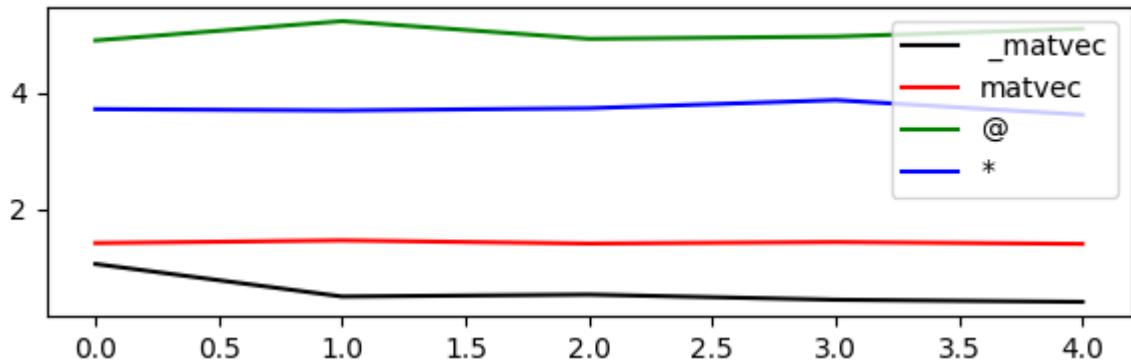
(continued from previous page)

```

    number=500, repeat=5))
t4 = 1.e3 * np.array(timeit.repeat(cmd4, setup=cmd_setup,
                                    number=500, repeat=5))

plt.figure(figsize=(7, 2))
plt.plot(t1, 'k', label='_matvec')
plt.plot(t2, 'r', label='matvec')
plt.plot(t3, 'g', label='@')
plt.plot(t4, 'b', label='*')
plt.axis('tight')
plt.legend();

```



Out:

```
<matplotlib.legend.Legend object at 0x7fcd6ddaae80>
```

Similarly we now consider the adjoint mode. This can be done in three different ways:

- `_rmatvec`: directly applies the method implemented for adjoint mode
- `rmatvec`: performs some checks before and after applying `_rmatvec`
- `.H*`: first applies the adjoint `.H` which creates a new `scipy.sparse.linalg._CustomLinearOperator` where `_matvec` and `_rmatvec` are swapped and then applies the new `_matvec`.

Once again, after timing these 3 different executions we can see how using `_rmatvec` (or `rmatvec`) will result in the faster computation while `.H*` is very unefficient and slow. Note that if the adjoint has to be applied multiple times it is at least advised to create the adjoint operator by applying `.H` only once upfront. Not surprisingly, the linear solvers in scipy as well as in PyLops actually use `matvec` and `rmatvec` when dealing with linear operators.

```

# __rmatvec
cmd1 = 'Dop.__rmatvec(x)'

# rmatvec
cmd2 = 'Dop.rmatvec(x)'

# .H* (pre-computed H)
cmd3 = 'DopH*x'

# .H*
cmd4 = 'Dop.H*x'

# timing

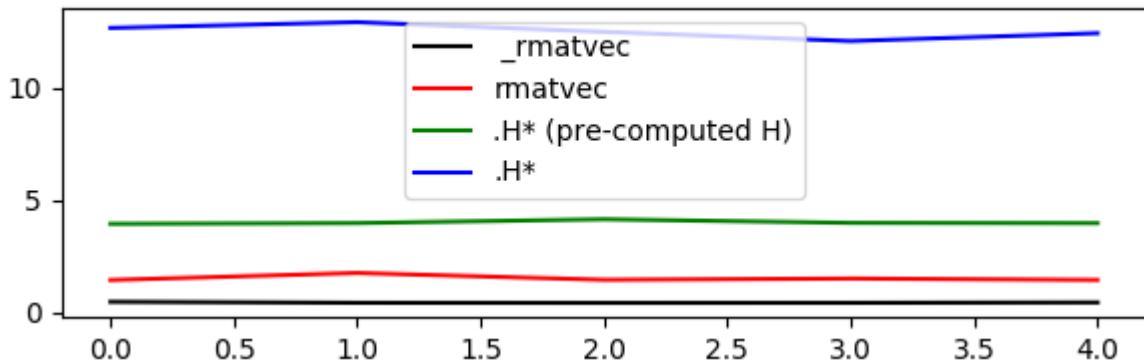
```

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```
t1 = 1.e3 * np.array(timeit.repeat(cmd1, setup=cmd_setup,
                                         number=500, repeat=5))
t2 = 1.e3 * np.array(timeit.repeat(cmd2, setup=cmd_setup,
                                         number=500, repeat=5))
t3 = 1.e3 * np.array(timeit.repeat(cmd3, setup=cmd_setup,
                                         number=500, repeat=5))
t4 = 1.e3 * np.array(timeit.repeat(cmd4, setup=cmd_setup,
                                         number=500, repeat=5))

plt.figure(figsize=(7, 2))
plt.plot(t1, 'k', label='_rmatvec')
plt.plot(t2, 'r', label='rmatvec')
plt.plot(t3, 'g', label='.H* (pre-computed H)')
plt.plot(t4, 'b', label='.H*')
plt.axis('tight')
plt.legend();
```



Out:

&lt;matplotlib.legend.Legend object at 0x7fc6e9c74e0&gt;

Just to reiterate once again, it is advised to call `matvec` and `rmatvec` unless PyLops linear operators are used for teaching purposes.

We now go through some other *methods* and *special methods* that are implemented in `scipy.sparse.linalg.LinearOperator` (and `pylops.LinearOperator`):

- `Op1+Op2`: maps the special method `__add__` and performs summation between two operators and returns a `pylops.LinearOperator`
- `-Op`: maps the special method `__neg__` and performs negation of an operators and returns a `pylops.LinearOperator`
- `Op1-Op2`: maps the special method `__sub__` and performs summation between two operators and returns a `pylops.LinearOperator`
- `Op1**N`: maps the special method `__pow__` and performs exponentiation of an operator and returns a `pylops.LinearOperator`
- `Op/y` (and `Op.div(y)`): maps the special method `__truediv__` and performs inversion of an operator
- `Op.eigs()`: estimates the eigenvalues of the operator
- `Op.cond()`: estimates the condition number of the operator
- `Op.conj()`: create complex conjugate operator

```
# +
print(Dop + Dop)

# -
print(-Dop)
print(Dop - 0.5 * Dop)

# **
print(Dop ** 3)

#* and /
y = Dop * x
print(Dop/y)

# eigs
print(Dop.eigs(neigs=3))

# cond
print(Dop.cond())

# conj
print(Dop.conj())
```

Out:

```
<10x10 LinearOperator with dtype=float64>
<10x10 LinearOperator with dtype=float64>
<10x10 LinearOperator with dtype=float64>
<10x10 LinearOperator with dtype=float64>
[1. 1. 1. 1. 1. 1. 1. 1. 1.]
[10.+0.j 9.+0.j 8.+0.j]
(9.9999999999986+0j)
<10x10 _ConjLinearOperator with dtype=float64>
```

To understand the effect of `conj` we need to look into a problem with an operator in the complex domain. Let's create again our `pylops.Diagonal` operator but this time we populate it with complex numbers. We will see that the action of the operator and its complex conjugate is different even if the model is real.

```
n = 5
d = 1j*(np.arange(n) + 1.)
x = np.ones(n)
Dop = pylops.Diagonal(d)

print('y = Dx = ', Dop*x)
print('y = conj(D)x = ', Dop.conj()*x)
```

Out:

```
y = Dx = [0.+1.j 0.+2.j 0.+3.j 0.+4.j 0.+5.j]
y = conj(D)x = [0.-1.j 0.-2.j 0.-3.j 0.-4.j 0.-5.j]
```

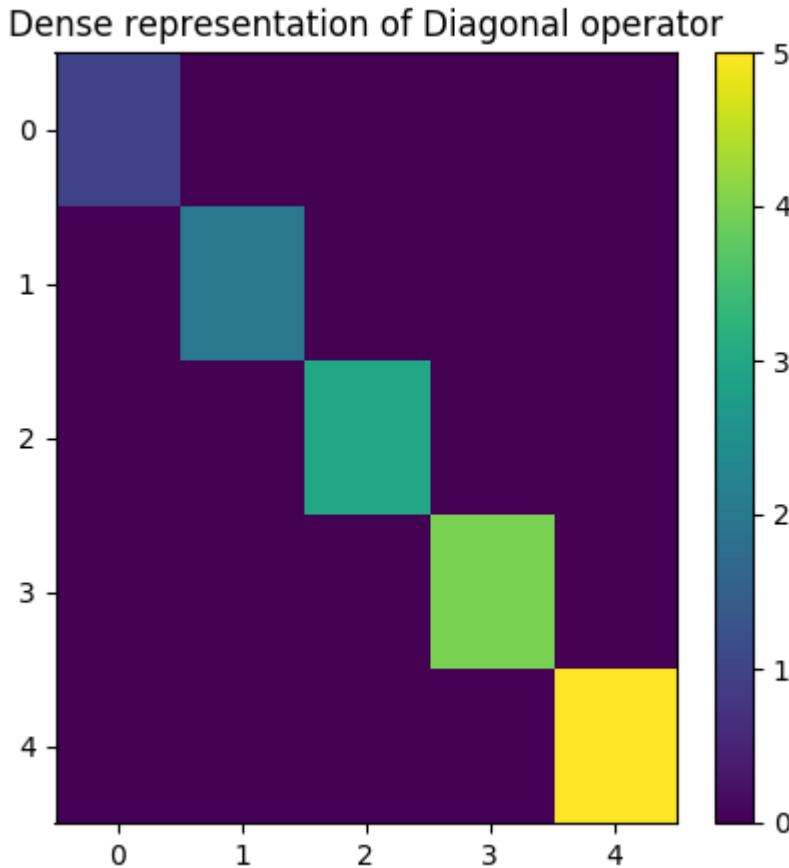
At this point, the concept of linear operator may sound abstract. The convinience method `pylops.LinearOperator.todense` can be used to create the equivalent dense matrix of any operator. In this case for example we expect to see a diagonal matrix with `d` values along the main diagonal

```
D = Dop.todense()
```

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```
plt.figure(figsize=(5, 5))
plt.imshow(np.abs(D))
plt.title('Dense representation of Diagonal operator')
plt.axis('tight')
plt.colorbar()
```



Out:

```
<matplotlib.colorbar.Colorbar object at 0x7fcd6e4c2278>
```

Finally it is worth reiterating that if two linear operators are combined by means of the algebraical operations shown above, the resulting operator is still a `pylops.LinearOperator` operator. This means that we can still apply any of the methods implemented in the original scipy class definition like `*`, as well as those in our class definition like `/`

```
Dop1 = Dop - Dop.conj()

y = Dop1 * x
print('x = (Dop - conj(Dop)) / y = ', Dop1 / y)

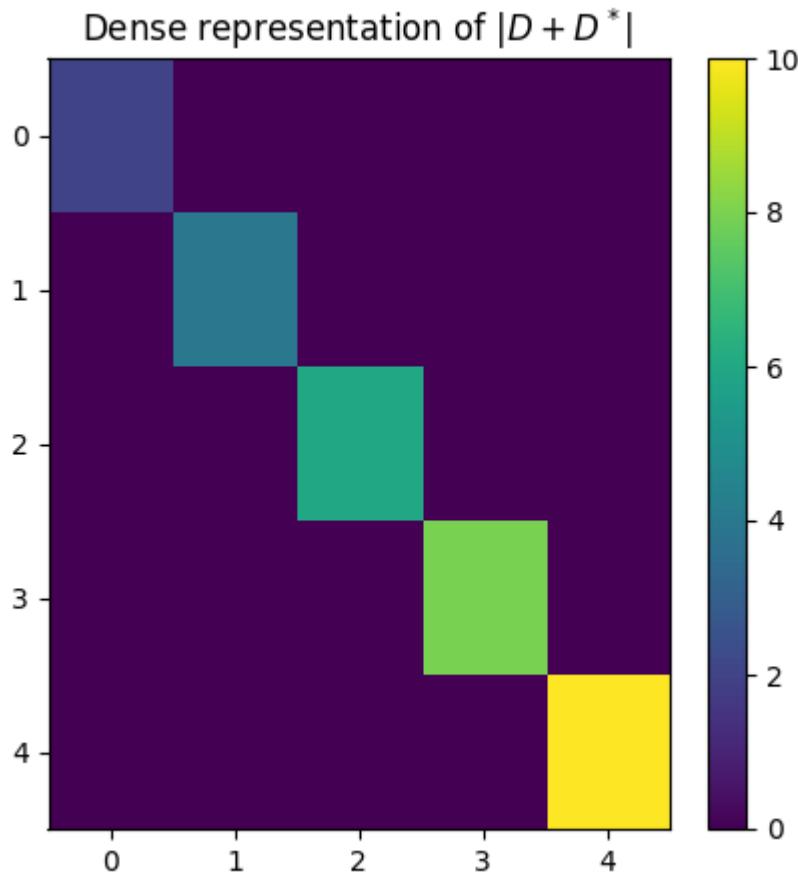
D1 = Dop1.todense()

plt.figure(figsize=(5, 5))
```

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```
plt.imshow(np.abs(D1))
plt.title(r'Dense representation of $|D + D^*|$')
plt.axis('tight')
plt.colorbar()
```



Out:

```
x = (Dop - conj(Dop))/y = [1.+0.j 1.+0.j 1.+0.j 1.+0.j 1.+0.j]
<matplotlib.colorbar.Colorbar object at 0x7fc6e9cd800>
```

This first tutorial is completed. You have seen the basic operations that can be performed using `scipy.sparse.linalg.LinearOperator` and our overload of such a class `pylops.LinearOperator` and you should be able to get started combining various PyLops operators and solving your own inverse problems.

**Total running time of the script:** ( 0 minutes 0.626 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.2 02. The Dot-Test

One of the most important aspect of writing a *Linear operator* is to be able to verify that the code implemented in *forward mode* and the code implemented in *adjoint mode* are effectively adjoint to each other. If this is the case, your Linear operator will successfully pass the so-called **dot-test**. Refer to the *Notes* section of [\*pylops.utils.dottest\*](#) for a more detailed description.

In this example, I will show you how to use the dot-test for a variety of operator when model and data are either real or complex numbers.

```
# pylint: disable=C0103
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.gridspec as pltgs

import pylops
from pylops.utils import dottest

plt.close('all')
```

Let's start with something very simple. We will make a [\*pylops.MatrixMult\*](#) operator and verify that its implementation passes the dot-test. For this time, we will do this step-by-step, replicating what happens in the [\*pylops.utils.dottest\*](#) routine.

```
N, M = 5, 3
Mat = np.arange(N*M).reshape(N, M)
Op = pylops.MatrixMult(Mat)

v = np.random.randn(N)
u = np.random.randn(M)

# Op * u
y = Op.matvec(u)
# Op' * v
x = Op.rmatvec(v)

yy = np.dot(y, v) # (Op * u)' * v
xx = np.dot(u, x) # u' * (Op' * v)

print('Dot-test %e' % np.abs((yy - xx) / ((yy + xx + 1e-15) / 2)))
```

Out:

```
Dot-test 1.366060e-16
```

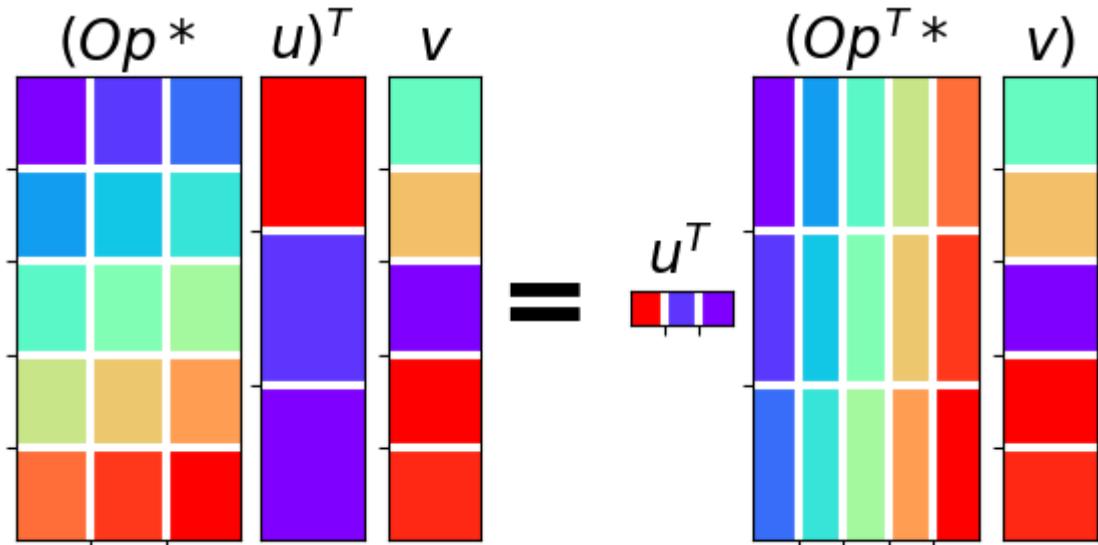
And here is a visual interpretation of what a dot-test is

```
gs = pltgs.GridSpec(1, 9)
fig = plt.figure(figsize=(7, 3))
ax = plt.subplot(gs[0, 0:2])
ax.imshow(Op.A, cmap='rainbow')
ax.set_title(r'$\mathbf{Op} \mathbf{A}$', size=20, fontweight='bold')
ax.set_xticks(np.arange(M-1)+0.5)
ax.set_yticks(np.arange(N-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
```

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```
ax.axis('tight')
ax = plt.subplot(gs[0, 2])
ax.imshow(u[:, np.newaxis], cmap='rainbow')
ax.set_title(r'$\mathbf{u}^T$', size=20, fontweight='bold')
ax.set_xticks([])
ax.set_yticks(np.arange(M-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis('tight')
ax = plt.subplot(gs[0, 3])
ax.imshow(v[:, np.newaxis], cmap='rainbow')
ax.set_title(r'$\mathbf{v}$', size=20, fontweight='bold')
ax.set_xticks([])
ax.set_yticks(np.arange(N-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 4])
ax.text(0.35, 0.5, '=', horizontalalignment='center',
        verticalalignment='center', size=40, fontweight='bold')
ax.axis('off')
ax = plt.subplot(gs[0, 5])
ax.imshow(u[:, np.newaxis].T, cmap='rainbow')
ax.set_title(r'$\mathbf{u}^T$', size=20, fontweight='bold')
ax.set_xticks(np.arange(M-1)+0.5)
ax.set_yticks([])
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 6:8])
ax.imshow(Op.A.T, cmap='rainbow')
ax.set_title(r'$\mathbf{Op}^T$', size=20, fontweight='bold')
ax.set_xticks(np.arange(N-1)+0.5)
ax.set_yticks(np.arange(M-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis('tight')
ax = plt.subplot(gs[0, 8])
ax.imshow(v[:, np.newaxis], cmap='rainbow')
ax.set_title(r'$\mathbf{v}$', size=20, fontweight='bold')
ax.set_xticks([])
ax.set_yticks(np.arange(N-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
```



Out:

```
[ ]
```

From now on, we can simply use the `pylops.utils.dottest` implementation of the dot-test and pass the operator we would like to validate, its size in the model and data spaces and optionally the tolerance we will be accepting for the dot-test to be considered successful. Finally we need to specify if our data or/and model vectors contain complex numbers using the `complexflag` parameter. While the dot-test will return `True` when successful and `False` otherwise, we can also ask to print its outcome putting the `verb` parameters to `True`.

```
N = 10
d = np.arange(N)
Dop = pylops.Diagonal(d)

dottest(Dop, N, N, tol=1e-6, complexflag=0, verb=True)
```

Out:

```
Dot test passed, v^T(Op)= -6.919654 - u^T(Op^Tv)= -6.919654
True
```

We move now to a more complicated operator, the `pylops.signalprocessing.FFT` operator. We use once again the `pylops.utils.dottest` to verify its implementation and since we are dealing with a transform that can be applied to both real and complex array, we try different combinations using the `complexflag` input.

```
dt = 0.005
nt = 100
nfft = 2**10

FFTop = pylops.signalprocessing.FFT(dims=(nt,), nfft=nfft, sampling=dt)
dottest(FFTop, nfft, nt, complexflag=2, verb=True)
dottest(FFTop, nfft, nt, complexflag=3, verb=True)
```

Out:

```
Dot test passed, v^T(Op_u)=-17.139104+3.828834i - u^T(Op^T_v)=-17.139104+3.828834i
Dot test passed, v^T(Op_u)=11.420800-3.688151i - u^T(Op^T_v)=11.420800-3.688151i

True
```

**Total running time of the script:** ( 0 minutes 0.228 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.3 03. Solvers

This tutorial will guide you through the `pylops.optimization` module and show how to use various solvers that are included in the PyLops library.

The main idea here is to provide the user of PyLops with very high-level functionalities to quickly and easily set up and solve complex systems of linear equations as well as include regularization and/or preconditioning terms (all of those constructed by means of PyLops linear operators).

To make this tutorial more interesting, we will present a real life problem and show how the choice of the solver and regularization/preconditioning terms is vital in many circumstances to successfully retrieve an estimate of the model. The problem that we are going to consider is generally referred to as the *data reconstruction* problem and aims at reconstructing a regularly sampled signal of size  $M$  from  $N$  randomly selected samples:

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where the restriction operator  $\mathbf{R}$  that selects the  $M$  elements from  $\mathbf{x}$  at random locations is implemented using `pylops.Restriction`, and

$$\mathbf{y} = [y_1, y_2, \dots, y_N]^T, \quad \mathbf{x} = [x_1, x_2, \dots, x_M]^T,$$

with  $M \gg N$ .

```
# pylint: disable=C0103
import numpy as np
import matplotlib.pyplot as plt

import pylops

plt.close('all')
np.random.seed(10)
```

Let's first create the data in the frequency domain. The data is composed by the superposition of 3 sinusoids with different frequencies.

```
# Signal creation in frequency domain
ifreqs = [41, 25, 66]
amps = [1., 1., 1.]
N = 200
nfft = 2**11
dt = 0.004
t = np.arange(N)*dt
f = np.fft.rfftfreq(nfft, dt)

FFTop = 10*pylops.signalprocessing.FFT(N, nfft=nfft, real=True)
```

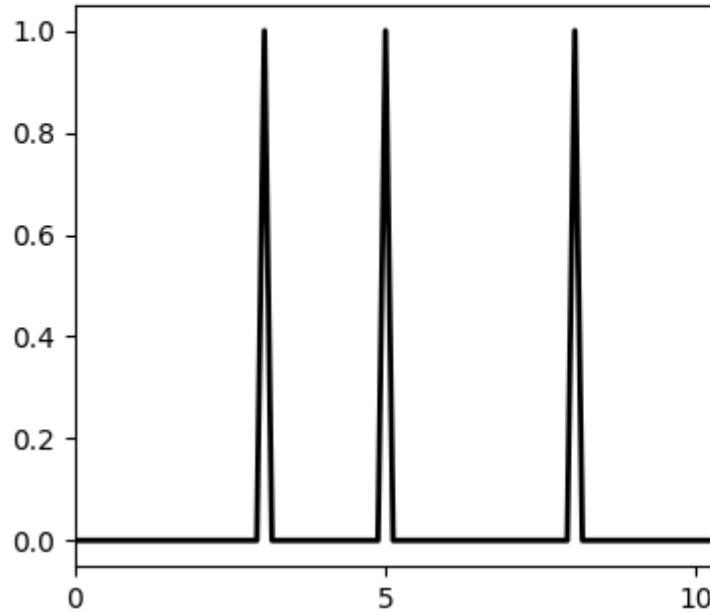
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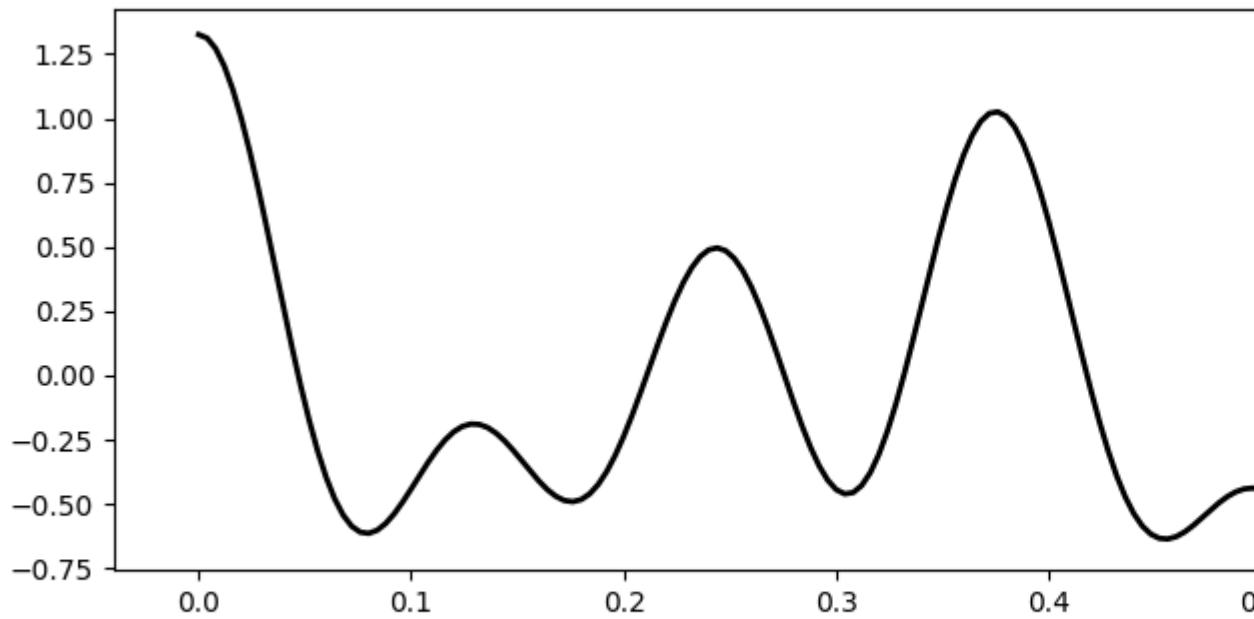
```
X = np.zeros(nfft//2+1, dtype='complex128')
X[ifreqs] = amps
x = FFTop.H*X

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
axs[0].plot(f, np.abs(X), 'k', LineWidth=2)
axs[0].set_xlim(0, 30)
axs[0].set_title('Data(frequency domain)')
axs[1].plot(t, x, 'k', LineWidth=2)
axs[1].set_title('Data(time domain)')
axs[1].axis('tight')
```

Data(frequency domain)



Data(time domain)



Out:

```
(-0.0398, 0.8358000000000001, -0.7554257721191237, 1.4249324045744862)
```

We now define the locations at which the signal will be sampled.

```
# subsampling locations
perc_subsampling = 0.2
Nsub = int(np.round(N*perc_subsampling))

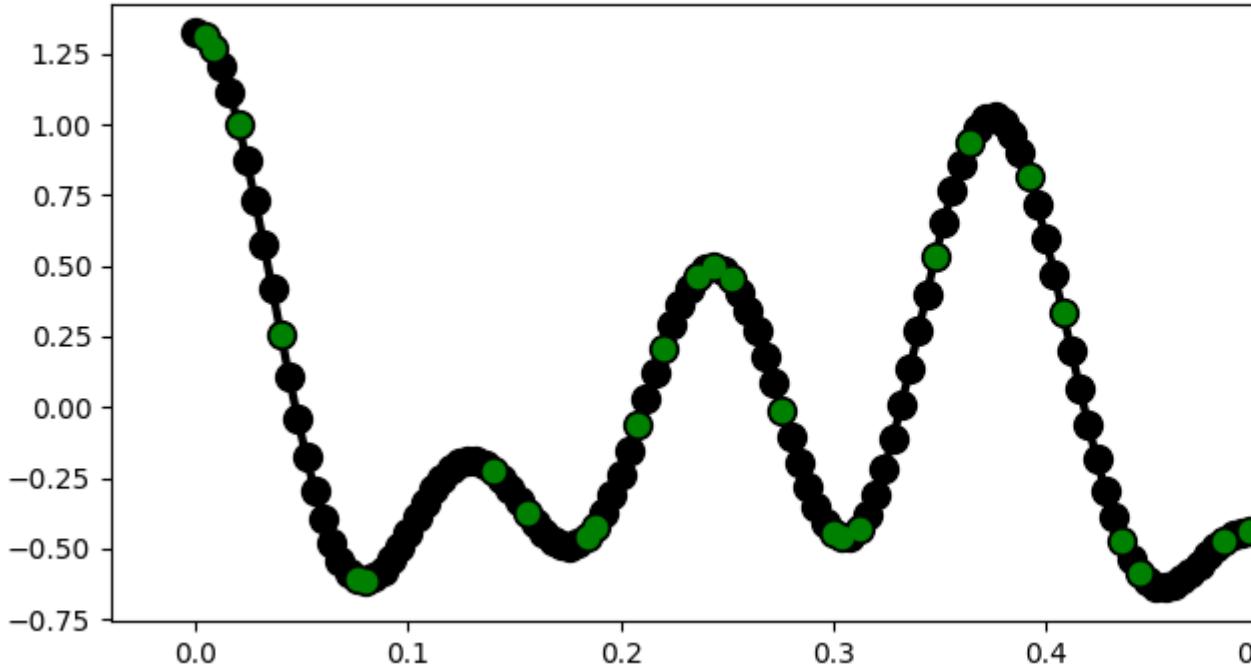
iava = np.sort(np.random.permutation(np.arange(N))[:Nsub])

# Create restriction operator
Rop = pylops.Restriction(N, iava, dtype='float64')

y = Rop*x
ymask = Rop.mask(x)

# Visualize data
fig = plt.figure(figsize=(12, 4))
plt.plot(t, x, 'k', lw=3)
plt.plot(t, x, '.k', ms=20, label='all samples')
plt.plot(t, ymask, '.g', ms=15, label='available samples')
plt.legend()
plt.title('Data restriction')
```

Data restriction



Out:

```
Text(0.5, 1.0, 'Data restriction')
```

To start let's consider the simplest 'solver', i.e., *least-square inversion without regularization*. We aim here to minimize the following cost function:

$$J = \|\mathbf{y} - \mathbf{Rx}\|_2$$

Depending on the choice of the operator  $\mathbf{R}$ , such problem can be solved using explicit matrix solvers as well as iterative solvers. In this case we will be using the latter approach (more specifically the scipy implementation of the *LSQR* solver - i.e., `scipy.sparse.linalg.lsqr`) as we do not want to explicitly create and invert a matrix. In most cases this will be the only viable approach as most of the large-scale optimization problems that we are interested to solve using PyLops do not lend naturally to the creation and inversion of explicit matrices.

This first solver can be very easily implemented using the `/` for PyLops operators, which will automatically call the `scipy.sparse.linalg.lsqr` with some default parameters.

```
xinv = Rop / y
```

We can also use `pylops.optimization.leastsquares.RegularizedInversion` (without regularization term for now) and customize our solvers using kwargs.

```
xinv = \
    pylops.optimization.leastsquares.RegularizedInversion(Rop, [], y,
                                                          **dict(damp=0,
                                                                 iter_lim=10,
                                                                 show=1))
```

Out:

```
LSQR          Least-squares solution of  Ax = b
The matrix A has      40 rows and      200 cols
damp = 0.00000000000000e+00  calc_var =      0
atol = 1.00e-08          conlim = 1.00e+08
btol = 1.00e-08          iter_lim =      10

Itn      x[0]      r1norm      r2norm  Compatible    LS      Norm A    Cond A
  0  0.00000e+00  3.759e+00  3.759e+00  1.0e+00  2.7e-01
  1  0.00000e+00  0.000e+00  0.000e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00

LSQR finished
Ax - b is small enough, given atol, btol

istop =      1  r1norm = 0.0e+00  anorm = 0.0e+00  arnorm = 0.0e+00
itn   =      1  r2norm = 0.0e+00  acond = 0.0e+00  xnorm  = 3.8e+00
```

Finally we can select a different starting guess from the null vector

```
xinv_fromx0 = \
    pylops.optimization.leastsquares.RegularizedInversion(Rop, [], y,
                                                          x0=np.ones(N),
                                                          **dict(damp=0,
                                                                 iter_lim=10,
                                                                 show=0))
```

The cost function above can be also expanded in terms of its *normal equations*

$$\mathbf{x}_{ne} = (\mathbf{R}^T \mathbf{R})^{-1} \mathbf{R}^T \mathbf{y}$$

The method `pylops.optimization.leastsquares.NormalEquationsInversion` implements such system of equations explicitly and solves them using an iterative scheme suitable for square matrices (i.e.,  $M = N$ ).

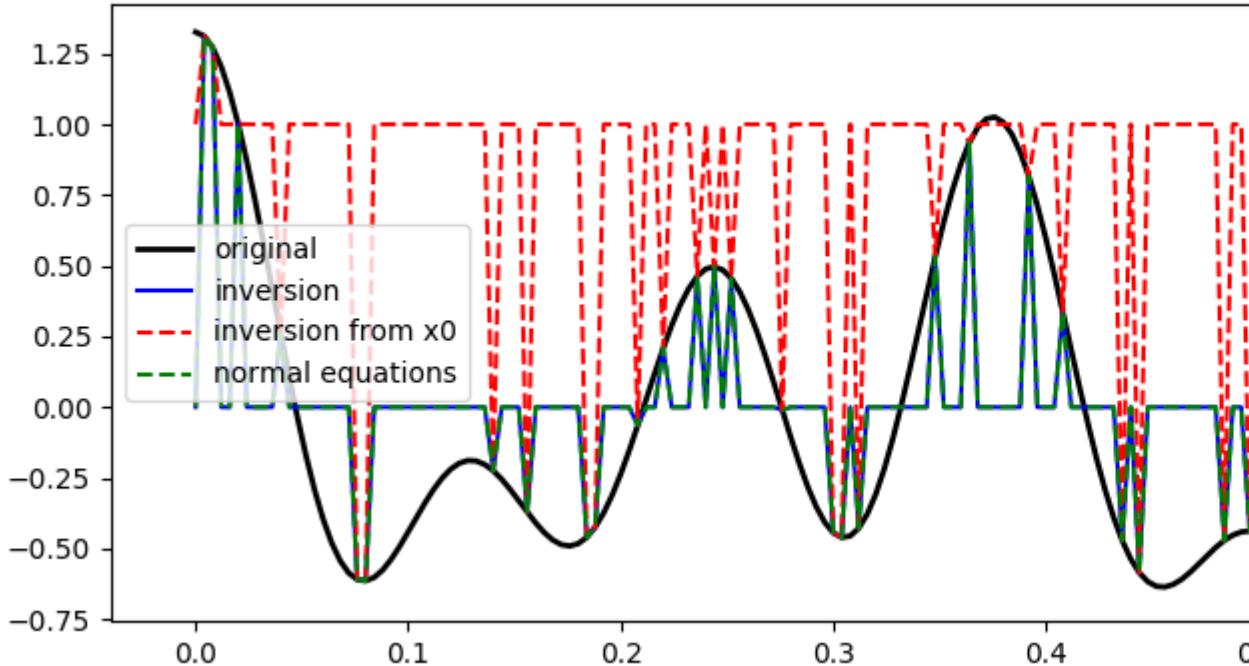
While this approach may seem not very useful, we will soon see how regularization terms could be easily added to the normal equations using this method.

```
xne = pylops.optimization.leastsquares.NormalEquationsInversion(Rop, [], y)
```

Let's now visualize the different inversion results

```
fig = plt.figure(figsize=(12, 4))
plt.plot(t, x, 'k', lw=2, label='original')
plt.plot(t, xinv, 'b', ms=10, label='inversion')
plt.plot(t, xinv_fromx0, '--r', ms=10, label='inversion from x0')
plt.plot(t, xne, '--g', ms=10, label='normal equations')
plt.legend()
plt.title('Data reconstruction without regularization')
```

Data reconstruction without regularization



Out:

```
Text(0.5, 1.0, 'Data reconstruction without regularization')
```

## Regularization

You may have noticed that none of the inversion has been successfull in recovering the original signal. This is a clear indication that the problem we are trying to solve is highly ill-posed and requires some prior knowledge from the user.

We will now see how to add prior information to the inverse process in the form of regularization (or preconditioning). This can be done in two different ways

- regularization via `pylops.optimization.leastsquares.NormalEquationsInversion` or `pylops.optimization.leastsquares.RegularizedInversion`)
- preconditioning via `pylops.optimization.leastsquares.PreconditionedInversion`

Let's start by regularizing the normal equations using a second derivative operator

$$\mathbf{x} = (\mathbf{R}^T \mathbf{R} + \epsilon_{\nabla}^2 \nabla^T \nabla)^{-1} \mathbf{R}^T \mathbf{y}$$

```
# Create regularization operator
D2op = pylops.SecondDerivative(N, dims=None, dtype='float64')

# Regularized inversion
epsR = np.sqrt(0.1)
epsI = np.sqrt(1e-4)

xne = \
    pylops.optimization.leastsquares.NormalEquationsInversion(Rop, [D2op], y,
                                                               epsI=epsI,
                                                               epsRs=[epsR],
                                                               returninfo=False,
                                                               **dict(maxiter=50))
```

We can do the same while using `pylops.optimization.leastsquares.RegularizedInversion` which solves the following augmented problem

$$\begin{bmatrix} \mathbf{R} \\ \epsilon_{\nabla} \nabla \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{y} \\ 0 \end{bmatrix}$$

```
xreg = \
    pylops.optimization.leastsquares.RegularizedInversion(Rop, [D2op], y,
                                                               epsRs=[np.sqrt(0.1)],
                                                               returninfo=False,
                                                               **dict(damp=np.sqrt(1e-4),
                                                               iter_lim=50,
                                                               show=0))
```

We can also write a preconditioned problem, whose cost function is

$$J = \|\mathbf{y} - \mathbf{R}\mathbf{P}\mathbf{p}\|_2$$

where  $\mathbf{P}$  is the preconditioned operator,  $\mathbf{p}$  is the projected model in the preconditioned space and  $\mathbf{x} = \mathbf{P}\mathbf{p}$  is the model in the original model space we want to solve for. Note that a preconditioned problem converges much faster to its solution than its corresponding regularized problem. This can be done using the routine `pylops.optimization.leastsquares.PreconditionedInversion`.

```
# Create regularization operator
Sop = pylops.Smoothing1D(nssmooth=11, dims=[N], dtype='float64')

# Invert for interpolated signal
xprec = \
```

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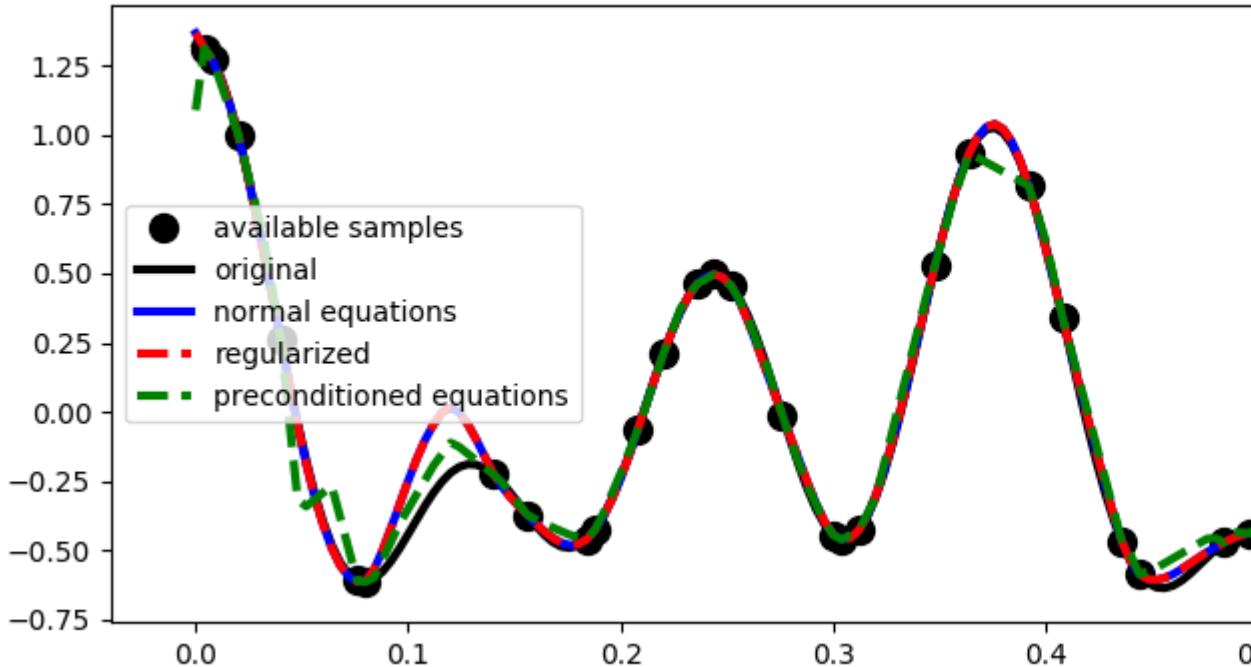
```
pylops.optimization.leastsquares.PreconditionedInversion(Rop, Sop, y,
                                                       returninfo=False,
                                                       **dict(damp=np.sqrt(1e-
→9)),
                                                       iter_lim=20,
                                                       show=0))
```

Let's finally visualize these solutions

```
# sphinx_gallery_thumbnail_number=4
fig = plt.figure(figsize=(12, 4))
plt.plot(t[iava], y, '.k', ms=20, label='available samples')
plt.plot(t, x, 'k', lw=3, label='original')
plt.plot(t, xne, 'b', lw=3, label='normal equations')
plt.plot(t, xreg, '--r', lw=3, label='regularized')
plt.plot(t, xprec, '--g', lw=3, label='preconditioned equations')
plt.legend()
plt.title('Data reconstruction with regularization')

subax = fig.add_axes([0.7, 0.2, 0.15, 0.6])
subax.plot(t[iava], y, '.k', ms=20)
subax.plot(t, x, 'k', lw=3)
subax.plot(t, xne, 'b', lw=3)
subax.plot(t, xreg, '--r', lw=3)
subax.plot(t, xprec, '--g', lw=3)
subax.set_xlim(0.05, 0.3)
```

Data reconstruction with regularization



Out:

```
(0.05, 0.3)
```

Much better estimates! We have seen here how regularization and/or preconditioning can be vital to successfully solve some ill-posed inverse problems.

We have however so far only considered solvers that can include additional norm-2 regularization terms. A very active area of research is that of *sparsity-promoting* solvers (also sometimes referred to as *compressive sensing*): the regularization term added to the cost function to minimize has norm-p ( $p \leq 1$ ) and the problem is generally recasted by considering the model to be sparse in some domain. We can follow this philosophy as our signal to invert was actually created as superposition of 3 sinusoids (i.e., three spikes in the Fourier domain). Our new cost function is:

$$J_1 = \|\mathbf{y} - \mathbf{R}\mathbf{F}\mathbf{p}\|_2 + \epsilon\|\mathbf{p}\|_1$$

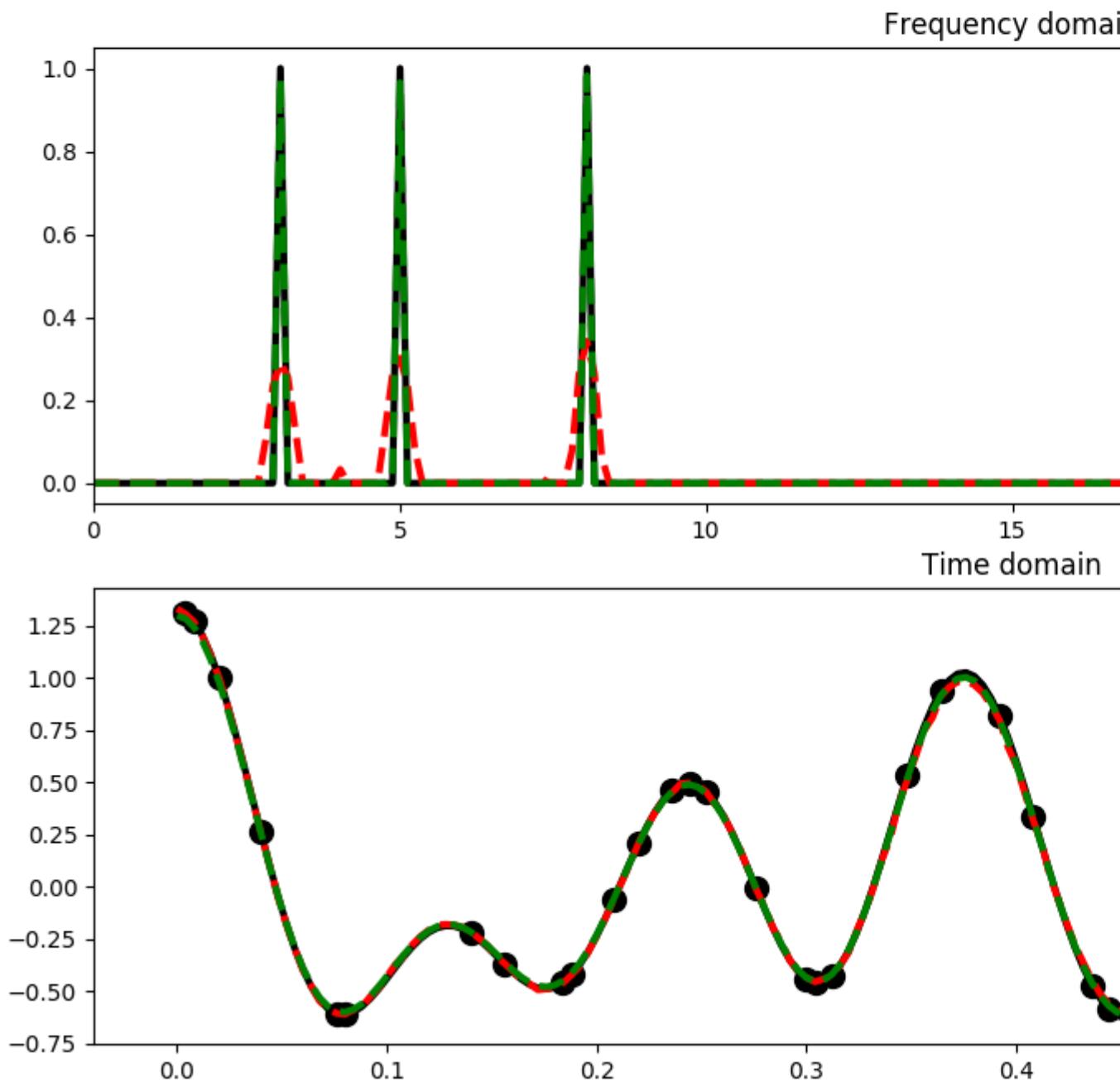
where  $\mathbf{F}$  is the FFT operator. We will thus use the `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` solvers to estimate our input signal.

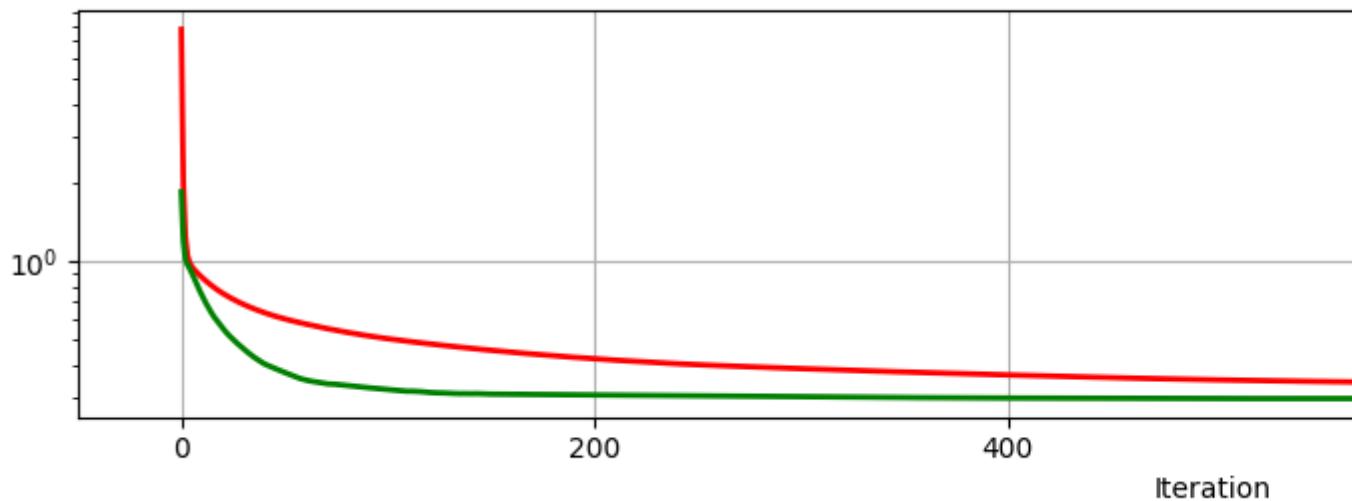
```
pista, niteri, costi = \
    pylops.optimization.sparsity.ISTA(Rop*FFTop.H, y, niter=1000,
                                      eps=0.1, tol=1e-7, returninfo=True)
xista = FFTop.H*pista

pfista, niterf, costf = \
    pylops.optimization.sparsity.FISTA(Rop*FFTop.H, y, niter=1000,
                                      eps=0.1, tol=1e-7, returninfo=True)
xfista = FFTop.H*pfista

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
fig.suptitle('Data reconstruction with sparsity', fontsize=14,
             fontweight='bold', y=0.9)
axs[0].plot(f, np.abs(X), 'k', lw=3)
axs[0].plot(f, np.abs(pista), '--r', lw=3)
axs[0].plot(f, np.abs(pfista), '--g', lw=3)
axs[0].set_xlim(0, 30)
axs[0].set_title('Frequency domain')
axs[1].plot(t[iava], y, '.k', ms=20, label='available samples')
axs[1].plot(t, x, 'k', lw=3, label='original')
axs[1].plot(t, xista, '--r', lw=3, label='ISTA')
axs[1].plot(t, xfista, '--g', lw=3, label='FISTA')
axs[1].set_title('Time domain')
axs[1].axis('tight')
axs[1].legend()
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, ax = plt.subplots(1, 1, figsize=(12, 3))
ax.semilogy(costi, 'r', lw=2, label='ISTA')
ax.semilogy(costf, 'g', lw=2, label='FISTA')
ax.set_title('Cost functions', size=15, fontweight='bold')
ax.set_xlabel('Iteration')
ax.legend()
ax.grid(True)
plt.tight_layout()
```

**Data reconstruction with**



As you can see, changing parametrization of the model and imposing sparsity in the Fourier domain has given an extra improvement to our ability of recovering the underlying densely sampled input signal. Moreover, FISTA converges much faster than ISTA as expected and should be preferred when using sparse solvers.

Finally we consider a slightly different cost function (note that in this case we try to solve a constrained problem):

$$J_1 = \|\mathbf{p}\|_1 \quad \text{subj.to} \quad \|\mathbf{y} - \mathbf{RFp}\|$$

A very popular solver to solve such kind of cost function is called *spgl1* and can be accessed via `pylops.optimization.sparsity.SPGL1`.

```
xspgll, pspgll, info = \
    pylops.optimization.sparsity.SPGL1(Rop, y, FFTop, tau=3, iter_lim=200)

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
fig.suptitle('Data reconstruction with SPGL1', fontsize=14,
             fontweight='bold', y=0.9)
axs[0].plot(f, np.abs(X), 'k', lw=3)
axs[0].plot(f, np.abs(pspgll), '--m', lw=3)
axs[0].set_xlim(0, 30)
axs[0].set_title('Frequency domain')
axs[1].plot(t[iaval], y, '.k', ms=20, label='available samples')
axs[1].plot(t, x, 'k', lw=3, label='original')
axs[1].plot(t, xspgll, '--m', lw=3, label='SPGL1')
axs[1].set_title('Time domain')
axs[1].axis('tight')
axs[1].legend()
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, ax = plt.subplots(1, 1, figsize=(12, 3))
ax.semilogy(info['rnorm2'], 'k', lw=2, label='ISTA')
ax.set_title('Cost functions', size=15, fontweight='bold')
ax.set_xlabel('Iteration')
ax.legend()
```

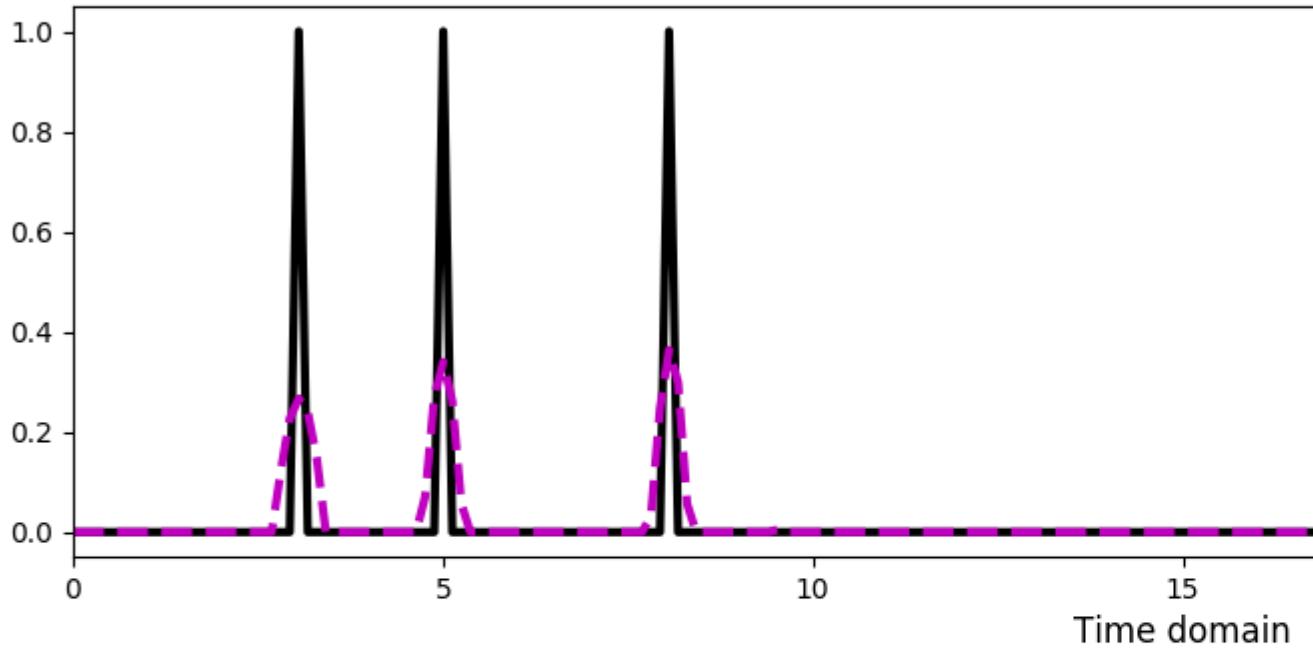
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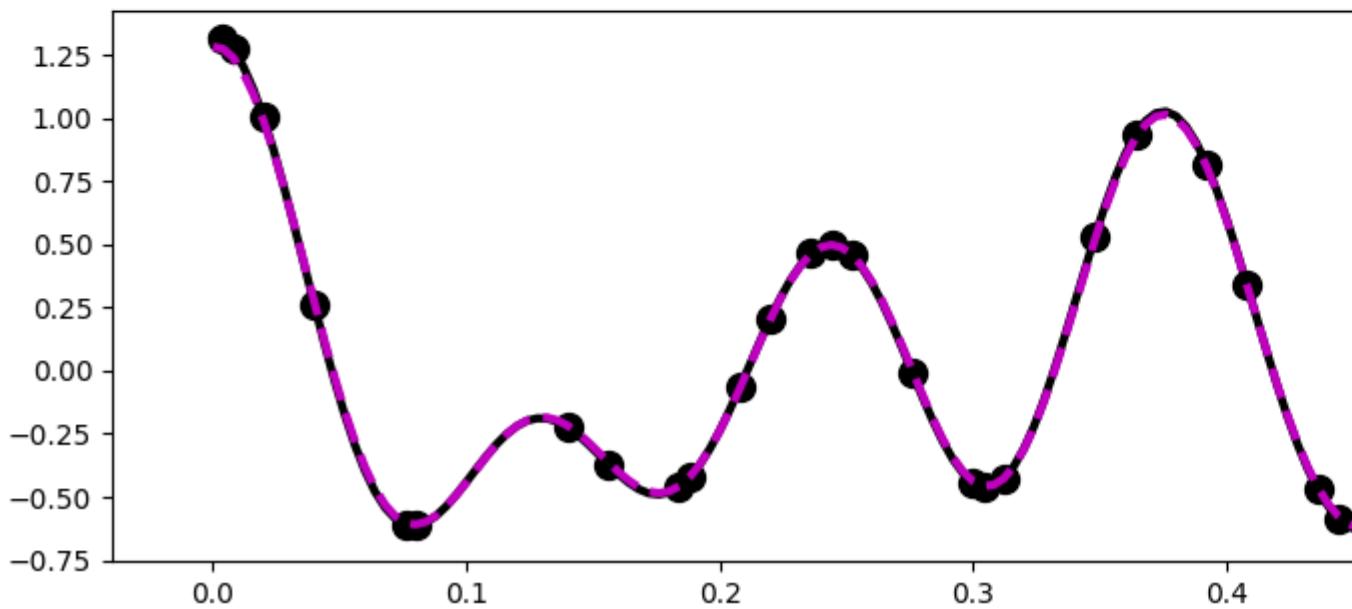
```
ax.grid(True)
plt.tight_layout()
```

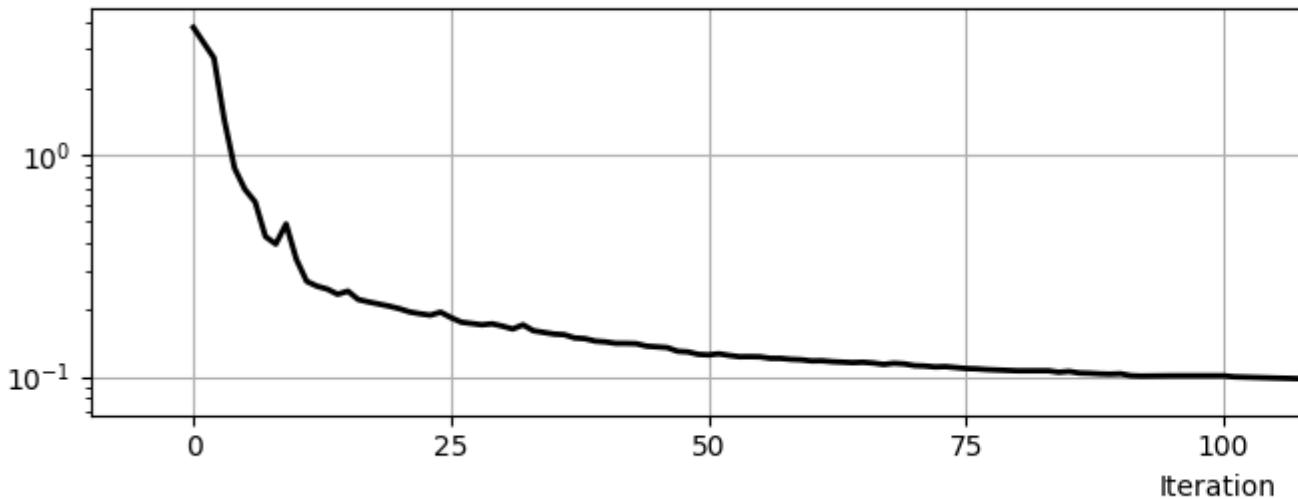
## Data reconstruction with PyLops

Frequency domain



Time domain





Total running time of the script: ( 0 minutes 2.902 seconds)

---

Note: Click [here](#) to download the full example code

---

### 3.4.4 04. Bayesian Inversion

This tutorial focuses on Bayesian inversion, a special type of inverse problem that aims at incorporating prior information in terms of model and data probabilities in the inversion process.

In this case we will be dealing with the same problem that we discussed in [03. Solvers](#), but instead of defining ad-hoc regularization or preconditioning terms we parametrize and model our input signal in the frequency domain in a probabilistic fashion: the central frequency, amplitude and phase of the three sinusoids have gaussian distributions as follows:

$$X(f) = \sum_{i=1}^3 a_i e^{j\phi_i} \delta(f - f_i)$$

where  $f_i \sim N(f_{0,i}, \sigma_{f,i})$ ,  $a_i \sim N(a_{0,i}, \sigma_{a,i})$ , and  $\phi_i \sim N(\phi_{0,i}, \sigma_{\phi,i})$ .

Based on the above definition, we construct some prior models in the frequency domain, convert each of them to the time domain and use such an ensemble to estimate the prior mean  $\mu_x$  and model covariance  $C_x$ .

We then create our data by sampling the true signal at certain locations and solve the reconstruction problem within a Bayesian framework. Since we are assuming gaussianity in our priors, the equation to obtain the posterior mean can be derived analytically:

$$\mathbf{x} = \mathbf{x}_0 + (\mathbf{R}\mathbf{C}_x\mathbf{R}^T + \mathbf{C}_y)^{-1}(\mathbf{y} - \mathbf{R}\mathbf{x}_0)$$

```
# sphinx_gallery_thumbnail_number = 2
import numpy as np
import matplotlib.pyplot as plt
import pyllops
```

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```
from scipy.sparse.linalg import lsqr

plt.close('all')
np.random.seed(10)
```

Let's start by creating our true model and prior realizations

```
def prior_realization(f0, a0, phi0, sigmaf, sigmaa, sigmaphi, dt, nt, nfft):
    """Create realization from prior mean and std for amplitude, frequency and
    phase
    """
    f = np.fft.rfftfreq(nfft, dt)
    df = f[1] - f[0]
    ifreqs = [int(np.random.normal(f, sigma)/df)
              for f, sigma in zip(f0, sigmaf)]
    amps = [np.random.normal(a, sigma) for a, sigma in zip(a0, sigmaa)]
    phis = [np.random.normal(phi, sigma) for phi, sigma in zip(phi0, sigmaphi)]

    # input signal in frequency domain
    X = np.zeros(nfft//2+1, dtype='complex128')
    X[ifreqs] = np.array(amps).squeeze() * \
        np.exp(1j * np.deg2rad(np.array(phis))).squeeze()

    # input signal in time domain
    FFTop = pylops.signalprocessing.FFT(nt, nfft=nfft, real=True)
    x = FFTop.H*X
    return x

# Priors
nreals = 100
f0 = [5, 3, 8]
sigmaf = [0.5, 1., 0.6]
a0 = [1., 1., 1.]
sigmaa = [0.1, 0.5, 0.6]
phi0 = [-90., 0., 0.]
sigmaphi = [0.1, 0.2, 0.4]
sigmad = 1e-2

# Prior models
nt = 200
nfft = 2**11
dt = 0.004
t = np.arange(nt)*dt
x = prior_realization(f0, a0, phi0, [0, 0, 0], [0, 0, 0],
                      [0, 0, 0], dt, nt, nfft)
xs = \
    np.array([prior_realization(f0, a0, phi0, sigmaf,
                                sigmaa, sigmaphi, dt, nt, nfft)
              for _ in range(nreals)])
```

We have now a set of prior models in time domain. We can easily use sample statistics to estimate the prior mean and covariance. For the covariance, we perform a second step where we average values around the main diagonal for each row and find a smooth, compact filter that we use to define a convolution linear operator that mimics the action of the covariance matrix on a vector

```

x0 = np.average(xs, axis=0)
Cm = ((xs - x0).T @ (xs - x0))/nreals

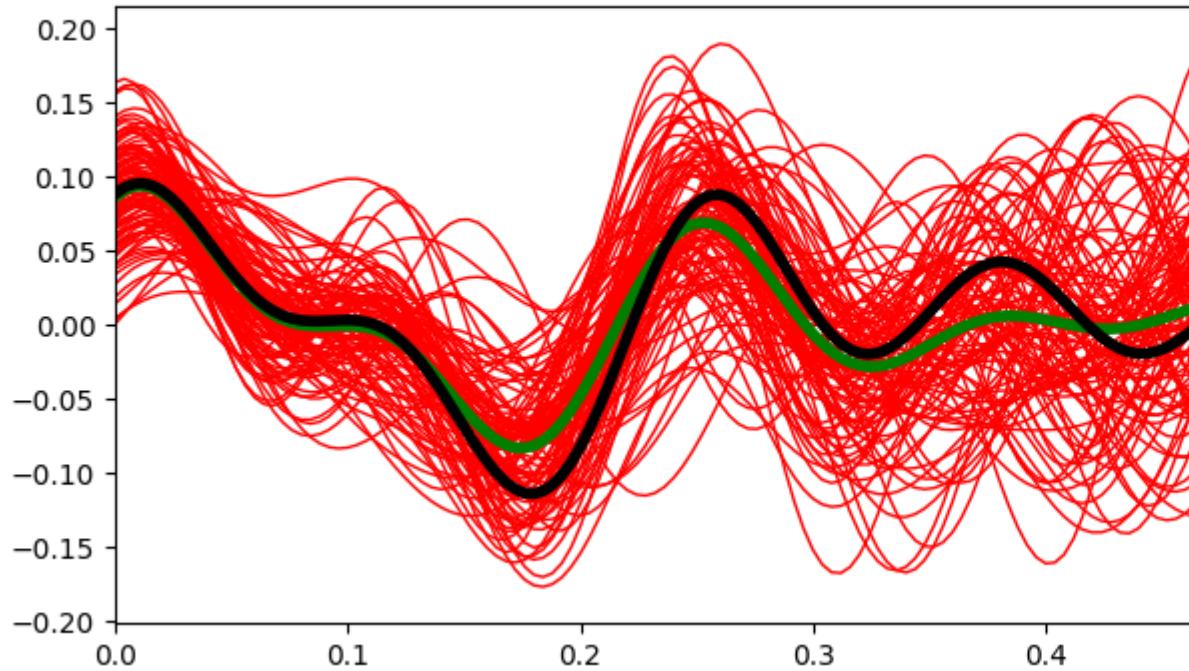
N = 30 # lenght of decorrelation
diags = np.array([Cm[i, i-N:i+N+1] for i in range(N, nt-N)])
diag_ave = np.average(diags, axis=0)
diag_ave *= np.hamming(2*N+1) # add a taper at the end to avoid edge effects

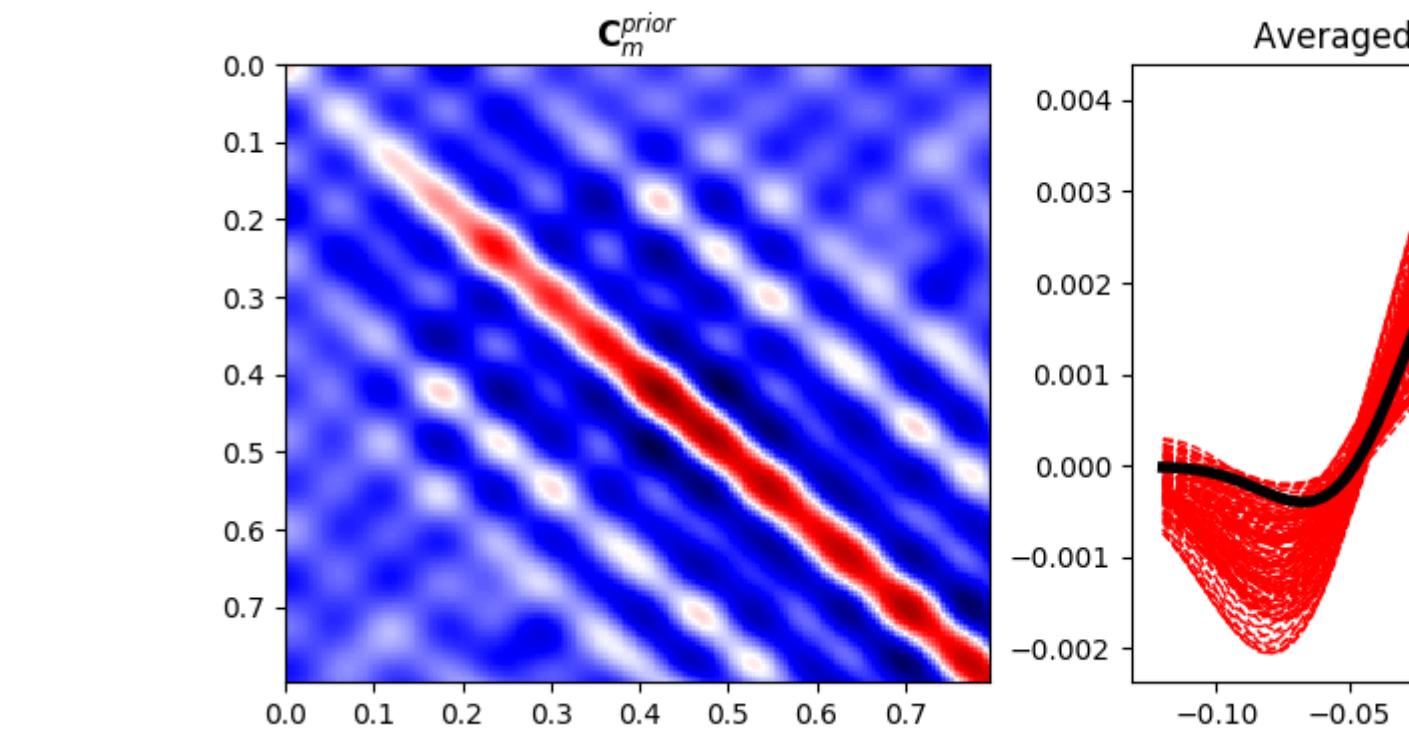
fig, ax = plt.subplots(1, 1, figsize=(12, 4))
ax.plot(t, xs.T, 'r', lw=1)
ax.plot(t, x0, 'g', lw=4)
ax.plot(t, x, 'k', lw=4)
ax.set_title('Prior realizations and mean')
ax.set_xlim(0, 0.8)

fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(10, 4))
im = ax1.imshow(Cm, interpolation='nearest', cmap='seismic',
                 extent=(t[0], t[-1], t[-1], t[0]))
ax1.set_title(r"\mathbf{C}_m^{\text{prior}}")
ax1.axis('tight')
ax2.plot(np.arange(-N, N + 1) * dt, diags.T, '--r', lw=1)
ax2.plot(np.arange(-N, N + 1) * dt, diag_ave, 'k', lw=4)
ax2.set_title("Averaged covariance 'filter'")

```

Prior realizations and mean





Out:

```
Text(0.5, 1.0, "Averaged covariance 'filter'")
```

Let's define now the sampling operator as well as create our covariance matrices in terms of linear operators. This may not be strictly necessary here but shows how even Bayesian-type of inversion can very easily scale to large model and data spaces.

```
# Sampling operator
perc_subsampling = 0.2
ntsub = int(np.round(nt*perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(nt))[:ntsub])
iava[-1] = nt-1 # assume we have the last sample to avoid instability
Rop = pylops.Restriction(nt, iava, dtype='float64')

# Covariance operators
Cm_op = \
    pylops.signalprocessing.Convolve1D(nt, diag_ave, offset=N)
Cd_op = sigmad**2 * pylops.Identity(ntsub)
```

We model now our data and add noise that respects our prior definition

```
n = np.random.normal(0, sigmad, nt)
y = Rop * x
yn = Rop * (x + n)
ymask = Rop.mask(x)
ynmask = Rop.mask(x + n)
```

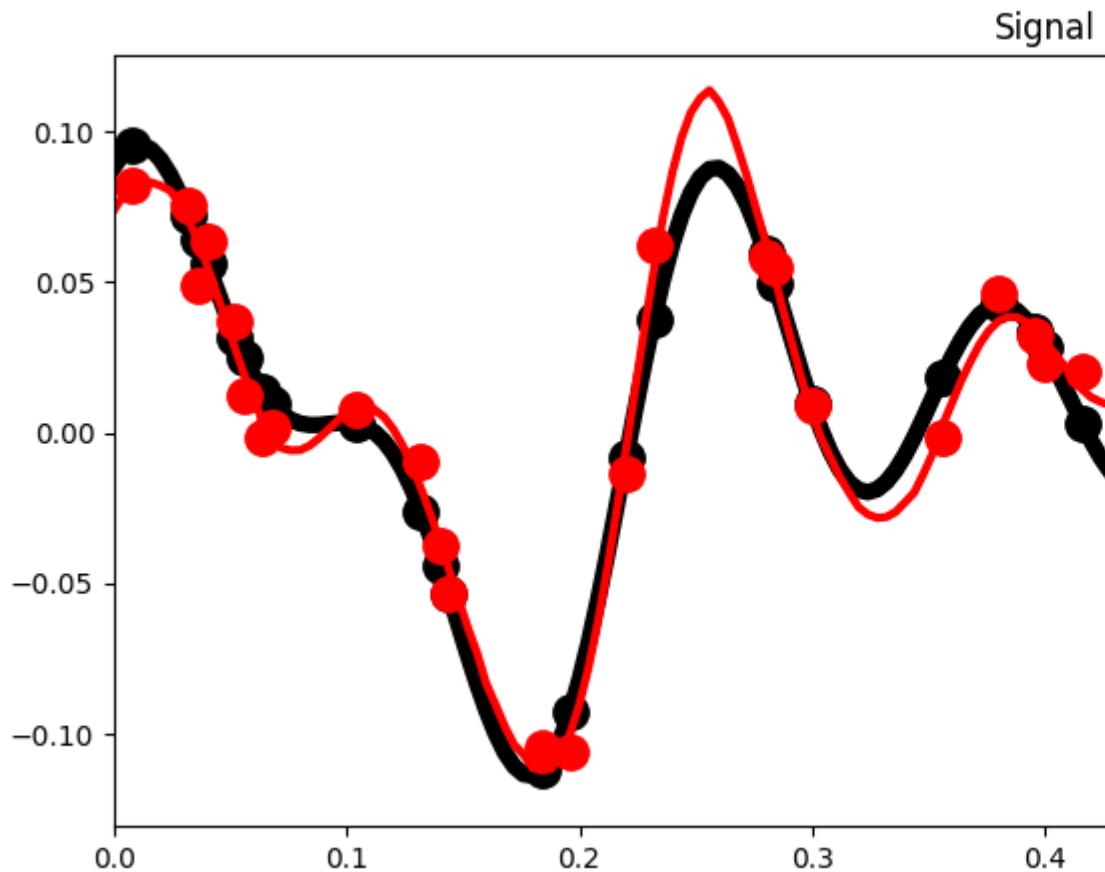
First we apply the Bayesian inversion equation

```

xbayes = x0 + Cm_op * Rop.H * (lsqr(Rop * Cm_op * Rop.H + Cd_op,
                                         yn - Rop*x0, iter_lim=400) [0])

# Visualize
fig, ax = plt.subplots(1, 1, figsize=(12, 5))
ax.plot(t, x, 'k', lw=6)
ax.plot(t, ymask, '.k', ms=25, label='available samples')
ax.plot(t, ynmask, '.r', ms=25, label='available noisy samples')
ax.plot(t, xbayes, 'r', lw=3, label='Bayesian inverse')
ax.legend()
ax.set_title('Signal')
ax.set_xlim(0, 0.8)

```



Out:

```
(0, 0.8)
```

So far we have been able to estimate our posterior mean. What about its uncertainties (i.e., posterior covariance)?

In real-life applications it is very difficult (if not impossible) to directly compute the posterior covariance matrix. It is much more useful to create a set of models that sample the posterior probability. We can do that by solving our problem several times using different prior realizations as starting guesses:

```

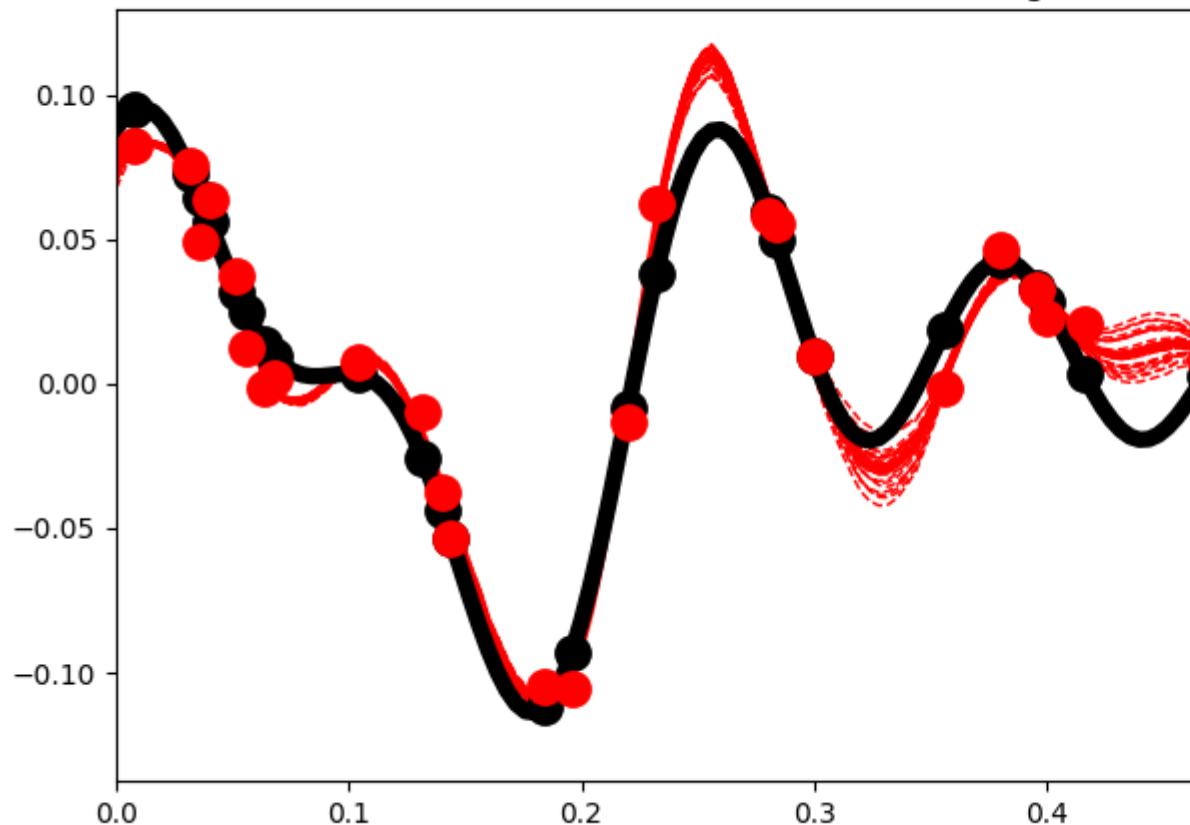
xpost = [x0 + Cm_op * Rop.H * (lsqr(Rop * Cm_op * Rop.H + Cd_op, yn - Rop * x0, iter_
    ↪lim=400)[0])
    ↪for x0 in xs[:30]]
xpost = np.array(xpost)

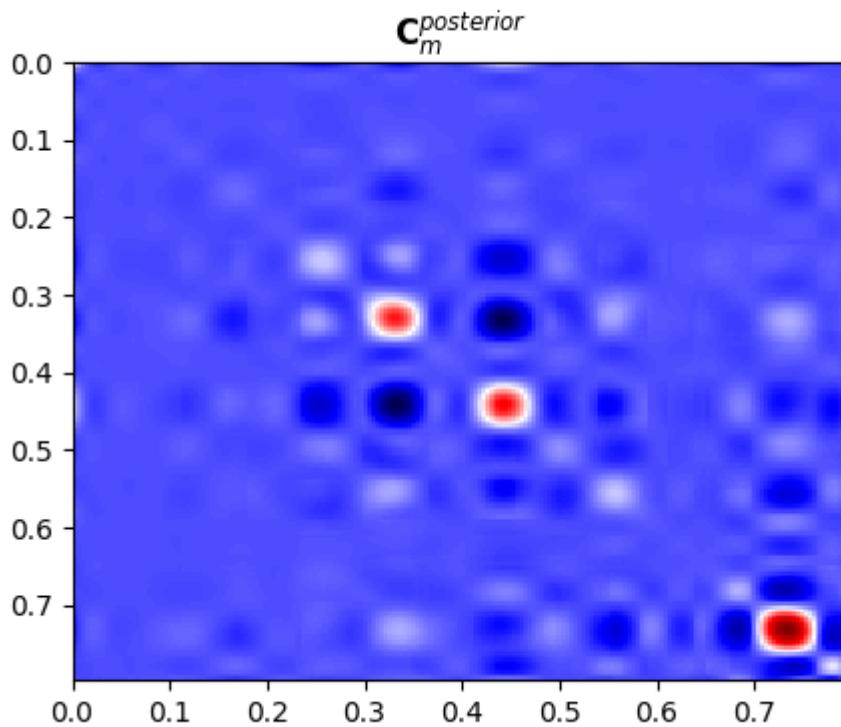
x0post = np.average(xpost, axis=0)
Cm_post = ((xpost - x0post).T @ (xpost - x0post)) / nreals

# Visualize
fig, ax = plt.subplots(1, 1, figsize=(12, 5))
ax.plot(t, xpost.T, '--r', lw=1)
ax.plot(t, x0post, 'r', lw=3, label='bayesian inverse')
ax.plot(t, x, 'k', lw=6)
ax.plot(t, ymask, '.k', ms=25, label='available samples')
ax.plot(t, ynmask, '.r', ms=25, label='available noisy samples')
ax.legend()
ax.set_title('Signal')
ax.set_xlim(0, 0.8)

fig, ax = plt.subplots(1, 1, figsize=(5, 4))
im = ax.imshow(Cm_post, interpolation='nearest', cmap='seismic',
    extent=(t[0], t[-1], t[-1], t[0]))
ax.set_title(r"\mathbf{C}_m^{posterior}")
ax.axis('tight')

```





Out:

```
(0.0, 0.796, 0.796, 0.0)
```

**Total running time of the script:** ( 0 minutes 1.592 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.5 05. Image deblurring

*Deblurring* is the process of removing blurring effects from images, caused for example by defocus aberration or motion blur.

In forward mode, such blurring effect is typically modelled as a 2-dimensional convolution between the so-called *point spread function* and a target sharp input image, where the sharp input image (which has to be recovered) is unknown and the point-spread function can be either known or unknown.

In this tutorial, an example of 2d blurring and deblurring will be shown using the `pylops.signalprocessing.Convolve2D` operator assuming knowledge of the point-spread function.

```
import numpy as np
import matplotlib.pyplot as plt
import pylops
```

Let's start by importing a 2d image and defining the blurring operator

```

im = np.load('../testdata/python.npy')[::5, ::5, 0]

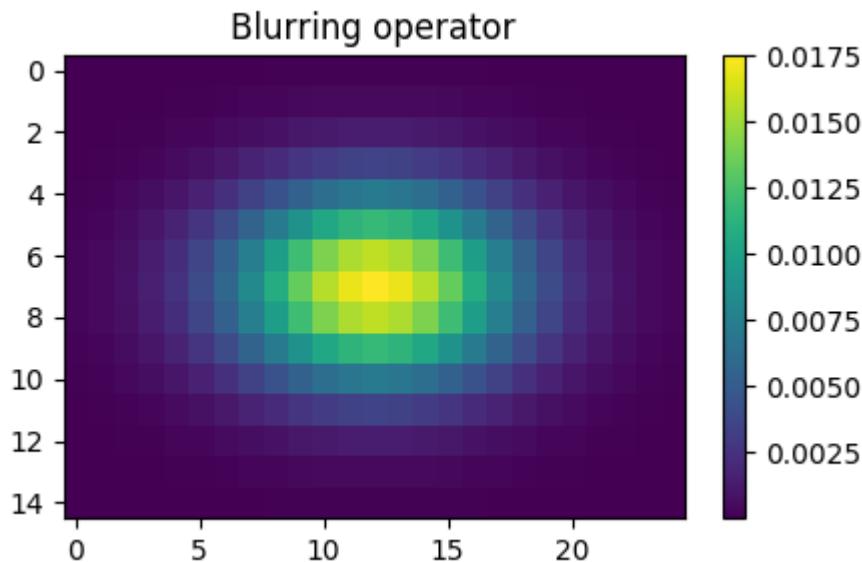
Nz, Nx = im.shape

# Blurring gaussian operator
nh = [15, 25]
hz = np.exp(-0.1*np.linspace(-(nh[0]//2), nh[0]//2, nh[0])**2)
hx = np.exp(-0.03*np.linspace(-(nh[1]//2), nh[1]//2, nh[1])**2)
hz /= np.trapz(hz) # normalize the integral to 1
hx /= np.trapz(hx) # normalize the integral to 1
h = hz[:, np.newaxis] * hx[np.newaxis, :]

fig, ax = plt.subplots(1, 1, figsize=(5, 3))
him = ax.imshow(h)
ax.set_title('Blurring operator')
fig.colorbar(him, ax=ax)
ax.axis('tight')

Cop = pylops.signalprocessing.Convolve2D(Nz * Nx, h=h,
                                         offset=(nh[0] // 2,
                                                  nh[1] // 2),
                                         dims=(Nz, Nx), dtype='float32')

```



We first apply the blurring operator to the sharp image. We then try to recover the sharp input image by inverting the convolution operator from the blurred image. Note that when we perform inversion without any regularization, the deblurred image will show some ringing due to the instabilities of the inverse process. Using a L1 solver with a DWT preconditioner or TV regularization allows to recover sharper contrasts.

```

imblur = Cop * im.flatten()

imdeblur = \
    pylops.optimization.leastsquares.NormalEquationsInversion(Cop, None,
                                                               imblur,
                                                               maxiter=50)

Wop = pylops.signalprocessing.DWT2D((Nz, Nx), wavelet='haar', level=3)

```

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```

Dop = [pylops.FirstDerivative(Nz * Nx, dims=(Nz, Nx), dir=0, edge=False),
       pylops.FirstDerivative(Nz * Nx, dims=(Nz, Nx), dir=1, edge=False)]
DWop = Dop + [Wop, ]

imdeblurfista = \
    pylops.optimization.sparsity.FISTA(Cop * Wop.H, imblur, eps=1e-1,
                                         niter=100)[0]
imdeblurfista = Wop.H * imdeblurfista

imdeblurtv = \
    pylops.optimization.sparsity.SplitBregman(Cop, Dop, imblur.flatten(),
                                                niter_outer=10, niter_inner=5,
                                                mu=1.5, epsRL1s=[2e0, 2e0],
                                                tol=1e-4, tau=1., show=False,
                                                ** dict(iter_lim=5, damp=1e-4))[0]

imdeblurtv1 = \
    pylops.optimization.sparsity.SplitBregman(Cop, DWop,
                                                imblur.flatten(),
                                                niter_outer=10, niter_inner=5,
                                                mu=1.5, epsRL1s=[1e0, 1e0, 1e0],
                                                tol=1e-4, tau=1., show=False,
                                                ** dict(iter_lim=5, damp=1e-4))[0]

# Reshape images
imblur = imblur.reshape((Nz, Nx))
imdeblur = imdeblur.reshape((Nz, Nx))
imdeblurfista = imdeblurfista.reshape((Nz, Nx))
imdeblurtv = imdeblurtv.reshape((Nz, Nx))
imdeblurtv1 = imdeblurtv1.reshape((Nz, Nx))

```

Finally we visualize the original, blurred, and recovered images.

```

# sphinx_gallery_thumbnail_number = 2
fig = plt.figure(figsize=(12, 6))
fig.suptitle('Deblurring', fontsize=14, fontweight='bold', y=0.95)
ax1 = plt.subplot2grid((2, 5), (0, 0))
ax2 = plt.subplot2grid((2, 5), (0, 1))
ax3 = plt.subplot2grid((2, 5), (0, 2))
ax4 = plt.subplot2grid((2, 5), (1, 0))
ax5 = plt.subplot2grid((2, 5), (1, 1))
ax6 = plt.subplot2grid((2, 5), (1, 2))
ax7 = plt.subplot2grid((2, 5), (0, 3), colspan=2)
ax8 = plt.subplot2grid((2, 5), (1, 3), colspan=2)
ax1.imshow(im, cmap='viridis', vmin=0, vmax=250)
ax1.axis('tight')
ax1.set_title('Original')
ax2.imshow(imblur, cmap='viridis', vmin=0, vmax=250)
ax2.axis('tight')
ax2.set_title('Blurred')
ax3.imshow(imdeblur, cmap='viridis', vmin=0, vmax=250)
ax3.axis('tight')
ax3.set_title('Deblurred')
ax4.imshow(imdeblurfista, cmap='viridis', vmin=0, vmax=250)
ax4.axis('tight')
ax4.set_title('FISTA deblurred')
ax5.imshow(imdeblurtv, cmap='viridis', vmin=0, vmax=250)

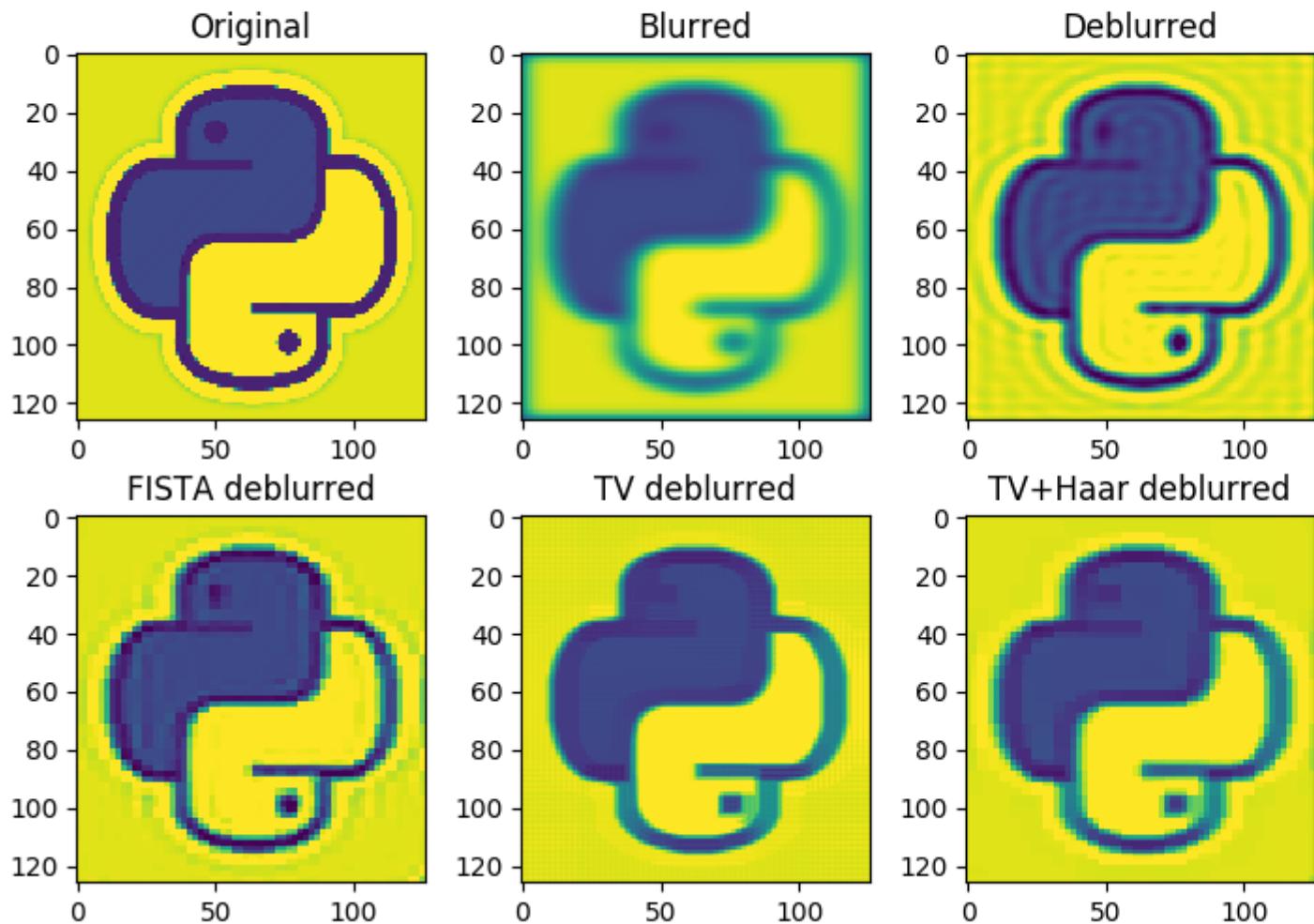
```

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```
ax5.axis('tight')
ax5.set_title('TV deblurred')
ax6.imshow(imdeblurtv1, cmap='viridis', vmin=0, vmax=250)
ax6.axis('tight')
ax6.set_title('TV+Haar deblurred')
ax7.plot(im[Nz//2], 'k')
ax7.plot(imblur[Nz//2], '--r')
ax7.plot(imdeblur[Nz//2], '--b')
ax7.plot(imdeblurfista[Nz//2], '--g')
ax7.plot(imdeblurtv[Nz//2], '--m')
ax7.plot(imdeblurtv1[Nz//2], '--y')
ax7.axis('tight')
ax7.set_title('Horizontal section')
ax8.plot(im[:, Nx//2], 'k', label='Original')
ax8.plot(imblur[:, Nx//2], '--r', label='Blurred')
ax8.plot(imdeblur[:, Nx//2], '--b', label='Deblurred')
ax8.plot(imdeblurfista[:, Nx//2], '--g', label='FISTA deblurred')
ax8.plot(imdeblurtv[:, Nx//2], '--m', label='TV deblurred')
ax8.plot(imdeblurtv1[:, Nx//2], '--y', label='TV+Haar deblurred')
ax8.axis('tight')
ax8.set_title('Vertical section')
ax8.legend(loc=5, fontsize='small')
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

## Deblurring



**Total running time of the script:** ( 0 minutes 5.064 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.6 06. 2D Interpolation

In the mathematical field of numerical analysis, interpolation is the problem of constructing new data points within the range of a discrete set of known data points. In signal and image processing, the data may be recorded at irregular locations and it is often required to *regularize* the data into a regular grid.

In this tutorial, an example of 2d interpolation of an image is carried out using a combination of PyLops operators (`pylops.Restriction` and `pylops.Laplacian`) and the `pylops.optimization` module.

Mathematically speaking, if we want to interpolate a signal using the theory of inverse problems, we can define the

following forward problem:

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where the restriction operator  $\mathbf{R}$  selects  $M$  elements from the regularly sampled signal  $\mathbf{x}$  at random locations. The input and output signals are:

$$\mathbf{y} = [y_1, y_2, \dots, y_N]^T, \quad \mathbf{x} = [x_1, x_2, \dots, x_M]^T,$$

with  $M >> N$ .

```
import numpy as np
import matplotlib.pyplot as plt
import pyllops

plt.close('all')
np.random.seed(0)
```

To start we import a 2d image and define our restriction operator to irregularly and randomly sample the image for 30% of the entire grid

```
im = np.load('../testdata/python.npy')[:, :, 0]

Nz, Nx = im.shape
N = Nz * Nx

# Subsample signal
perc_subsampling = 0.2

Nsub2d = int(np.round(N*perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(N))[:Nsub2d])

# Create operators and data
Rop = pyllops.Restriction(N, iava, dtype='float64')
D2op = pyllops.Laplacian((Nz, Nx), weights=(1, 1), dtype='float64')

x = im.flatten()
y = Rop * x
y1 = Rop.mask(x)
```

We will now use two different routines from our optimization toolbox to estimate our original image in the regular grid.

```
xcg_reg_lop = \
    pyllops.optimization.leastsquares.NormalEquationsInversion(Rop, [D2op], y,
                                                               epsRs=[np.sqrt(0.1)], 
                                                               returninfo=False,
                                                               **dict(maxiter=200))

# Invert for interpolated signal, lsqr
xlsqr_reg_lop, istop, itn, r1norm, r2norm = \
    pyllops.optimization.leastsquares.RegularizedInversion(Rop, [D2op], y,
                                                             epsRs=[np.sqrt(0.1)], 
                                                             returninfo=True,
                                                             **dict(damp=0,
                                                                    iter_lim=200,
                                                                    show=0))
```

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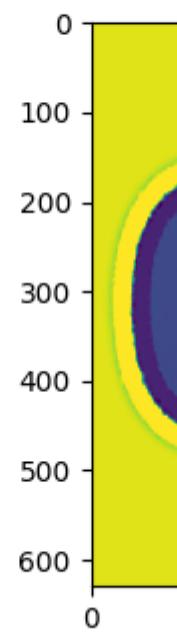
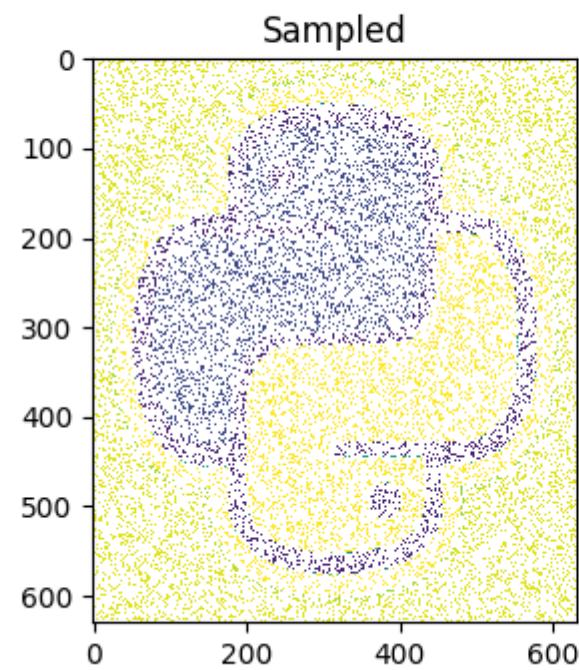
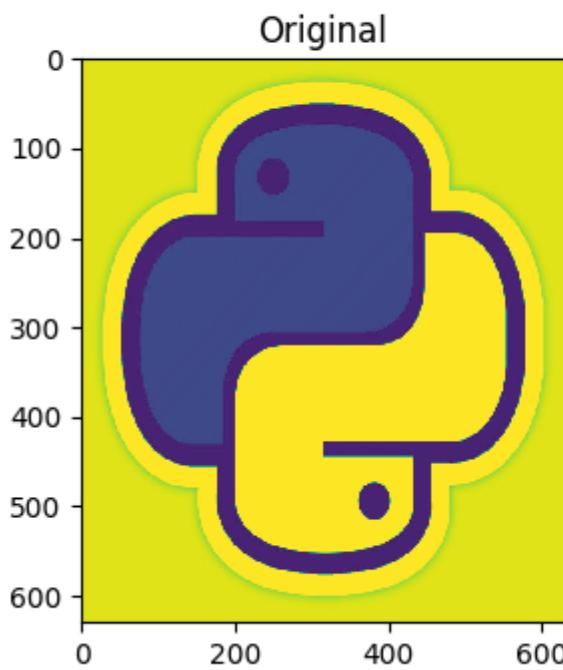
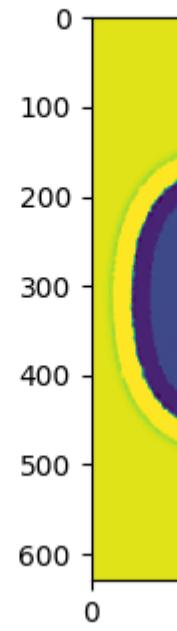
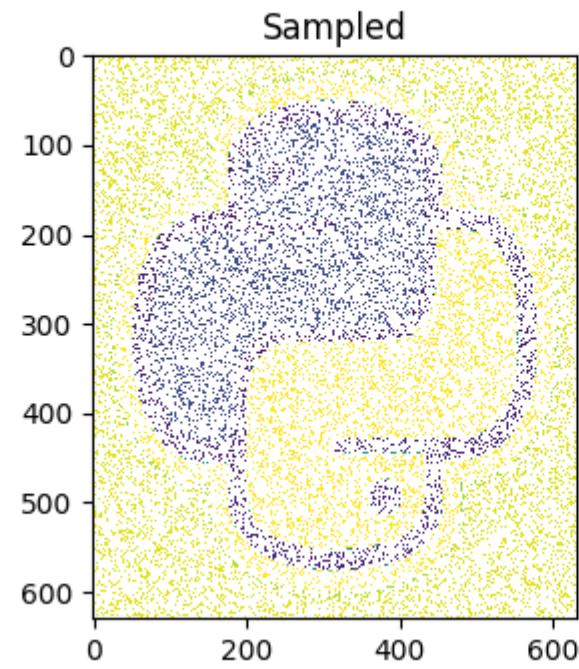
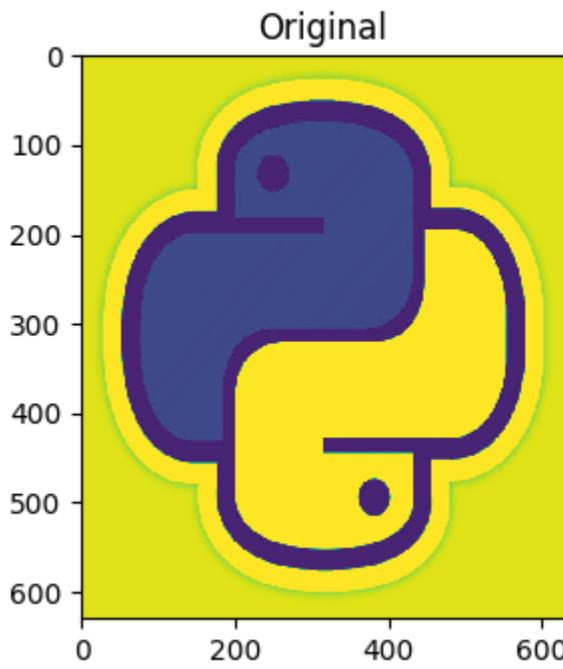
(continued from previous page)

```
# Reshape estimated images
im_sampled = y1.reshape((Nz, Nx))
im_rec_lap_cg = xcg_reg_lop.reshape((Nz, Nx))
im_rec_lap_lsqr = xlsqr_reg_lop.reshape((Nz, Nx))
```

Finally we visualize the original image, the reconstructed images and their error

```
fig, axs = plt.subplots(1, 4, figsize=(12, 4))
fig.suptitle('Data reconstruction - normal eqs', fontsize=14,
             fontweight='bold', y=0.95)
axs[0].imshow(im, cmap='viridis', vmin=0, vmax=250)
axs[0].axis('tight')
axs[0].set_title('Original')
axs[1].imshow(im_sampled, cmap='viridis', vmin=0, vmax=250)
axs[1].axis('tight')
axs[1].set_title('Sampled')
axs[2].imshow(im_rec_lap_cg, cmap='viridis', vmin=0, vmax=250)
axs[2].axis('tight')
axs[2].set_title('2D Regularization')
axs[3].imshow(im - im_rec_lap_cg, cmap='gray', vmin=-80, vmax=80)
axs[3].axis('tight')
axs[3].set_title('2D Regularization Error')
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, axs = plt.subplots(1, 4, figsize=(12, 4))
fig.suptitle('Data reconstruction - regularized eqs', fontsize=14,
             fontweight='bold', y=0.95)
axs[0].imshow(im, cmap='viridis', vmin=0, vmax=250)
axs[0].axis('tight')
axs[0].set_title('Original')
axs[1].imshow(im_sampled, cmap='viridis', vmin=0, vmax=250)
axs[1].axis('tight')
axs[1].set_title('Sampled')
axs[2].imshow(im_rec_lap_lsqr, cmap='viridis', vmin=0, vmax=250)
axs[2].axis('tight')
axs[2].set_title('2D Regularization')
axs[3].imshow(im - im_rec_lap_lsqr, cmap='gray', vmin=-80, vmax=80)
axs[3].axis('tight')
axs[3].set_title('2D Regularization Error')
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

**Data reconstruction - no****Data reconstruction - regul**

Total running time of the script: ( 0 minutes 9.490 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.7 07. Post-stack inversion

Estimating subsurface properties from band-limited seismic data represents an important task for geophysical subsurface characterization.

In this tutorial, the `pylops.avo.poststack.PoststackLinearModelling` operator is used for modelling of both 1d and 2d synthetic post-stack seismic data from a profile or 2d model of the subsurface acoustic impedance.

$$d(t, \theta = 0) = \frac{1}{2}w(t) * \frac{d \ln(AI(t))}{dt}$$

where  $AI(t)$  is the acoustic impedance profile and  $w(t)$  is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{WDai}$$

where  $\mathbf{W}$  is a convolution operator,  $\mathbf{D}$  is a first derivative operator, and  $\mathbf{ai}$  is the input model. Subsequently the acoustic impedance model is estimated via the `pylops.avo.poststack.PoststackInversion` module. A two-steps inversion strategy is finally presented to deal with the case of noisy data.

```
# sphinx_gallery_thumbnail_number = 4
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close('all')
np.random.seed(10)
```

Let's start with a 1d example. A synthetic profile of acoustic impedance is created and data is modelled using both the dense and linear operator version of `pylops.avo.poststack.PoststackLinearModelling` operator.

```
# model
nt0 = 301
dt0 = 0.004
t0 = np.arange(nt0)*dt0
vp = 1200 + np.arange(nt0) +
    filtfilt(np.ones(5)/5., 1, np.random.normal(0, 80, nt0))
rho = 1000 + vp +
    filtfilt(np.ones(5)/5., 1, np.random.normal(0, 30, nt0))
vp[131:] += 500
rho[131:] += 100
m = np.log(vp*rho)

# smooth model
nsmooth = 100
mback = filtfilt(np.ones(nsmooth)/float(nsmooth), 1, m)

# wavelet
ntwav = 41
wav, twav, wavc = ricker(t0[:ntwav//2+1], 20)
```

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```
# dense operator
PPop_dense = \
    pylops.avo.poststack.PoststackLinearModelling(wav / 2, nt0=nt0, explicit=True)

# lop operator
PPop = pylops.avo.poststack.PoststackLinearModelling(wav / 2, nt0=nt0)

# data
d_dense = PPop_dense*m.flatten()
d = PPop*m

# add noise
dn_dense = d_dense + np.random.normal(0, 2e-2, d_dense.shape)
```

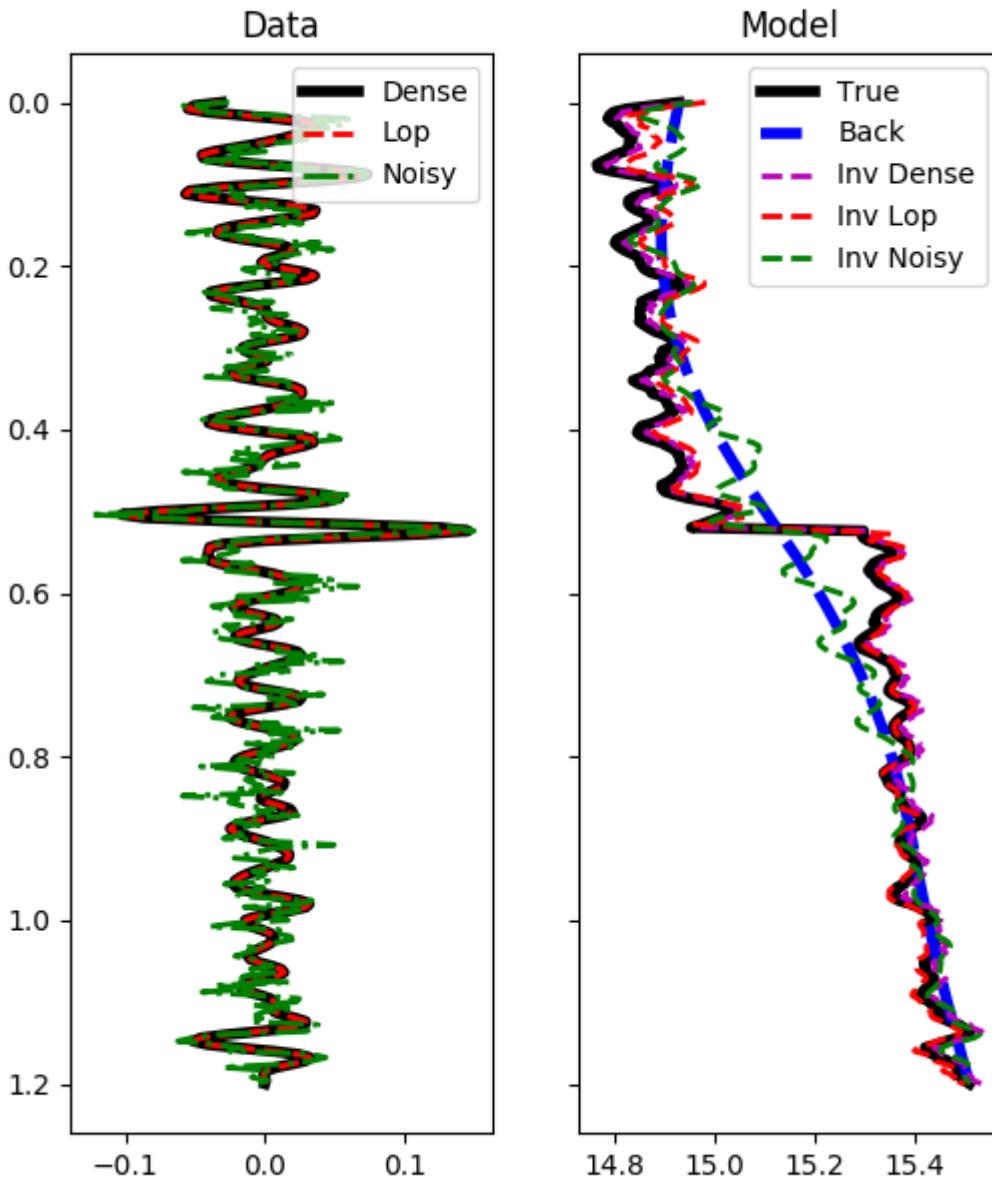
We can now estimate the acoustic profile from band-limited data using either the dense operator or linear operator.

```
# solve dense
minv_dense = \
    pylops.avo.poststack.PoststackInversion(d, wav / 2, m0=mback, explicit=True,
                                             simultaneous=False) [0]

# solve lop
minv = \
    pylops.avo.poststack.PoststackInversion(d_dense, wav / 2, m0=mback,
                                             explicit=False,
                                             simultaneous=False,
                                             **dict(iter_lim=2000)) [0]

# solve noisy
mn = \
    pylops.avo.poststack.PoststackInversion(dn_dense, wav / 2, m0=mback,
                                             explicit=True,
                                             epsR=1e0, **dict(damp=1e-1)) [0]

fig, axs = plt.subplots(1, 2, figsize=(6, 7), sharey=True)
axs[0].plot(d_dense, t0, 'k', lw=4, label='Dense')
axs[0].plot(d, t0, '--r', lw=2, label='Lop')
axs[0].plot(dn_dense, t0, '-.g', lw=2, label='Noisy')
axs[0].set_title('Data')
axs[0].invert_yaxis()
axs[0].axis('tight')
axs[0].legend(loc=1)
axs[1].plot(m, t0, 'k', lw=4, label='True')
axs[1].plot(mback, t0, '--b', lw=4, label='Back')
axs[1].plot(minv_dense, t0, '--m', lw=2, label='Inv Dense')
axs[1].plot(minv, t0, '--r', lw=2, label='Inv Lop')
axs[1].plot(mn, t0, '--g', lw=2, label='Inv Noisy')
axs[1].set_title('Model')
axs[1].axis('tight')
axs[1].legend(loc=1)
```



Out:

```
<matplotlib.legend.Legend object at 0x7fcd6e8ba6a0>
```

We see how inverting a dense matrix is in this case faster than solving for the linear operator (a good estimate of the model is in fact obtained only after 2000 iterations of lsqr). Nevertheless, having a linear operator is useful when we deal with larger dimensions (2d or 3d) and we want to couple our modelling operator with different types of spatial regularizations or preconditioning.

Before we move onto a 2d example, let's consider the case of non-stationary wavelet and see how we can easily use

the same routines in this case

```
# wavelet
ntwav = 41
f0s = np.flip(np.arange(nt0) * 0.05 + 3)
wavs = np.array([ricker(t0[:ntwav], f0)[0] for f0 in f0s])
wavc = np.argmax(wavs[0])

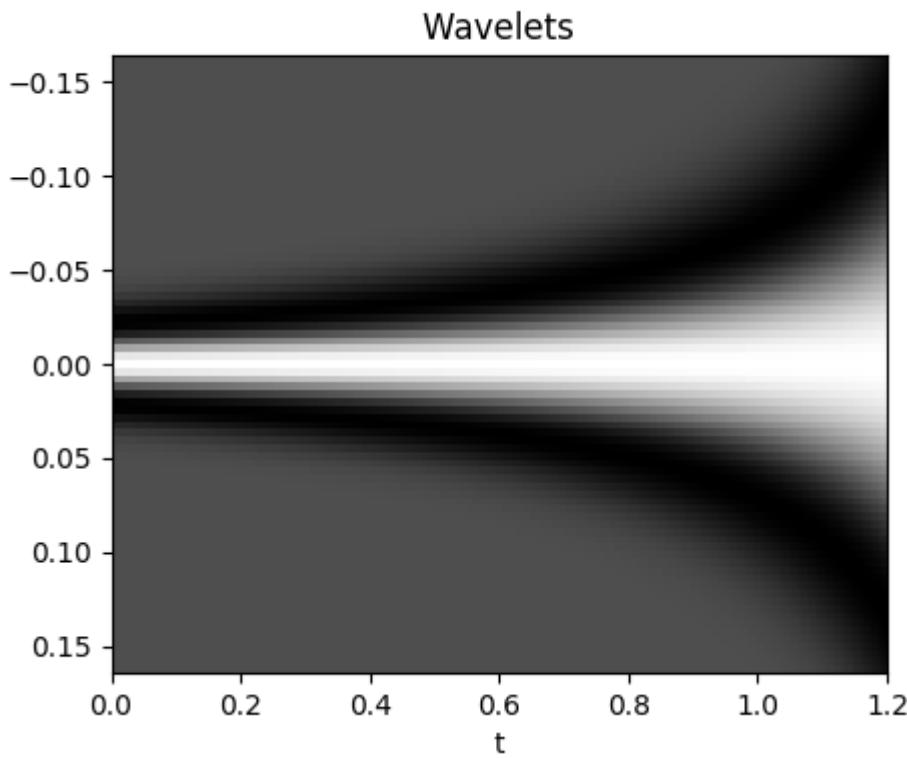
plt.figure(figsize=(5, 4))
plt.imshow(wavs.T, cmap='gray', extent=(t0[0], t0[-1], t0[ntwav], -t0[ntwav]))
plt.xlabel('t')
plt.title('Wavelets')
plt.axis('tight')

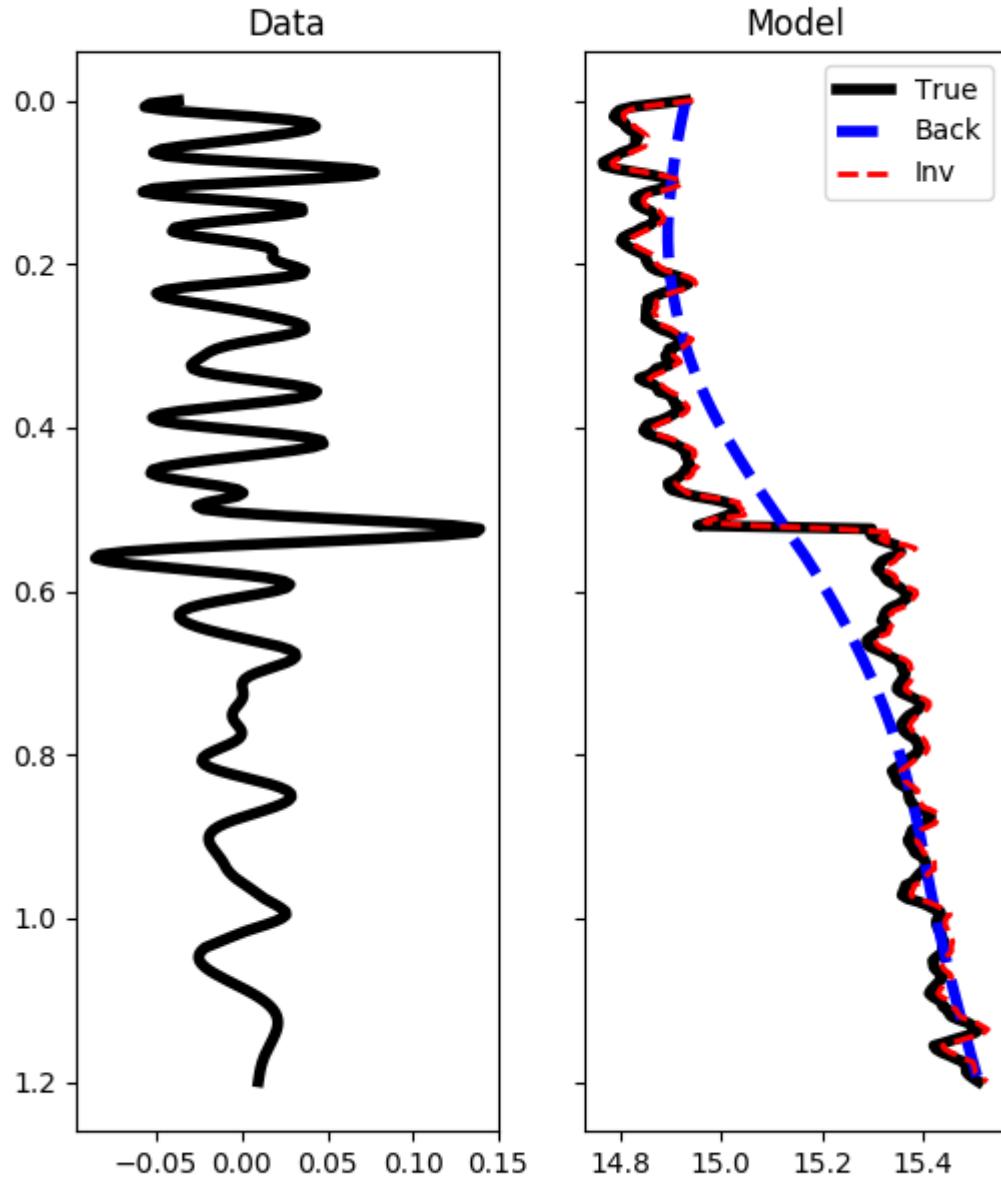
# operator
PPop = \
    pylops.avo.poststack.PoststackLinearModelling(wavs / 2, nt0=nt0, explicit=True)

# data
d = PPop * m

# solve
minv = \
    pylops.avo.poststack.PoststackInversion(d, wavs / 2, m0=mback, explicit=True,
                                              **dict(cond=1e-10))[0]

fig, axs = plt.subplots(1, 2, figsize=(6, 7), sharey=True)
axs[0].plot(d, t0, 'k', lw=4)
axs[0].set_title('Data')
axs[0].invert_yaxis()
axs[0].axis('tight')
axs[1].plot(m, t0, 'k', lw=4, label='True')
axs[1].plot(mback, t0, '--b', lw=4, label='Back')
axs[1].plot(minv, t0, '--r', lw=2, label='Inv')
axs[1].set_title('Model')
axs[1].axis('tight')
axs[1].legend(loc=1)
```





Out:

```
<matplotlib.legend.Legend object at 0x7fcd8b759320>
```

We move now to a 2d example. First of all the model is loaded and data generated.

```
# model
inputfile = '../testdata/avo/poststack_model.npz'
```

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```

model = np.load(inputfile)
m = np.log(model['model'][:, ::3])
x, z = model['x'][:, ::3]/1000., model['z']/1000.
nx, nz = len(x), len(z)

# smooth model
nsmoothz, nsmoothx = 60, 50
mback = filtfilt(np.ones(nsmoothz)/float(nsmoothz), 1, m, axis=0)
mback = filtfilt(np.ones(nsmoothx)/float(nsmoothx), 1, mback, axis=1)

# dense operator
PPop_dense = \
    pylops.avo.poststack.PoststackLinearModelling(wav / 2, nt0=nz,
                                                    spatdims=nx, explicit=True)

# lop operator
PPop = pylops.avo.poststack.PoststackLinearModelling(wav / 2, nt0=nz,
                                                       spatdims=nx)

# data
d = (PPop_dense * m.flatten()).reshape(nz, nx)
n = np.random.normal(0, 1e-1, d.shape)
dn = d + n

```

Finally we perform 4 different inversions:

- trace-by-trace inversion with explicit solver and dense operator with noise-free data
- trace-by-trace inversion with explicit solver and dense operator with noisy data
- multi-trace regularized inversion with iterative solver and linear operator using the result of trace-by-trace inversion as starting guess

$$J = \|\Delta\mathbf{d} - \mathbf{W}\Delta\mathbf{ai}\|_2 + \epsilon_{\nabla}^2 \|\nabla\mathbf{ai}\|_2$$

where  $\Delta\mathbf{d} = \mathbf{d} - \mathbf{W}\mathbf{A}\mathbf{I}_0$  is the residual data

- multi-trace blocky inversion with iterative solver and linear operator

```

# dense inversion with noise-free data
minv_dense = \
    pylops.avo.poststack.PoststackInversion(d, wav / 2, m0=mback,
                                             explicit=True,
                                             simultaneous=False) [0]

# dense inversion with noisy data
minv_dense_noisy = \
    pylops.avo.poststack.PoststackInversion(dn, wav / 2, m0=mback,
                                             explicit=True, epsI=4e-2,
                                             simultaneous=False) [0]

# spatially regularized lop inversion with noisy data
minv_lop_reg = \
    pylops.avo.poststack.PoststackInversion(dn, wav / 2, m0=minv_dense_noisy,
                                             explicit=False, epsR=5e1,
                                             **dict(damp=np.sqrt(1e-4),
                                                    iter_lim=80)) [0]

```

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```
# blockiness promoting inversion with noisy data
minv_lop_blocky = \
    pylops.avo.poststack.PoststackInversion(dn, wav / 2, m0=mback,
                                              explicit=False,
                                              epsR=[0.4], epsRL1=[0.1],
                                              **dict(mu=0.1,
                                                     niter_outer=5,
                                                     niter_inner=10,
                                                     iter_lim=5, damp=1e-3))[0]

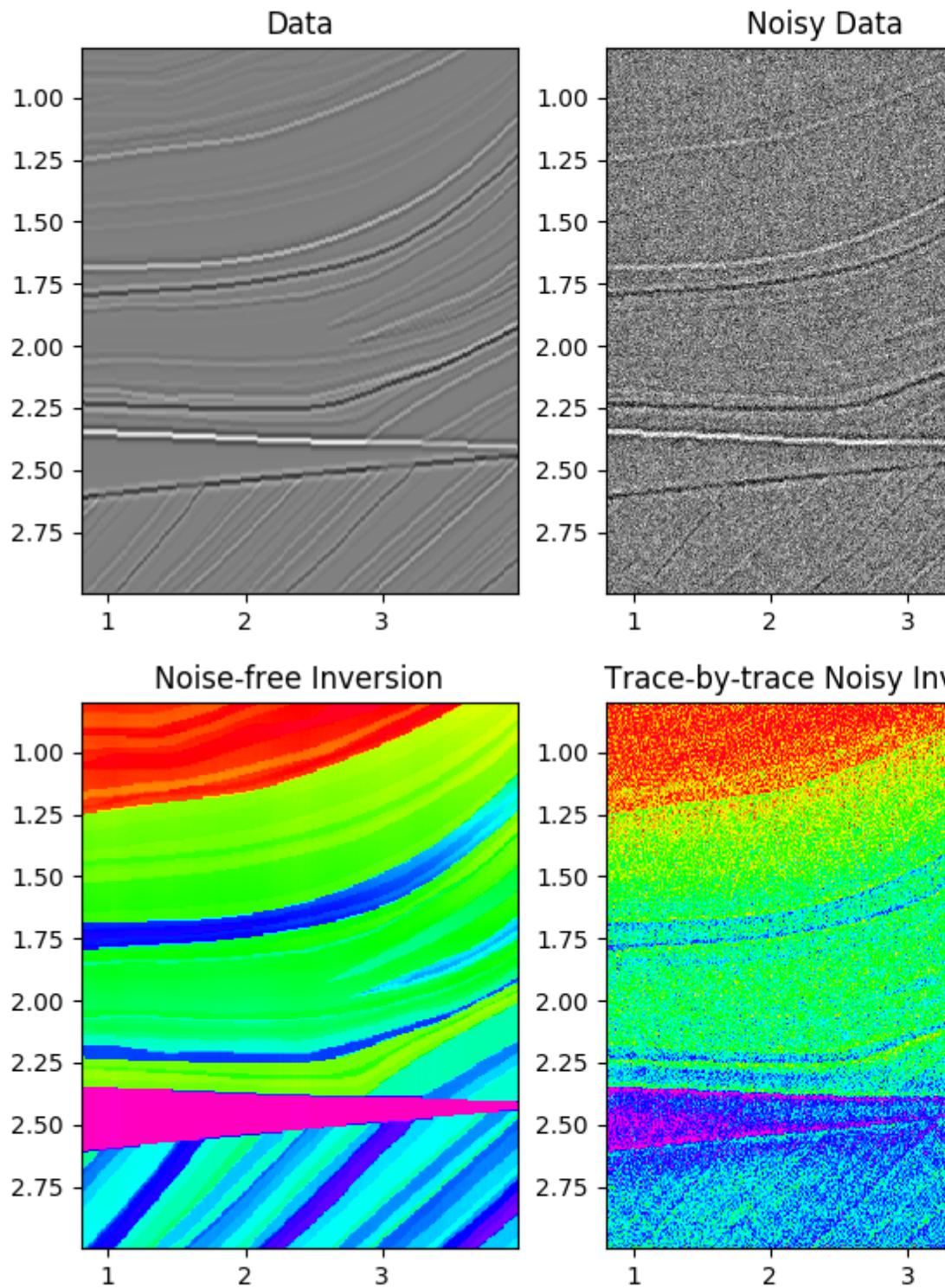
fig, axs = plt.subplots(2, 4, figsize=(15, 9))
axs[0][0].imshow(d, cmap='gray',
                  extent=(x[0], x[-1], z[-1], z[0]),
                  vmin=-0.4, vmax=0.4)
axs[0][0].set_title('Data')
axs[0][0].axis('tight')
axs[0][1].imshow(dn, cmap='gray',
                  extent=(x[0], x[-1], z[-1], z[0]),
                  vmin=-0.4, vmax=0.4)
axs[0][1].set_title('Noisy Data')
axs[0][1].axis('tight')
axs[0][2].imshow(m, cmap='gist_rainbow',
                  extent=(x[0], x[-1], z[-1], z[0]),
                  vmin=m.min(), vmax=m.max())
axs[0][2].set_title('Model')
axs[0][2].axis('tight')
axs[0][3].imshow(mback, cmap='gist_rainbow',
                  extent=(x[0], x[-1], z[-1], z[0]),
                  vmin=m.min(), vmax=m.max())
axs[0][3].set_title('Smooth Model')
axs[0][3].axis('tight')
axs[1][0].imshow(minv_dense, cmap='gist_rainbow',
                  extent=(x[0], x[-1], z[-1], z[0]),
                  vmin=m.min(), vmax=m.max())
axs[1][0].set_title('Noise-free Inversion')
axs[1][0].axis('tight')
axs[1][1].imshow(minv_dense_noisy, cmap='gist_rainbow',
                  extent=(x[0], x[-1], z[-1], z[0]),
                  vmin=m.min(), vmax=m.max())
axs[1][1].set_title('Trace-by-trace Noisy Inversion')
axs[1][1].axis('tight')
axs[1][2].imshow(minv_lop_reg, cmap='gist_rainbow',
                  extent=(x[0], x[-1], z[-1], z[0]),
                  vmin=m.min(), vmax=m.max())
axs[1][2].set_title('Regularized Noisy Inversion - lop ')
axs[1][2].axis('tight')
axs[1][3].imshow(minv_lop_blocky, cmap='gist_rainbow',
                  extent=(x[0], x[-1], z[-1], z[0]),
                  vmin=m.min(), vmax=m.max())
axs[1][3].set_title('Blocky Noisy Inversion - lop ')
axs[1][3].axis('tight')

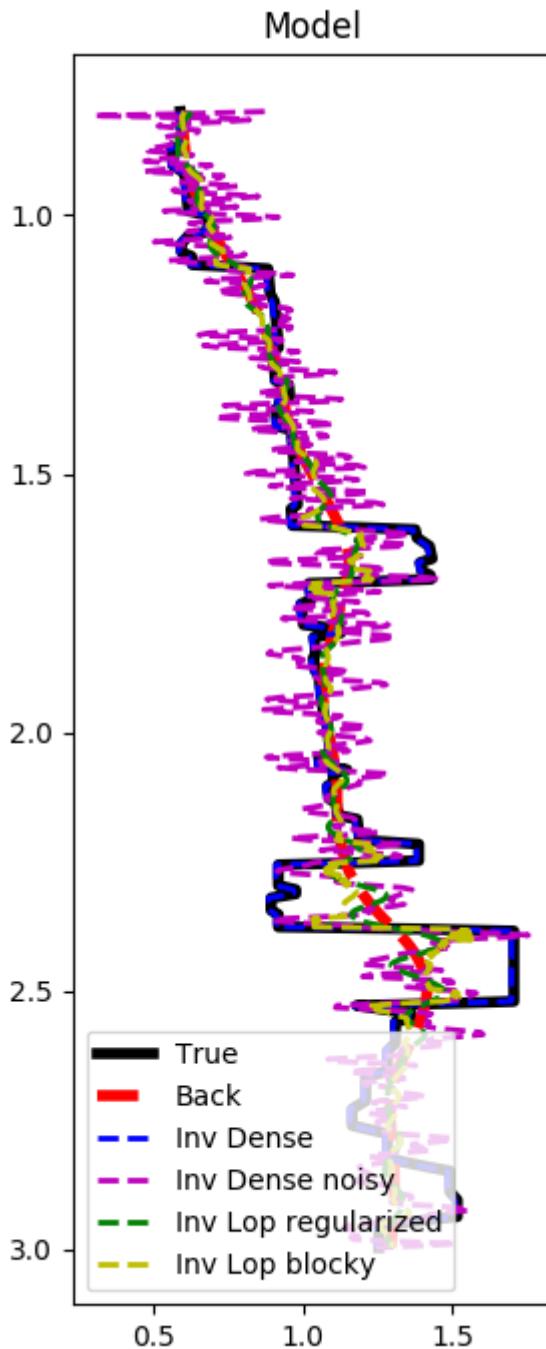
fig, ax = plt.subplots(1, 1, figsize=(3, 7))
ax.plot(m[:, nx//2], z, 'k', lw=4, label='True')
ax.plot(mback[:, nx//2], z, '--r', lw=4, label='Back')
ax.plot(minv_dense[:, nx//2], z, '--b', lw=2, label='Inv Dense')
ax.plot(minv_dense_noisy[:, nx//2], z, '--m', lw=2, label='Inv Dense noisy')
```

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```
ax.plot(minv_lop_reg[:, nx//2], z, '--g', lw=2, label='Inv Lop regularized')
ax.plot(minv_lop_blocky[:, nx//2], z, '--y', lw=2, label='Inv Lop blocky')
ax.set_title('Model')
ax.invert_yaxis()
ax.axis('tight')
ax.legend()
plt.tight_layout()
```





That's almost it. If you wonder how this can be applied to real data, head over to the following [notebook](#) where the open-source `segyio` library is used alongside pylops to create an end-to-end open-source seismic inversion workflow with SEG-Y input data.

**Total running time of the script:** ( 0 minutes 10.140 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.8 08. Pre-stack (AVO) inversion

Pre-stack inversion represents one step beyond post-stack inversion in that not only the profile of acoustic impedance can be inferred from seismic data, rather a set of elastic parameters is estimated from pre-stack data (i.e., angle gathers) using the information contained in the so-called AVO (amplitude versus offset) response. Such elastic parameters represent vital information for more sophisticated geophysical subsurface characterization than it would be possible to achieve working with post-stack seismic data.

In this tutorial, the `pylops.avo.prestack.PrestackLinearModelling` operator is used for modelling of both 1d and 2d synthetic pre-stack seismic data using 1d profiles or 2d models of different subsurface elastic parameters (P-wave velocity, S-wave velocity, and density) as input.

$$d(t, \theta) = w(t) * \sum_{i=1}^N G_i(t, \theta) \frac{d \ln(m_i(t))}{dt}$$

where  $\mathbf{m}(t) = [V_P(t), V_S(t), \rho(t)]$  is a vector containing three elastic parameters at time  $t$ ,  $G_i(t, \theta)$  are the coefficients of the AVO parametrization used to model pre-stack data and  $w(t)$  is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{WG}\mathbf{D}\mathbf{m}$$

where  $\mathbf{W}$  is a convolution operator,  $\mathbf{G}$  is the AVO modelling operator,  $\mathbf{D}$  is a block-diagonal derivative operator, and  $\mathbf{m}$  is the input model. Subsequently the elastic parameters are estimated via the `pylops.avo.prestack.PrestackInversion` module. Once again, a two-steps inversion strategy can also be used to deal with the case of noisy data.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close('all')
np.random.seed(0)
```

Let's start with a 1d example. A synthetic profile of acoustic impedance is created and data is modelled using both the dense and linear operator version of `pylops.avo.prestack.PrestackLinearModelling` operator

```
# sphinx_gallery_thumbnail_number = 5

# model
nt0 = 301
dt0 = 0.004

t0 = np.arange(nt0)*dt0
vp = 1200 + np.arange(nt0) + \
    filtfilt(np.ones(5)/5., 1, np.random.normal(0, 80, nt0))
vs = 600 + vp/2 + \
    filtfilt(np.ones(5)/5., 1, np.random.normal(0, 20, nt0))
rho = 1000 + vp + \
    filtfilt(np.ones(5)/5., 1, np.random.normal(0, 30, nt0))
vp[131:] += 500
vs[131:] += 200
rho[131:] += 100
vsvp = 0.5
```

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```

m = np.stack((np.log(vp), np.log(vs), np.log(rho)), axis=1)

# background model
nsmooth = 50
mback = filtfilt(np.ones(nsmooth)/float(nsmooth), 1, m, axis=0)

# angles
ntheta = 21
thetamin, thetamax = 0, 40
theta = np.linspace(thetamin, thetamax, ntheta)

# wavelet
ntwav = 41
wav = ricker(t0[:ntwav//2+1], 15)[0]

# lop
PPop = \
    pylops.avo.prestack.PrestackLinearModelling(wav, theta,
                                                   vsvp=vsvp, nt0=nt0,
                                                   linearization='akirich')

# dense
PPop_dense = \
    pylops.avo.prestack.PrestackLinearModelling(wav, theta,
                                                   vsvp=vsvp, nt0=nt0,
                                                   linearization='akirich',
                                                   explicit=True)

# data lop
dPP = PPop*m.flatten()
dPP = dPP.reshape(nt0, ntheta)

# data dense
dPP_dense = PPop_dense*m.T.flatten()
dPP_dense = dPP_dense.reshape(ntheta, nt0).T

# noisy data
dPPn_dense = dPP_dense + np.random.normal(0, 1e-2, dPP_dense.shape)

```

We can now invert our data and retrieve elastic profiles for both noise-free and noisy data using `pylops.avo.prestack.PrestackInversion`.

```

# dense
minv_dense, dPP_dense_res = \
    pylops.avo.prestack.PrestackInversion(dPP_dense, theta, wav, m0=mback,
                                           linearization='akirich',
                                           explicit=True, returnres=True,
                                           **dict(cond=1e-10))

# lop
minv, dPP_res = \
    pylops.avo.prestack.PrestackInversion(dPP, theta, wav, m0=mback,
                                           linearization='akirich',
                                           explicit=False, returnres=True,
                                           **dict(damp=1e-10, iter_lim=2000))

```

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```
# dense noisy
minv_dense_noise, dPPn_dense_res = \
    pylops.avo.prestack.PrestackInversion(dPPn_dense, theta, wav, m0=mback,
                                           linearization='akirich',
                                           explicit=True,
                                           returnres=True, **dict(cond=1e-1))

# lop noisy (with vertical smoothing)
minv_noise, dPPn_res = \
    pylops.avo.prestack.PrestackInversion(dPPn_dense, theta, wav, m0=mback,
                                           linearization='akirich',
                                           explicit=False,
                                           epsR=5e-1, returnres=True,
                                           **dict(damp=1e-1, iter_lim=100))
```

The data, inverted models and residuals are now displayed.

```
# Data and model
fig, (axd, axdn, axvp, axvs, axrho) = \
    plt.subplots(1, 5, figsize=(8, 5), sharey=True)
axd.imshow(dPP_dense, cmap='gray',
           extent=(theta[0], theta[-1], t0[-1], t0[0]),
           vmin=-np.abs(dPP_dense).max(), vmax=np.abs(dPP_dense).max())
axd.set_title('Data')
axd.axis('tight')
axdn.imshow(dPPn_dense, cmap='gray',
            extent=(theta[0], theta[-1], t0[-1], t0[0]),
            vmin=-np.abs(dPPn_dense).max(), vmax=np.abs(dPPn_dense).max())
axdn.set_title('Noisy Data')
axdn.axis('tight')
axvp.plot(vp, t0, 'k', lw=4, label='True')
axvp.plot(np.exp(mback[:, 0]), t0, '--r', lw=4, label='Back')
axvp.plot(np.exp(minv_dense[:, 0]), t0, '--b', lw=2, label='Inv Dense')
axvp.plot(np.exp(minv[:, 0]), t0, '--m', lw=2, label='Inv Lop')
axvp.plot(np.exp(minv_dense_noise[:, 0]), t0, '--c', lw=2, label='Noisy Dense')
axvp.plot(np.exp(minv_noise[:, 0]), t0, '--g', lw=2, label='Noisy Lop')
axvp.set_title(r'$V_P$')
axvs.plot(vs, t0, 'k', lw=4, label='True')
axvs.plot(np.exp(mback[:, 1]), t0, '--r', lw=4, label='Back')
axvs.plot(np.exp(minv_dense[:, 1]), t0, '--b', lw=2, label='Inv Dense')
axvs.plot(np.exp(minv[:, 1]), t0, '--m', lw=2, label='Inv Lop')
axvs.plot(np.exp(minv_dense_noise[:, 1]), t0, '--c', lw=2, label='Noisy Dense')
axvs.plot(np.exp(minv_noise[:, 1]), t0, '--g', lw=2, label='Noisy Lop')
axvs.set_title(r'$V_S$')
axrho.plot(rho, t0, 'k', lw=4, label='True')
axrho.plot(np.exp(mback[:, 2]), t0, '--r', lw=4, label='Back')
axrho.plot(np.exp(minv_dense[:, 2]), t0, '--b', lw=2, label='Inv Dense')
axrho.plot(np.exp(minv[:, 2]), t0, '--m', lw=2, label='Inv Lop')
axrho.plot(np.exp(minv_dense_noise[:, 2]),
           t0, '--c', lw=2, label='Noisy Dense')
axrho.plot(np.exp(minv_noise[:, 2]), t0, '--g', lw=2, label='Noisy Lop')
axrho.set_title(r'$\rho$')
axrho.legend(loc='center left', bbox_to_anchor=(1, 0.5))
axd.axis('tight')
plt.tight_layout()

# Residuals
```

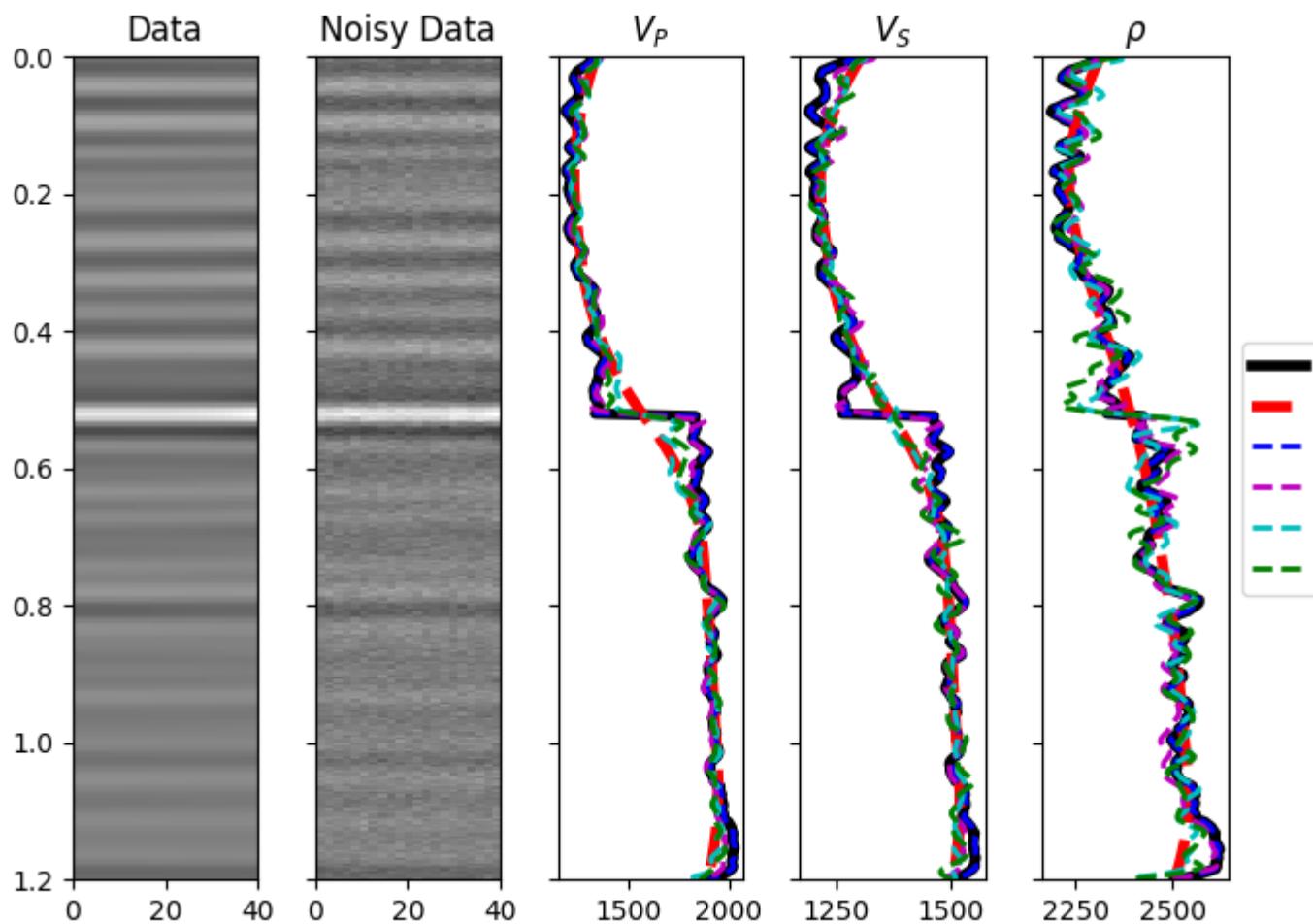
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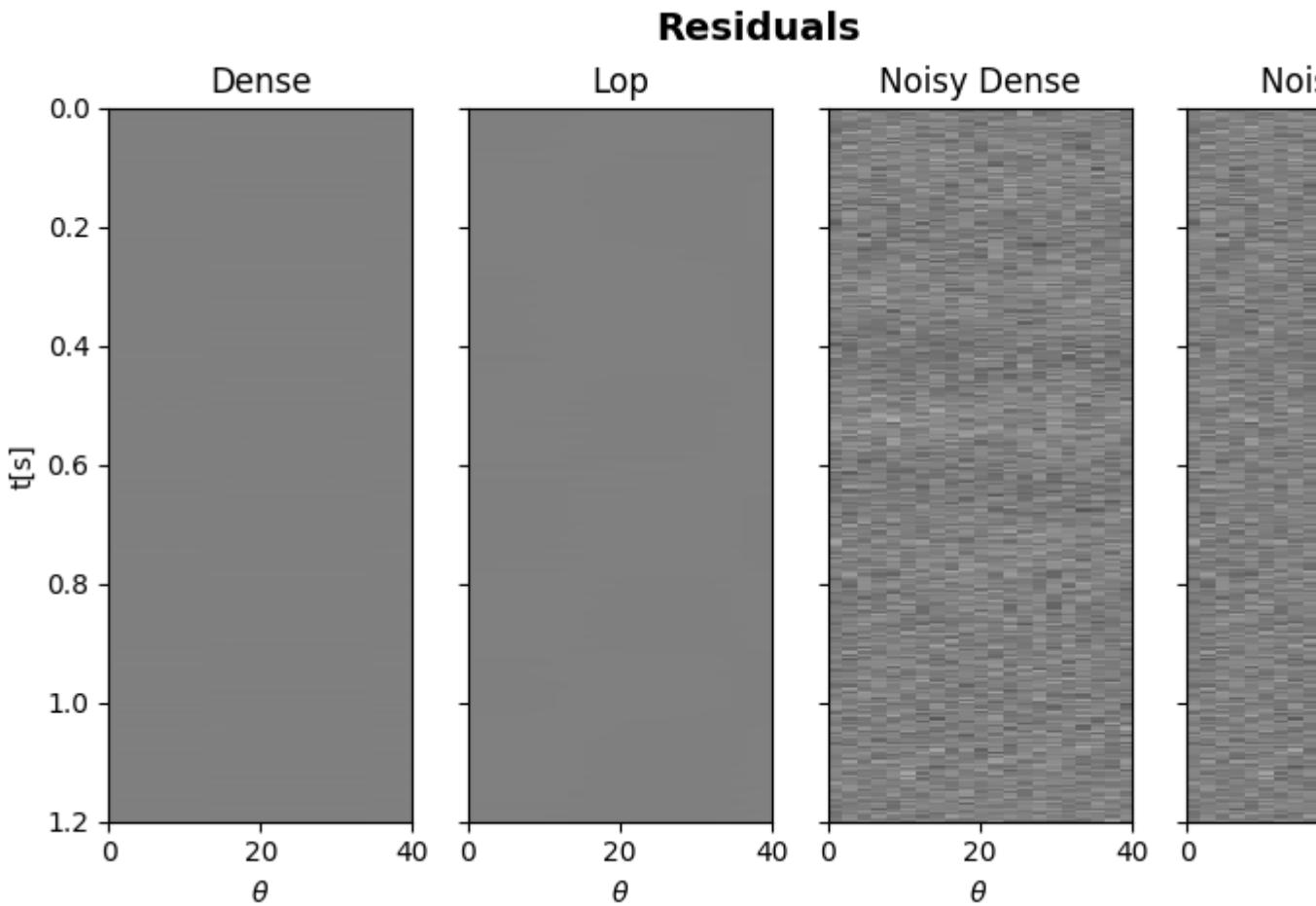
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```

fig, axs = plt.subplots(1, 4, figsize=(8, 5), sharey=True)
fig.suptitle('Residuals', fontsize=14,
             fontweight='bold', y=0.95)
im = axs[0].imshow(dPP_dense_res, cmap='gray',
                    extent=(theta[0], theta[-1], t0[-1], t0[0]),
                    vmin=-0.1, vmax=0.1)
axs[0].set_title('Dense')
axs[0].set_xlabel(r'$\theta$')
axs[0].set_ylabel('t[s]')
axs[0].axis('tight')
axs[1].imshow(dPP_res, cmap='gray',
               extent=(theta[0], theta[-1], t0[-1], t0[0]),
               vmin=-0.1, vmax=0.1)
axs[1].set_title('Lop')
axs[1].set_xlabel(r'$\theta$')
axs[1].axis('tight')
axs[2].imshow(dPPn_dense_res, cmap='gray',
               extent=(theta[0], theta[-1], t0[-1], t0[0]),
               vmin=-0.1, vmax=0.1)
axs[2].set_title('Noisy Dense')
axs[2].set_xlabel(r'$\theta$')
axs[2].axis('tight')
axs[3].imshow(dPPn_res, cmap='gray',
               extent=(theta[0], theta[-1], t0[-1], t0[0]),
               vmin=-0.1, vmax=0.1)
axs[3].set_title('Noisy Lop')
axs[3].set_xlabel(r'$\theta$')
axs[3].axis('tight')
plt.tight_layout()
plt.subplots_adjust(top=0.85)

```





We move now to a 2d example. First of all the model is loaded and data generated.

```
# model
inputfile = '../testdata/avo/poststack_model.npz'

model = np.load(inputfile)
x, z = model['x'][::6]/1000., model['z'][:300]/1000.
nx, nz = len(x), len(z)
m = 1000*model['model'][:300, ::6]

mvp = m.copy()
mvs = m/2
mrho = m/3+300
m = np.log(np.stack((mvp, mvs, mrho), axis=1))

# smooth model
nsmoothz, nsMOOTHx = 30, 25
mback = filtfilt(np.ones(nsmoothz)/float(nsmoothz), 1, m, axis=0)
mback = filtfilt(np.ones(nsMOOTHx)/float(nsMOOTHx), 1, mback, axis=2)

# dense operator
PPop_dense = \
    pylops.avo.prestack.PrestackLinearModelling(wav, theta, vsvp=vsvp,
```

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```

        nt0=nz, spatdims=(nx,),  

        linearization='akirich',  

        explicit=True)

# lop operator
PPop = pylops.avo.prestack.PrestackLinearModelling(wav, theta, vsvp=vsvp,
                                                      nt0=nz, spatdims=(nx,),  

                                                      linearization='akirich')

# data
dPP = PPop_dense*m.swapaxes(0, 1).flatten()
dPP = dPP.reshape(ntheta, nz, nx).swapaxes(0, 1)
dPPn = dPP + np.random.normal(0, 5e-2, dPP.shape)

```

Finally we perform the same 4 different inversions as in the post-stack tutorial (see [07. Post-stack inversion](#) for more details).

```

# dense inversion with noise-free data
minv_dense = \
    pylops.avo.prestack.PrestackInversion(dPP, theta, wav, m0=mback,  

                                            explicit=True,  

                                            simultaneous=False)

# dense inversion with noisy data
minv_dense_noisy = \
    pylops.avo.prestack.PrestackInversion(dPPn, theta, wav, m0=mback,  

                                            explicit=True, epsI=4e-2,  

                                            simultaneous=False)

# spatially regularized lop inversion with noisy data
minv_lop_reg = \
    pylops.avo.prestack.PrestackInversion(dPPn, theta, wav,  

                                           m0=minv_dense_noisy,  

                                           explicit=False, epsR=1e1,  

                                           **dict(damp=np.sqrt(1e-4),  

                                                 iter_lim=20))

# blockiness promoting inversion with noisy data
minv_blocky = \
    pylops.avo.prestack.PrestackInversion(dPPn, theta, wav,  

                                           m0=mback,  

                                           explicit=False,  

                                           epsR=0.4, epsRL1=0.1,  

                                           **dict(mu=0.1,  

                                                 niter_outer=3,  

                                                 niter_inner=3,  

                                                 iter_lim=5, damp=1e-3))

```

Let's now visualize the inverted elastic parameters for the different scenarios

```

def plotmodel(axes, m, x, z, vmin, vmax,  

              params=('VP', 'VS', 'Rho'),  

              cmap='gist_rainbow', title=None):  

    """Quick visualization of model  

    """  

    for ip, param in enumerate(params):

```

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```

    axs[ip].imshow(m[:, ip],
                   extent=(x[0], x[-1], z[-1], z[0]),
                   vmin=vmin, vmax=vmax, cmap=cmap)
    axs[ip].set_title('%s - %s' %(param, title))
    axs[ip].axis('tight')
plt.setp(axs[1].get_yticklabels(), visible=False)
plt.setp(axs[2].get_yticklabels(), visible=False)

# data
fig = plt.figure(figsize=(8, 9))
ax1 = plt.subplot2grid((2, 3), (0, 0), colspan=3)
ax2 = plt.subplot2grid((2, 3), (1, 0))
ax3 = plt.subplot2grid((2, 3), (1, 1), sharey=ax2)
ax4 = plt.subplot2grid((2, 3), (1, 2), sharey=ax2)
ax1.imshow(dPP[:, 0], cmap='gray',
           extent=(x[0], x[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax1.vlines([x[nx//5], x[nx//2], x[4*nx//5]], ymin=z[0], ymax=z[-1],
           colors='w', linestyles='--')
ax1.set_xlabel('x [km]')
ax1.set_ylabel('z [km]')
ax1.set_title(r'Stack ($\theta=0$)')
ax1.axis('tight')
ax2.imshow(dPP[:, :, nx//5], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax2.set_xlabel(r'$\theta$')
ax2.set_ylabel('z [km]')
ax2.set_title(r'Gather (x=% .2f)' % x[nx//5])
ax2.axis('tight')
ax3.imshow(dPP[:, :, nx//2], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax3.set_xlabel(r'$\theta$')
ax3.set_title(r'Gather (x=% .2f)' % x[nx//2])
ax3.axis('tight')
ax4.imshow(dPP[:, :, 4*nx//5], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax4.set_xlabel(r'$\theta$')
ax4.set_title(r'Gather (x=% .2f)' % x[4*nx//5])
ax4.axis('tight')
plt.setp(ax3.get_yticklabels(), visible=False)
plt.setp(ax4.get_yticklabels(), visible=False)

# noisy data
fig = plt.figure(figsize=(8, 9))
ax1 = plt.subplot2grid((2, 3), (0, 0), colspan=3)
ax2 = plt.subplot2grid((2, 3), (1, 0))
ax3 = plt.subplot2grid((2, 3), (1, 1), sharey=ax2)
ax4 = plt.subplot2grid((2, 3), (1, 2), sharey=ax2)
ax1.imshow(dPPn[:, 0], cmap='gray',
           extent=(x[0], x[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax1.vlines([x[nx//5], x[nx//2], x[4*nx//5]], ymin=z[0], ymax=z[-1],
           colors='w', linestyles='--')
ax1.set_xlabel('x [km]')

```

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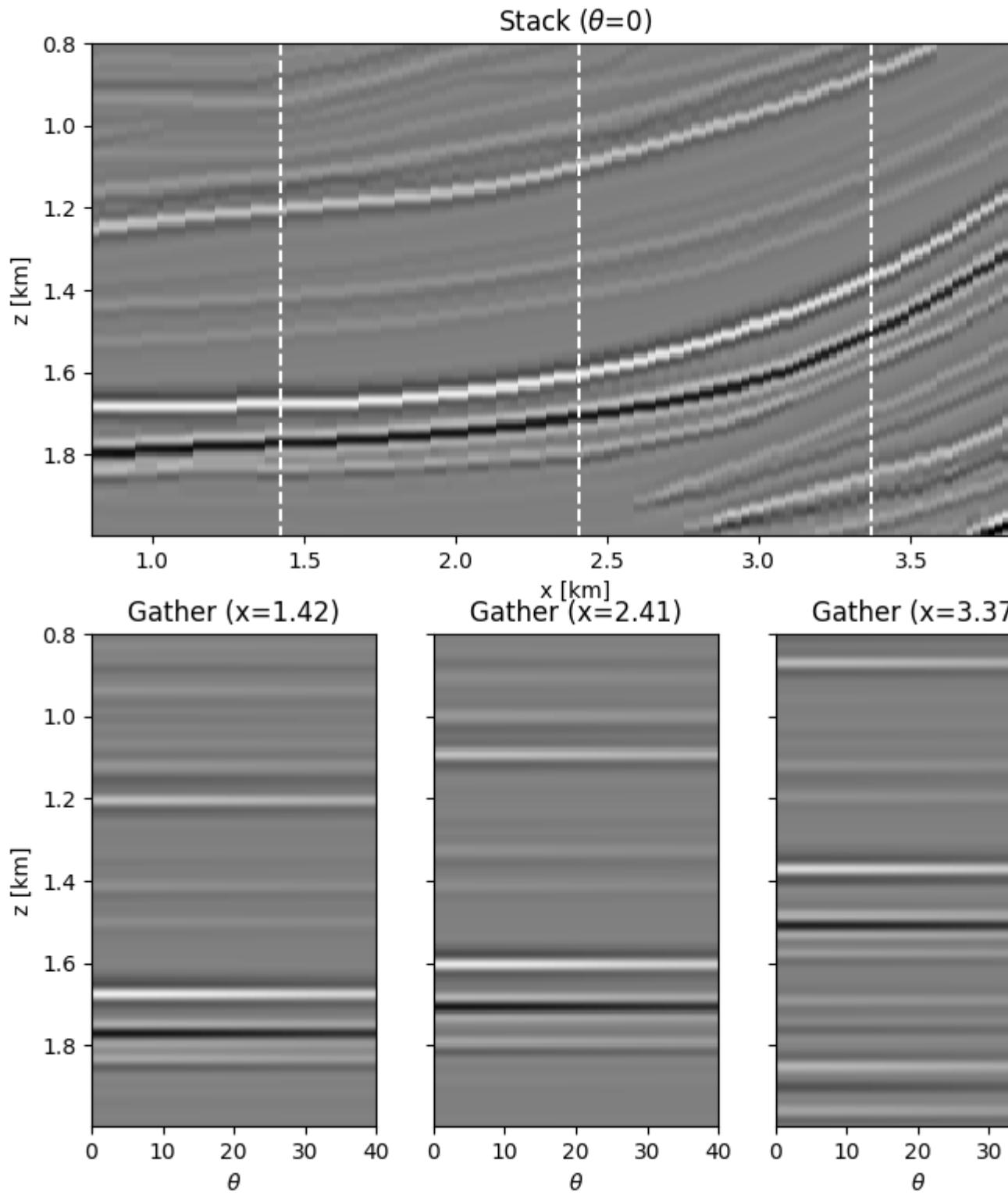
```

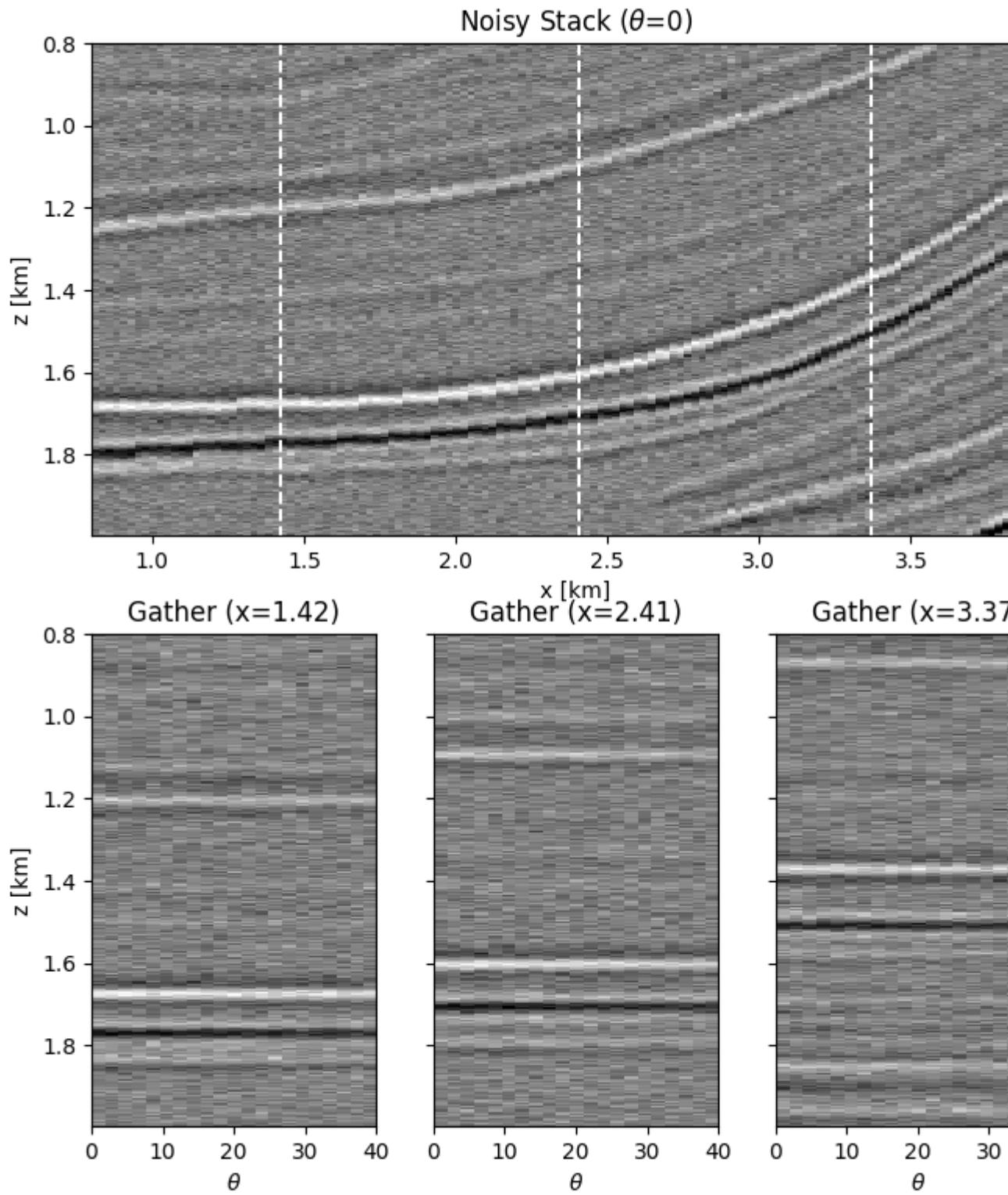
ax1.set_ylabel('z [km]')
ax1.set_title(r'Noisy Stack ($\theta=0$)')
ax1.axis('tight')
ax2.imshow(dPPn[:, :, nx//5], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax2.set_xlabel(r'$\theta$')
ax2.set_ylabel('z [km]')
ax2.set_title(r'Gather (x=%.2f)' % x[nx//5])
ax2.axis('tight')
ax3.imshow(dPPn[:, :, nx//2], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax3.set_title(r'Gather (x=%.2f)' % x[nx//2])
ax3.set_xlabel(r'$\theta$')
ax3.axis('tight')
ax4.imshow(dPPn[:, :, 4*nx//5], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax4.set_xlabel(r'$\theta$')
ax4.set_title(r'Gather (x=%.2f)' % x[4*nx//5])
ax4.axis('tight')
plt.setp(ax3.get_yticklabels(), visible=False)
plt.setp(ax4.get_yticklabels(), visible=False)

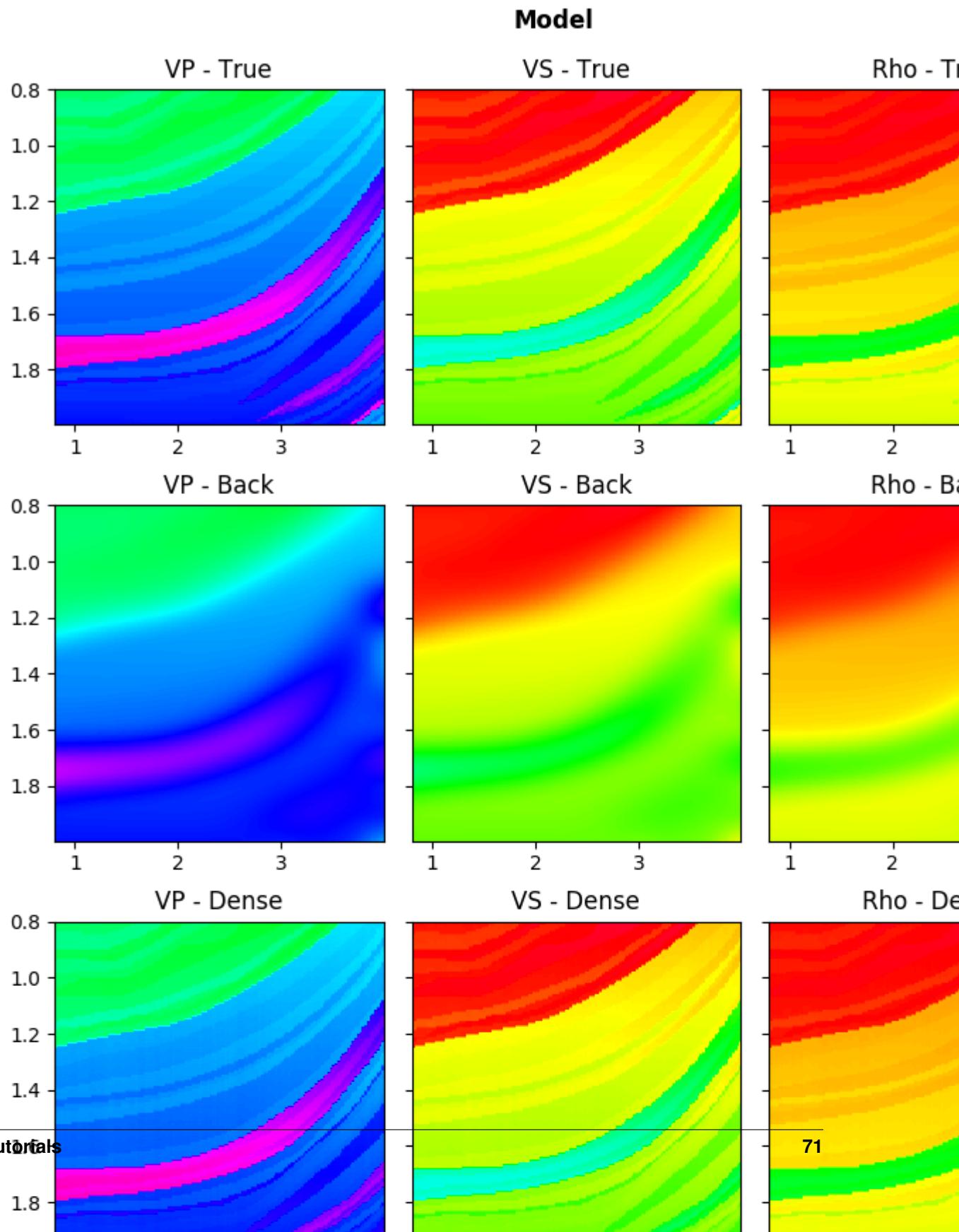
# inverted models
fig, axs = plt.subplots(6, 3, figsize=(8, 19))
fig.suptitle('Model', fontsize=12, fontweight='bold', y=0.95)
plotmodel(axs[0], m, x, z, m.min(),
          m.max(), title='True')
plotmodel(axs[1], mback, x, z, m.min(),
          m.max(), title='Back')
plotmodel(axs[2], minv_dense, x, z,
          m.min(), m.max(), title='Dense')
plotmodel(axs[3], minv_dense_noisy, x, z,
          m.min(), m.max(), title='Dense noisy')
plotmodel(axs[4], minv_lop_reg, x, z,
          m.min(), m.max(), title='Lop regularized')
plotmodel(axs[5], minv_blocky, x, z,
          m.min(), m.max(), title='Lop blocky')
plt.tight_layout()
plt.subplots_adjust(top=0.92)

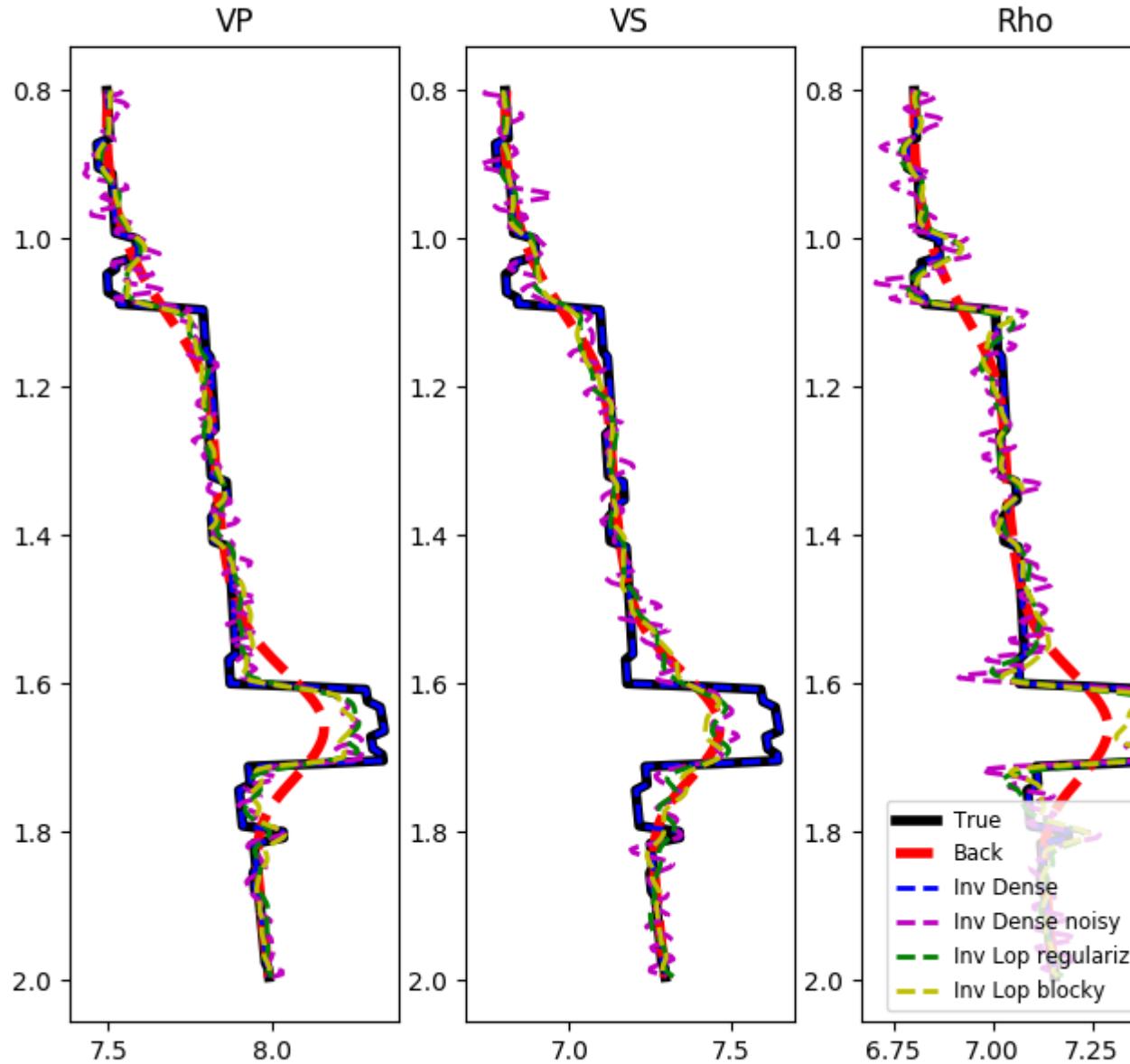
fig, axs = plt.subplots(1, 3, figsize=(8, 7))
for ip, param in enumerate(['VP', 'VS', 'Rho']):
    axs[ip].plot(m[:, ip, nx//2], z, 'k', lw=4, label='True')
    axs[ip].plot(mback[:, ip, nx//2], z, '--r', lw=4, label='Back')
    axs[ip].plot(minv_dense[:, ip, nx//2], z, '--b', lw=2, label='Inv Dense')
    axs[ip].plot(minv_dense_noisy[:, ip, nx//2], z, '--m', lw=2,
                 label='Inv Dense noisy')
    axs[ip].plot(minv_lop_reg[:, ip, nx//2], z, '--g', lw=2,
                 label='Inv Lop regularized')
    axs[ip].plot(minv_blocky[:, ip, nx // 2], z, '--y', lw=2,
                 label='Inv Lop blocky')
    axs[ip].set_title(param)
    axs[ip].invert_yaxis()
axs[2].legend(loc=8, fontsize='small')

```









Out:

```
<matplotlib.legend.Legend object at 0x7fcd6c9f95f8>
```

While the background model `m0` has been provided in all the examples so far, it is worth showing that the module `pylops.avo.prestack.PrestackInversion` can also produce so-called relative elastic parameters (i.e., variations from an average medium property) when the background model `m0` is not available.

```
dminv = \
```

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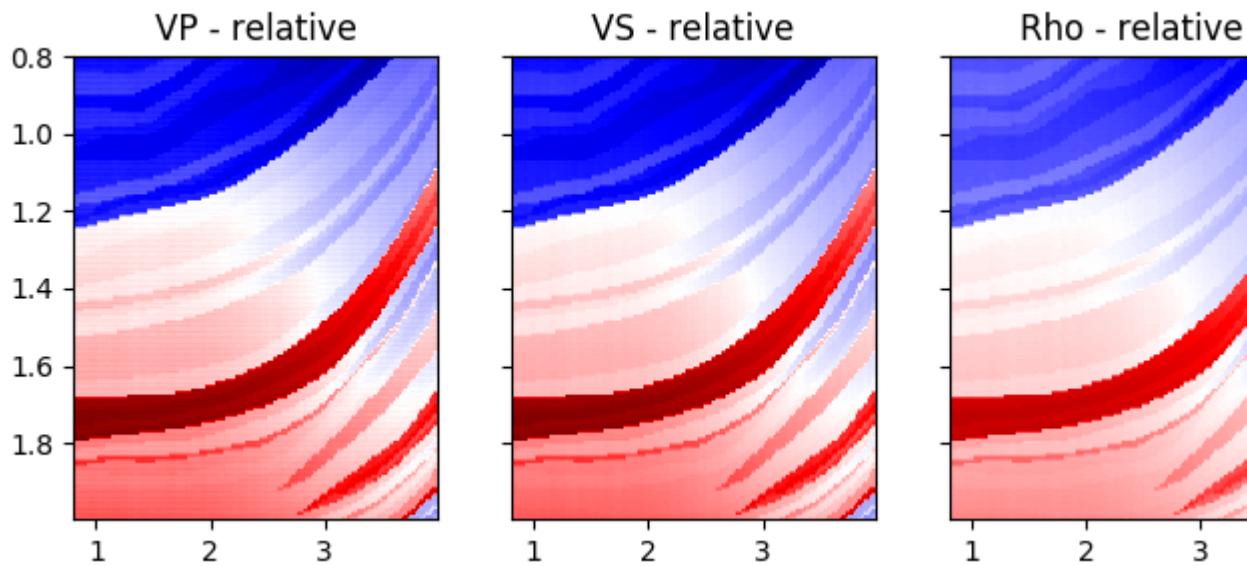
```

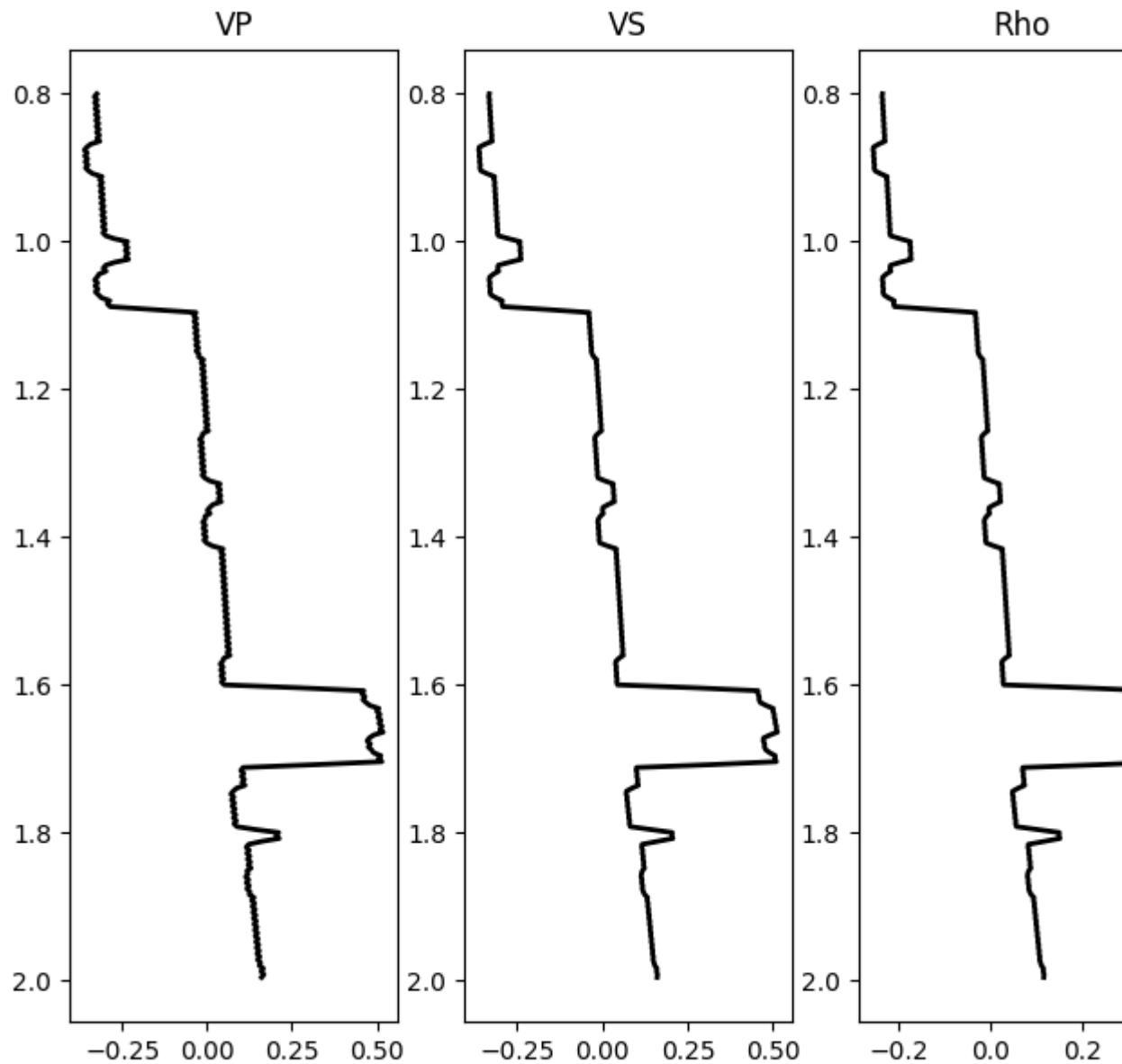
pylops.avo.prestack.PrestackInversion(dPP, theta, wav, m0=None,
                                       explicit=True,
                                       simultaneous=False)

fig, axs = plt.subplots(1, 3, figsize=(8, 3))
plotmodel(axs, dminv, x, z, -dminv.max(), dminv.max(),
           cmap='seismic', title='relative')

fig, axs = plt.subplots(1, 3, figsize=(8, 7))
for ip, param in enumerate(['VP', 'VS', 'Rho']):
    axs[ip].plot(dminv[:, ip, nx//2], z, 'k', lw=2)
    axs[ip].set_title(param)
    axs[ip].invert_yaxis()

```





Total running time of the script: ( 0 minutes 23.437 seconds)

---

Note: Click [here](#) to download the full example code

---

### 3.4.9 09. Multi-Dimensional Deconvolution

This example shows how to set-up and run the `pylops.waveeqprocessing.MDD` inversion using synthetic data.

```

import warnings
import numpy as np
import matplotlib.pyplot as plt
import pylops

from pylops.utils.tapers import taper3d
from pylops.utils.wavelets import ricker
from pylops.utils.seismicevents import makeaxis, hyperbolic2d

warnings.filterwarnings('ignore')
plt.close('all')

```

Let's start by creating a set of hyperbolic events to be used as our MDC kernel

```

# Input parameters
par = {'ox':-150, 'dx':10, 'nx':31,
       'oy':-250, 'dy':10, 'ny':51,
       'ot':0, 'dt':0.004, 'nt':300,
       'f0': 20, 'nfmax': 200}

t0_m = [0.2]
vrms_m = [700.]
amp_m = [1.]

t0_G = [0.2, 0.5, 0.7]
vrms_G = [800., 1200., 1500.]
amp_G = [1., 0.6, 0.5]

# Taper
tap = taper3d(par['nt'], [par['ny'], par['nx']],
               (5, 5), tapertype='hanning')

# Create axis
t, t2, x, y = makeaxis(par)

# Create wavelet
wav = ricker(t[:41], f0=par['f0'])[0]

# Generate model
m, mwav = hyperbolic2d(x, t, t0_m, vrms_m, amp_m, wav)

# Generate operator
G, Gwav = np.zeros((par['ny'], par['nx'], par['nt'])), \
            np.zeros((par['ny'], par['nx'], par['nt']))
for iy, y0 in enumerate(y):
    G[iy], Gwav[iy] = hyperbolic2d(x-y0, t, t0_G, vrms_G, amp_G, wav)
G, Gwav = G*tap, Gwav*tap

# Add negative part to data and model
m = np.concatenate((np.zeros((par['nx'], par['nt']-1)), m), axis=-1)
mwav = np.concatenate((np.zeros((par['nx'], par['nt']-1)), mwav), axis=-1)
Gwav2 = np.concatenate((np.zeros((par['ny'], par['nx'], par['nt']-1)), Gwav),
                       axis=-1)

# Define MDC linear operator
Gwav_fft = np.fft.rfft(Gwav2, 2*par['nt']-1, axis=-1)
Gwav_fft = Gwav_fft[..., :par['nfmax']]

```

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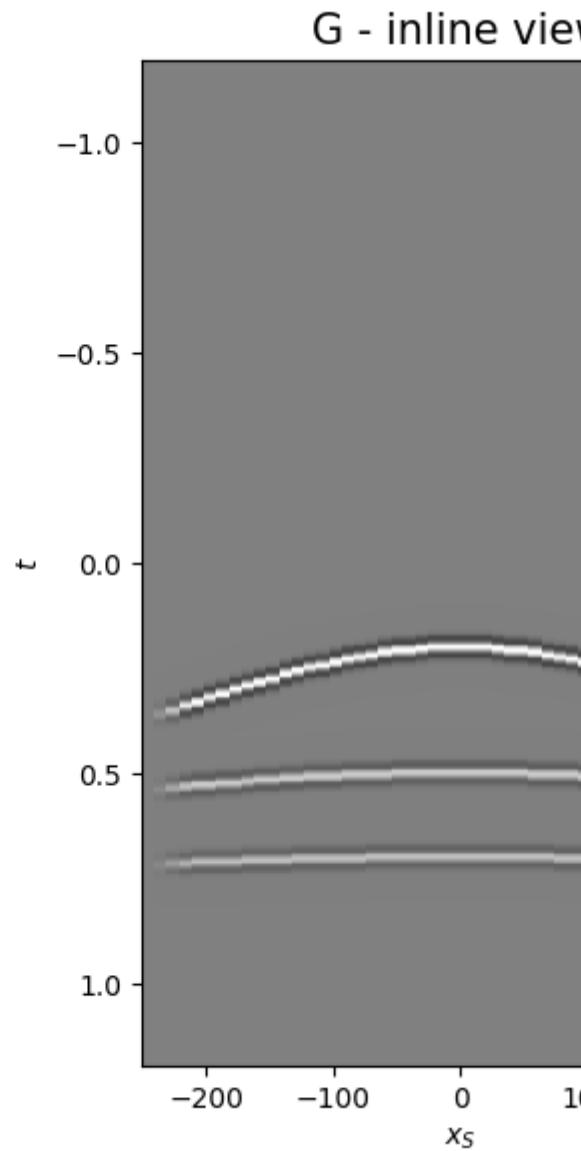
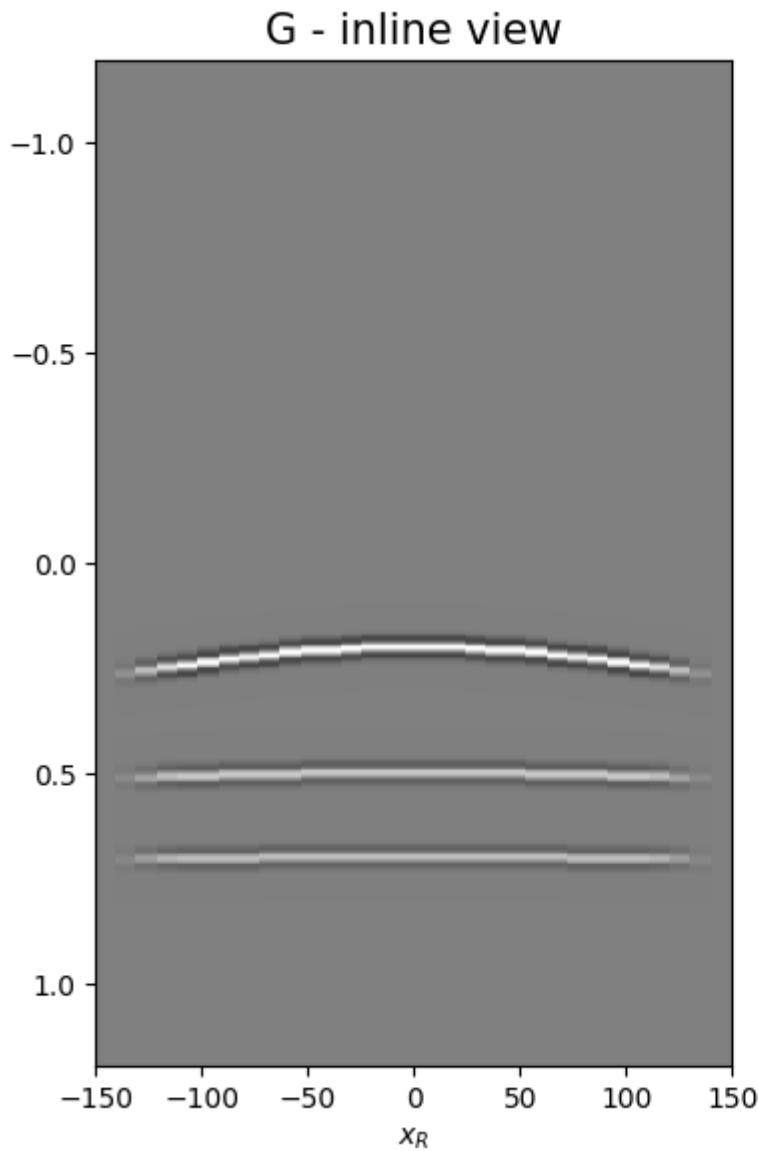
```
MDCop = pylops.waveeqprocessing.MDC(Gwav_fft, nt=2 * par['nt']-1, nv=1,
                                    dt=0.004, dr=1., dtype='float32')

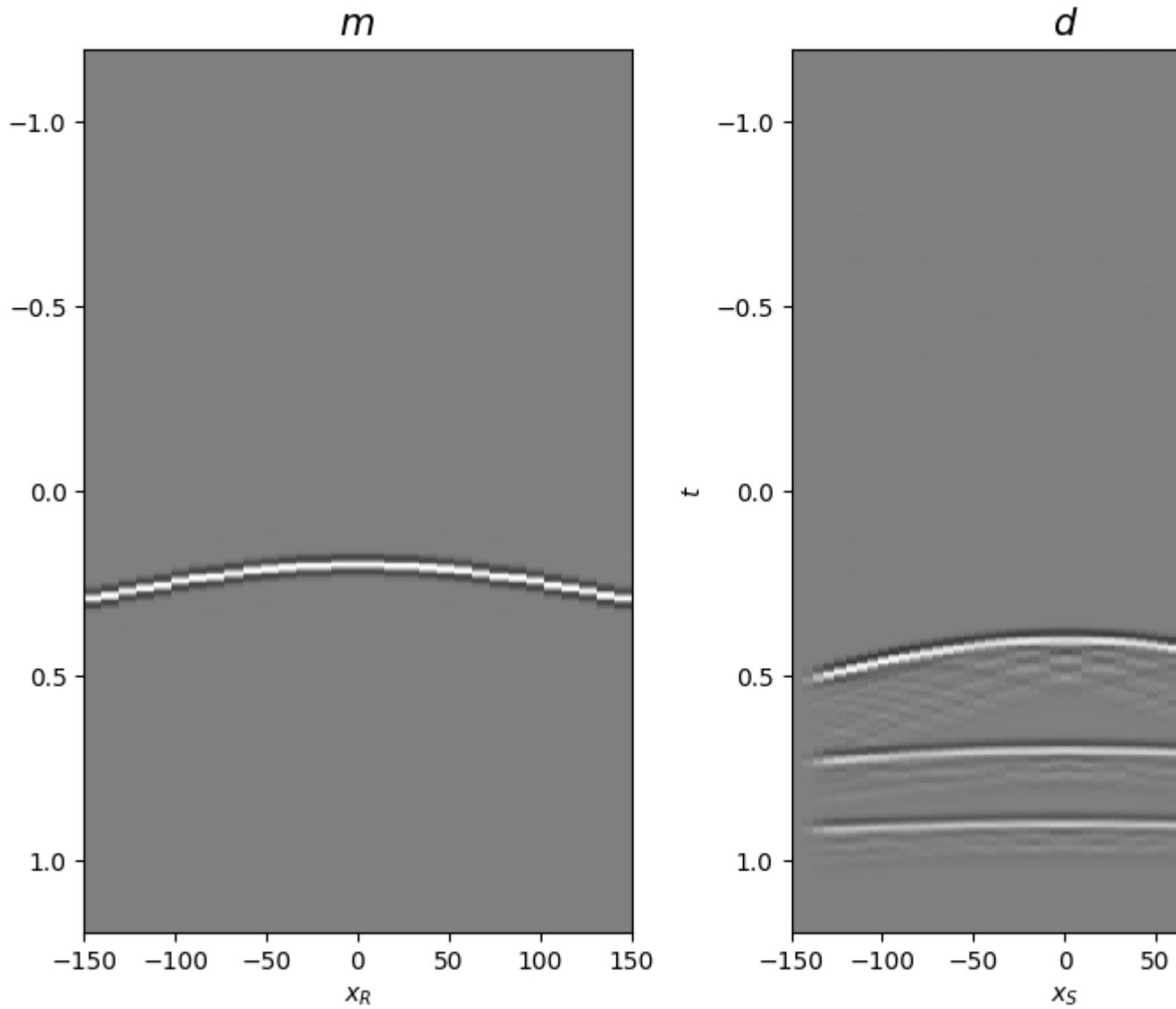
# Create data
d = MDCop*m.flatten()
d = d.reshape(par['ny'], 2*par['nt']-1)
```

Let's display what we have so far: operator, input model, and data

```
fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(Gwav2[int(par['ny']/2)].T, aspect='auto',
               interpolation='nearest', cmap='gray',
               vmin=-np.abs(Gwav2.max()), vmax=np.abs(Gwav2.max()),
               extent=(x.min(), x.max(), t2.max(), t2.min()))
axs[0].set_title('G - inline view', fontsize=15)
axs[0].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].imshow(Gwav2[:, int(par['nx']/2)].T, aspect='auto',
               interpolation='nearest', cmap='gray',
               vmin=-np.abs(Gwav2.max()), vmax=np.abs(Gwav2.max()),
               extent=(y.min(), y.max(), t2.max(), t2.min()))
axs[1].set_title('G - inline view', fontsize=15)
axs[1].set_xlabel(r'$x_S$')
axs[1].set_ylabel(r'$t$')
fig.tight_layout()

fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(mwav.T, aspect='auto', interpolation='nearest', cmap='gray',
               vmin=-np.abs(mwav.max()), vmax=np.abs(mwav.max()),
               extent=(x.min(), x.max(), t2.max(), t2.min()))
axs[0].set_title(r'$m$', fontsize=15)
axs[0].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].imshow(d.T, aspect='auto', interpolation='nearest', cmap='gray',
               vmin=-np.abs(d.max()), vmax=np.abs(d.max()),
               extent=(x.min(), x.max(), t2.max(), t2.min()))
axs[1].set_title(r'$d$', fontsize=15)
axs[1].set_xlabel(r'$x_S$')
axs[1].set_ylabel(r'$t$')
fig.tight_layout()
```





We are now ready to feed our operator to `pylops.waveeqprocessing.MDD` and invert back for our input model

```
minv, madj, psfinv, psfadj = \
    pylops.waveeqprocessing.MDD(Gwav, d[:, par['nt'] - 1:], \
                                dt=par['dt'], dr=par['dx'], \
                                nfmax=par['nfmax'], wav=wav, \
                                twosided=True, add_negative=True, \
                                adjoint=True, psf=True, \
                                dtype='complex64', dottest=False, \
                                **dict(damp=1e-4, iter_lim=20, show=0))

fig = plt.figure(figsize=(8, 6))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(madj.T, aspect='auto', interpolation='nearest', cmap='gray',
```

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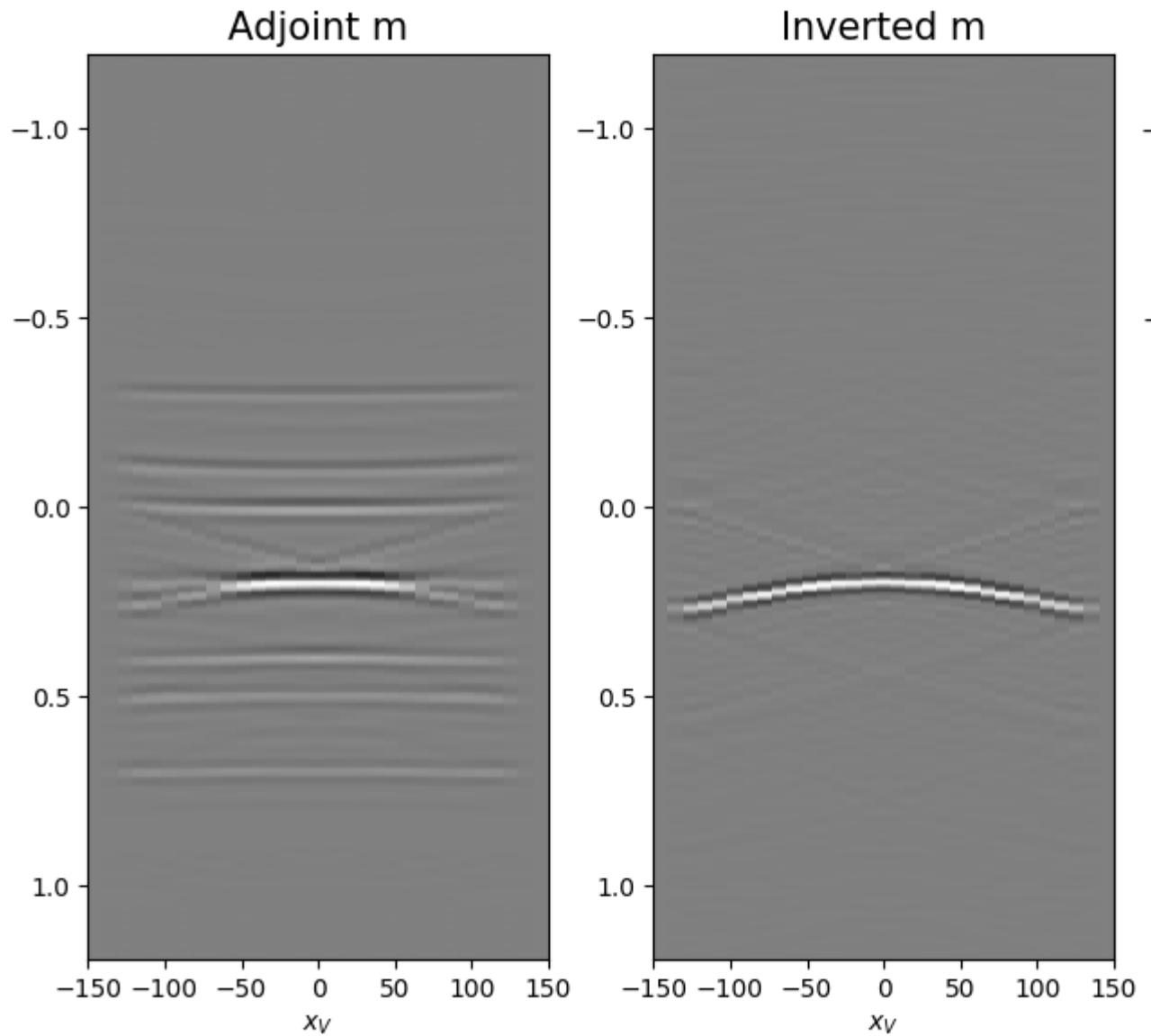
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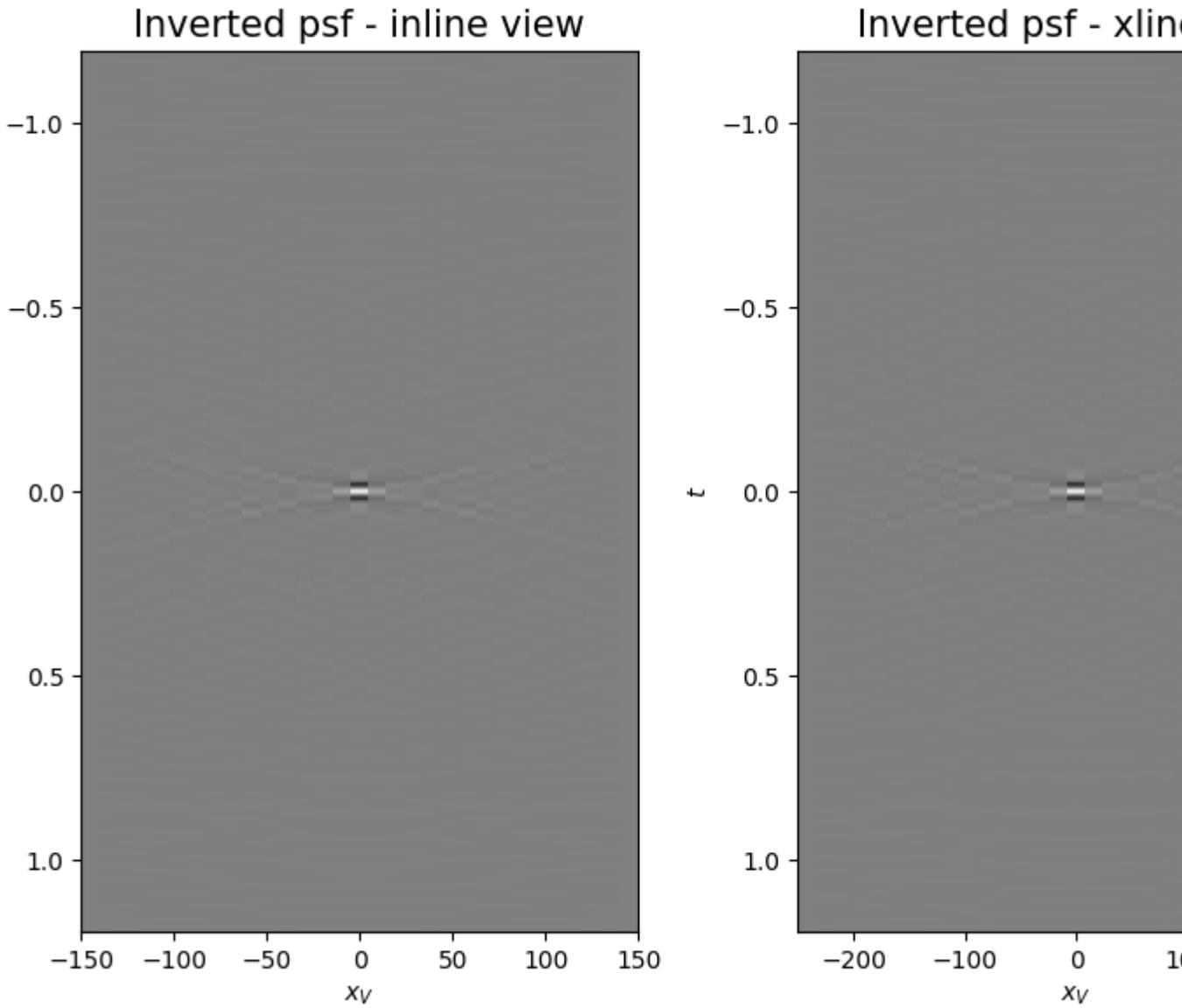
```

vmin=-np.abs(madj.max()), vmax=np.abs(madj.max()))
extent=(x.min(), x.max(), t2.max(), t2.min()))
ax1.set_title('Adjoint m', fontsize=15)
ax1.set_xlabel(r'$x_V$')
ax1.set_ylabel(r'$t$')
ax2.imshow(minv.T, aspect='auto', interpolation='nearest', cmap='gray',
           vmin=-np.abs(minv.max()), vmax=np.abs(minv.max()),
           extent=(x.min(), x.max(), t2.max(), t2.min()))
ax2.set_title('Inverted m', fontsize=15)
ax2.set_xlabel(r'$x_V$')
ax2.set_ylabel(r'$t$')
ax3.plot(madj[int(par['nx']/2)]/np.abs(madj[int(par['nx']/2)]).max(),
          t2, 'r', lw=5)
ax3.plot(minv[int(par['nx']/2)]/np.abs(minv[int(par['nx']/2)]).max(),
          t2, 'k', lw=3)
ax3.set_ylim([t2[-1], t2[0]])
fig.tight_layout()

fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(psfinv[int(par['nx']/2)].T,
              aspect='auto', interpolation='nearest',
              vmin=-np.abs(psfinv.max()), vmax=np.abs(psfinv.max()),
              cmap='gray', extent=(x.min(), x.max(), t2.max(), t2.min()))
axs[0].set_title('Inverted psf - inline view', fontsize=15)
axs[0].set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
axs[1].imshow(psfinv[:, int(par['nx']/2)].T,
              aspect='auto', interpolation='nearest',
              vmin=-np.abs(psfinv.max()), vmax=np.abs(psfinv.max()),
              cmap='gray', extent=(y.min(), y.max(), t2.max(), t2.min()))
axs[1].set_title('Inverted psf - xline view', fontsize=15)
axs[1].set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
fig.tight_layout()

```





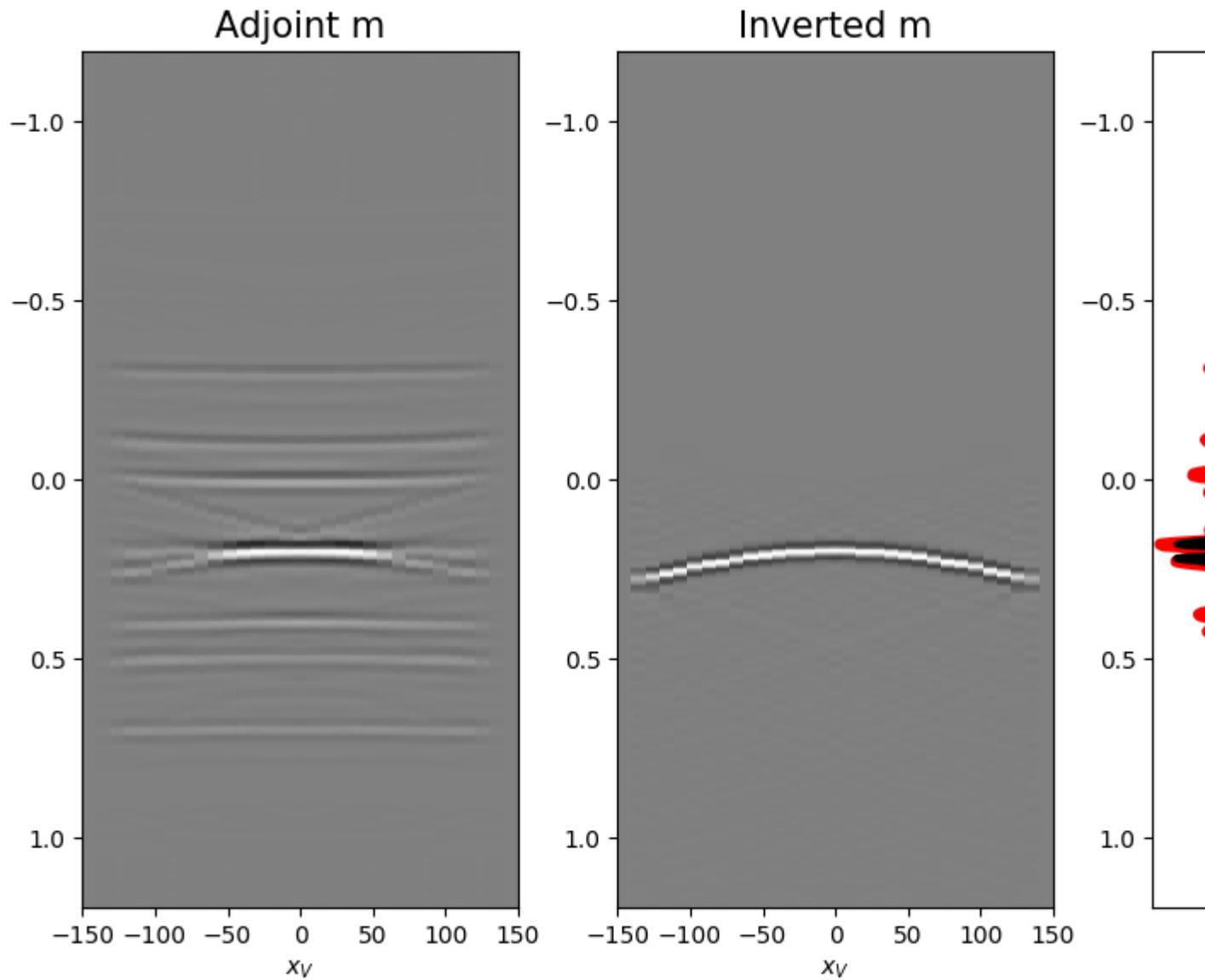
We repeat the same procedure but this time we will add a preconditioning by means of `causality_precond` parameter, which enforces the inverted model to be zero in the negative part of the time axis (as expected by theory). This preconditioning will have the effect of speeding up the convergence of the iterative solver and thus reduce the computation time of the deconvolution

```
minvprec = pylops.waveeqprocessing.MDD(Gwav, d[:, par['nt'] - 1:],  
                                      dt=par['dt'], dr=par['dx'],  
                                      nfmax=par['nfmax'], wav=wav,  
                                      twosided=True, add_negative=True,  
                                      adjoint=False, psf=False,  
                                      causality_precond=True,  
                                      dtype='complex64',  
                                      dottest=False,  
                                      **dict(damp=1e-4, iter_lim=50, show=0))  
  
# sphinx_gallery_thumbnail_number = 5
```

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```
fig = plt.figure(figsize=(8, 6))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(madj.T, aspect='auto', interpolation='nearest', cmap='gray',
           vmin=-np.abs(madj.max()), vmax=np.abs(madj.max()),
           extent=(x.min(), x.max(), t2.max(), t2.min()))
ax1.set_title('Adjoint m', fontsize=15)
ax1.set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
ax2.imshow(minvprec.T, aspect='auto', interpolation='nearest', cmap='gray',
           vmin=-np.abs(minvprec.max()), vmax=np.abs(minvprec.max()),
           extent=(x.min(), x.max(), t2.max(), t2.min()))
ax2.set_title('Inverted m', fontsize=15)
ax2.set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
ax3.plot(madj[int(par['nx']/2)]/np.abs(madj[int(par['nx']/2)]).max(),
          t2, 'r', lw=5)
ax3.plot(minvprec[int(par['nx']/2)]/np.abs(minvprec[int(par['nx']/2)]).max(),
          t2, 'k', lw=3)
ax3.set_xlim([t2[-1], t2[0]])
fig.tight_layout()
```



**Total running time of the script:** ( 0 minutes 7.721 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.10 10. Marchenko redatuming by inversion

This example shows how to set-up and run the `pylops.waveeqprocessing.Marchenko` inversion using synthetic data.

```
# sphinx_gallery_thumbnail_number = 5
# pylint: disable=C0103
import warnings
import numpy as np
```

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```

import matplotlib.pyplot as plt

from scipy.signal import convolve
from pyllops.waveeqprocessing import Marchenko

warnings.filterwarnings('ignore')
plt.close('all')

```

Let's start by defining some input parameters and loading the test data

```

# Input parameters
inputfile = '../testdata/marchenko/input.npz'

vel = 2400.0          # velocity
toff = 0.045           # direct arrival time shift
nsmooth = 10            # time window smoothing
nfmax = 1000            # max frequency for MDC (#samples)
niter = 10              # iterations

inputdata = np.load(inputfile)

# Receivers
r = inputdata['r']
nr = r.shape[1]
dr = r[0, 1]-r[0, 0]

# Sources
s = inputdata['s']
ns = s.shape[1]
ds = s[0, 1]-s[0, 0]

# Virtual points
vs = inputdata['vs']

# Density model
rho = inputdata['rho']
z, x = inputdata['z'], inputdata['x']

# Reflection data (R[s, r, t]) and subsurface fields
R = inputdata['R'][:, :, :-100]
R = np.swapaxes(R, 0, 1) # just because of how the data was saved

Gsub = inputdata['Gsub'][:-100]
G0sub = inputdata['G0sub'][:-100]
wav = inputdata['wav']
wav_c = np.argmax(wav)

t = inputdata['t'][:-100]
ot, dt, nt = t[0], t[1]-t[0], len(t)

Gsub = np.apply_along_axis(convolve, 0, Gsub, wav, mode='full')
Gsub = Gsub[wav_c:][:nt]
G0sub = np.apply_along_axis(convolve, 0, G0sub, wav, mode='full')
G0sub = G0sub[wav_c:][:nt]

plt.figure(figsize=(10, 5))
plt.imshow(rho, cmap='gray', extent=(x[0], x[-1], z[-1], z[0]))

```

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```

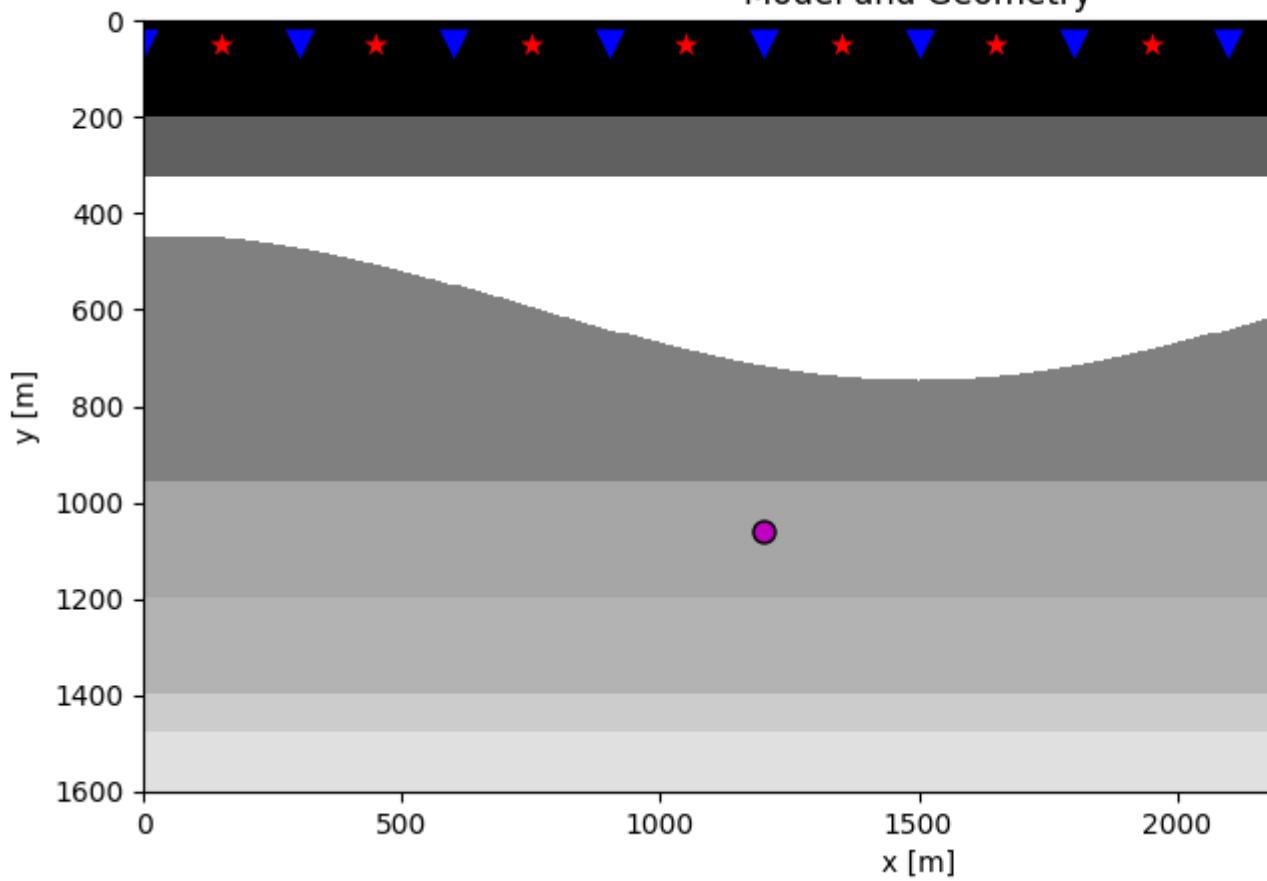
plt.scatter(s[0, 5::10], s[1, 5::10], marker='*', s=150, c='r', edgecolors='k')
plt.scatter(r[0, ::10], r[1, ::10], marker='v', s=150, c='b', edgecolors='k')
plt.scatter(vs[0], vs[1], marker='.', s=250, c='m', edgecolors='k')
plt.axis('tight')
plt.xlabel('x [m]')
plt.ylabel('y [m]')
plt.title('Model and Geometry')
plt.xlim(x[0], x[-1])

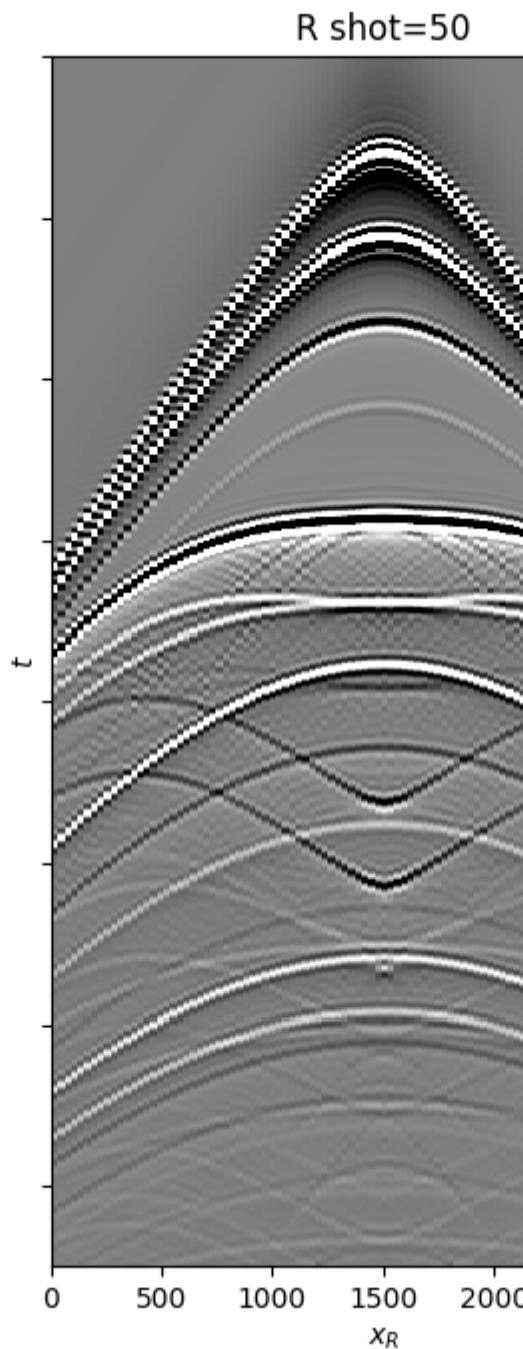
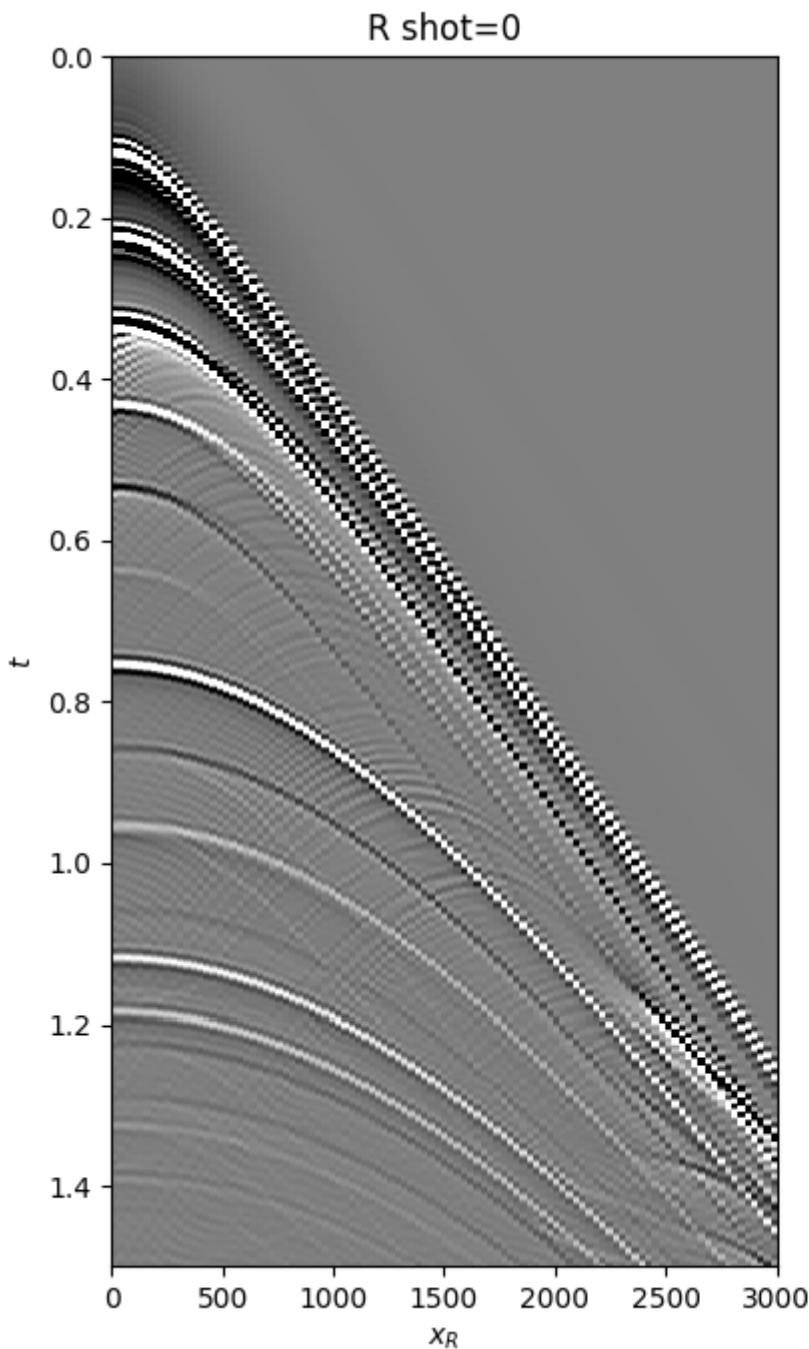
fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 7))
axs[0].imshow(R[0].T, cmap='gray', vmin=-1e-2, vmax=1e-2,
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[0].set_title('R shot=0')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
axs[0].axis('tight')
axs[0].set_ylim(1.5, 0)
axs[1].imshow(R[ns//2].T, cmap='gray', vmin=-1e-2, vmax=1e-2,
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[1].set_title('R shot=%d' %(ns//2))
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].axis('tight')
axs[1].set_ylim(1.5, 0)
axs[2].imshow(R[-1].T, cmap='gray', vmin=-1e-2, vmax=1e-2,
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[2].set_title('R shot=%d' %ns)
axs[2].set_xlabel(r'$x_R$')
axs[2].axis('tight')
axs[2].set_ylim(1.5, 0)
fig.tight_layout()

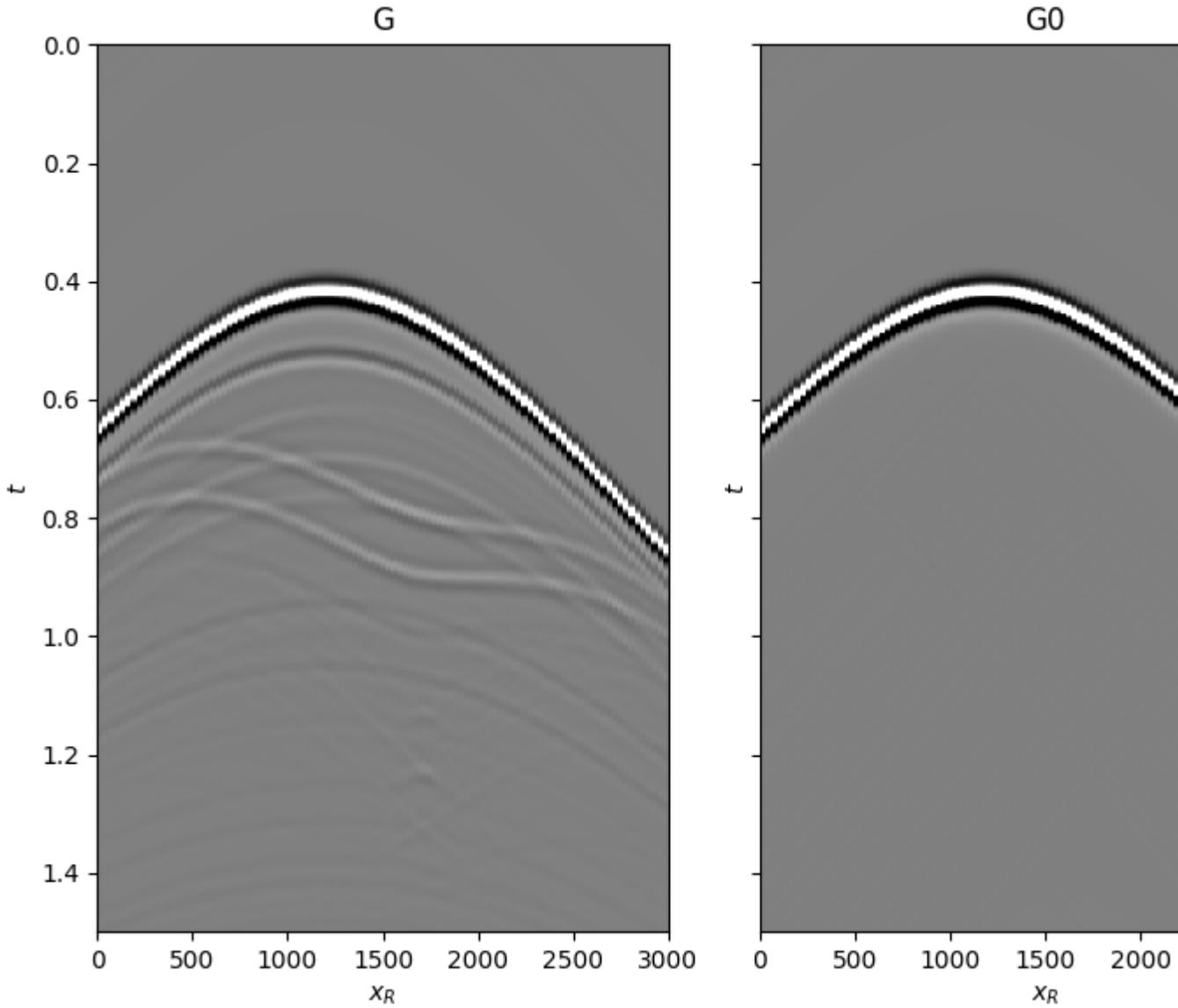
fig, axs = plt.subplots(1, 2, sharey=True, figsize=(8, 6))
axs[0].imshow(Gsub, cmap='gray', vmin=-1e6, vmax=1e6,
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[0].set_title('G')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
axs[0].axis('tight')
axs[0].set_ylim(1.5, 0)
axs[1].imshow(G0sub, cmap='gray', vmin=-1e6, vmax=1e6,
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[1].set_title('G0')
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].axis('tight')
axs[1].set_ylim(1.5, 0)
fig.tight_layout()

```

Model and Geometry







Let's now create an object of the `pylops.waveeqprocessing.Marchenko` class and apply redatuming for a single subsurface point `vs`.

```
# direct arrival window
trav = np.sqrt((vs[0]-r[0])**2+(vs[1]-r[1])**2)/vel

MarchenkoWM = Marchenko(R, dt=dt, dr=dr, nfmax=nfmax, wav=wav,
                         toff=toff, nsmooth=nsmooth)

f1_inv_minus, f1_inv_plus, p0_minus, g_inv_minus, g_inv_plus = \
    MarchenkoWM.apply_onepoint(trav, G0=G0sub.T, rtm=True, greens=True,
                                dottest=True, **dict(iter_lim=niter, show=True))
g_inv_tot = g_inv_minus + g_inv_plus
```

Out:

```
Dot test passed, v^T(Op_u)=491.131956 - u^T(Op^Tv)=491.131956
Dot test passed, v^T(Op_u)=464.113195 - u^T(Op^Tv)=464.113195
```

```
LSQR          Least-squares solution of Ax = b
The matrix A has 282598 rows and 282598 cols
damp = 0.00000000000000e+00 calc_var =      0
atol = 1.00e-08           condlim = 1.00e+08
btol = 1.00e-08           iter_lim =     10

    Itn      x[0]      r1norm      r2norm  Compatible    LS      Norm A   Cond A
    0  0.00000e+00  3.134e+07  3.134e+07  1.0e+00  3.3e-08
    1  0.00000e+00  1.374e+07  1.374e+07  4.4e-01  9.3e-01  1.1e+00  1.0e+00
    2  0.00000e+00  7.770e+06  7.770e+06  2.5e-01  3.9e-01  1.8e+00  2.2e+00
    3  0.00000e+00  5.750e+06  5.750e+06  1.8e-01  3.3e-01  2.1e+00  3.4e+00
    4  0.00000e+00  3.930e+06  3.930e+06  1.3e-01  3.4e-01  2.5e+00  5.1e+00
    5  0.00000e+00  3.042e+06  3.042e+06  9.7e-02  2.6e-01  2.9e+00  6.8e+00
    6  0.00000e+00  2.423e+06  2.423e+06  7.7e-02  2.2e-01  3.3e+00  8.6e+00
    7  0.00000e+00  1.675e+06  1.675e+06  5.3e-02  2.5e-01  3.6e+00  1.1e+01
    8  0.00000e+00  1.248e+06  1.248e+06  4.0e-02  2.0e-01  3.9e+00  1.3e+01
    9  0.00000e+00  1.004e+06  1.004e+06  3.2e-02  1.5e-01  4.2e+00  1.4e+01
   10 0.00000e+00  7.762e+05  7.762e+05  2.5e-02  1.8e-01  4.4e+00  1.6e+01

LSQR finished
The iteration limit has been reached

istop =      7      r1norm = 7.8e+05      anorm = 4.4e+00      arnorm = 6.1e+05
itn   =     10      r2norm = 7.8e+05      acond = 1.6e+01      xnorm  = 3.6e+07
```

We can now compare the result of Marchenko redatuming via LSQR with standard redatuming

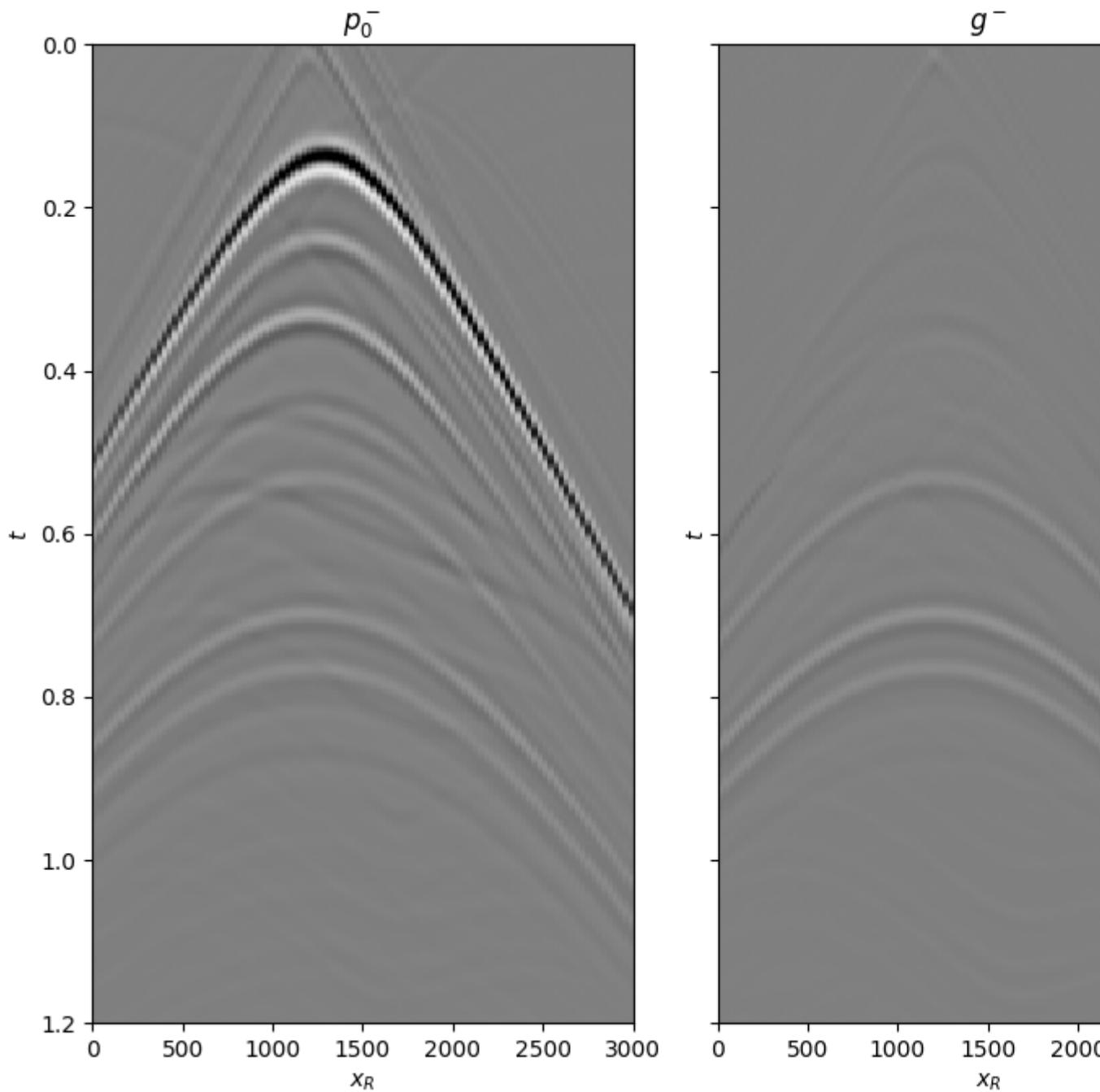
```
fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 7))
axs[0].imshow(p0_minus.T, cmap='gray', vmin=-5e5, vmax=5e5,
              extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[0].set_title(r'$p_0^{-}$')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
axs[0].axis('tight')
axs[0].set_xlim(1.2, 0)
axs[1].imshow(g_inv_minus.T, cmap='gray', vmin=-5e5, vmax=5e5,
              extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[1].set_title(r'$g^{-}$')
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].axis('tight')
axs[1].set_xlim(1.2, 0)
axs[2].imshow(g_inv_plus.T, cmap='gray', vmin=-5e5, vmax=5e5,
              extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[2].set_title(r'$g^{+}$')
axs[2].set_xlabel(r'$x_R$')
axs[2].set_ylabel(r'$t$')
axs[2].axis('tight')
axs[2].set_xlim(1.2, 0)
fig.tight_layout()

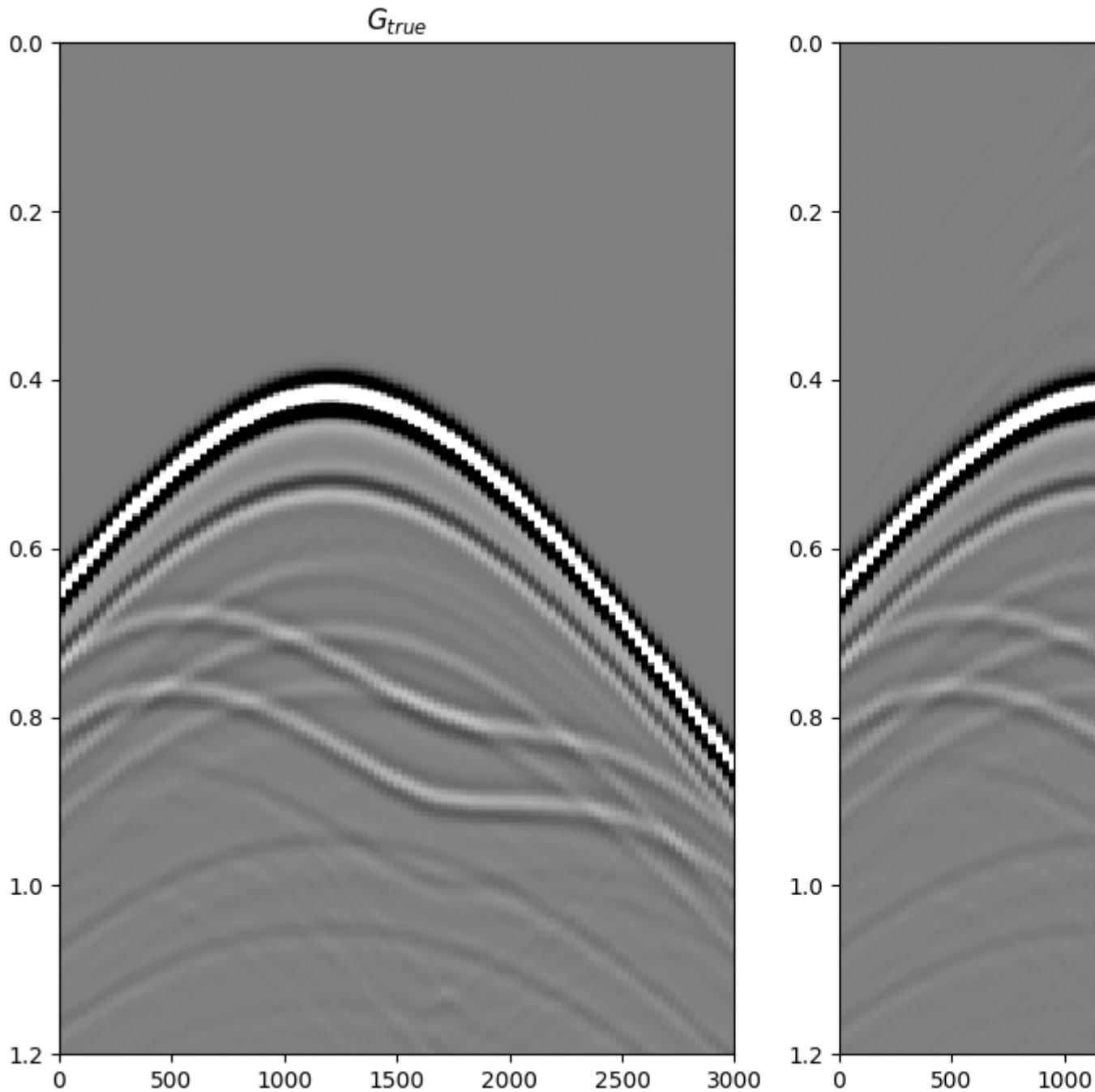
fig = plt.figure(figsize=(12, 7))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
```

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```
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(Gsub, cmap='gray', vmin=-5e5, vmax=5e5,
           extent=(r[0, 0], r[0, -1], t[-1], t[0]))
ax1.set_title(r'$G_{\text{true}}$')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
ax1.axis('tight')
ax1.set_ylim(1.2, 0)
ax2.imshow(g_inv_tot.T, cmap='gray', vmin=-5e5, vmax=5e5,
           extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
ax2.set_title(r'$G_{\text{est}}$')
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
ax2.axis('tight')
ax2.set_ylim(1.2, 0)
ax3.plot(Gsub[:, nr//2]/Gsub.max(), t, 'r', lw=5)
ax3.plot(g_inv_tot[nr//2, nt-1:]/g_inv_tot.max(), t, 'k', lw=3)
ax3.set_ylim(1.2, 0)
fig.tight_layout()
```





Note that Marchenko redatuming can also be applied simultaneously to multiple subsurface points. Use `pylops.waveeqprocessing.Marchenko.apply_multiplepoints` instead of `pylops.waveeqprocessing.Marchenko.apply_onepoint`.

**Total running time of the script:** ( 0 minutes 15.627 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.11 11. Radon filtering

In this example we will be taking advantage of the `pylops.signalprocessing.Radon2D` operator to perform filtering of unwanted events from a seismic data. For those of you not familiar with seismic data, let's imagine that we have a data composed of a certain number of flat events and a parabolic event, we are after a transform that allows us to separate such an event from the others and filter it out. Those of you with a geophysics background may immediately realize this is the case of seismic angle (or offset) gathers after migration and those events with parabolic moveout are generally residual multiples that we would like to suppress prior to performing further analysis of our data.

The Radon transform is actually a very good transform to perform such a separation. We can thus devise a simple workflow that takes our data as input, applies a Radon transform, filters some of the events out and goes back to the original domain.

```
import numpy as np
import matplotlib.pyplot as plt

import pylops
from pylops.utils.wavelets import ricker

plt.close('all')
np.random.seed(0)
```

Let's first create a data composed on 3 linear events and a parabolic event.

```
par = {'ox':0, 'dx':2, 'nx':121,
       'ot':0, 'dt':0.004, 'nt':100,
       'f0': 30}

# linear events
v = 1500
t0 = [0.1, 0.2, 0.3]
theta = [0, 0, 0]
amp = [1., -2, 0.5]

# parabolic event
tp0 = [0.13]
px = [0]
pxx = [5e-7]
ampp = [0.7]

# create axis
taxis, taxis2, xaxis, yaxis = pylops.utils.seismicevents.makeaxis(par)

# create wavelet
wav = ricker(taxis[:41], f0=par['f0'])[0]

# generate model
y = pylops.utils.seismicevents.linear2d(xaxis, taxis, v, t0,
                                         theta, amp, wav)[1] + \
    pylops.utils.seismicevents.parabolic2d(xaxis, taxis, tp0,
                                            px, pxx, ampp, wav)[1]
```

We can now create the `pylops.signalprocessing.Radon2D` operator. We also apply its adjoint to the data to obtain a representation of those 3 linear events overlapping to a parabolic event in the Radon domain. Similarly, we feed the operator to a sparse solver like `pylops.optimization.sparsity.FISTA` to obtain a sparse representation of the data in the Radon domain. At this point we try to filter out the unwanted event. We can see how this is much easier for the sparse transform as each event has a much more compact representation in the Radon domain than for the adjoint transform.

```
# radon operator
npx = 61
pxmax = 5e-4
px = np.linspace(-pxmax, pxmax, npx)

Rop = pylops.signalprocessing.Radon2D(taxis, xaxis, px, kind='linear',
                                         interp='nearest', centereddh=False,
                                         dtype='float64')

# adjoint Radon transform
xadj = Rop.H * y.flatten()
xadj = xadj.reshape(npx, par['nt'])

# sparse Radon transform
xinv, niter, cost = \
    pylops.optimization.sparsity.FISTA(Rop, y.flatten(), 15,
                                         eps=1e1, returninfo=True)
xinv = xinv.reshape(npx, par['nt'])

# filtering
xfilt = np.zeros_like(xadj)
xfilt[npx//2-3:npx//2+4] = xadj[npx//2-3:npx//2+4]

yfilt = Rop * xfilt.flatten()
yfilt = yfilt.reshape(par['nx'], par['nt'])

# filtering on sparse transform
xinvfilt = np.zeros_like(xinv)
xinvfilt[npx//2-3:npx//2+4] = xinv[npx//2-3:npx//2+4]

yinvfilt = Rop * xinvfilt.flatten()
yinvfilt = yinvfilt.reshape(par['nx'], par['nt'])
```

Finally we visualize our results.

```
fig, axs = plt.subplots(1, 5, sharey=True, figsize=(12, 5))
axs[0].imshow(y.T, cmap='gray',
               vmin=-np.abs(y).max(), vmax=np.abs(y).max(),
               extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0].set_title('Data')
axs[0].axis('tight')
axs[1].imshow(xadj.T, cmap='gray',
               vmin=-np.abs(xadj).max(), vmax=np.abs(xadj).max(),
               extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1].axvline(px[npx//2-3], color='r', linestyle='--')
axs[1].axvline(px[npx//2+3], color='r', linestyle='--')
axs[1].set_title('Radon')
axs[1].axis('tight')
axs[2].imshow(yfilt.T, cmap='gray',
               vmin=-np.abs(yfilt).max(), vmax=np.abs(yfilt).max(),
               extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[2].set_title('Filtered data')
axs[2].axis('tight')
axs[3].imshow(xinv.T, cmap='gray',
               vmin=-np.abs(xinv).max(), vmax=np.abs(xinv).max(),
               extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[3].axvline(px[npx//2-3], color='r', linestyle='--')
```

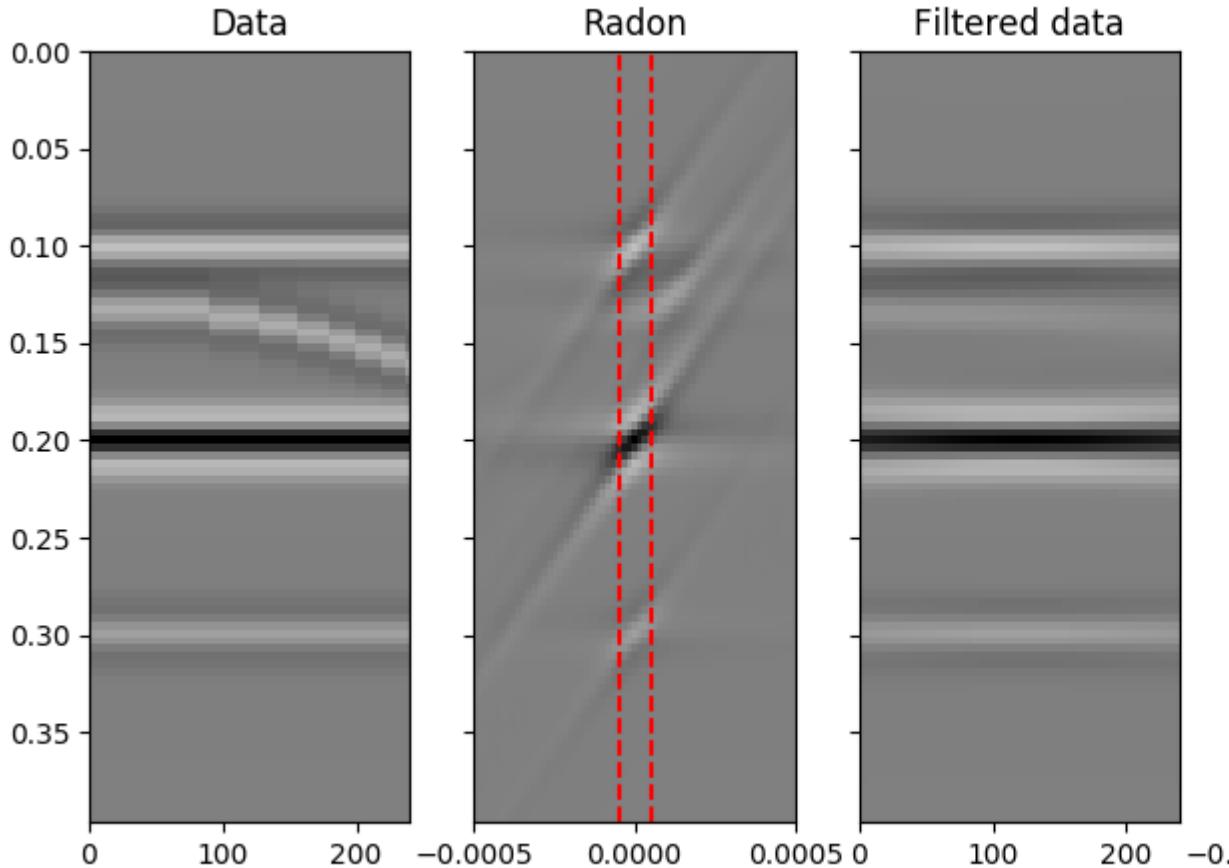
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```

axs[3].axvline(px[npx//2+3], color='r', linestyle='--')
axs[3].set_title('Sparse Radon')
axs[3].axis('tight')
axs[4].imshow(yinvfilt.T, cmap='gray',
              vmin=-np.abs(y).max(), vmax=np.abs(y).max(),
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[4].set_title('Sparse filtered data')
axs[4].axis('tight')

```



Out:

```
(0.0, 240.0, 0.396, 0.0)
```

As expected, the Radon domain is a suitable domain for this type of filtering and the sparse transform improves the ability to filter out parabolic events with small curvature.

On the other hand, it is important to note that we have not been able to correctly preserve the amplitudes of each event. This is because the sparse Radon transform can only identify a sparsest response that explain the data within a certain threshold. For this reason an more suitable approach for preserving amplitudes could be to apply a parabolic Radon transform with the aim of reconstructing only the unwanted event and apply an adaptive subtraction between the input data and the reconstructed unwanted event.

**Total running time of the script:** ( 0 minutes 31.691 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.12 12. Seismic regularization

The problem of *seismic data regularization* (or interpolation) is a very simple one to write, yet ill-posed and very hard to solve.

The forward modelling operator is a simple `pylops.Restriction` operator which is applied along the spatial direction(s).

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

Here  $\mathbf{y} = [\mathbf{y}_{R1}^T, \mathbf{y}_{R2}^T, \dots, \mathbf{y}_{RN^T}]^T$  where each vector  $\mathbf{y}_{Ri}$  contains all time samples recorded in the seismic data at the specific receiver  $R_i$ . Similarly,  $\mathbf{x} = [\mathbf{x}_{r1}^T, \mathbf{x}_{r2}^T, \dots, \mathbf{x}_{rM}^T]$ , contains all traces at the regularly and finely sampled receiver locations  $r_i$ .

By inverting such an equation we can create a regularized data with densely and regularly spatial direction(s).

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import convolve

import pylops
from pylops.utils.wavelets import ricker
from pylops.utils.seismicevents import makeaxis, linear2d

np.random.seed(0)
plt.close('all')
```

Let's start by creating a very simple 2d data composed of 3 linear events input parameters

```
par = {'ox':0, 'dx':2, 'nx':70,
       'ot':0, 'dt':0.004, 'nt':80,
       'f0': 20}

v = 1500
t0_m = [0.1, 0.2, 0.28]
theta_m = [0, 30, -80]
phi_m = [0]
amp_m = [1., -2, 0.5]

# axis
taxis, t2, xaxis, y = makeaxis(par)

# wavelet
wav = ricker(taxis[:41], f0=par['f0'])[0]

# model
_, x = linear2d(xaxis, taxis, v, t0_m, theta_m, amp_m, wav)
```

We can now define the spatial locations along which the data has been sampled. In this specific example we will assume that we have access only to 40% of the ‘original’ locations.

```
perc_subsampling = 0.6
nbsub = int(np.round(par['nx']*perc_subsampling))
```

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```
iava = np.sort(np.random.permutation(np.arange(par['nx']))[:nbsub])

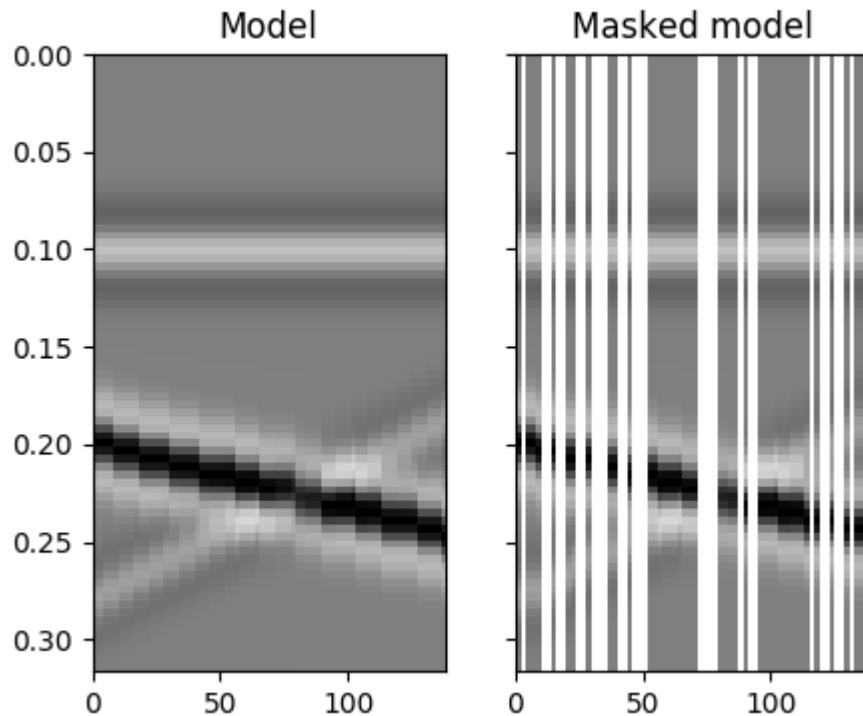
# restriction operator
Rop = pylops.Restriction(par['nx']*par['nt'],
                           iava, dims=(par['nx'], par['nt']),
                           dir=0, dtype='float64')

# data
y = Rop*x.ravel()
y = y.reshape(nbsub, par['nt'])

# mask
ymask = Rop.mask(x.flatten())

# inverse
xinv = Rop / y.ravel()
xinv = xinv.reshape(par['nx'], par['nt'])

fig, axs = plt.subplots(1, 2, sharey=True, figsize=(5, 4))
axs[0].imshow(x.T, cmap='gray', vmin=-2, vmax=2,
               extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0].set_title('Model')
axs[0].axis('tight')
axs[1].imshow(ymask.T, cmap='gray', vmin=-2, vmax=2,
               extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[1].set_title('Masked model')
axs[1].axis('tight')
```



Out:

```
(0.0, 138.0, 0.316, 0.0)
```

As we can see, inverting the restriction operator is not possible without adding any prior information into the inverse problem. In the following we will consider two possible routes:

- regularized inversion with second derivative along the spatial axis

$$J = \|\mathbf{y} - \mathbf{R}\mathbf{x}\|_2 + \epsilon_{\nabla}^2 \|\nabla \mathbf{x}\|_2$$

- sparsity-promoting inversion with `pylops.FFT2` operator used as sparsifying transform

$$J = \|\mathbf{y} - \mathbf{R}\mathbf{F}^H\mathbf{x}\|_2 + \epsilon \|\mathbf{F}^H\mathbf{x}\|_1$$

```
# smooth inversion
D2op = pylops.SecondDerivative(par['nx']*par['nt'],
                                dims=(par['nx'], par['nt']),
                                dir=0, dtype='float64')

xsmooth, _, _ = \
    pylops.waveeqprocessing.SeismicInterpolation(y, par['nx'], iava,
                                                kind='spatial',
                                                **dict(epsRs=[np.sqrt(0.1)],
                                                       damp=np.sqrt(1e-4),
                                                       iter_lim=50, show=0))

# sparse inversion with FFT2
nfft = 2**8
FFTop = pylops.signalprocessing.FFT2D(dims=[par['nx'], par['nt']],
                                       nffts=[nfft, nfft],
                                       sampling=[par['dx'], par['dt']])

X = FFTop*x.flatten()
X = np.reshape(X, (nfft, nfft))

x11, X11, cost = \
    pylops.waveeqprocessing.SeismicInterpolation(y, par['nx'], iava, kind='fk',
                                                nffts=(nfft, nfft),
                                                sampling=(par['dx'],
                                                          par['dt']),
                                                **dict(niter=50, eps=1e-1,
                                                       returninfo=True))

fig, axs = plt.subplots(1, 4, sharey=True, figsize=(13, 4))
axs[0].imshow(x.T, cmap='gray', vmin=-2, vmax=2,
               extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0].set_title('Model')
axs[0].axis('tight')
axs[1].imshow(ymask.T, cmap='gray', vmin=-2, vmax=2,
               extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[1].set_title('Masked model')
axs[1].axis('tight')
axs[2].imshow(xsmooth.T, cmap='gray', vmin=-2, vmax=2,
               extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[2].set_title('Smoothed model')
axs[2].axis('tight')
axs[3].imshow(x11.T, cmap='gray', vmin=-2, vmax=2,
```

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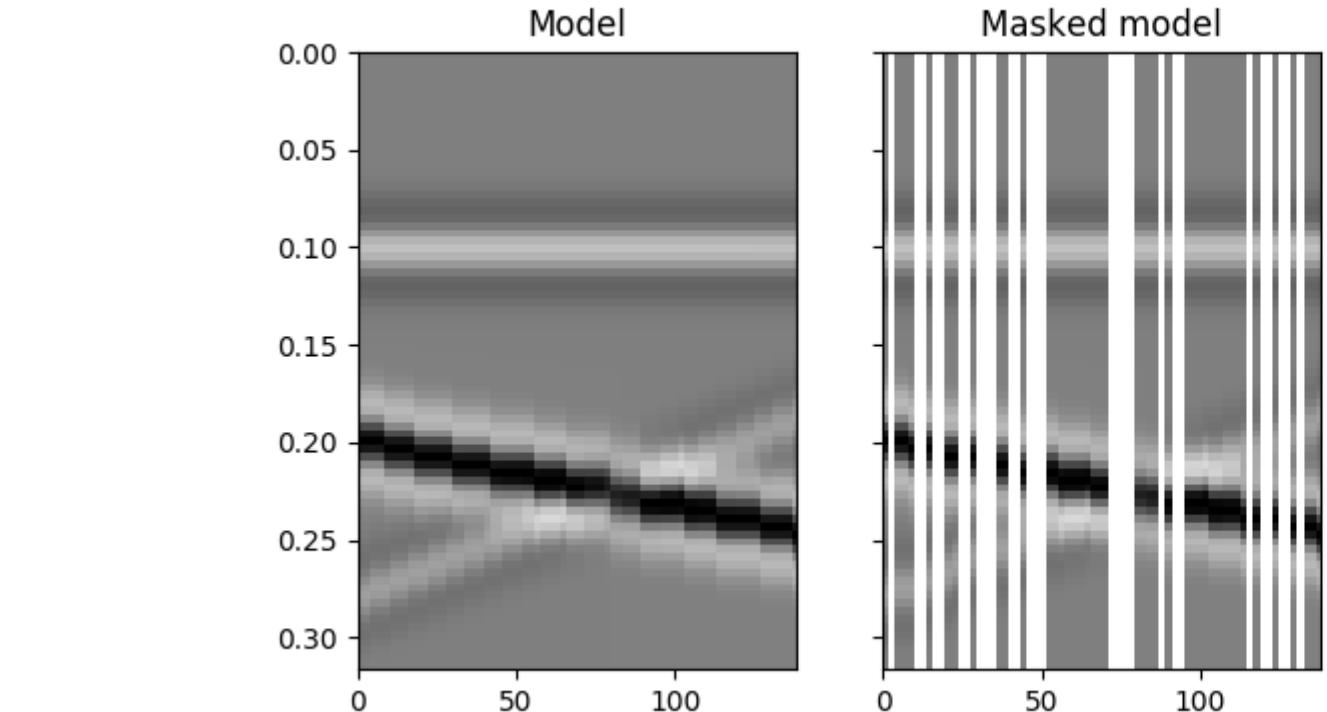
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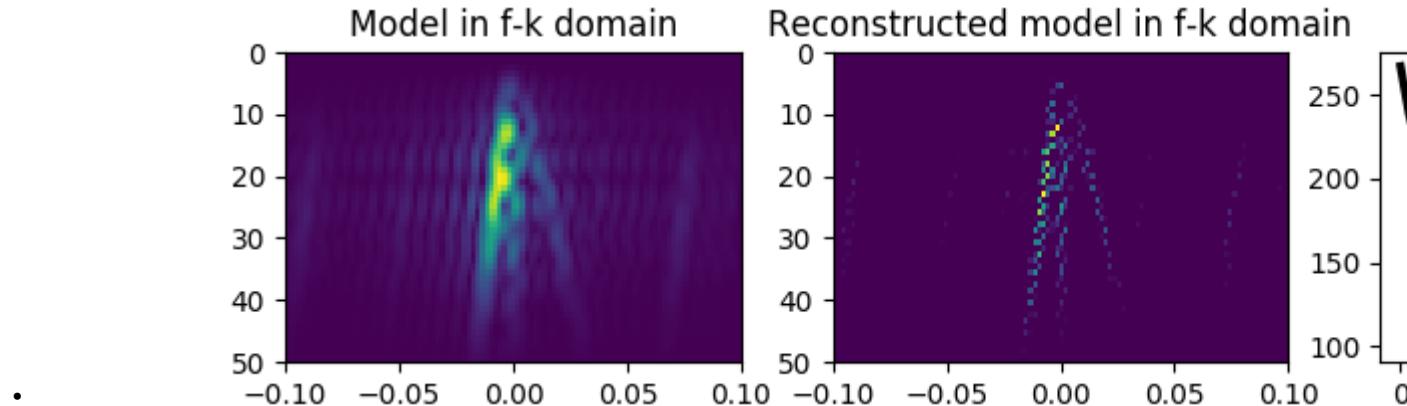
```

        extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[3].set_title('L1 model')
axs[3].axis('tight')

fig, axs = plt.subplots(1, 3, figsize=(10, 2))
axs[0].imshow(np.fft.fftshift(np.abs(X[:, :nfft//2-1])), axes=0).T,
              extent=(np.fft.fftshift(FFTop.f1)[0],
                      np.fft.fftshift(FFTop.f1)[-1],
                      FFTop.f2[nfft // 2 - 1], FFTop.f2[0]))
axs[0].set_title('Model in f-k domain')
axs[0].axis('tight')
axs[0].set_xlim(-0.1, 0.1)
axs[0].set_ylim(50, 0)
axs[1].imshow(np.fft.fftshift(np.abs(X11[:, :nfft // 2 - 1])), axes=0).T,
              extent=(np.fft.fftshift(FFTop.f1)[0],
                      np.fft.fftshift(FFTop.f1)[-1],
                      FFTop.f2[nfft // 2 - 1], FFTop.f2[0],))
axs[1].set_title('Reconstructed model in f-k domain')
axs[1].axis('tight')
axs[1].set_xlim(-0.1, 0.1)
axs[1].set_ylim(50, 0)
axs[2].plot(cost, 'k', lw=3)
axs[2].set_title('FISTA convergence')

```





Out:

```
Text(0.5, 1.0, 'FISTA convergence')
```

We see how adding prior information to the inversion can help improving the estimate of the regularized seismic data. Nevertheless, in both cases the reconstructed data is not perfect. A better sparsifying transform could in fact be chosen here to be the linear `pylops.signalprocessing.Radon2D` transform in spite of the `pylops.FFT2` transform.

```
npx = 40
pxmax = 1e-3
px = np.linspace(-pxmax, pxmax, npx)
Radop = pylops.signalprocessing.Radon2D(taxis, xaxis, px, engine='numba')

RRop = Rop*Radop

# adjoint
Xadj_fromx = Radop.H*x.flatten()
Xadj_fromx = Xadj_fromx.reshape(npx, par['nt'])

Xadj = RRop.H*y.flatten()
Xadj = Xadj.reshape(npx, par['nt'])

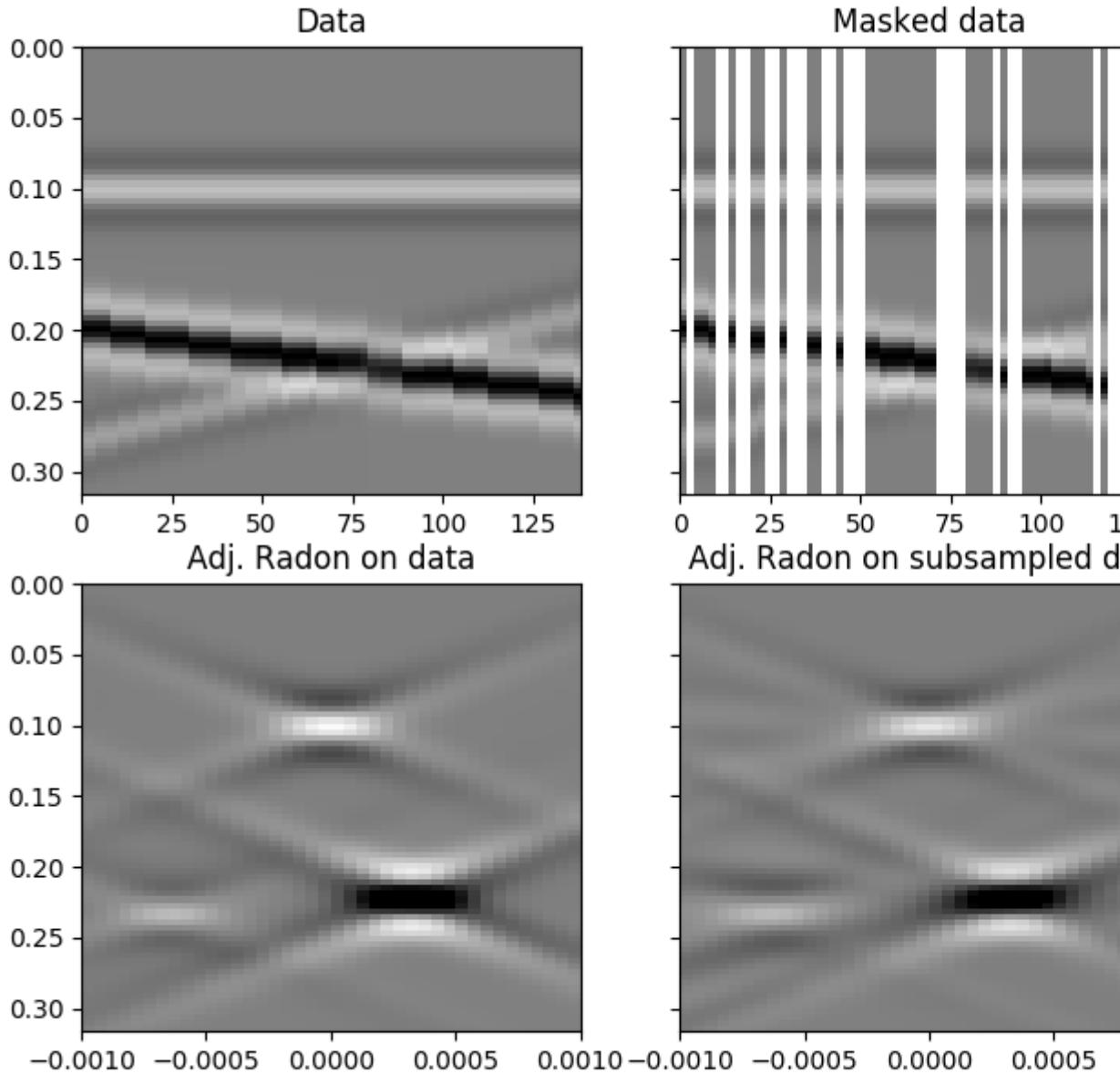
# L1 inverse
x11, X11, cost = \
    pylops.waveeqprocessing.SeismicInterpolation(y, par['nx'], iava,
                                                kind='radon-linear',
                                                spataxis=xaxis,
                                                taxis=taxis, paxis=px,
                                                centereddh=True,
                                                **dict(niter=50, eps=1e-1,
                                                       returninfo=True))

fig, axs = plt.subplots(2, 3, sharey=True, figsize=(12, 7))
axs[0][0].imshow(x.T, cmap='gray', vmin=-2, vmax=2,
                  extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][0].set_title('Data', fontsize=12)
axs[0][0].axis('tight')
axs[0][1].imshow(ymask.T, cmap='gray', vmin=-2, vmax=2,
                  extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][1].set_title('Masked data', fontsize=12)
axs[0][1].axis('tight')
axs[0][2].imshow(x11.T, cmap='gray', vmin=-2, vmax=2,
```

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```
        extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][2].set_title('Reconstructed data', fontsize=12)
axs[0][2].axis('tight')
axs[1][0].imshow(Xadj_fromx.T, cmap='gray', vmin=-70, vmax=70,
                  extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][0].set_title('Adj. Radon on data', fontsize=12)
axs[1][0].axis('tight')
axs[1][1].imshow(Xadj.T, cmap='gray', vmin=-50, vmax=50,
                  extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][1].set_title('Adj. Radon on subsampled data', fontsize=12)
axs[1][1].axis('tight')
axs[1][2].imshow(Xll.T, cmap='gray', vmin=-0.2, vmax=0.2,
                  extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][2].set_title('Inverse Radon on subsampled data', fontsize=12)
axs[1][2].axis('tight')
```



Out:

```
(-0.001, 0.001, 0.316, 0.0)
```

Finally, let's take now a more realistic dataset. We will use once again the linear `pylops.signalprocessing.Radon2D` transform but we will take advantage of the `pylops.signalprocessing.Sliding2D` operator to perform such a transform locally instead of globally to the entire dataset.

```
inputfile = '../testdata/marchenko/input.npz'
```

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```

inputdata = np.load(inputfile)

x = inputdata['R'][50, :, ::2]
x = x/np.abs(x).max()
taxis, xaxis = inputdata['t'][::2], inputdata['r'][0]

par = {}
par['nx'], par['nt'] = x.shape
par['dx'] = inputdata['r'][0, 1] - inputdata['r'][0, 0]
par['dt'] = inputdata['t'][1] - inputdata['t'][0]

# add wavelet
wav = inputdata['wav'][::2]
wav_c = np.argmax(wav)
x = np.apply_along_axis(convolve, 1, x, wav, mode='full')
x = x[:, wav_c:][:, :par['nt']]

# gain
gain = np.tile((taxis**2)[:, np.newaxis], (1, par['nx'])).T
x = x*gain

# subsampling locations
perc_subsampling = 0.5
Nsub = int(np.round(par['nx']*perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(par['nx']))[:Nsub])

# restriction operator
Rop = pylops.Restriction(par['nx']*par['nt'], iava,
                           dims=(par['nx'], par['nt']),
                           dir=0, dtype='float64')

y = Rop*x.flatten()
xadj = Rop.H*y.flatten()

y = y.reshape(Nsub, par['nt'])
xadj = xadj.reshape(par['nx'], par['nt'])

# apply mask
ymask = Rop.mask(x.flatten())

# sliding windows with radon transform
dx = par['dx']
nwins = 4
nwin = 27
nover = 3
npx = 31
pxmax = 5e-4
px = np.linspace(-pxmax, pxmax, npx)
dimsd = x.shape
dims = (nwins*npx, dimsd[1])

Op = \
    pylops.signalprocessing.Radon2D(taxis, np.linspace(-par['dx']*nwin//2,
                                                       par['dx']*nwin//2,
                                                       nwin),
                                      px, centereddh=True, kind='linear',
                                      engine='numba')

```

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```

Slidop = pylops.signalprocessing.Sliding2D(Op, dims, dimssd, nwin, nover,
                                            tapertype='cosine', design=True)

# adjoint
RSop = Rop*Slidop

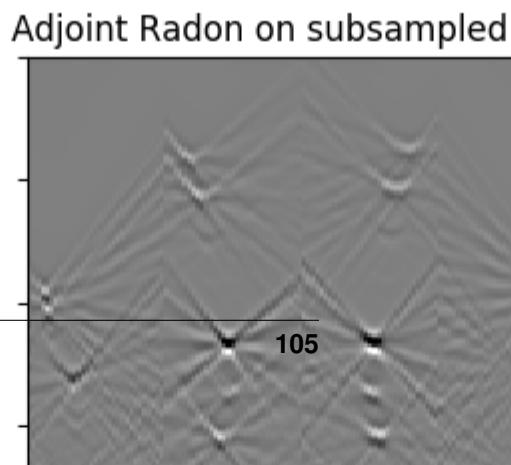
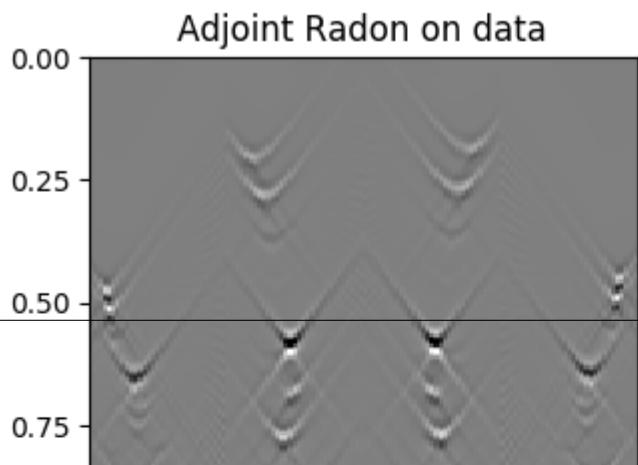
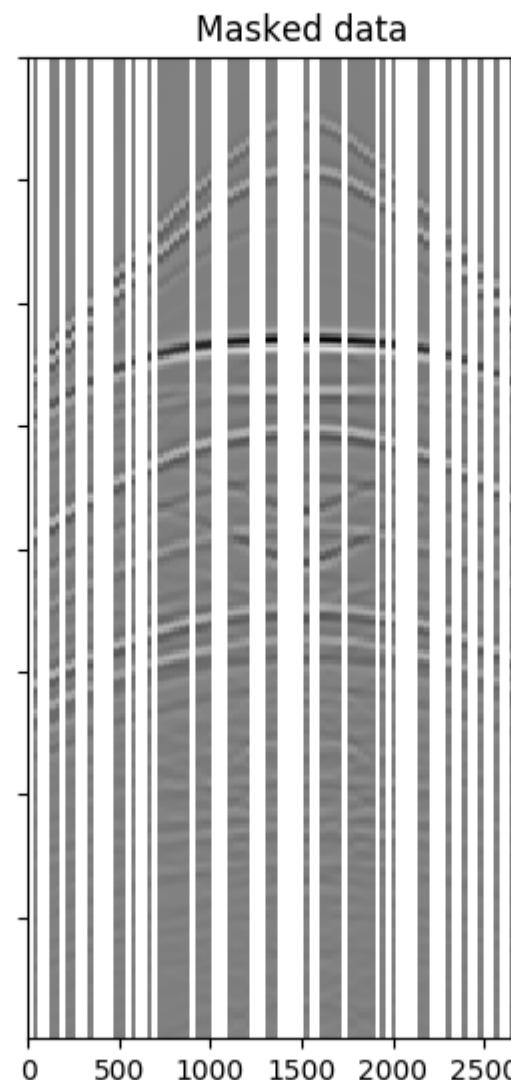
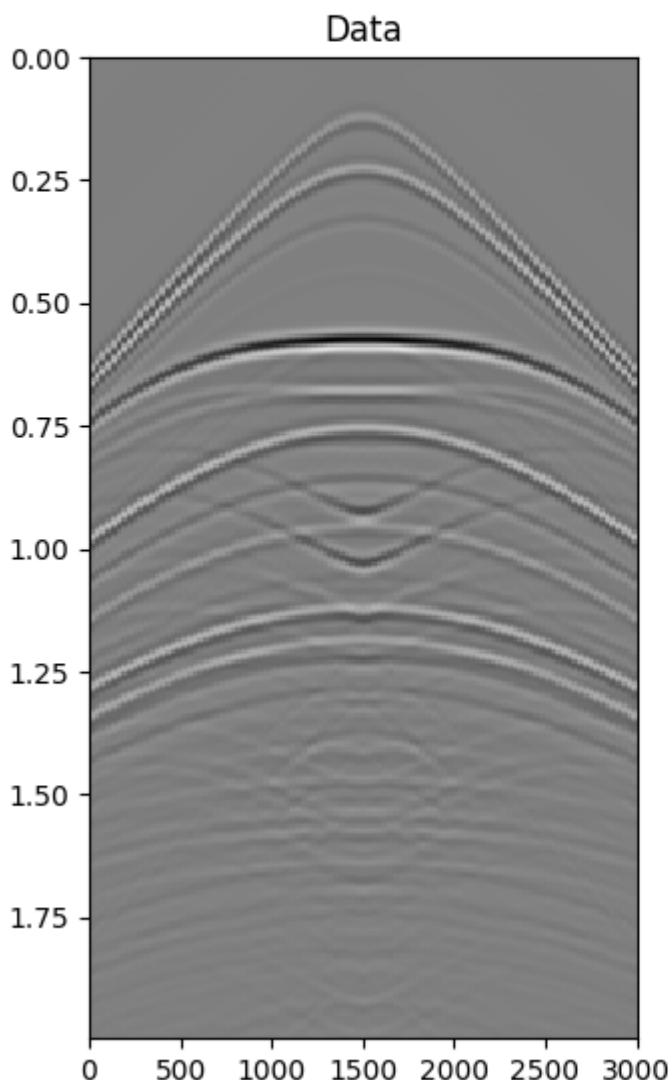
Xadj_fromx = Slidop.H*x.flatten()
Xadj_fromx = Xadj_fromx.reshape(npx*nwins, par['nt'])

Xadj = RSop.H*y.flatten()
Xadj = Xadj.reshape(npx*nwins, par['nt'])

# inverse
x11, X11, _ = \
    pylops.waveeqprocessing.SeismicInterpolation(y, par['nx'], iava,
                                                kind='sliding',
                                                spataxis=xaxis,
                                                taxis=taxis, paxis=px,
                                                nwins=nwins, nwin=nwin,
                                                nover=nover,
                                                **dict(niter=50, eps=1e-2))

fig, axs = plt.subplots(2, 3, sharey=True, figsize=(12, 14))
axs[0][0].imshow(x.T, cmap='gray', vmin=-0.1, vmax=0.1,
                  extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][0].set_title('Data')
axs[0][0].axis('tight')
axs[0][1].imshow(ymask.T, cmap='gray', vmin=-0.1, vmax=0.1,
                  extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][1].set_title('Masked data')
axs[0][1].axis('tight')
axs[0][2].imshow(x11.T, cmap='gray', vmin=-0.1, vmax=0.1,
                  extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][2].set_title('Reconstructed data')
axs[0][2].axis('tight')
axs[1][0].imshow(Xadj_fromx.T, cmap='gray', vmin=-1, vmax=1,
                  extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][0].set_title('Adjoint Radon on data')
axs[1][0].axis('tight')
axs[1][1].imshow(Xadj.T, cmap='gray', vmin=-0.6, vmax=0.6,
                  extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][1].set_title('Adjoint Radon on subsampled data')
axs[1][1].axis('tight')
axs[1][2].imshow(X11.T, cmap='gray', vmin=-0.03, vmax=0.03,
                  extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][2].set_title('Inverse Radon on subsampled data')
axs[1][2].axis('tight')

```



Out:

```
(-0.0005, 0.0005, 1.995, 0.0)
```

As expected the linear `pylops.signalprocessing.Radon2D` is able to locally explain events in the input data and leads to a satisfactory recovery. Note that increasing the number of iterations and sliding windows can further refine the result, especially the accuracy of weak events, as shown in this companion [notebook](#).

**Total running time of the script:** ( 0 minutes 5.112 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.13 13. Deghosting

Single-component seismic data can be decomposed in their up- and down-going constituents in a model driven fashion. This task can be achieved by defining an f-k propagator (or ghost model) and solving an inverse problem as described in `pylops.waveeqprocessing.Deghosting`.

```
# sphinx_gallery_thumbnail_number = 3
import numpy as np
import matplotlib.pyplot as plt
from scipy.sparse.linalg import lsqr

import pylops

np.random.seed(0)
plt.close('all')
```

Let's start by loading the input dataset and geometry

```
inputfile = '../testdata/updown/input.npz'
inputdata = np.load(inputfile)

vel_sep = 2400.0 # velocity at separation level
clip = 1e-1 # plotting clip

# Receivers
r = inputdata['r']
nr = r.shape[1]
dr = r[0, 1]-r[0, 0]

# Sources
s = inputdata['s']

# Model
rho = inputdata['rho']

# Axes
t = inputdata['t']
nt, dt = len(t), t[1]-t[0]
x, z = inputdata['x'], inputdata['z']
dx, dz = x[1] - x[0], z[1] - z[0]

# Data
```

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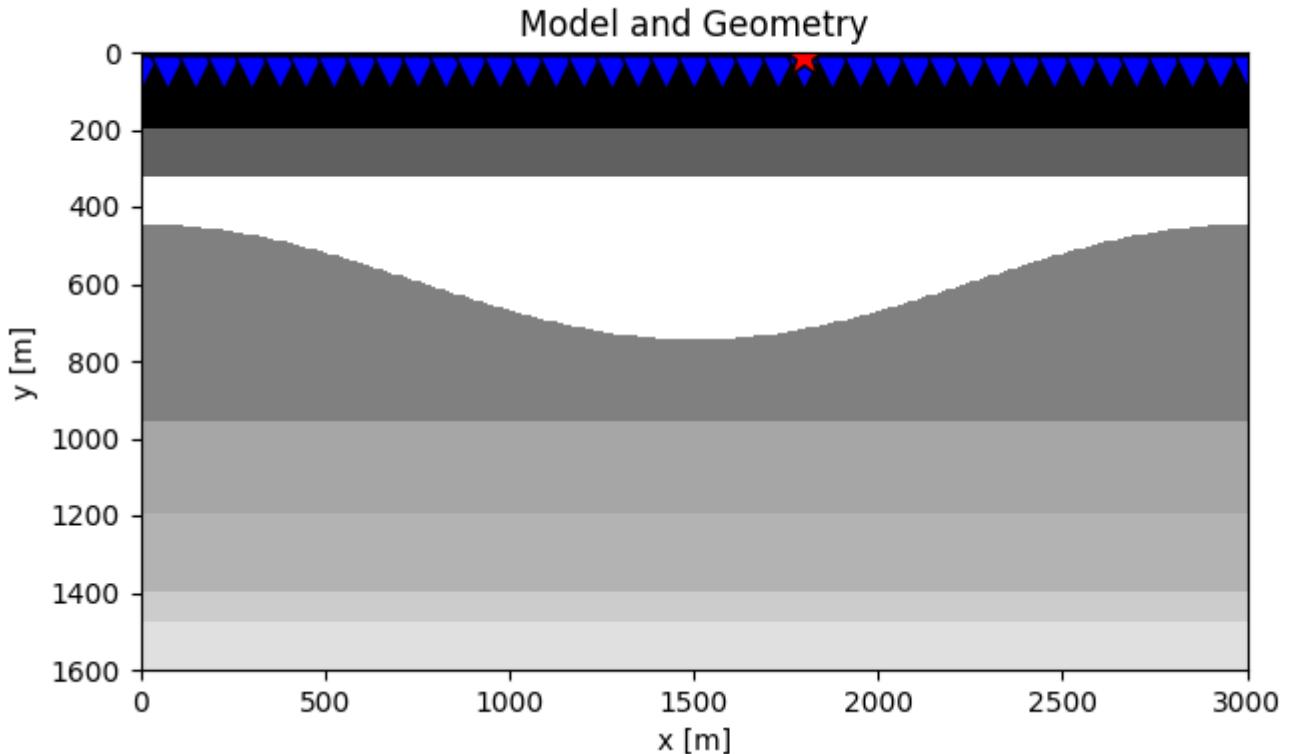
(continued from previous page)

```

p = inputdata['p'].T
p /= p.max()

fig = plt.figure(figsize=(9, 4))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=4)
ax2 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(rho, cmap='gray', extent=(x[0], x[-1], z[-1], z[0]))
ax1.scatter(r[0, ::5], r[1, ::5], marker='v', s=150, c='b', edgecolors='k')
ax1.scatter(s[0], s[1], marker='*', s=250, c='r', edgecolors='k')
ax1.axis('tight')
ax1.set_xlabel('x [m]')
ax1.set_ylabel('y [m]')
ax1.set_title('Model and Geometry')
ax1.set_xlim(x[0], x[-1])
ax1.set_ylim(z[-1], z[0])
ax2.plot(rho[:, len(x)//2], z, 'k', lw=2)
ax2.set_ylim(z[-1], z[0])
ax2.set_yticks([], [])

```



Out:

[]

To be able to deghost the input dataset, we need to remove its direct arrival. In this example we will create a mask based on the analytical travelttime of the direct arrival.

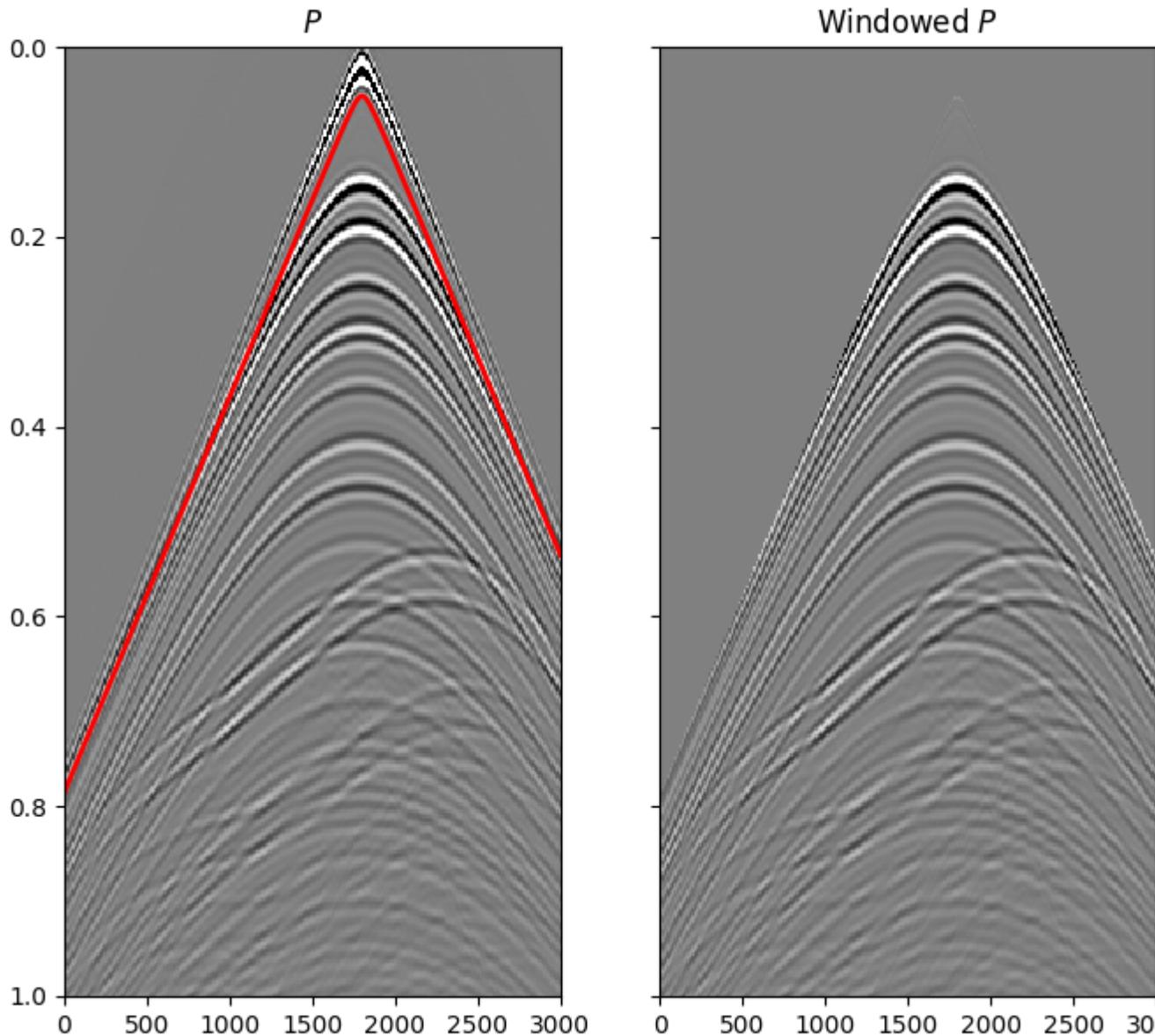
```
direct = np.sqrt(np.sum((s[:, np.newaxis]-r)**2, axis=0))/vel_sep
```

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```
# Window
off = 0.035
direct_off = direct + off
win = np.zeros((nt, nr))
iwin = np.round(direct_off/dt).astype(np.int)
for i in range(nr):
    win[iwin[i]:, i] = 1

fig, axs = plt.subplots(1, 2, sharey=True, figsize=(8, 7))
axs[0].imshow(p.T, cmap='gray', vmin=-clip*np.abs(p).max(),
               vmax=clip*np.abs(p).max(),
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[0].plot(r[0], direct_off, 'r', lw=2)
axs[0].set_title(r'$P$')
axs[0].axis('tight')
axs[1].imshow(win * p.T, cmap='gray', vmin=-clip*np.abs(p).max(),
               vmax=clip*np.abs(p).max(),
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[1].set_title(r'Windowed $P$')
axs[1].axis('tight')
axs[1].set_ylim(1, 0)
```



Out:

```
(1, 0)
```

We can now perform deghosting

```
pup, pdown = \
    pylops.waveeqprocessing.Deghosting(p.T, nt, nr, dt, dr, vel_sep,
                                      r[1, 0] + dz, win=win,
```

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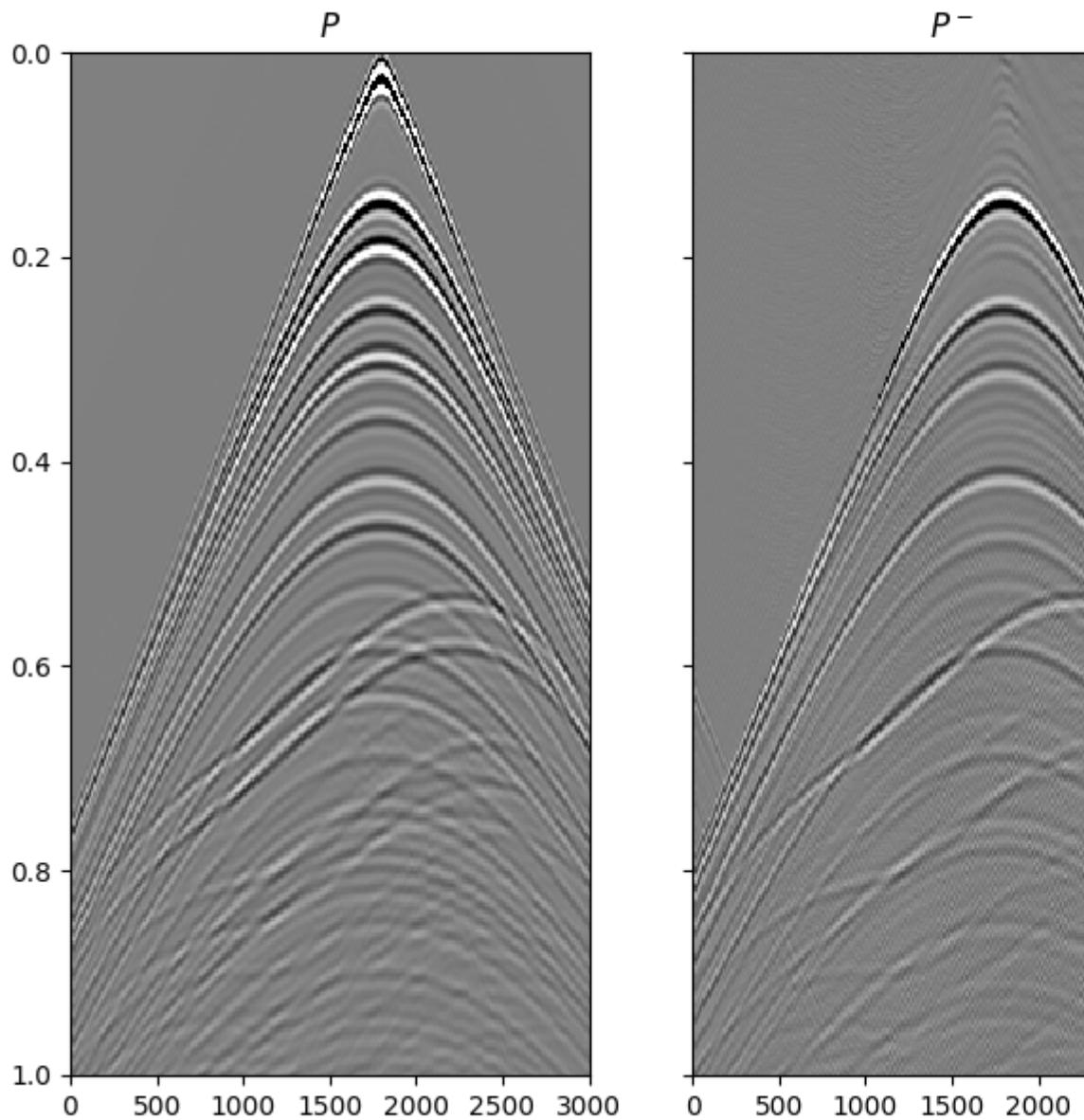
(continued from previous page)

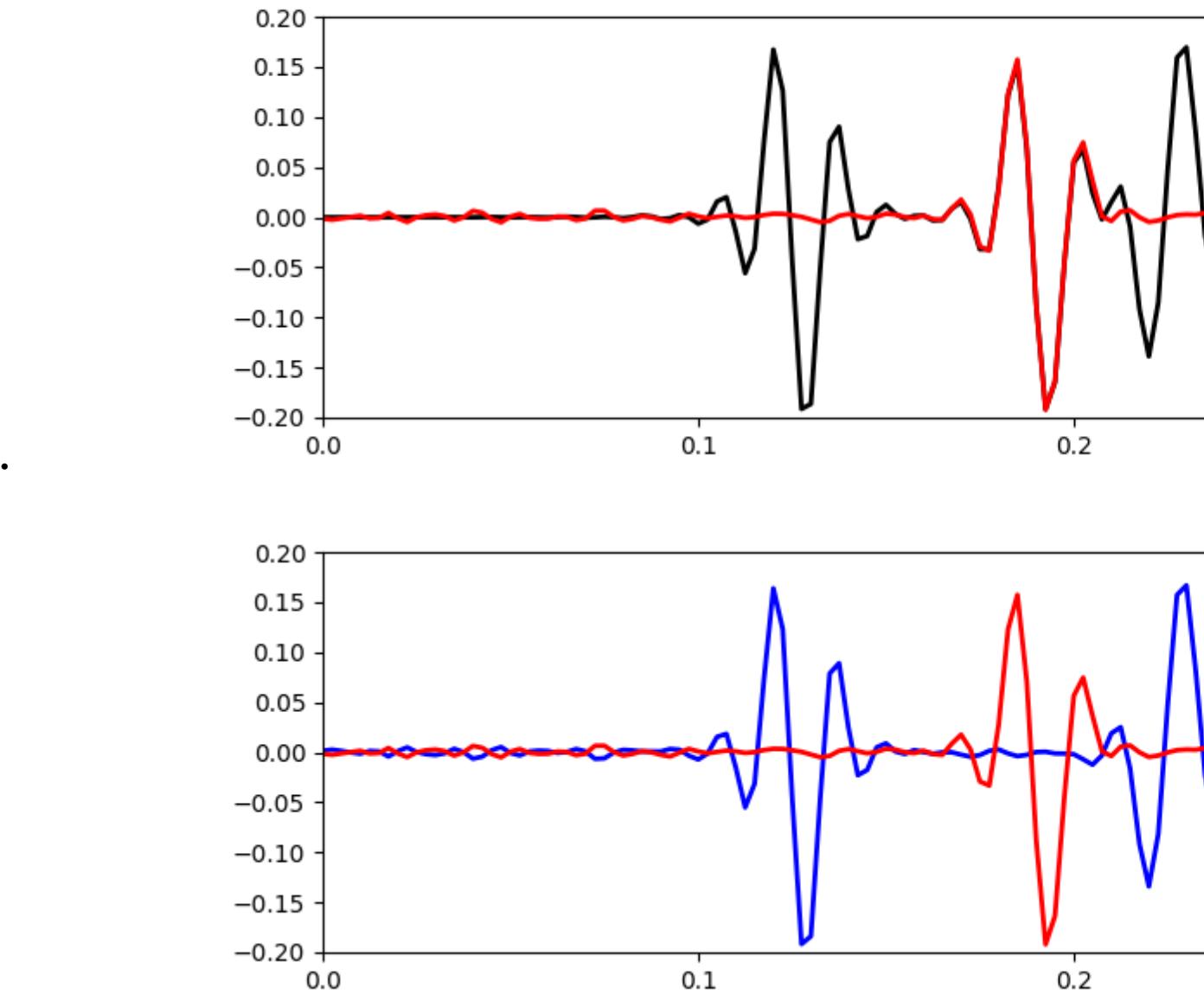
```
npad=5, ntaper=11, solver=lsqr,
dottest=False, dtype='complex128',
**dict(damp=1e-10, iter_lim=60))

fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 7))
axs[0].imshow(p.T, cmap='gray', vmin=-clip * np.abs(p).max(),
               vmax=clip * np.abs(p).max(),
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[0].set_title(r'$P$')
axs[0].axis('tight')
axs[1].imshow(pup, cmap='gray', vmin=-clip * np.abs(p).max(),
               vmax=clip * np.abs(p).max(),
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[1].set_title(r'$P^-$')
axs[1].axis('tight')
axs[2].imshow(pdown, cmap='gray', vmin=-clip * np.abs(p).max(),
               vmax=clip * np.abs(p).max(),
               extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[2].set_title(r'$P^+$')
axs[2].axis('tight')
axs[2].set_ylim(1, 0)

plt.figure(figsize=(14, 3))
plt.plot(t, p[nr // 2], 'k', lw=2, label=r'$p$')
plt.plot(t, pup[:, nr // 2], 'r', lw=2, label=r'$P^-$')
plt.xlim(0, t[200])
plt.ylim(-0.2, 0.2)
plt.legend()

plt.figure(figsize=(14, 3))
plt.plot(t, pdown[:, nr // 2], 'b', lw=2, label=r'$P^+$')
plt.plot(t, pup[:, nr // 2], 'r', lw=2, label=r'$P^-$')
plt.xlim(0, t[200])
plt.ylim(-0.2, 0.2)
plt.legend()
```





Out:

```
<matplotlib.legend.Legend object at 0x7fcd6ea56390>
```

To see more examples head over to the following notebook: [notebook1](#).

**Total running time of the script:** ( 0 minutes 5.885 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.14 14. Seismic wavefield decomposition

Multi-component seismic data can be decomposed in their up- and down-going constituents in a purely data driven fashion. This task can be accurately achieved by linearly combining the input pressure and particle velocity data

in the frequency-wavenumber described in details in [pylops.waveeqprocessing.UpDownComposition2D](#) and [pylops.waveeqprocessing.WavefieldDecomposition](#).

In this tutorial we will consider a simple synthetic data composed of six events (three up-going and three down-going). We will first combine them to create pressure and particle velocity data and then show how we can retrieve their directional constituents both by directly combining the input data as well as by setting an inverse problem. The latter approach results vital in case of spatial aliasing, as applying simple scaled summation in the frequency-wavenumber would result in sub-optimal decomposition due to the superposition of different frequency-wavenumber pairs at some (aliased) locations.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker
from pylops.utils.seismicevents import makeaxis, hyperbolic2d

np.random.seed(0)
plt.close('all')
```

Let's first the input up- and down-going wavefields

```
par = {'ox':-220, 'dx':5, 'nx':89,
       'ot':0, 'dt':0.004, 'nt':200,
       'f0': 40}

t0_plus = np.array([0.2, 0.5, 0.7])
t0_minus = t0_plus + 0.04
vrms = np.array([1400., 1500., 2000.])
amp = np.array([1., -0.6, 0.5])
vel_sep = 1000.0 # velocity at separation level
rho_sep = 1000.0 # density at separation level

# Create axis
t, t2, x, y = makeaxis(par)

# Create wavelet
wav = ricker(t[:41], f0=par['f0'])[0]

# Create data
_, p_minus = hyperbolic2d(x, t, t0_minus, vrms, amp, wav)
_, p_plus = hyperbolic2d(x, t, t0_plus, vrms, amp, wav)
```

We can now combine them to create pressure and particle velocity data

```
critical = 1.1
ntaper = 51
nfft = 2**10

# 2d fft operator
FFTop = pylops.signalprocessing.FFT2D(dims=[par['nx'], par['nt']],
                                         nffts=[nfft, nfft],
                                         sampling=[par['dx'], par['dt']])

#obliquity factor
[Kx, F] = np.meshgrid(FFTop.f1, FFFtop.f2, indexing='ij')
k = F/vel_sep
```

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```

Kz = np.sqrt((k**2-Kx**2).astype(np.complex))
Kz[np.isnan(Kz)] = 0
OBL = rho_sep*(np.abs(F)/Kz)
OBL[Kz == 0] = 0

mask = np.abs(Kx) < critical*np.abs(F)/vel_sep
OBL *= mask
OBL = filtfilt(np.ones(ntaper)/float(ntaper), 1, OBL, axis=0)
OBL = filtfilt(np.ones(ntaper)/float(ntaper), 1, OBL, axis=1)

# composition operator
UPop = \
    pylops.waveeqprocessingUpDownComposition2D(par['nt'], par['nx'],
                                                par['dt'], par['dx'],
                                                rho_sep, vel_sep,
                                                nffts=(nfft, nfft),
                                                critical=critical*100.,
                                                ntaper=ntaper,
                                                dtype='complex128')

# wavefield modelling
d = UPop * np.concatenate((p_plus.flatten(), p_minus.flatten())).flatten()
d = np.real(d.reshape(2*par['nx'], par['nt']))
p, vz = d[:par['nx']], d[par['nx']:]

# obliquity scaled vz
VZ = FFTop * vz.flatten()
VZ = VZ.reshape(nfft, nfft)

VZ_obl = OBL * VZ
vz_obl = FFTop.H*VZ_obl.flatten()
vz_obl = np.real(vz_obl.reshape(par['nx'], par['nt']))

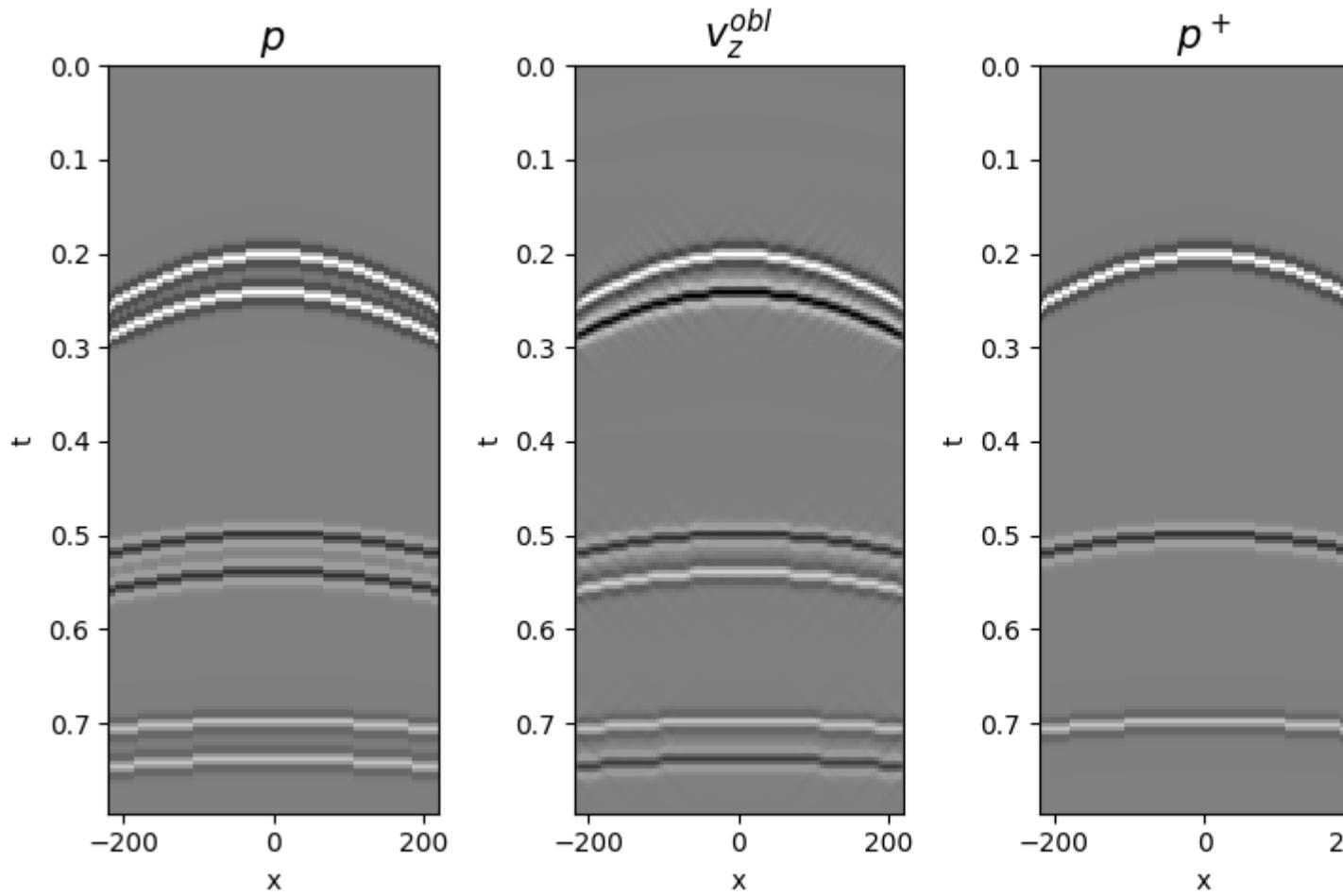
fig, axs = plt.subplots(1, 4, figsize=(10, 5))
axs[0].imshow(p.T, aspect='auto', vmin=-1, vmax=1,
               interpolation='nearest', cmap='gray',
               extent=(x.min(), x.max(), t.max(), t.min()))
axs[0].set_title(r'$p$', fontsize=15)
axs[0].set_xlabel('x')
axs[0].set_ylabel('t')
axs[1].imshow(vz_obl.T, aspect='auto', vmin=-1, vmax=1,
               interpolation='nearest', cmap='gray',
               extent=(x.min(), x.max(), t.max(), t.min()))
axs[1].set_title(r'$v_z^{obl}$', fontsize=15)
axs[1].set_xlabel('x')
axs[1].set_ylabel('t')
axs[2].imshow(p_plus.T, aspect='auto', vmin=-1, vmax=1,
               interpolation='nearest', cmap='gray',
               extent=(x.min(), x.max(), t.max(), t.min()))
axs[2].set_title(r'$p^+$', fontsize=15)
axs[2].set_xlabel('x')
axs[2].set_ylabel('t')
axs[3].imshow(p_minus.T, aspect='auto',
               interpolation='nearest', cmap='gray',
               extent=(x.min(), x.max(), t.max(), t.min()),
               vmin=-1, vmax=1)
axs[3].set_title(r'$p^-$', fontsize=15)

```

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```
axs[3].set_xlabel('x')
axs[3].set_ylabel('t')
plt.tight_layout()
```



Wavefield separation is first performed using the analytical expression for combining pressure and particle velocity data in the wavenumber-frequency domain

```
pup_sep, pdown_sep = \
    pylops.waveeqprocessing.WavefieldDecomposition(p, vz, par['nt'], par['nx'],
                                                   par['dt'], par['dx'],
                                                   rho_sep, vel_sep,
                                                   nffts=(nfft, nfft),
                                                   kind='analytical',
                                                   critical=critical*100,
                                                   ntaper=ntaper,
                                                   dtype='complex128')

fig = plt.figure(figsize=(12, 5))
axs0 = plt.subplot2grid((2, 5), (0, 0), rowspan=2)
axs1 = plt.subplot2grid((2, 5), (0, 1), rowspan=2)
axs2 = plt.subplot2grid((2, 5), (0, 2), colspan=3)
axs3 = plt.subplot2grid((2, 5), (1, 2), colspan=3)
axs0.imshow(pup_sep.T, cmap='gray', vmin=-1, vmax=1,
```

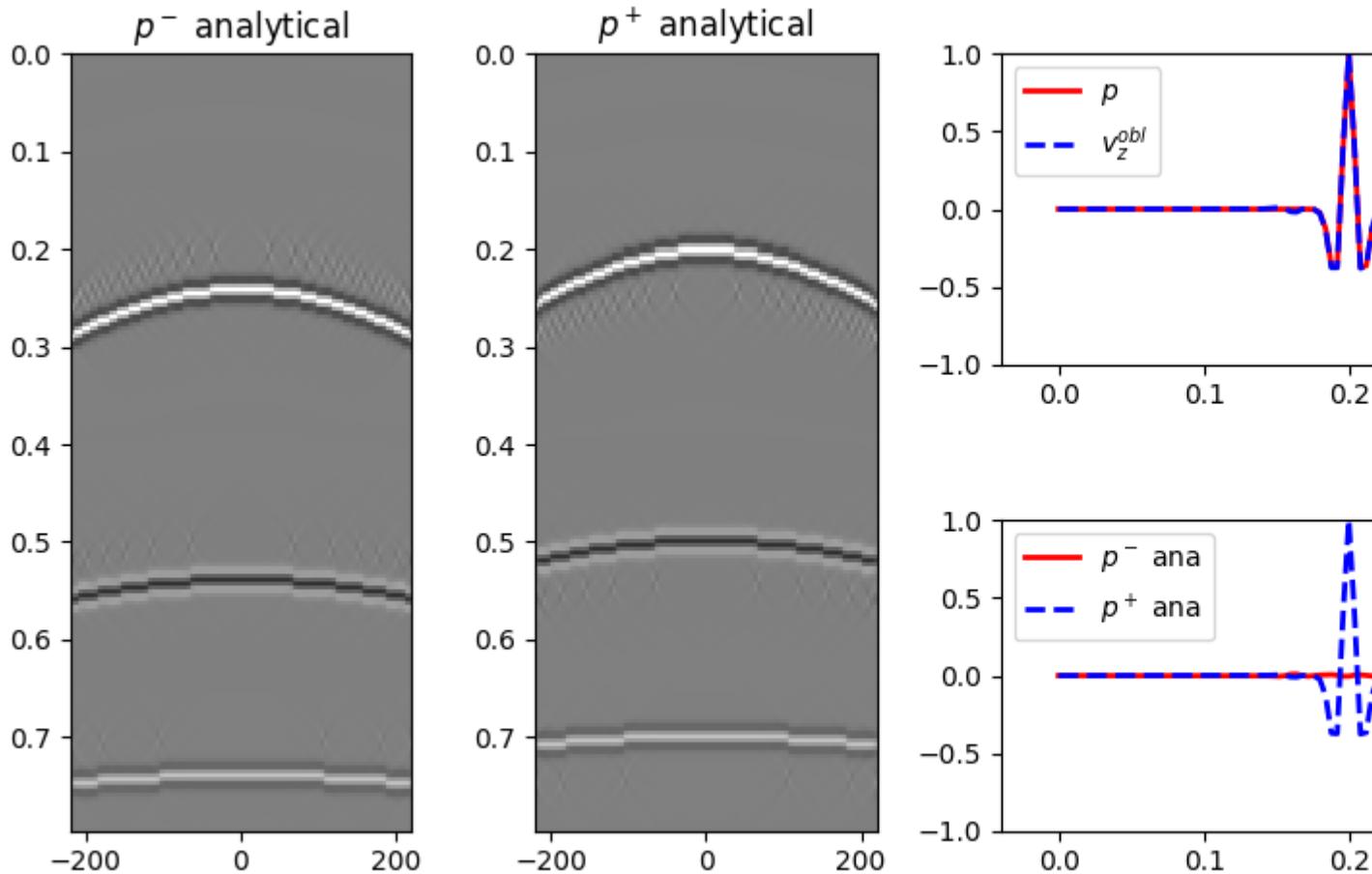
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```

    extent=(x.min(), x.max(), t.max(), t.min())))
axs0.set_title(r'$p^-$ analytical')
axs0.axis('tight')
axs1.imshow(pdown_sep.T, cmap='gray', vmin=-1, vmax=1,
            extent=(x.min(), x.max(), t.max(), t.min())))
axs1.set_title(r'$p^+$ analytical')
axs1.axis('tight')
axs2.plot(t, p[par['nx']]//2, 'r', lw=2, label=r'$p$')
axs2.plot(t, vz_obl[par['nx']]//2, '--b', lw=2, label=r'$v_z^{obl}$')
axs2.set_xlim(-1, 1)
axs2.set_title('Data at x=% .2f' % x[par['nx']]//2)
axs2.set_xlabel('t [s]')
axs2.legend()
axs3.plot(t, pup_sep[par['nx']]//2, 'r', lw=2, label=r'$p^-$ ana')
axs3.plot(t, pdown_sep[par['nx']]//2, '--b', lw=2, label=r'$p^+$ ana')
axs3.set_title('Separated wavefields at x=% .2f' % x[par['nx']]//2)
axs3.set_xlabel('t [s]')
axs3.set_xlim(-1, 1)
axs3.legend()
plt.tight_layout()

```

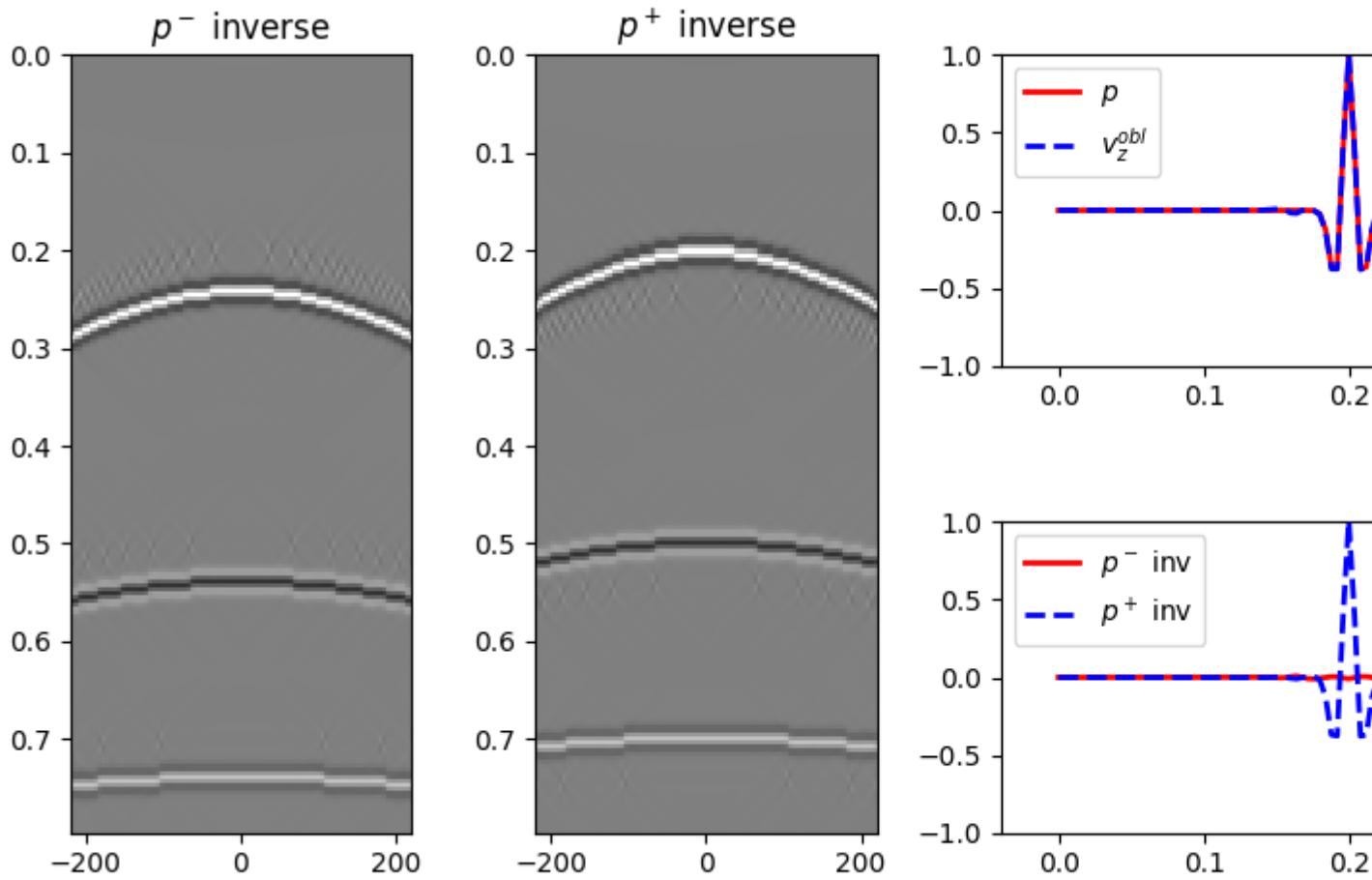


We repeat the same exercise but this time we invert the composition operator `pylops.waveeqprocessing`.

*UpDownComposition2D*

```
pup_inv, pdown_inv = \
    pylops.waveeqprocessing.WavefieldDecomposition(p, vz, par['nt'], par['nx'],
                                                   par['dt'], par['dx'],
                                                   rho_sep, vel_sep,
                                                   nffts=(nfft, nfft),
                                                   kind='inverse',
                                                   critical=critical*100,
                                                   ntaper=ntaper,
                                                   dtype='complex128',
                                                   **dict(damp=1e-10,
                                                       iter_lim=20))

fig = plt.figure(figsize=(12, 5))
axs0 = plt.subplot2grid((2, 5), (0, 0), rowspan=2)
axs1 = plt.subplot2grid((2, 5), (0, 1), rowspan=2)
axs2 = plt.subplot2grid((2, 5), (0, 2), colspan=3)
axs3 = plt.subplot2grid((2, 5), (1, 2), colspan=3)
axs0.imshow(pup_inv.T, cmap='gray', vmin=-1, vmax=1,
            extent=(x.min(), x.max(), t.max(), t.min()))
axs0.set_title(r'$p^{-}$ inverse')
axs0.axis('tight')
axs1.imshow(pdown_inv.T, cmap='gray', vmin=-1, vmax=1,
            extent=(x.min(), x.max(), t.max(), t.min()))
axs1.set_title(r'$p^{+}$ inverse')
axs1.axis('tight')
axs2.plot(t, p[par['nx']//2], 'r', lw=2, label=r'$p$')
axs2.plot(t, vz_obl[par['nx']//2], '--b', lw=2, label=r'$v_z^{obl}$')
axs2.set_xlim(-1, 1)
axs2.set_title('Data at x=% .2f' % x[par['nx']//2])
axs2.set_xlabel('t [s]')
axs2.legend()
axs3.plot(t, pup_inv[par['nx']//2], 'r', lw=2, label=r'$p^{-} inv$')
axs3.plot(t, pdown_inv[par['nx']//2], '--b', lw=2, label=r'$p^{+} inv$')
axs3.set_title('Separated wavefields at x=% .2f' % x[par['nx']//2])
axs3.set_xlabel('t [s]')
axs3.set_xlim(-1, 1)
axs3.legend()
plt.tight_layout()
```



The up- and down-going constituents have been successfully separated in both cases. Finally, we use the [pylops.waveeqprocessing.UpDownComposition2D](#) operator to reconstruct the particle velocity wavefield from its up- and down-going pressure constituents

```
PtoVop = \
    pylops.waveeqprocessing.PressureToVelocity(par['nt'], par['nx'],
                                                par['dt'], par['dx'],
                                                rho_sep, vel_sep,
                                                nffts=(nfft, nfft),
                                                critical=critical * 100.,
                                                ntaper=ntaper,
                                                topressure=False)

vdown_rec = (PtoVop * pdown_inv.ravel()).reshape(par['nx'], par['nt'])
vup_rec = (PtoVop * pup_inv.ravel()).reshape(par['nx'], par['nt'])
vz_rec = np.real(vdown_rec - vup_rec)

fig, axs = plt.subplots(1, 3, figsize=(13, 6))
axs[0].imshow(vz.T, cmap='gray', vmin=-1e-6, vmax=1e-6,
               extent=(x.min(), x.max(), t.max(), t.min()))
axs[0].set_title(r'$v_z$')
axs[0].axis('tight')
axs[1].imshow(vz_rec.T, cmap='gray', vmin=-1e-6, vmax=1e-6,
```

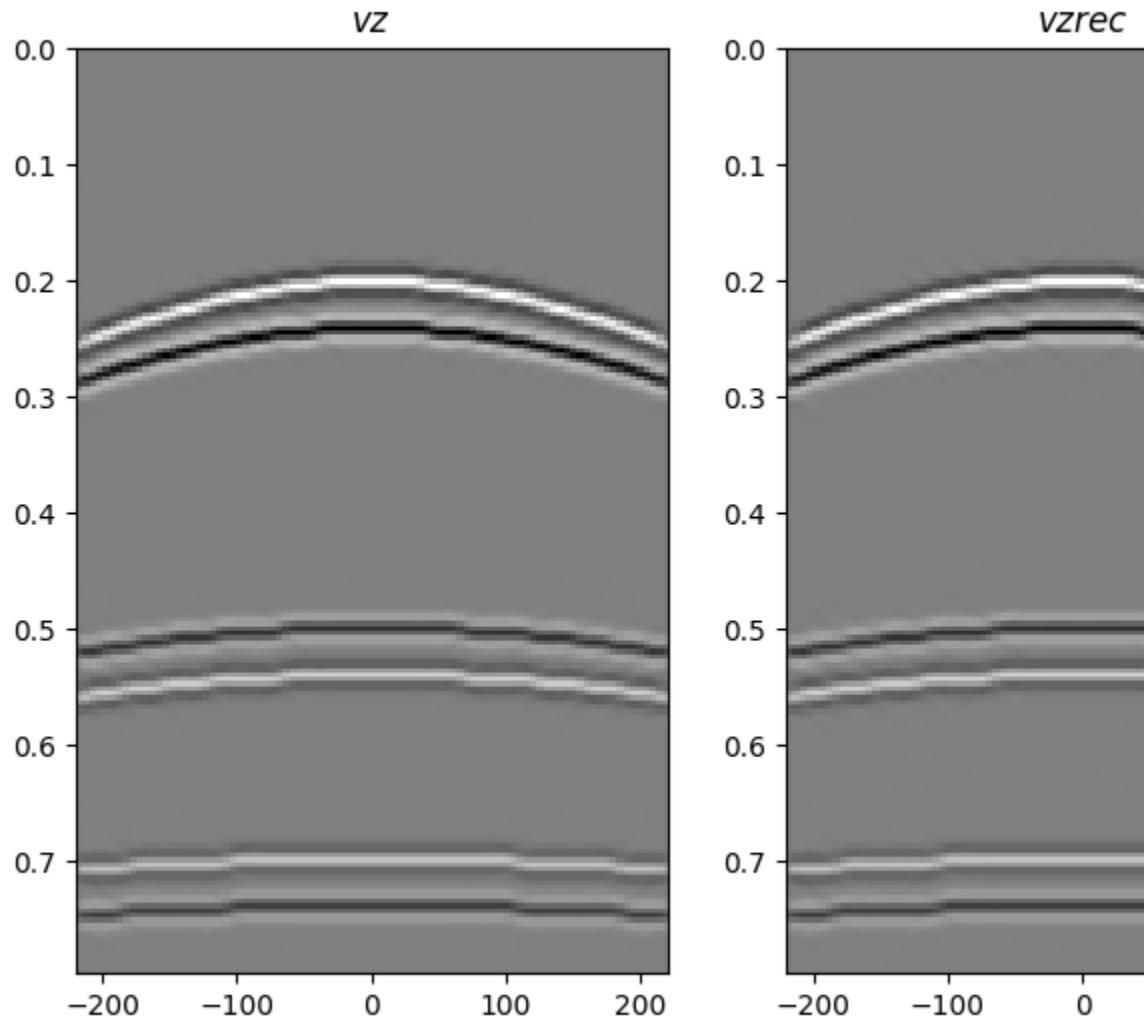
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```

        extent=(x.min(), x.max(), t[-1], t[0]))
axs[1].set_title(r'$vz$ rec$')
axs[1].axis('tight')
axs[2].imshow(vz.T - vz_rec.T, cmap='gray', vmin=-1e-6, vmax=1e-6,
              extent=(x.min(), x.max(), t[-1], t[0]))
axs[2].set_title(r'$error$')
axs[2].axis('tight')

```



Out:

(-220.0, 220.0, 0.796, 0.0)

To see more examples, including applying wavefield separation and regularization simultaneously, as well as 3D examples, head over to the following notebooks: [notebook1](#) and [notebook2](#)

**Total running time of the script:** ( 0 minutes 7.395 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.15 15. Least-squares migration

Seismic migration is the process by which seismic data are manipulated to create an image of the subsurface reflectivity.

While traditionally solved as the adjoint of the demigration operator, it is becoming more and more common to solve the underlying inverse problem in the quest for more accurate and detailed subsurface images.

Independently of the choice of the modelling operator (i.e., ray-based or full wavefield-based), the demigration/migration process can be expressed as a linear operator of such a kind:

$$d(\mathbf{x}_r, \mathbf{x}_s, t) = w(t) * \int_V G(\mathbf{x}, \mathbf{x}_s, t)G(\mathbf{x}_r, \mathbf{x}, t)m(\mathbf{x})d\mathbf{x}$$

where  $m(\mathbf{x})$  is the reflectivity at every location in the subsurface,  $G(\mathbf{x}, \mathbf{x}_s, t)$  and  $G(\mathbf{x}_r, \mathbf{x}, t)$  are the Green's functions from source-to-subsurface-to-receiver and finally  $w(t)$  is the wavelet. Ultimately, while the Green's functions can be computed in many different ways, solving this system of equations for the reflectivity model is what we generally refer to as Least-squares migration (LSM).

In this tutorial we will consider the most simple scenario where we use an eikonal solver to compute the Green's functions and show how we can use the `pylops.waveeqprocessing.LSM` operator to perform LSM.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.sparse.linalg import lsqr

import pylops

plt.close('all')
np.random.seed(0)
```

To start we create a simple model with 2 interfaces

```
# Velocity Model
nx, nz = 81, 60
dx, dz = 4, 4
x, z = np.arange(nx)*dx, np.arange(nz)*dz
v0 = 1000 # initial velocity
kv = 0. # gradient
vel = np.outer(np.ones(nx), v0 + kv*z)

# Reflectivity Model
refl = np.zeros((nx, nz))
refl[:, 30] = -1
refl[:, 50] = 0.5

# Receivers
nr = 11
rx = np.linspace(10*dx, (nx-10)*dx, nr)
rz = 20*np.ones(nr)
recs = np.vstack((rx, rz))
dr = recs[0,1]-recs[0,0]
```

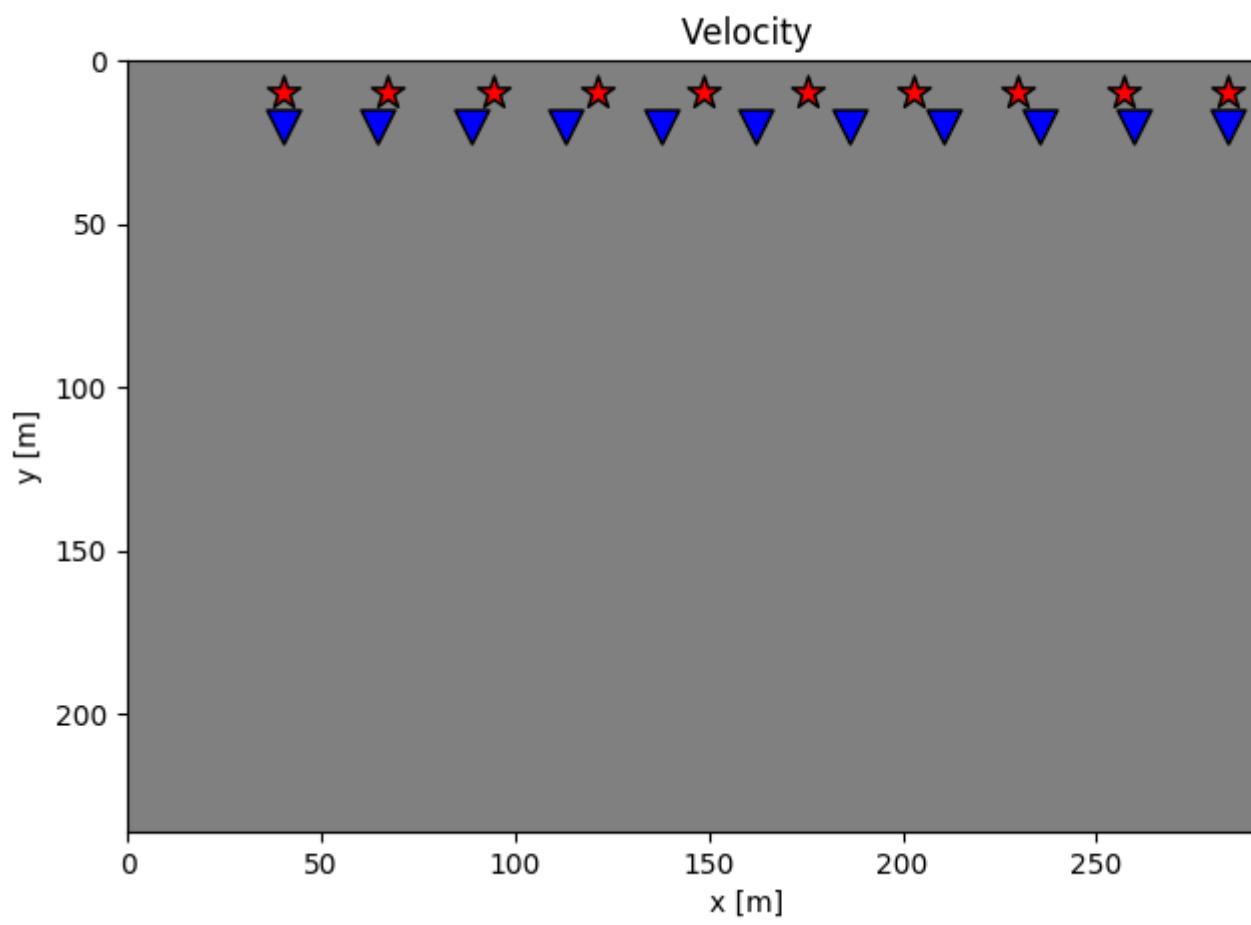
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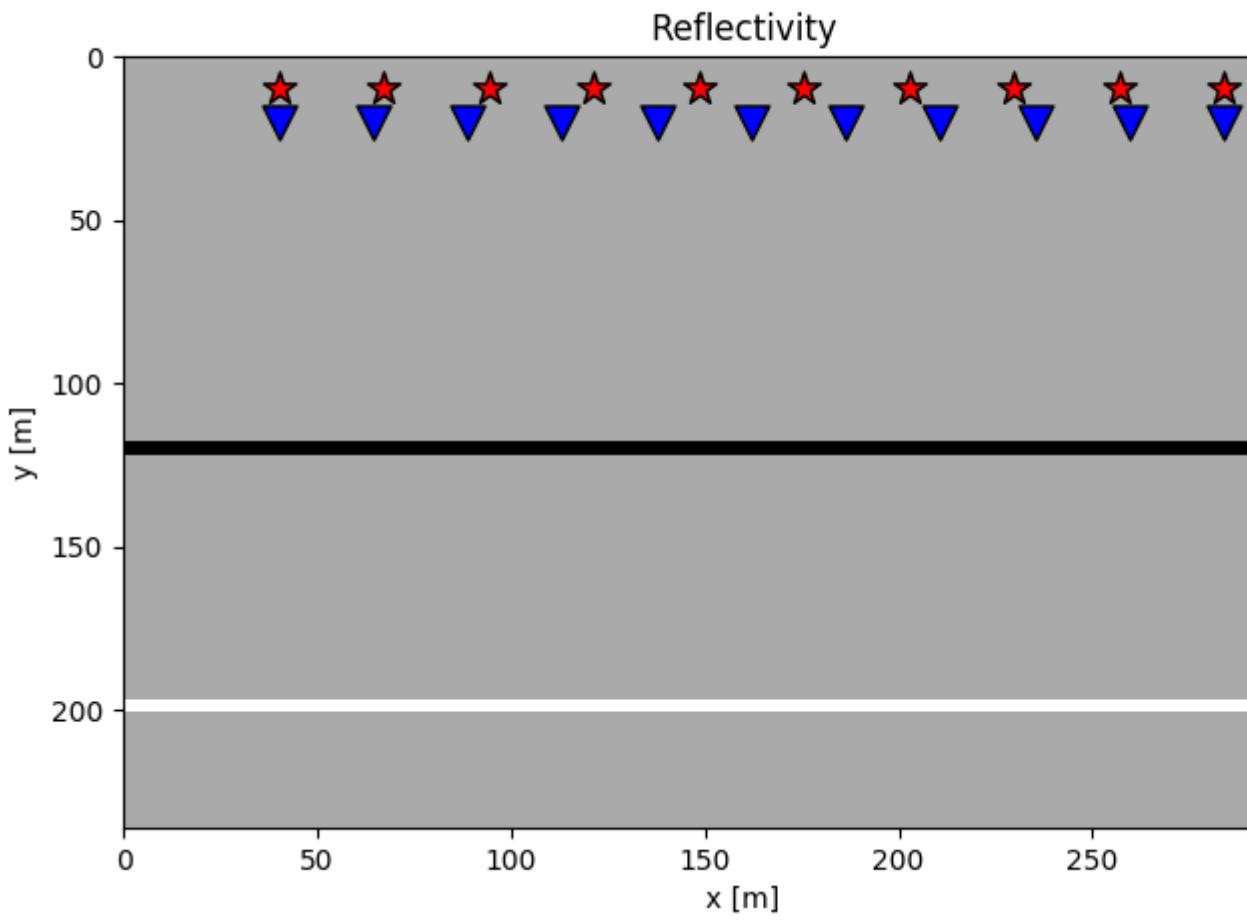
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```
# Sources
ns = 10
sx = np.linspace(dx*10, (nx-10)*dx, ns)
sz = 10*np.ones(ns)
sources = np.vstack((sx, sz))
ds = sources[0,1]-sources[0,0]

plt.figure(figsize=(10, 5))
im = plt.imshow(vel.T, cmap='gray', extent = (x[0], x[-1], z[-1], z[0]))
plt.scatter(recs[0], recs[1], marker='v', s=150, c='b', edgecolors='k')
plt.scatter(sources[0], sources[1], marker='*', s=150, c='r', edgecolors='k')
plt.colorbar(im)
plt.axis('tight')
plt.xlabel('x [m]'),plt.ylabel('y [m]')
plt.title('Velocity')
plt.xlim(x[0], x[-1])

plt.figure(figsize=(10, 5))
im = plt.imshow(refl.T, cmap='gray', extent = (x[0], x[-1], z[-1], z[0]))
plt.scatter(recs[0], recs[1], marker='v', s=150, c='b', edgecolors='k')
plt.scatter(sources[0], sources[1], marker='*', s=150, c='r', edgecolors='k')
plt.colorbar(im)
plt.axis('tight')
plt.xlabel('x [m]'),plt.ylabel('y [m]')
plt.title('Reflectivity')
plt.xlim(x[0], x[-1])
```





Out:

```
(0, 320)
```

We can now create our LSM object and invert for the reflectivity using two different solvers: `scipy.sparse.linalg.lsqr` (LS solution) and `pylops.optimization.sparsity.FISTA` (LS solution with sparse model).

```
nt = 651
dt = 0.004
t = np.arange(nt)*dt
wav, wavy, wavc = pylops.utils.wavelets.ricker(t[:41], f0=20)

lsm = pylops.waveeqprocessing.LSM(z, x, t, sources, recs, v0, wav, wavc,
                                  mode='analytic')

d = lsm.Demop * refl.ravel()
d = d.reshape(ns, nr, nt)

madj = lsm.Demop.H * d.ravel()
madj = madj.reshape(nx, nz)
```

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```

minv = lsm.solve(d.ravel(), solver=lsqr, **dict(iter_lim=100))
minv = minv.reshape(nx, nz)

minv_sparse = lsm.solve(d.ravel(),
                        solver=pylops.optimization.sparsity.FISTA,
                        **dict(eps=1e2, niter=100))
minv_sparse = minv_sparse.reshape(nx, nz)

# demigration
dadj = lsm.Demop * madj.ravel()
dadj = dadj.reshape(ns, nr, nt)

dinv = lsm.Demop * minv.ravel()
dinv = dinv.reshape(ns, nr, nt)

dinv_sparse = lsm.Demop * minv_sparse.ravel()
dinv_sparse = dinv_sparse.reshape(ns, nr, nt)

# sphinx_gallery_thumbnail_number = 2
fig, axs = plt.subplots(2, 2, figsize=(10, 8))
axs[0][0].imshow(refl.T, cmap='gray', vmin=-1, vmax=1)
axs[0][0].axis('tight')
axs[0][0].set_title(r'$m$')
axs[0][1].imshow(madj.T, cmap='gray', vmin=-madj.max(), vmax=madj.max())
axs[0][1].set_title(r'$m_{adj}$')
axs[0][1].axis('tight')
axs[1][0].imshow(minv.T, cmap='gray', vmin=-1, vmax=1)
axs[1][0].axis('tight')
axs[1][0].set_title(r'$m_{inv}$')
axs[1][1].imshow(minv_sparse.T, cmap='gray', vmin=-1, vmax=1)
axs[1][1].axis('tight')
axs[1][1].set_title(r'$m_{FISTA}$')

fig, axs = plt.subplots(1, 4, figsize=(10, 4))
axs[0].imshow(d[0, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[0].set_title(r'$d$')
axs[0].axis('tight')
axs[1].imshow(dadj[0, :, :300].T, cmap='gray',
              vmin=-dadj.max(), vmax=dadj.max())
axs[1].set_title(r'$d_{adj}$')
axs[1].axis('tight')
axs[2].imshow(dinv[0, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[2].set_title(r'$d_{inv}$')
axs[2].axis('tight')
axs[3].imshow(dinv_sparse[0, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[3].set_title(r'$d_{fista}$')
axs[3].axis('tight')

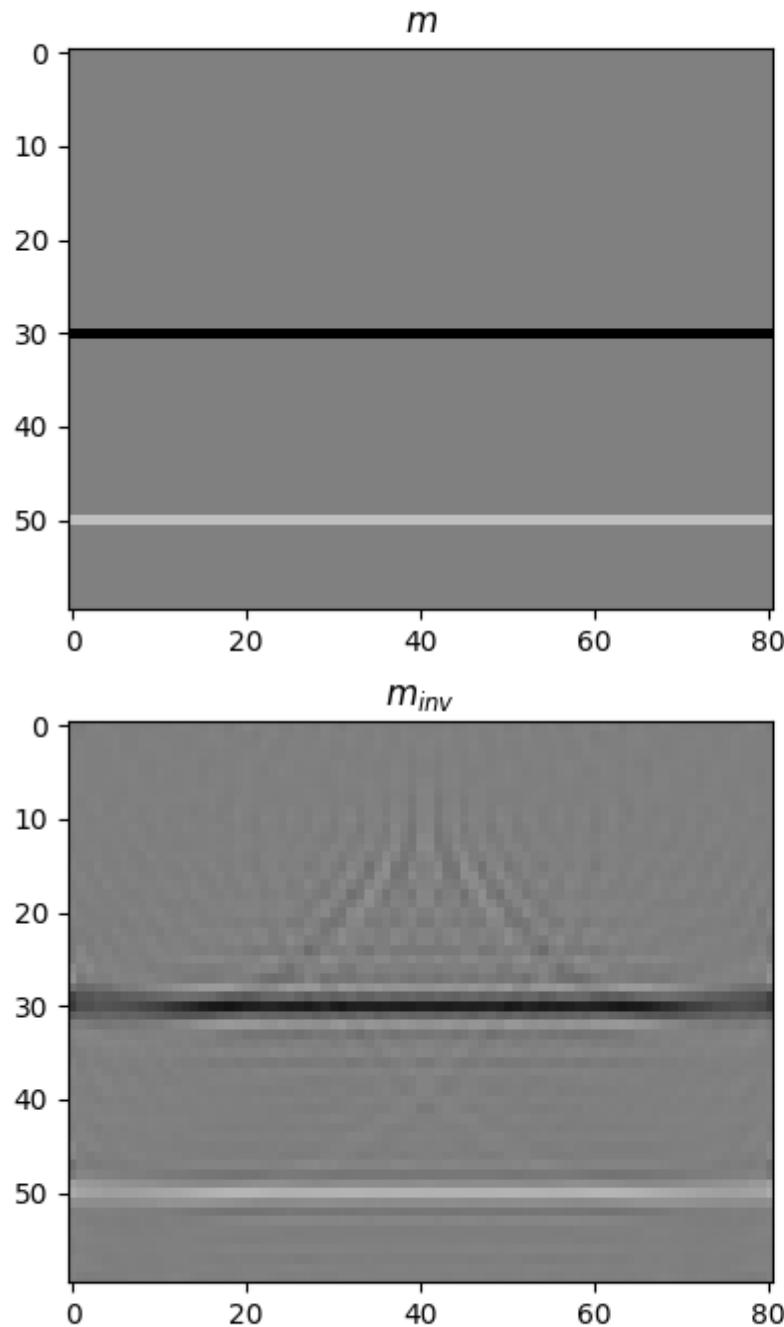
fig, axs = plt.subplots(1, 4, figsize=(10, 4))
axs[0].imshow(d[ns//2, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[0].set_title(r'$d$')
axs[0].axis('tight')
axs[1].imshow(dadj[ns//2, :, :300].T, cmap='gray',
              vmin=-dadj.max(), vmax=dadj.max())

```

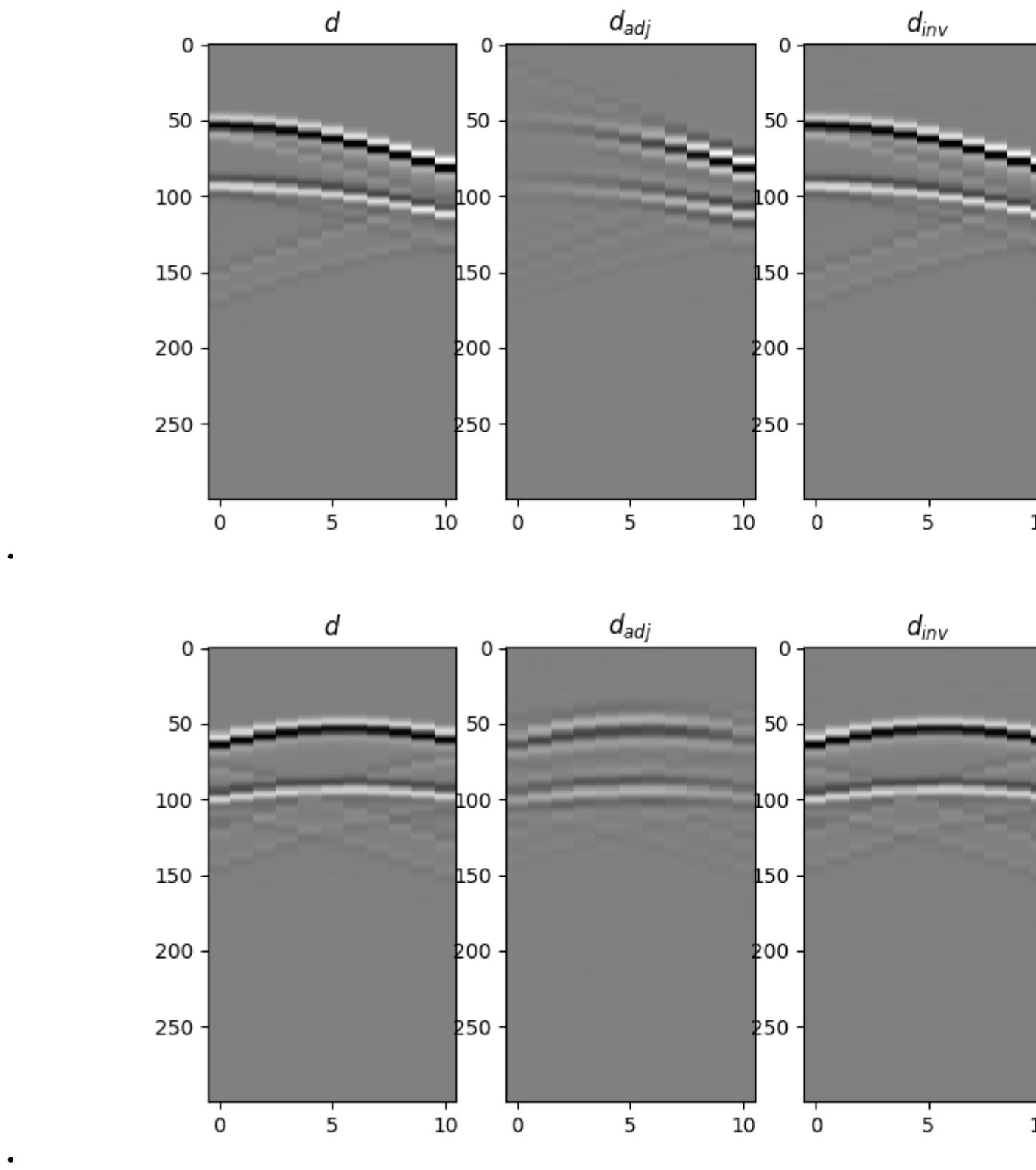
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```
vmin=-dadj.max(), vmax=dadj.max())
axs[1].set_title(r'$d_{adj}$')
axs[1].axis('tight')
axs[2].imshow(dinv[ns//2, :, :300].T, cmap='gray',
               vmin=-d.max(), vmax=d.max())
axs[2].set_title(r'$d_{inv}$')
axs[2].axis('tight')
axs[3].imshow(dinv_sparse[ns//2, :, :300].T, cmap='gray',
               vmin=-d.max(), vmax=d.max())
axs[3].set_title(r'$d_{fista}$')
axs[3].axis('tight')
```



•



Out:

```
(-0.5, 10.5, 299.5, -0.5)
```

This was just a short teaser, for a more advanced set of examples of 2D and 3D traveltime-based LSM head over to this [notebook](#).

**Total running time of the script:** ( 0 minutes 4.503 seconds)

---

**Note:** Click [here](#) to download the full example code

---

### 3.4.16 16. CT Scan Imaging

This tutorial considers a very well-known inverse problem from the field of medical imaging.

We will be using the `pylops.signalprocessing.Radon2D` operator to model a *sinogram*, which is a graphic representation of the raw data obtained from a CT scan. The sinogram is further inverted using both a L2 solver and a TV-regularized solver like Split-Bregman.

```
# sphinx_gallery_thumbnail_number = 2
import numpy as np
import matplotlib.pyplot as plt
import pylops

from numba import jit

plt.close('all')
np.random.seed(10)
```

Let's start by loading the Shepp-Logan phantom model. We can then construct the sinogram by providing a custom-made function to the `pylops.signalprocessing.Radon2D` that samples parametric curves of such a type:

$$t(r, \theta; x) = \tan(90 - \theta) * x + r / \sin(\theta)$$

where  $\theta$  is the angle between the x-axis ( $x$ ) and the perpendicular to the summation line and  $r$  is the distance from the origin of the summation line.

```
@jit(nopython=True)
def radoncurve(x, r, theta):
    return (r - ny//2)/(np.sin(np.deg2rad(theta))+1e-15) + np.tan(np.deg2rad(90 - theta))*x + ny//2

x = np.load('../testdata/optimization/shepp_logan_phantom.npy')
x = x / x.max()
ny, nx = x.shape

ntheta = 150
theta = np.linspace(0., 180., ntheta, endpoint=False)

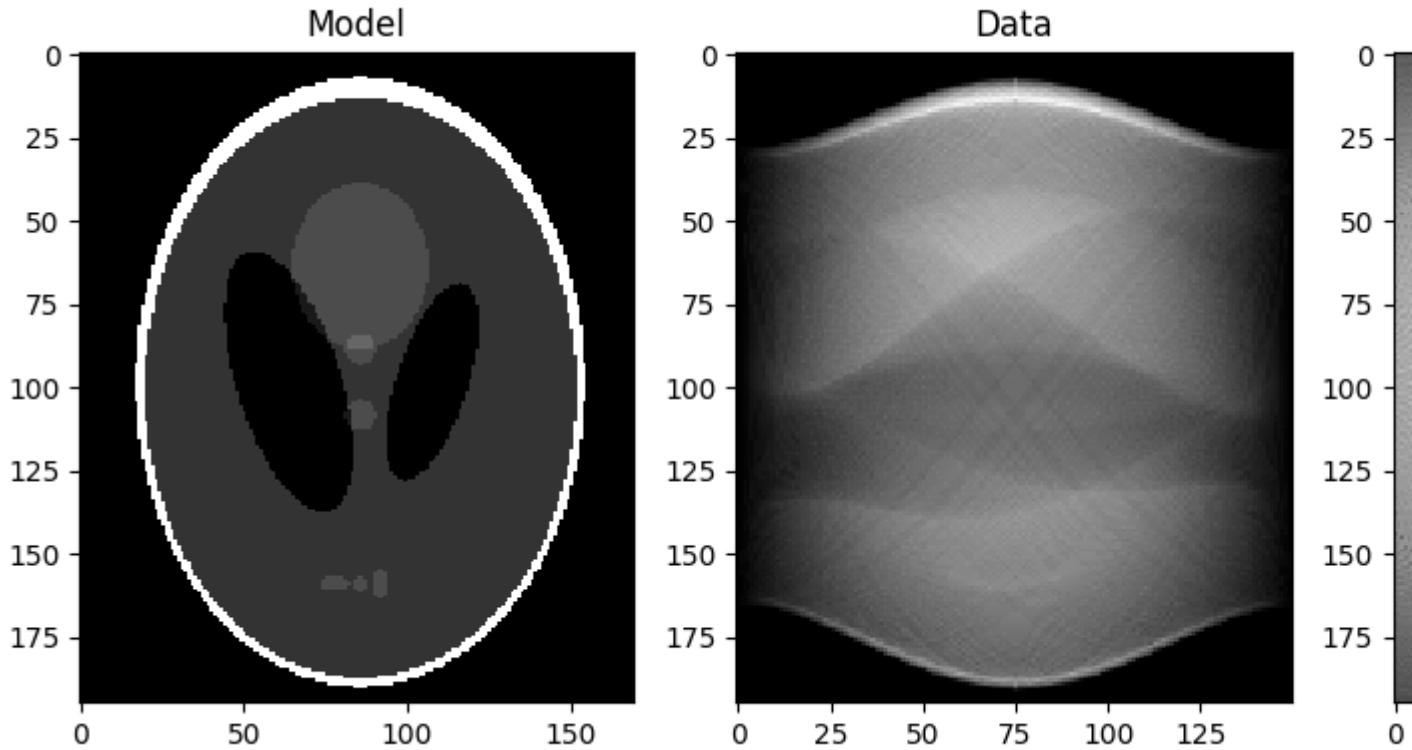
Rlop = \
    pylops.signalprocessing.Radon2D(np.arange(ny), np.arange(nx),
                                    theta, kind=radoncurve,
                                    centeredh=True, interp=False,
                                    engine='numba', dtype='float64')

y = Rlop.H * x.T.ravel()
y = y.reshape(ntheta, ny).T
```

We can now first perform the adjoint, which in the medical imaging literature is also referred to as back-projection. This is the first step of a common reconstruction technique, named filtered back-projection, which simply applies a correction filter in the frequency domain to the adjoint model.

```
xrec = RLoc*y.T.ravel()
xrec = xrec.reshape(nx, ny).T

fig, axs = plt.subplots(1, 3, figsize=(10, 4))
axs[0].imshow(x, vmin=0, vmax=1, cmap='gray')
axs[0].set_title('Model')
axs[0].axis('tight')
axs[1].imshow(y, cmap='gray')
axs[1].set_title('Data')
axs[1].axis('tight')
axs[2].imshow(xrec, cmap='gray')
axs[2].set_title('Adjoint model')
axs[2].axis('tight')
fig.tight_layout()
```



Finally we take advantage of our different solvers and try to invert the modelling operator both in a least-squares sense and using TV-reg.

```
Dop = [
    pylops.FirstDerivative(ny * nx, dims=(nx, ny), dir=0, edge=True, dtype=np.float),
    pylops.FirstDerivative(ny * nx, dims=(nx, ny), dir=1, edge=True, dtype=np.float)]
D2op = pylops.Laplacian(dims=(nx, ny), edge=True, dtype=np.float)

# L2
xinv_sm = \
```

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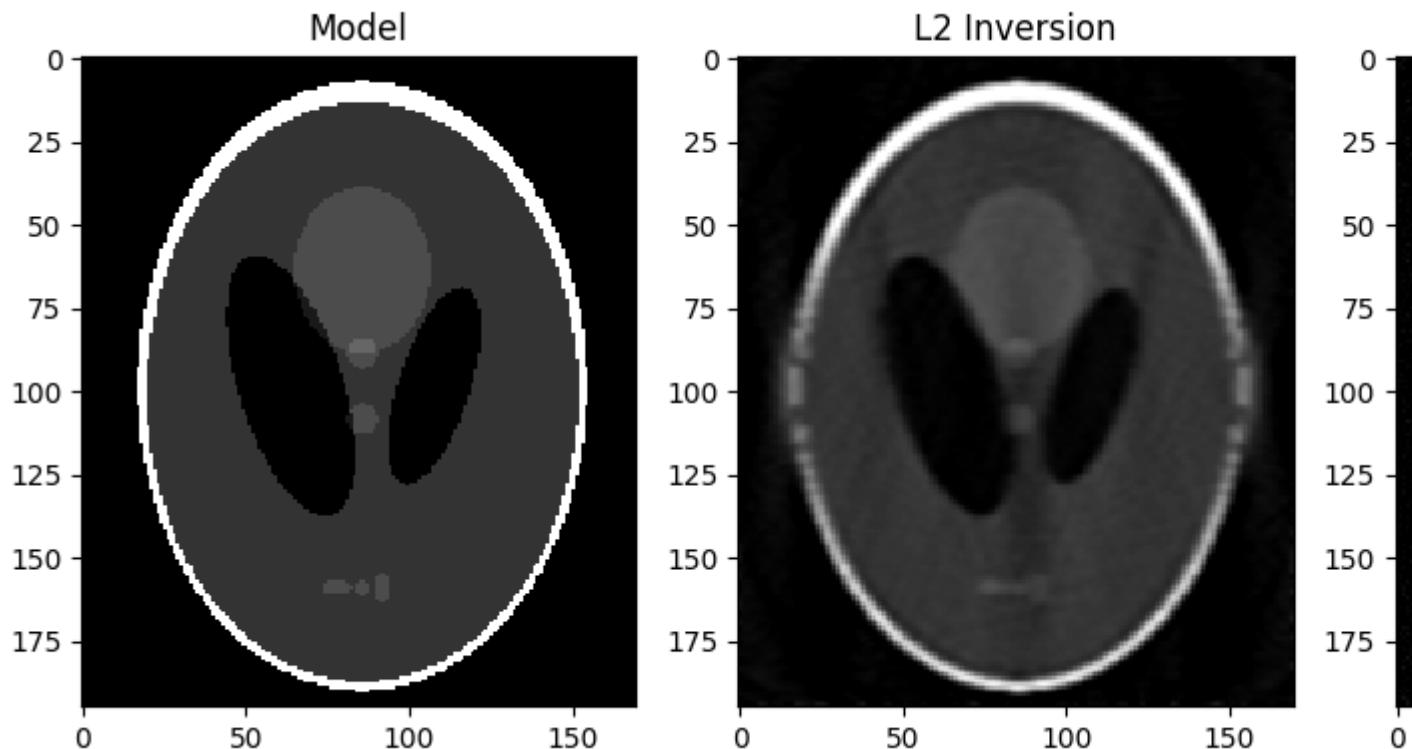
```
pylops.optimization.leastsquares.RegularizedInversion(RLoc.H,
                                                       [D2op],
                                                       y.T.flatten(),
                                                       epsRs=[1e1],
                                                       **dict(iter_lim=20))

xinv_sm = np.real(xinv_sm.reshape(nx, ny)).T

# TV
mu = 1.5
lamda = [1., 1.]
niter = 3
niterinner = 4

xinv, niter = pylops.optimization.sparsity.SplitBregman(RLoc.H, Dop, y.T.flatten(),  
             niter, niterinner,  
             mu=mu, epsRL1s=lamda, tol=1e-4, tau=1., show=False,  
             **dict(iter_lim=20, damp=1e-2))
xinv = np.real(xinv.reshape(nx, ny)).T

fig, axs = plt.subplots(1, 3, figsize=(10, 4))
axs[0].imshow(x, vmin=0, vmax=1, cmap='gray')
axs[0].set_title('Model')
axs[0].axis('tight')
axs[1].imshow(xinv_sm, vmin=0, vmax=1, cmap='gray')
axs[1].set_title('L2 Inversion')
axs[1].axis('tight')
axs[2].imshow(xinv, vmin=0, vmax=1, cmap='gray')
axs[2].set_title('TV-Reg Inversion')
axs[2].axis('tight')
fig.tight_layout()
```



**Total running time of the script:** ( 0 minutes 10.976 seconds)

## 3.5 Frequently Asked Questions

### 1. Can I visualize my operator?

Yes, you can. Every operator has a method called `todense` that will return the dense matrix equivalent of the operator. Note, however, that in order to do so we need to allocate a numpy array of the size of your operator and apply the operator N times, where N is the number of columns of the operator. The allocation can be very heavy on your memory and the computation may take long time, so use it with care only for small toy examples to understand what your operator looks like. This method should however not be abused, as the reason of working with linear operators is indeed that you don't really need to access the explicit matrix representation of an operator.

## 3.6 PyLops API

The Application Programming Interface (API) of PyLops can be loosely seen as composed of a stack of three main layers:

- *Linear operators*: building blocks for the setting up of inverse problems
- *Solvers*: interfaces to a variety of solvers, providing an easy way to augment an inverse problem with additional regularization and/or preconditioning term
- *Applications*: high-level interfaces allowing users to easily setup and solve specific problems (while hiding the non-needed details - i.e., creation and setup of linear operators and solvers).

### 3.6.1 Linear operators

#### Templates

<code>LinearOperator([Op, explicit])</code>	Common interface for performing matrix-vector products.
<code>FunctionOperator(f, *args, **kwargs)</code>	Function Operator.

#### pylops.LinearOperator

```
class pylops.LinearOperator(Op=None, explicit=False)
```

Common interface for performing matrix-vector products.

This class is an overload of the `scipy.sparse.linalg.LinearOperator` class. It adds functionalities by overloading standard operators such as `__truediv__` as well as creating convenience methods such as `eigs`, `cond`, and `conj`.

---

**Note:** End users of PyLops should not use this class directly but simply use operators that are already implemented. This class is meant for developers and it has to be used as the parent class of any new operator developed within PyLops. Find more details regarding implementation of new operators at [Implementing new operators](#).

---

#### Parameters

<b>Op</b> [scipy.sparse.linalg.LinearOperator linalg._ProductLinearOperator _SumLinearOperator] Operator	or	scipy.sparse. scipy.sparse.linalg.
<b>explicit</b> [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)		

#### Methods

<code>__init__(self[, Op, explicit])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**div** (`self, y, niter=100`)

Solve the linear problem  $y = Ax$ .

Overloading of operator / to improve expressivity of *Pylops* when solving inverse problems.

**Parameters**

**y** [np.ndarray] Data

**niter** [int, optional] Number of iterations (to be used only when `explicit=False`)

**Returns**

**xest** [np.ndarray] Estimated model

**todense** (*self*)

Return dense matrix.

The operator is converted into its dense matrix equivalent. In order to do so, the operator is applied to an identity matrix whose number of rows and columns is equivalent to the number of columns of the operator. Note that this operation may be costly for very large operators and it is only suggest it to use as a way to inspect the structure of the matricial equivalent of the operator.

**Returns**

**matrix** [numpy.ndarray] Dense matrix.

**eigs** (*self*, `neigs=None`, `symmetric=False`, `niter=None`, `**kwargs_eig`)

Most significant eigenvalues of linear operator.

Return an estimate of the most significant eigenvalues of the linear operator. If the operator has rectangular shape (`shape[0] != shape[1]`), eigenvalues are first computed for the square operator  $\mathbf{A}^H \mathbf{A}$  and the square-root values are returned.

**Parameters**

**neigs** [int] Number of eigenvalues to compute (if `None`, return all). Note that for `explicit=False`, only  $N - 1$  eigenvalues can be computed where  $N$  is the size of the operator in the model space

**symmetric** [bool, optional] Operator is symmetric (`True`) or not (`False`). User should set this parameter to `True` only when it is guaranteed that the operator is real-symmetric or complex-hermitian matrices

**niter** [int, optional] Number of iterations for eigenvalue estimation

**\*\*kwargs\_eig** Arbitrary keyword arguments for `scipy.sparse.linalg.eigs` or `scipy.sparse.linalg.eigsh`

**Returns**

**eigenvalues** [numpy.ndarray] Operator eigenvalues.

**Notes**

Eigenvalues are estimated using `scipy.sparse.linalg.eigs` (`explicit=True`) or `scipy.sparse.linalg.eigsh` (`explicit=False`).

This is a port of ARPACK [1], a Fortran package which provides routines for quickly finding eigenvalues/eigenvectors of a matrix. As ARPACK requires only left-multiplication by the matrix in question, eigenvalues/eigenvectors can also be estimated for linear operators when the dense matrix is not available.

**cond** (*self*, `**kwargs_eig`)

Condition number of linear operator.

Return an estimate of the condition number of the linear operator as the ratio of the largest and lowest estimated eigenvalues.

**Parameters**

**\*\*kwargs\_eig** Arbitrary keyword arguments for `scipy.sparse.linalg.eigs` or `scipy.sparse.linalg.eigsh`

## Returns

**eigenvalues** [numpy.ndarray] Operator eigenvalues.

## Notes

The condition number of a matrix (or linear operator) can be estimated as the ratio of the largest and lowest estimated eigenvalues:

$$k = \frac{\lambda_{max}}{\lambda_{min}}$$

The condition number provides an indication of the rate at which the solution of the inversion of the linear operator  $A$  will change with respect to a change in the data  $y$ .

Thus, if the condition number is large, even a small error in  $y$  may cause a large error in  $x$ . On the other hand, if the condition number is small then the error in  $x$  is not much bigger than the error in  $y$ . A problem with a low condition number is said to be *well-conditioned*, while a problem with a high condition number is said to be *ill-conditioned*.

## `conj(self)`

Complex conjugate operator

## Returns

**conjp** [`pylops.LinearOperator`] Complex conjugate operator

## `apply_columns(self, cols)`

Apply subset of columns of operator

This method can be used to wrap a LinearOperator and mimic the action of a subset of columns of the operator on a reduced model in forward mode, and retrieve only the result of a subset of rows in adjoint mode.

Note that unless the operator has `explicit=True`, this is not optimal as the entire forward and adjoint passes of the original operator will have to be performed. It can however be useful for the implementation of solvers such as Orthogonal Matching Pursuit (OMP) that iteratively build a solution by evaluating only a subset of the columns of the operator.

## Parameters

**cols** [list] Columns to be selected

## Returns

**colop** [`pylops.LinearOperator`] Apply column operator

## Examples using `pylops.LinearOperator`

- sphx\_glr\_gallery\_plot\_sliding.py
- sphx\_glr\_gallery\_plot\_bilinear.py
- sphx\_glr\_gallery\_plot\_causalintegration.py
- sphx\_glr\_gallery\_plot\_derivative.py
- sphx\_glr\_gallery\_plot\_linearregr.py

- sphx\_glr\_gallery\_plot\_mdc.py
- sphx\_glr\_gallery\_plot\_stacking.py
- sphx\_glr\_gallery\_plot\_phaseshift.py
- sphx\_glr\_gallery\_plot\_regr.py
- sphx\_glr\_gallery\_plot\_prestack.py
- sphx\_glr\_gallery\_plot\_restriction.py
- [01. The LinearOperator](#)
- [03. Solvers](#)
- [04. Bayesian Inversion](#)
- [06. 2D Interpolation](#)
- [07. Post-stack inversion](#)
- [08. Pre-stack \(AVO\) inversion](#)
- [09. Multi-Dimensional Deconvolution](#)
- [12. Seismic regularization](#)
- [14. Seismic wavefield decomposition](#)
- [16. CT Scan Imaging](#)

## pylops.FunctionOperator

```
class pylops.FunctionOperator(f, *args, **kwargs)
```

Function Operator.

Simple wrapper to functions for forward *f* and adjoint *fc* multiplication.

Functions *f* and *fc* are such that  $f : \mathbb{F}^m \rightarrow \mathbb{F}^n$  and  $fc : \mathbb{F}^n \rightarrow \mathbb{F}^m$  where  $\mathbb{F}$  is the appropriate underlying type (e.g.,  $\mathbb{R}$  for real or  $\mathbb{C}$  for complex)

FunctionOperator can be called in the following ways: `FunctionOperator(f, n)`, `FunctionOperator(f, n, m)`, `FunctionOperator(f, fc, n)`, and `FunctionOperator(f, fc, n, m)`.

The first two methods can only be used for forward modelling and will return *NotImplementedError* if the adjoint is called. The first and third method assume the matrix (or matrices) to be square. All methods can be called with the *dtype* keyword argument.

### Parameters

- f** [`callable`] Function for forward multiplication.
- fc** [`callable`, optional] Function for adjoint multiplication.
- n** [`int`, optional] Number of rows (length of data vector).
- m** [`int`, optional] Number of columns (length of model vector).
- dtype** [`str`, optional] Type of elements in input array.

## Examples

```
>>> from pyllops.basicoperators import FunctionOperator
>>> def forward(v):
...     return np.array([2*v[0], 3*v[1]])
...
>>> A = FunctionOperator(forward, 2)
>>> A
<2x2 FunctionOperator with dtype=float64>
>>> A.matvec(np.ones(2))
array([2., 3.])
>>> A @ np.ones(2)
array([2., 3.])
```

## Attributes

- shape** [tuple] Operator shape [ $n \times m$ ]  
**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, f, *args, **kwargs)</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, **kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Basic operators

<code>MatrixMult(A[, dims, dtype])</code>	Matrix multiplication.
<code>Identity(N[, M, dtype, inplace])</code>	Identity operator.
<code>Zero(N[, M, dtype])</code>	Zero operator.
<code>Diagonal(diag[, dims, dir, dtype])</code>	Diagonal operator.
<code>Transpose(dims, axes[, dtype])</code>	Transpose operator.
<code>Flip(N[, dims, dir, dtype])</code>	Flip along an axis.
<code>Roll(N[, dims, dir, shift, dtype])</code>	Roll along an axis.
<code>Pad(dims, pad[, dtype])</code>	Pad operator.
<code>Sum(dims, dir[, dtype])</code>	Sum operator.
<code>Symmetrize(N[, dims, dir, dtype])</code>	Symmetrize along an axis.
<code>Restriction(M, iava[, dims, dir, dtype, inplace])</code>	Restriction (or sampling) operator.
<code>Regression(taxis, order[, dtype])</code>	Polynomial regression.

Continued on next page

Table 4 – continued from previous page

<code>LinearRegression(taxis[, dtype])</code>	Linear regression.
<code>CausalIntegration(N[, dims, dir, sampling, ...])</code>	Causal integration.
<code>Spread(dims, dimsd[, table, dtable, fh, ...])</code>	Spread operator.
<code>VStack(ops[, dtype])</code>	Vertical stacking.
<code>HStack(ops[, dtype])</code>	Horizontal stacking.
<code>Block(ops[, dtype])</code>	Block operator.
<code>BlockDiag(ops[, dtype])</code>	Block-diagonal operator.
<code>Kronecker(Op1, Op2[, dtype])</code>	Kronecker operator.

## pylops.MatrixMult

```
class pylops.MatrixMult (A, dims=None, dtype='float64')
    Matrix multiplication.

    Simple wrapper to numpy.dot and numpy.vdot for an input matrix A.
```

### Parameters

**A** [numpy.ndarray or scipy.sparse matrix] Matrix.  
**dims** [tuple, optional] Number of samples for each other dimension of model (model/data will be reshaped and A applied multiple times to each column of the model/data).  
**dtype** [str, optional] Type of elements in input array.

### Attributes

**shape** [tuple] Operator shape  
**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)  
**complex** [bool] Matrix has complex numbers (True) or not (False)

### Methods

<code>__init__(self, A[, dims, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\nkwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>inv(self)</code>	Return the inverse of A.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**inv(self)**  
 Return the inverse of A.

### Returns

**Ainv** [numpy.ndarray] Inverse matrix.

### Examples using `pylops.MatrixMult`

- sphx\_glr\_gallery\_plot\_matrixmult.py
- sphx\_glr\_gallery\_plot\_ista.py
- sphx\_glr\_gallery\_plot\_stacking.py
- 02. The Dot-Test
- 07. Post-stack inversion
- 08. Pre-stack (AVO) inversion

## pylops.Identity

**class** pylops.Identity(*N, M=None, dtype='float64', inplace=True*)  
Identity operator.

Simply move model to data in forward model and viceversa in adjoint mode if  $M = N$ . If  $M > N$  removes last  $M - N$  elements from model in forward and pads with 0 in adjoint. If  $N > M$  removes last  $N - M$  elements from data in adjoint and pads with 0 in forward.

### Parameters

- N** [int] Number of samples in data (and model, if **M** is not provided).
- M** [int, optional] Number of samples in model.
- dtype** [str, optional] Type of elements in input array.
- inplace** [bool, optional] Work inplace (`True`) or make a new copy (`False`). By default, data is a reference to the model (in forward) and model is a reference to the data (in adjoint).

### Notes

For  $M = N$ , an *Identity* operator simply moves the model **x** to the data **y** in forward mode and viceversa in adjoint mode:

$$y_i = x_i \quad \forall i = 1, 2, \dots, N$$

or in matrix form:

$$\mathbf{y} = \mathbf{I}\mathbf{x} = \mathbf{x}$$

and

$$\mathbf{x} = \mathbf{I}\mathbf{y} = \mathbf{y}$$

For  $M > N$ , the *Identity* operator takes the first  $M$  elements of the model **x** into the data **y** in forward mode

$$y_i = x_i \quad \forall i = 1, 2, \dots, N$$

and all the elements of the data **y** into the first  $M$  elements of model in adjoint mode (other elements are 0):

$$\begin{aligned} x_i &= y_i \quad \forall i = 1, 2, \dots, M \\ x_i &= 0 \quad \forall i = M + 1, \dots, N \end{aligned}$$

**Attributes**

- shape** [`tuple`] Operator shape  
**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

**Methods**

<code>__init__(self, N[, M, dtype, inplace])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**Examples using `pylops.Identity`**

- sphx\_glr\_gallery\_plot\_identity.py
- sphx\_glr\_gallery\_plot\_stacking.py
- sphx\_glr\_gallery\_plot\_tvreg.py
- *04. Bayesian Inversion*

**pylops.Zero**

```
class pylops.Zero(N, M=None, dtype='float64')
```

Zero operator.

Transform model into array of zeros of size  $N$  in forward and transform data into array of zeros of size  $N$  in adjoint.

**Parameters**

- N** [`int`] Number of samples in data (and model in M is not provided).  
**M** [`int`, optional] Number of samples in model.  
**dtype** [`str`, optional] Type of elements in input array.

**Notes**

An `Zero` operator simply creates a null data vector  $y$  in forward mode:

$$\mathbf{0}\mathbf{x} = \mathbf{0}_N$$

and a null model vector  $\mathbf{x}$  in forward mode:

$$\mathbf{0y} = \mathbf{0}_M$$

## Attributes

**shape** [tuple] Operator shape

**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, N[, M, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\nkwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $\mathbf{y} = \mathbf{Ax}$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.Zero`

- sphx\_glr\_gallery\_plot\_zero.py

## pylops.Diagonal

```
class pylops.Diagonal(diag, dims=None, dir=0, dtype='float64')  
    Diagonal operator.
```

Applies element-wise multiplication of the input vector with the vector `diag` in forward and with its complex conjugate in adjoint mode.

This operator can also broadcast; in this case the input vector is reshaped into its dimensions `dims` and the element-wise multiplication with `diag` is performed on the direction `dir`. Note that the vector `diag` will need to have size equal to `dims[dir]`.

### Parameters

**diag** [numpy.ndarray] Vector to be used for element-wise multiplication.

**dims** [list, optional] Number of samples for each dimension (None if only one dimension is available)

**dir** [int, optional] Direction along which multiplication is applied.

**dtype** [str, optional] Type of elements in input array.

## Notes

Element-wise multiplication between the model **x** and/or data **y** vectors and the array **d** can be expressed as

$$y_i = d_i x_i \quad \forall i = 1, 2, \dots, N$$

This is equivalent to a matrix-vector multiplication with a matrix containing the vector **d** along its main diagonal. For real-valued `diag`, the Diagonal operator is self-adjoint as the adjoint of a diagonal matrix is the diagonal matrix itself. For complex-valued `diag`, the adjoint is equivalent to the element-wise multiplication with the complex conjugate elements of `diag`.

## Attributes

`shape` [tuple] Operator shape

`explicit` [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, diag[, dims, dir, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matrix(self)</code>	Return diagonal matrix as dense <code>numpy.ndarray</code>
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

`matrix(self)`

Return diagonal matrix as dense `numpy.ndarray`

### Returns

`densem` [`numpy.ndarray`] Dense matrix.

## Examples using `pylops.Diagonal`

- sphx\_glr\_gallery\_plot\_diagonal.py
- 01. The LinearOperator
- 02. The Dot-Test

**pylops.Transpose**

```
class pylops.Transpose(dims, axes, dtype='float64')  
    Transpose operator.
```

Transpose axes of a multi-dimensional array. This operator works with flattened input model (or data), which are however multi-dimensional in nature and will be reshaped and treated as such in both forward and adjoint modes.

**Parameters**

- dims** [`tuple`, optional] Number of samples for each dimension
- axes** [`tuple`, optional] Direction along which transposition is applied
- dtype** [`str`, optional] Type of elements in input array

**Raises**

**ValueError** If `axes` contains repeated dimensions (or a dimension is missing)

**Notes**

The Transpose operator reshapes the input model into a multi-dimensional array of size `dims` and transposes (or swaps) its axes as defined in `axes`.

Similarly, in adjoint mode the data is reshaped into a multi-dimensional array whose size is a permuted version of `dims` defined by `axes`. The array is then rearranged into the original model dimensions `dims`.

**Attributes**

- shape** [`tuple`] Operator shape
- explicit** [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

**Methods**

<code>__init__(self, dims, axes[, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *kargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**Examples using `pylops.Transpose`**

- sphx\_glr\_gallery\_plot\_transpose.py

## pylops.Flip

```
class pylops.Flip(N, dims=None, dir=0, dtype='float64')
    Flip along an axis.
```

Flip a multi-dimensional array along a specified direction `dir`.

### Parameters

`N` [`int`] Number of samples in model.

`dims` [`list`, optional] Number of samples for each dimension (`None` if only one dimension is available)

`dir` [`int`, optional] Direction along which flipping is applied.

`dtype` [`str`, optional] Type of elements in input array.

### Notes

The Flip operator flips the input model (and data) along any chosen direction. For simplicity, given a one dimensional array, in forward mode this is equivalent to:

$$y[i] = x[N - i] \quad \forall i = 0, 1, 2, \dots, N - 1$$

where  $N$  is the length of the input model. As this operator is self-adjoint,  $x$  and  $y$  in the equation above are simply swapped in adjoint mode.

### Attributes

`shape` [`tuple`] Operator shape

`explicit` [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

### Methods

<code>__init__(self, N[, dims, dir, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *kargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

### Examples using `pylops.Flip`

- sphx\_glr\_gallery\_plot\_flip.py

**pylops.Roll**

```
class pylops.Roll(N, dims=None, dir=0, shift=1, dtype='float64')
```

Roll along an axis.

Roll a multi-dimensional array along a specified direction `dir` for a chosen number of samples (`shift`).

**Parameters**

`N` [`int`] Number of samples in model.

`dims` [`list`, optional] Number of samples for each dimension (`None` if only one dimension is available)

`dir` [`int`, optional] Direction along which rolling is applied.

`shift` [`int`, optional] Number of samples by which elements are shifted

`dtype` [`str`, optional] Type of elements in input array.

**Notes**

The Roll operator is a thin wrapper around `numpy.roll` and shifts elements in a multi-dimensional array along a specified direction for a chosen number of samples.

**Attributes**

`shape` [`tuple`] Operator shape

`explicit` [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

**Methods**

<code>__init__(self, N[, dims, dir, shift, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\nkwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**Examples using `pylops.Roll`**

- sphx\_glr\_gallery\_plot\_roll.py

## pylops.Pad

```
class pylops.Pad(dims, pad, dtype='float64')  
    Pad operator.
```

Zero-pad model in forward model and extract non-zero subsequence in adjoint. Padding can be performed in one or multiple directions to any multi-dimensional input arrays.

## Parameters

**dims** [int or tuple] Number of samples for each dimension

**pad** [`tuple`] Number of samples to pad. If `dims` is a scalar, `pad` is a single tuple (`pad_in`, `pad_end`). If `dims` is a tuple, `pad` is a tuple of tuples where each inner tuple contains the number of samples to pad in each dimension

**dtype** [str, optional] Type of elements in input array.

## Raises

**ValueError** If any element of pad is negative.

## Notes

Given an array of size  $N$ , the *Pad* operator simply adds  $pad_{in}$  at the start and  $pad_{end}$  at the end in forward mode:

$$y_i = x_{i - pad_{in}} \quad \forall i = pad_{in}, pad_{in} + 1, \dots, pad_{in} + N - 1$$

and  $y_i = 0 \quad \forall i = 0, \dots, pad_{in} - 1, pad_{in} + N - 1, \dots, N + pad_{in} + pad_{end}$

In adjoint mode, values from  $pad_{in}$  to  $N - pad_{end}$  are extracted from the data:

$$x_i = y_{pad_{in}+i} \quad \forall i = 0, N-1$$

## Attributes

**shape** [tuple] Operator shape

**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, dims, pad[, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.Pad`

- sphx\_glr\_gallery\_plot\_pad.py

## pylops.Sum

```
class pylops.Sum(dims, dir, dtype='float64')
```

Sum operator.

Sum along an axis of a multi-dimensional array (at least 2 dimensions are required) in forward model, and spread along the same axis in adjoint mode.

## Parameters

**dims** [tuple] Number of samples for each dimension

**dir** [*int*] Direction along which summation is performed.

**dtype** [str, optional] Type of elements in input array.

## Notes

Given a two dimensional array, the *Sum* operator re-arranges the input model into a multi-dimensional array of size `dims` and sums values along direction `dir`:

$$y_j = \sum_i x_{i,j}$$

In adjoint mode, the data is spread along the same direction:

$$x_{i,j} = y_j \quad \forall i = 0, N - 1$$

## Attributes

**shape** [tuple] Operator shape

**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, dims, dir[, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.Sum`

- sphx\_glr\_gallery\_plot\_sum.py

## `pylops.Symmetrize`

```
class pylops.Symmetrize(N, dims=None, dir=0, dtype='float64')
    Symmetrize along an axis.
```

Symmetrize a multi-dimensional array along a specified direction `dir`.

### Parameters

`N` [`int`] Number of samples in model. Symmetric data has  $2N - 1$  samples

`dims` [`list`, optional] Number of samples for each dimension (`None` if only one dimension is available)

`dir` [`int`, optional] Direction along which symmetrization is applied

`dtype` [`str`, optional] Type of elements in input array

### Notes

The Symmetrize operator constructs a symmetric array given an input model in forward mode, by pre-pending the input model in reversed order.

For simplicity, given a one dimensional array, the forward operation can be expressed as:

$$y[i] = \begin{cases} x[i - N], & i \geq N \\ x[N - i], & \text{otherwise} \end{cases}$$

for  $i = 0, 1, 2, \dots, 2N - 2$ , where  $N$  is the lenght of the input model.

In adjoint mode, the Symmetrize operator assigns the sums of the elements in position  $N - i$  and  $N + i$  to position  $i$  as follows:

apart from the central sample where  $x[0] = y[N]$ .

### Attributes

`shape` [`tuple`] Operator shape

`explicit` [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

### Methods

<code>__init__(self, N[, dims, dir, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\nkwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .

Continued on next page

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<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.Symmetrize`

- sphx\_glr\_gallery\_plot\_symmetrize.py
- sphx\_glr\_gallery\_plot\_wavest.py

## `pylops.Restriction`

**class** `pylops.Restriction(M, iava, dims=None, dir=0, dtype='float64', inplace=True)`  
 Restriction (or sampling) operator.

Extract subset of values from input vector at locations `iava` in forward mode and place those values at locations `iava` in an otherwise zero vector in adjoint mode.

### Parameters

**M** [`int`] Number of samples in model.  
**iava** [`list` or `numpy.ndarray`] Integer indices of available samples for data selection.  
**dims** [`list`] Number of samples for each dimension (`None` if only one dimension is available)  
**dir** [`int`, optional] Direction along which restriction is applied.  
**dtype** [`str`, optional] Type of elements in input array.  
**inplace** [`bool`, optional] Work inplace (`True`) or make a new copy (`False`). By default, data is a reference to the model (in forward) and model is a reference to the data (in adjoint).

### See also:

`pylops.signalprocessing.Interp` Interpolation operator

### Notes

Extraction (or *sampling*) of a subset of  $N$  values at locations `iava` from an input (or model) vector `x` of size  $M$  can be expressed as:

$$y_i = x_{l_i} \quad \forall i = 1, 2, \dots, N$$

where  $\mathbf{l} = [l_1, l_2, \dots, l_N]$  is a vector containing the indeces of the original array at which samples are taken.

Conversely, in adjoint mode the available values in the data vector `y` are placed at locations  $\mathbf{l} = [l_1, l_2, \dots, l_M]$  in the model vector:

$$x_{l_i} = y_i \quad \forall i = 1, 2, \dots, N$$

and  $x_j = 0$  (*j* ≠ *i* (i.e., at all other locations in input vector)).

**Attributes****shape** [tuple] Operator shape**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)**Methods**

<code>__init__(self, M, iava[, dims, dir, dtype, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\nkwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>mask(self, x)</code>	Apply mask to input signal returning a signal of same size with values at <code>iava</code> locations and 0 at other locations
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**mask (self, x)**

Apply mask to input signal returning a signal of same size with values at `iava` locations and 0 at other locations

**Parameters****x** [numpy.ndarray] Input array (can be either flattened or not)**Returns****y** [numpy.ma.core.MaskedArray] Masked array.**Examples using `pylops.Restriction`**

- sphx\_glr\_gallery\_plot\_restriction.py
- sphx\_glr\_gallery\_plot\_tvreg.py
- *03. Solvers*
- *04. Bayesian Inversion*
- *06. 2D Interpolation*
- *12. Seismic regularization*

**pylops.Regression**

```
class pylops.Regression(taxis, order, dtype='float64')
    Polynomial regression.
```

Creates an operator that applies polynomial regression to a set of points. Values along the t-axis must be provided while initializing the operator. The coefficients of the polynomial regression form the model vector to be provided in forward mode, while the values of the regression curve shall be provided in adjoint mode.

**Parameters**

- taxis** [numpy.ndarray] Elements along the t-axis.
- order** [int] Order of the regressed polynomial.
- dtype** [str, optional] Type of elements in input array.

**Raises**

**TypeError** If t is not numpy.ndarray.

**See also:**

[LinearRegression](#) Linear regression

**Notes**

The Regression operator solves the following problem:

$$y_i = \sum_{n=0}^{\text{order}} x_n t_i^n \quad \forall i = 1, 2, \dots, N$$

where  $N$  represents the order of the chosen polynomial. We can express this problem in a matrix form

$$\mathbf{y} = \mathbf{Ax}$$

where

$$\mathbf{y} = [y_1, y_2, \dots, y_N]^T, \quad \mathbf{x} = [x_0, x_1, \dots, x_{\text{order}}]^T$$

and

$$\mathbf{A} = \begin{bmatrix} 1 & t_1 & t_1^2 & .. & t_1^{\text{order}} \\ 1 & t_2 & t_2^2 & .. & t_2^{\text{order}} \\ .. & .. & .. & .. & .. \\ 1 & t_N & t_N^2 & .. & t_N^{\text{order}} \end{bmatrix}$$

**Attributes**

- shape** [tuple] Operator shape
- explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

**Methods**


---

<code>__init__(self, taxis, order[, dtype])</code>	Initialize this LinearOperator.
	Continued on next page

---

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<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply(self, t, x)</code>	Return values along y-axis given certain t location(s) along t-axis and regression coefficients x
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**apply (self, t, x)**

Return values along y-axis given certain t location(s) along t-axis and regression coefficients x

**Parameters****taxis** [numpy.ndarray] Elements along the t-axis.**x** [numpy.ndarray] Regression coefficients**dtype** [str, optional]**Returns****y** [numpy.ndarray] Values along y-axis**Examples using `pylops.LinearRegression`**

- sphx\_glr\_gallery\_plot\_linearregr.py
- sphx\_glr\_gallery\_plot\_regr.py

**pylops.LinearRegression**

```
pylops.LinearRegression(taxis, dtype='float64')
Linear regression.
```

Creates an operator that applies linear regression to a set of points. Values along the t-axis must be provided while initializing the operator. Intercept and gradient form the model vector to be provided in forward mode, while the values of the regression line curve shall be provided in adjoint mode.

**Parameters****taxis** [numpy.ndarray] Elements along the t-axis.**dtype** [str, optional] Type of elements in input array.**Raises****TypeError** If t is not numpy.ndarray.

**See also:**

[Regression](#) Polynomial regression

**Notes**

The LinearRegression operator solves the following problem:

$$y_i = x_0 + x_1 t_i \quad \forall i = 1, 2, \dots, N$$

We can express this problem in a matrix form

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

where

$$\mathbf{y} = [y_1, y_2, \dots, y_N]^T, \quad \mathbf{x} = [x_0, x_1]^T$$

and

$$\mathbf{A} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \ddots \\ 1 & t_N \end{bmatrix}$$

Note that this is a particular case of the `pylops.Regression` operator and it is in fact just a lazy call of that operator with `order=1`.

**Attributes**

**shape** [`tuple`] Operator shape

**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

**Examples using `pylops.LinearRegression`**

- sphx\_glr\_gallery\_plot\_linearregr.py

**pylops.CausalIntegration**

```
class pylops.CausalIntegration(N, dims=None, dir=-1, sampling=1, halfcurrent=True,
                               dtype='float64')
Causal integration.
```

Apply causal integration to a multi-dimensional array along `dir` axis.

**Parameters**

**N** [`int`] Number of samples in model.

**dims** [`list`, optional] Number of samples for each dimension (`None` if only one dimension is available)

**dir** [`int`, optional] Direction along which smoothing is applied.

**sampling** [`float`, optional] Sampling step `dx`.

**halfcurrent** [`float`, optional] Add half of current value (`True`) or the entire value (`False`)

**dtype** [`str`, optional] Type of elements in input array.

## Notes

The CausalIntegration operator applies a causal integration to any chosen direction of a multi-dimensional array. For simplicity, given a one dimensional array, the causal integration is:

$$y(t) = \int x(t)dt$$

which can be discretised as :

$$y[i] = \sum_{j=0}^i x[j]dt$$

or

$$y[i] = \left( \sum_{j=0}^{i-1} x[j] + 0.5x[i] \right) dt$$

where  $dt$  is the sampling interval. In our implementation, the choice to add  $x[i]$  or just  $0.5x[i]$  is made by selecting the `halfcurrent` parameter.

Note that the integral of a signal has no unique solution, as any constant  $c$  can be added to  $y$ , for example if  $x(t) = t^2$  the resulting integration is:

$$y(t) = \int t^2 dt = \frac{t^3}{3} + c$$

If we apply a first derivative to  $y$  we in fact obtain:

$$x(t) = \frac{dy}{dt} = t^2$$

no matter the choice of  $c$ .

## Attributes

**shape** [tuple] Operator shape

**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, N[, dims, dir, sampling, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.CausalIntegration`

- sphx\_glr\_gallery\_plot\_causalintegration.py

## pylops.Spread

```
class pylops.Spread(dims, dimsd, table=None, dtable=None, fh=None, interp=False, engine='numpy', dtype='float64')
```

Spread operator.

Spread values from the input model vector arranged as a 2-dimensional array of size  $[n_{x0} \times n_{t0}]$  into the data vector of size  $[n_x \times n_t]$ . Spreading is performed along parametric curves provided as look-up table of pre-computed indices (`table`) or computed on-the-fly using a function handle (`fh`).

In adjoint mode, values from the data vector are instead stacked along the same parametric curves.

### Parameters

**dims** [tuple] Dimensions of model vector (vector will be reshaped internally into a two-dimensional array of size  $[n_{x0} \times n_{t0}]$ , where the first dimension is the spreading/stacking direction)

**dimsd** [tuple] Dimensions of model vector (vector will be reshaped internal into a two-dimensional array of size  $[n_x \times n_t]$ )

**table** [np.ndarray, optional] Look-up table of indeces of size  $[n_{x0} \times n_{t0} \times n_x]$  (if None use function handle `fh`)

**dtable** [np.ndarray, optional] Look-up table of decimals remainders for linear interpolation of size  $[n_{x0} \times n_{t0} \times n_x]$  (if None use function handle `fh`)

**fh** [np.ndarray, optional] Function handle that returns an index (and a fractional value in case of `interp=True`) to be used for spreading/stacking given indices in  $x0$  and  $t$  axes (if None use look-up table `table`)

**interp** [bool, optional] Apply linear interpolation (True) or nearest interpolation (False) during stacking/spreading along parametric curve. To be used only if `engine='numba'`, inferred directly from the number of outputs of `fh` for `engine='numpy'`

**engine** [str, optional] Engine used for fft computation (numpy or numba). Note that numba can only be used when providing a look-up table

**dtype** [str, optional] Type of elements in input array.

### Raises

**KeyError** If `engine` is neither numpy nor numba

**NotImplementedError** If both `table` and `fh` are not provided

**ValueError** If `table` has shape different from  $[n_{x0} \times n_{t0} \times n_x]$

## Notes

The Spread operator applies the following linear transform in forward mode to the model vector after reshaping it into a 2-dimensional array of size  $[n_x \times n_t]$ :

$$m(x_0, t_0) \rightarrow d(x, t = f(x_0, x, t_0))$$

where  $f(x_0, x, t)$  is a mapping function that returns a value  $t$  given values  $x_0$ ,  $x$ , and  $t_0$ .

In adjoint mode, the model is reconstructed by means of the following stacking operation:

$$m(x_0, t_0) = \int d(x, t = f(x_0, x, t_0))dx$$

Note that `table` (or `fh`) must return integer numbers representing indices in the axis  $t$ . However it also possible to perform linear interpolation as part of the spreading/stacking process by providing the decimal part of the mapping function ( $t - \lfloor t \rfloor$ ) either in `dtable` input parameter or as second value in the return of `fh` function.

### Attributes

`shape` [`tuple`] Operator shape

`explicit` [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

### Methods

<code>__init__(self, dims, dimsd[], table, dtable, ...)</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *kargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

### Examples using `pylops.Spread`

- `sphx_glr_gallery_plot_sliding.py`
- `sphx_glr_gallery_plot_radon.py`
- [11. Radon filtering](#)
- [12. Seismic regularization](#)
- [16. CT Scan Imaging](#)

### `pylops.VStack`

**class** `pylops.VStack(ops, dtype=None)`  
Vertical stacking.

Stack a set of N linear operators vertically.

#### Parameters

`ops` [`list`] Linear operators to be stacked. Alternatively, `numpy.ndarray` or `scipy.sparse` matrices can be passed in place of one or more operators.

**dtype** [str, optional] Type of elements in input array.

## Raises

**ValueError** If `ops` have different number of rows

## Notes

A vertical stack of  $N$  linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \dots \\ \mathbf{L}_N \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{L}_1 \mathbf{x} \\ \mathbf{L}_2 \mathbf{x} \\ \dots \\ \mathbf{L}_N \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \dots \\ \mathbf{y}_N \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_1^H & \mathbf{L}_2^H & \dots & \mathbf{L}_N^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \mathbf{L}_1^H \mathbf{y}_1 + \mathbf{L}_2^H \mathbf{y}_2 + \dots + \mathbf{L}_N^H \mathbf{y}_N$$

### Attributes

**shape** [tuple] Operator shape

**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, ops[, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.vStack`

- sphx\_glr\_gallery\_plot\_derivative.py
  - sphx\_glr\_gallery\_plot\_stacking.py
  - sphx\_glr\_gallery\_plot\_wavest.py

## pylops.HStack

```
class pylops.HStack(ops, dtype='float64')
    Horizontal stacking.
```

Stack a set of N linear operators horizontally.

### Parameters

**ops** [`list`] Linear operators to be stacked. Alternatively, `numpy.ndarray` or `scipy.sparse` matrices can be passed in place of one or more operators.

**dtype** [`str`, optional] Type of elements in input array.

### Raises

**ValueError** If `ops` have different number of columns

### Notes

An horizontal stack of N linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 & \dots & \mathbf{L}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} = \mathbf{L}_1\mathbf{x}_1 + \mathbf{L}_2\mathbf{x}_2 + \dots + \mathbf{L}_N\mathbf{x}_N$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_1^H \\ \mathbf{L}_2^H \\ \vdots \\ \mathbf{L}_N^H \end{bmatrix} \mathbf{y} = \begin{bmatrix} \mathbf{L}_1^H \mathbf{y} \\ \mathbf{L}_2^H \mathbf{y} \\ \vdots \\ \mathbf{L}_N^H \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix}$$

### Attributes

**shape** [`tuple`] Operator shape

**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

### Methods

<code>__init__(self, ops[, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

### Examples using `pylops.HStack`

- sphx\_glr\_gallery\_plot\_stacking.py

### pylops.Block

`pylops.Block(ops, dtype=None)`

Block operator.

Create a block operator from N lists of M linear operators each.

#### Parameters

`ops` [`list`] List of lists of operators to be combined in block fashion. Alternatively, `numpy.ndarray` or `scipy.sparse` matrices can be passed in place of one or more operators.

`dtype` [`str`, optional] Type of elements in input array.

#### Notes

In mathematics, a block or a partitioned matrix is a matrix that is interpreted as being broken into sections called blocks or submatrices. Similarly a block operator is composed of N sets of M linear operators each such that its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_{1,1} & \mathbf{L}_{1,2} & \dots & \mathbf{L}_{1,M} \\ \mathbf{L}_{2,1} & \mathbf{L}_{2,2} & \dots & \mathbf{L}_{2,M} \\ \dots & \dots & \dots & \dots \\ \mathbf{L}_{N,1} & \mathbf{L}_{N,2} & \dots & \mathbf{L}_{N,M} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_M \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{1,1}\mathbf{x}_1 + \mathbf{L}_{1,2}\mathbf{x}_2 + \mathbf{L}_{1,M}\mathbf{x}_M \\ \mathbf{L}_{2,1}\mathbf{x}_1 + \mathbf{L}_{2,2}\mathbf{x}_2 + \mathbf{L}_{2,M}\mathbf{x}_M \\ \vdots \\ \mathbf{L}_{N,1}\mathbf{x}_1 + \mathbf{L}_{N,2}\mathbf{x}_2 + \mathbf{L}_{N,M}\mathbf{x}_M \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_{1,1}^H & \mathbf{L}_{2,1}^H & \dots & \mathbf{L}_{N,1}^H \\ \mathbf{L}_{1,2}^H & \mathbf{L}_{2,2}^H & \dots & \mathbf{L}_{N,2}^H \\ \dots & \dots & \dots & \dots \\ \mathbf{L}_{1,M}^H & \mathbf{L}_{2,M}^H & \dots & \mathbf{L}_{N,M}^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{1,1}^H\mathbf{y}_1 + \mathbf{L}_{2,1}^H\mathbf{y}_2 + \mathbf{L}_{N,1}^H\mathbf{y}_N \\ \mathbf{L}_{1,2}^H\mathbf{y}_1 + \mathbf{L}_{2,2}^H\mathbf{y}_2 + \mathbf{L}_{N,2}^H\mathbf{y}_N \\ \vdots \\ \mathbf{L}_{1,M}^H\mathbf{y}_1 + \mathbf{L}_{2,M}^H\mathbf{y}_2 + \mathbf{L}_{N,M}^H\mathbf{y}_N \end{bmatrix}$$

#### Attributes

`shape` [`tuple`] Operator shape

`explicit` [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

### Examples using `pylops.Block`

- sphx\_glr\_gallery\_plot\_stacking.py

### pylops.BlockDiag

`class pylops.BlockDiag(ops, dtype=None)`

Block-diagonal operator.

Create a block-diagonal operator from N linear operators.

#### Parameters

**ops** [`list`] Linear operators to be stacked. Alternatively, `numpy.ndarray` or `scipy.sparse` matrices can be passed in place of one or more operators.

**dtype** [`str`, optional] Type of elements in input array.

## Notes

A block-diagonal operator composed of N linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_2 & \dots & \mathbf{0} \\ \dots & \dots & \dots & \dots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{L}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1 \mathbf{x}_1 \\ \mathbf{L}_2 \mathbf{x}_2 \\ \vdots \\ \mathbf{L}_N \mathbf{x}_N \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_1^H & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_2^H & \dots & \mathbf{0} \\ \dots & \dots & \dots & \dots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{L}_N^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1^H \mathbf{y}_1 \\ \mathbf{L}_2^H \mathbf{y}_2 \\ \vdots \\ \mathbf{L}_N^H \mathbf{y}_N \end{bmatrix}$$

## Attributes

**shape** [`tuple`] Operator shape

**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

## Methods

<code>__init__(self, ops[, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, [*kwargs_eig])</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.BlockDiag`

- `sphx_glr_gallery_plot_stacking.py`

**pylops.Kronecker**

```
class pylops.Kronecker(Op1, Op2, dtype='float64')
    Kronecker operator.
```

Perform Kronecker product of two operators. Note that the combined operator is never created explicitly, rather the product of this operator with the model vector is performed in forward mode, or the product of the adjoint of this operator and the data vector in adjoint mode.

**Parameters**

- Op1** [*pylops.LinearOperator*] First operator
- Op2** [*pylops.LinearOperator*] Second operator
- dtype** [*str*, optional] Type of elements in input array.

**Notes**

The Kronecker product (denoted with  $\otimes$ ) is an operation on two operators **Op<sub>1</sub>** and **Op<sub>2</sub>** of sizes  $[n_1 \times m_1]$  and  $[n_2 \times m_2]$  respectively, resulting in a block matrix of size  $[n_1 n_2 \times m_1 m_2]$ .

$$\mathbf{Op}_1 \otimes \mathbf{Op}_2 = \begin{bmatrix} \mathbf{Op}_1^{1,1} \mathbf{Op}_2 & \dots & \mathbf{Op}_1^{1,m_1} \mathbf{Op}_2 \\ \dots & \dots & \dots \\ \mathbf{Op}_1^{n_1,1} \mathbf{Op}_2 & \dots & \mathbf{Op}_1^{n_1,m_1} \mathbf{Op}_2 \end{bmatrix}$$

The application of the resulting matrix to a vector **x** of size  $[m_1 m_2 \times 1]$  is equivalent to the application of the second operator **Op<sub>2</sub>** to the rows of a matrix of size  $[m_2 \times m_1]$  obtained by reshaping the input vector **x**, followed by the application of the first operator to the transposed matrix produced by the first operator. In adjoint mode the same procedure is followed but the adjoint of each operator is used.

**Attributes**

- shape** [*tuple*] Operator shape
- explicit** [*bool*] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

**Methods**

<code>__init__(self, Op1, Op2[, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\`*kargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.Kronecker`

- sphx\_glr\_gallery\_plot\_stacking.py

## Smoothing and derivatives

<code>Smoothing1D(nsMOOTH, dims[, dir, dtype])</code>	1D Smoothing.
<code>Smoothing2D(nsMOOTH, dims[, nodir, dtype])</code>	2D Smoothing.
<code>FirstDerivative(N[, dims, dir, sampling, ...])</code>	First derivative.
<code>SecondDerivative(N[, dims, dir, sampling, ...])</code>	Second derivative.
<code>Laplacian(dims[, dirs, weights, sampling, ...])</code>	Laplacian.
<code>Gradient(dims[, sampling, edge, dtype])</code>	Gradient.
<code>FirstDirectionalDerivative(dims, v[, ...])</code>	First Directional derivative.
<code>SecondDirectionalDerivative(dims, v[, ...])</code>	Second Directional derivative.

### pylops.Smoothing1D

`pylops.Smoothing1D(nsMOOTH, dims, dir=0, dtype='float64')`  
1D Smoothing.

Apply smoothing to model (and data) along a specific direction of a multi-dimensional array depending on the choice of `dir`.

#### Parameters

**nsMOOTH** [`int`] Length of smoothing operator (must be odd)  
**dIMS** [`tuple` or `int`] Number of samples for each dimension  
**dir** [`int`, optional] Direction along which smoothing is applied  
**dtype** [`str`, optional] Type of elements in input array.

#### Notes

The Smoothing1D operator is a special type of convolutional operator that convolves the input model (or data) with a constant filter of size  $n_{smooth}$ :

$$\mathbf{f} = [1/n_{smooth}, 1/n_{smooth}, \dots, 1/n_{smooth}]$$

When applied to the first direction:

$$y[i, j, k] = 1/n_{smooth} \sum_{l=-(n_{smooth}-1)/2}^{(n_{smooth}-1)/2} x[l, j, k]$$

Similarly when applied to the second direction:

$$y[i, j, k] = 1/n_{smooth} \sum_{l=-(n_{smooth}-1)/2}^{(n_{smooth}-1)/2} x[i, l, k]$$

and the third direction:

$$y[i, j, k] = 1/n_{smooth} \sum_{l=-(n_{smooth}-1)/2}^{(n_{smooth}-1)/2} x[i, j, l]$$

Note that since the filter is symmetrical, the *Smoothing1D* operator is self-adjoint.

## Attributes

**shape** [`tuple`] Operator shape

**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

## Examples using `pylops.Smoothing1D`

- sphx\_glr\_gallery\_plot\_smoothing1d.py
- sphx\_glr\_gallery\_plot\_causalintegration.py
- sphx\_glr\_gallery\_plot\_wavest.py
- *03. Solvers*

## `pylops.Smoothing2D`

`pylops.Smoothing2D` (`nsmooth`, `dims`, `nodir=None`, `dtype='float64'`)  
2D Smoothing.

Apply smoothing to model (and data) along two directions of a multi-dimensional array depending on the choice of `nodir`.

## Parameters

**nsmooth** [`tuple` or `list`] Length of smoothing operator in 1st and 2nd dimensions (must be odd)

**dims** [`tuple`] Number of samples for each dimension

**nodir** [`int`, optional] Direction along which smoothing is NOT applied (set to `None` for 2d arrays)

**dtype** [`str`, optional] Type of elements in input array.

See also:

`lops.signalprocessing.Convolve2D` 2D convolution

## Notes

The 2D Smoothing operator is a special type of convolutional operator that convolves the input model (or data) with a constant 2d filter of size  $n_{smooth,1} \times n_{smooth,2}$ :

Its application to a two dimensional input signal is:

$$y[i, j] = 1/(n_{smooth,1} * n_{smooth,2}) \sum_{l=-(n_{smooth,1}-1)/2}^{(n_{smooth,1}-1)/2} \sum_{m=-(n_{smooth,2}-1)/2}^{(n_{smooth,2}-1)/2} x[l, m]$$

Note that since the filter is symmetrical, the *Smoothing2D* operator is self-adjoint.

## Attributes

**shape** [`tuple`] Operator shape

**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

## Examples using `pylops.Smoothing2D`

- sphx\_glr\_gallery\_plot\_smoothing2d.py
- sphx\_glr\_gallery\_plot\_causalintegration.py

## `pylops.FirstDerivative`

```
class pylops.FirstDerivative(N, dims=None, dir=0, sampling=1.0, edge=False,
                             dtype='float64')
```

First derivative.

Apply second-order centered first derivative.

### Parameters

**N** [`int`] Number of samples in model.

**dims** [`tuple`, optional] Number of samples for each dimension (None if only one dimension is available)

**dir** [`int`, optional] Direction along which smoothing is applied.

**sampling** [`float`, optional] Sampling step  $\text{dx}$ .

**edge** [`bool`, optional] Use reduced order derivative at edges (True) or ignore them (False)

**dtype** [`str`, optional] Type of elements in input array.

### Notes

The FirstDerivative operator applies a first derivative to any chosen direction of a multi-dimensional array.

For simplicity, given a one dimensional array, the second-order centered first derivative is:

$$y[i] = (0.5x[i + 1] - 0.5x[i - 1])/\text{dx}$$

### Attributes

**shape** [`tuple`] Operator shape

**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (True) or not (False)

### Methods

<code>__init__(self, N[, dims, dir, sampling, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.

Continued on next page

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<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.FirstDerivative`

- sphx\_glr\_gallery\_plot\_causalintegration.py
- sphx\_glr\_gallery\_plot\_derivative.py
- sphx\_glr\_gallery\_plot\_stacking.py
- sphx\_glr\_gallery\_plot\_tvreg.py
- *05. Image deblurring*
- *16. CT Scan Imaging*

## `pylops.SecondDerivative`

```
class pylops.SecondDerivative(N, dims=None, dir=0, sampling=1, edge=False, dtype='float64')
```

Second derivative.

Apply second-order second derivative.

### Parameters

**N** [`int`] Number of samples in model.

**dims** [`tuple`, optional] Number of samples for each dimension (None if only one dimension is available)

**dir** [`int`, optional] Direction along which smoothing is applied.

**sampling** [`float`, optional] Sampling step dx.

**edge** [`bool`, optional] Use reduced order derivative at edges (True) or ignore them (False)

**dtype** [`str`, optional] Type of elements in input array.

### Notes

The SecondDerivative operator applies a second derivative to any chosen direction of a multi-dimensional array.

For simplicity, given a one dimensional array, the second-order centered first derivative is:

$$y[i] = (x[i+1] - 2x[i] + x[i-1])/dx^2$$

### Attributes

**shape** [`tuple`] Operator shape

**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (True) or not (False)

### Methods

<code>__init__(self, N[, dims, dir, sampling, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.SecondDerivative`

- sphx\_glr\_gallery\_plot\_causalintegration.py
- sphx\_glr\_gallery\_plot\_derivative.py
- sphx\_glr\_gallery\_plot\_stacking.py
- sphx\_glr\_gallery\_plot\_tvreg.py
- sphx\_glr\_gallery\_plot\_wavest.py
- *03. Solvers*
- *12. Seismic regularization*

## pylops.Laplacian

`pylops.Laplacian(dims, dirs=(0, 1), weights=(1, 1), sampling=(1, 1), edge=False, dtype='float64')`  
 Laplacian.

Apply second-order centered Laplacian operator to a multi-dimensional array (at least 2 dimensions are required)

### Parameters

- dims** [tuple] Number of samples for each dimension.
- dirs** [tuple, optional] Directions along which laplacian is applied.
- weights** [tuple, optional] Weight to apply to each direction (real laplacian operator if `weights=[1, 1]`)
- sampling** [tuple, optional] Sampling steps for each direction
- edge** [bool, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`)
- dtype** [str, optional] Type of elements in input array.

### Returns

- l2op** [`pylops.LinearOperator`] Laplacian linear operator

## Notes

The Laplacian operator applies a second derivative along two directions of a multi-dimensional array.

For simplicity, given a two dimensional array, the Laplacian is:

$$y[i, j] = (x[i + 1, j] + x[i - 1, j] + x[i, j - 1] + x[i, j + 1] - 4x[i, j]) / (dx * dy)$$

## Examples using `pylops.Laplacian`

- sphx\_glr\_gallery\_plot\_bilinear.py
- sphx\_glr\_gallery\_plot\_causalintegration.py
- sphx\_glr\_gallery\_plot\_derivative.py
- [06. 2D Interpolation](#)
- [16. CT Scan Imaging](#)

## `pylops.Gradient`

`pylops.Gradient(dims, sampling=1, edge=False, dtype='float64')`  
Gradient.

Apply gradient operator to a multi-dimensional array (at least 2 dimensions are required).

### Parameters

- dims** [`tuple`] Number of samples for each dimension.
- sampling** [`tuple`, optional] Sampling steps for each direction.
- edge** [`bool`, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`).
- dtype** [`str`, optional] Type of elements in input array.

### Returns

`I2op` [`pylops.LinearOperator`] Gradient linear operator

## Notes

The Gradient operator applies a first-order derivative to each dimension of a multi-dimensional array in forward mode.

For simplicity, given a three dimensional array, the Gradient in forward mode is:

$$g_{i,j,k} = (f_{i+1,j,k} - f_{i-1,j,k})/d_1 \mathbf{i}_1 + (f_{i,j+1,k} - f_{i,j-1,k})/d_2 \mathbf{i}_2 + (f_{i,j,k+1} - f_{i,j,k-1})/d_3 \mathbf{i}_3$$

which is discretized as follows:

$$\mathbf{g} = \begin{bmatrix} df_1 \\ df_2 \\ df_3 \end{bmatrix}$$

In adjoint mode, the adjoints of the first derivatives along different axes are instead summed together.

## Examples using `pylops.Gradient`

- sphx\_glr\_gallery\_plot\_derivative.py

## `pylops.FirstDirectionalDerivative`

`pylops.FirstDirectionalDerivative(dims, v, sampling=1, edge=False, dtype='float64')`

First Directional derivative.

Apply directional derivative operator to a multi-dimensional array (at least 2 dimensions are required) along either a single common direction or different directions for each point of the array.

### Parameters

`dims` [`tuple`] Number of samples for each dimension.

`v` [`np.ndarray`, optional] Single direction (array of size  $n_{\text{dims}}$ ) or group of directions (array of size  $[n_{\text{dims}} \times n_{d0} \times \dots \times n_{ndims}]$ )

`sampling` [`tuple`, optional] Sampling steps for each direction.

`edge` [`bool`, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`).

`dtype` [`str`, optional] Type of elements in input array.

### Returns

`ddop` [`pylops.LinearOperator`] First directional derivative linear operator

## Notes

The `FirstDirectionalDerivative` applies a first-order derivative to a multi-dimensional array along the direction defined by the unitary vector  $\mathbf{v}$ :

$$df_{\mathbf{v}} = \nabla f \mathbf{v}$$

or along the directions defined by the unitary vectors  $\mathbf{v}(x, y)$ :

$$df_{\mathbf{v}}(x, y) = \nabla f(x, y) \mathbf{v}(x, y)$$

where we have here considered the 2-dimensional case.

This operator can be easily implemented as the concatenation of the `pylops.Gradient` operator and the `pylops.Diagonal` operator with `:math:\mathbf{v}` along the main diagonal.

## Examples using `pylops.FirstDirectionalDerivative`

- sphx\_glr\_gallery\_plot\_derivative.py

## `pylops.SecondDirectionalDerivative`

`pylops.SecondDirectionalDerivative(dims, v, sampling=1, edge=False, dtype='float64')`

Second Directional derivative.

Apply second directional derivative operator to a multi-dimensional array (at least 2 dimensions are required) along either a single common direction or different directions for each point of the array.

## Parameters

**dims** [`tuple`] Number of samples for each dimension.

**v** [`np.ndarray`, optional] Single direction (array of size  $n_{dims}$ ) or group of directions (array of size  $[n_{dims} \times n_{d0} \times \dots \times n_{ndims}]$ )

**sampling** [`tuple`, optional] Sampling steps for each direction.

**edge** [`bool`, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`).

**dtype** [`str`, optional] Type of elements in input array.

## Returns

**ddop** [`pylops.LinearOperator`] Second directional derivative linear operator

## Notes

The `SecondDirectionalDerivative` applies a second-order derivative to a multi-dimensional array along the direction defined by the unitary vector  $\mathbf{v}$ :

$$d^2 f_{\mathbf{v}} = -D_{\mathbf{v}}^T [D_{\mathbf{v}} f]$$

where  $D_{\mathbf{v}}$  is the first-order directional derivative implemented by `pylops.SecondDirectionalDerivative`.

This operator is sometimes also referred to as directional Laplacian in the literature.

## Examples using `pylops.SecondDirectionalDerivative`

- `sphx_glr_gallery_plot_derivative.py`

## Signal processing

<code>Convolve1D(N, h[, offset, dims, dir, dtype, ...])</code>	1D convolution operator.
<code>Convolve2D(N, h, dims[, offset, nodir, ...])</code>	2D convolution operator.
<code>ConvolveND(N, h, dims[, offset, dirs, ...])</code>	ND convolution operator.
<code>Interp(M, iava[, dims, dir, kind, dtype])</code>	Interpolation operator.
<code>Bilinear(iava, dims[, dtype])</code>	Bilinear interpolation operator.
<code>FFT(dims[, dir, nfft, sampling, real, ...])</code>	One dimensional Fast-Fourier Transform.
<code>FFT2D(dims[, dirs, nffts, sampling, dtype])</code>	Two dimensional Fast-Fourier Transform.
<code>FFTND(dims[, dirs, nffts, sampling, dtype])</code>	N-dimensional Fast-Fourier Transform.
<code>DWT(dims[, dir, wavelet, level, dtype])</code>	One dimensional Wavelet operator.
<code>DWT2D(dims[, dirs, wavelet, level, dtype])</code>	Two dimensional Wavelet operator.
<code>Radon2D(taxis, haxis, pxaxis[, kind, ...])</code>	Two dimensional Radon transform.
<code>Radon3D(taxis, hyaxis, hxaxis, pyaxis, pxaxis)</code>	Three dimensional Radon transform.
<code>Sliding2D(Op, dims, dimsd, nwin, nover[, ...])</code>	2D Sliding transform operator.
<code>Sliding3D(Op, dims, dimsd, nwin, nover, nop)</code>	3D Sliding transform operator.
<code>Fredholm1(G[, nz, saveGt, usematmul, dtype])</code>	Fredholm integral of first kind.

## pylops.signalprocessing.Convolve1D

```
class pylops.signalprocessing.Convolve1D(N, h, offset=0, dims=None, dir=0,
                                         dtype='float64', method='direct')
```

1D convolution operator.

Apply one-dimensional convolution with a compact filter to model (and data) along a specific direction of a multi-dimensional array depending on the choice of `dir`.

### Parameters

- N** [`int`] Number of samples in model.
- h** [`numpy.ndarray`] 1d compact filter to be convolved to input signal
- offset** [`int`] Index of the center of the compact filter
- dims** [`tuple`] Number of samples for each dimension (`None` if only one dimension is available)
- dir** [`int`, optional] Direction along which convolution is applied
- method** [`str`, optional] Method used to calculate the convolution (`direct` or `fft`). Note that `fft` approach is always used if `dims=None`.
- dtype** [`str`, optional] Type of elements in input array.

### Raises

**ValueError** If `offset` is bigger than `len(h) - 1`

### Notes

The Convolve1D operator applies convolution between the input signal  $x(t)$  and a compact filter kernel  $h(t)$  in forward model:

$$y(t) = \int_{-\infty}^{\infty} h(t - \tau)x(\tau)d\tau$$

This operation can be discretized as follows

$$y[n] = \sum_{m=-\infty}^{\infty} h[n - m]x[m]$$

as well as performed in the frequency domain.

$$Y(f) = F(h(t)) * F(x(t))$$

Convolve1D operator uses `scipy.signal.convolve` that automatically chooses the best domain for the operation to be carried out for one dimensional inputs. The fft implementation `scipy.signal.fftconvolve` is however enforced for signals in 2 or more dimensions as this routine efficiently operates on multi-dimensional arrays.

As the adjoint of convolution is correlation, Convolve1D operator applies correlation in the adjoint mode.

In time domain:

$$x(t) = \int_{-\infty}^{\infty} h(t + \tau)x(\tau)d\tau$$

or in frequency domain:

$$y(t) = F^{-1}(H(f)^* * X(f))$$

## Attributes

<code>shape</code> [ <code>tuple</code> ]	Operator shape
<code>explicit</code> [ <code>bool</code> ]	Operator contains a matrix that can be solved explicitly ( <code>True</code> ) or not ( <code>False</code> )

## Methods

<code>__init__(self, N, h[, offset, dims, dir, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.signalprocessing.Convolve1D`

- `sphx_glr_gallery_plot_smoothing1d.py`
- `sphx_glr_gallery_plot_convolve.py`
- `sphx_glr_gallery_plot_ista.py`
- `sphx_glr_gallery_plot_wavest.py`
- *03. Solvers*
- *04. Bayesian Inversion*

## `pylops.signalprocessing.Convolve2D`

`pylops.signalprocessing.Convolve2D(N, h, dims, offset=(0, 0), nodir=None, dtype='float64', method='fft')`

2D convolution operator.

Apply two-dimensional convolution with a compact filter to model (and data) along a pair of specific directions of a two or three-dimensional array depending on the choice of `nodir`.

### Parameters

<code>N</code> [ <code>int</code> ]	Number of samples in model
<code>h</code> [ <code>numpy.ndarray</code> ]	2d compact filter to be convolved to input signal
<code>dims</code> [ <code>list</code> ]	Number of samples for each dimension
<code>offset</code> [ <code>tuple</code> , optional]	Indeces of the center of the compact filter
<code>nodir</code> [ <code>int</code> , optional]	Direction along which convolution is NOT applied (set to None for 2d arrays)

**dtype** [`str`, optional] Type of elements in input array.

**method** [`str`, optional] Method used to calculate the convolution (`direct` or `fft`).

### Returns

**cop** [`pylops.LinearOperator`] Convolve2D linear operator

### Notes

The Convolve2D operator applies two-dimensional convolution between the input signal  $d(t, x)$  and a compact filter kernel  $h(t, x)$  in forward model:

$$y(t, x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(t - \tau, x - \chi) d(\tau, \chi) d\tau d\chi$$

This operation can be discretized as follows

$$y[i, n] = \sum_{j=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h[i - j, n - m] d[j, m]$$

as well as performed in the frequency domain.

$$Y(f, k_x) = F(h(t, x)) * F(d(t, x))$$

Convolve2D operator uses `scipy.signal.convolve2d` that automatically chooses the best domain for the operation to be carried out.

As the adjoint of convolution is correlation, Convolve2D operator applies correlation in the adjoint mode.

In time domain:

$$y(t, x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(t + \tau, x + \chi) d(\tau, \chi) d\tau d\chi$$

or in frequency domain:

$$y(t, x) = F^{-1}(H(f, k_x)^* * X(f, k_x))$$

### Examples using `pylops.signalprocessing.Convolve2D`

- `sphx_glr_gallery_plot_convolve.py`
- [05. Image deblurring](#)

### `pylops.signalprocessing.ConvolveND`

```
class pylops.signalprocessing.ConvolveND(N, h, dims, offset=(0, 0, 0), dirs=None,
                                         method='fft', dtype='float64')
```

ND convolution operator.

Apply n-dimensional convolution with a compact filter to model (and data) along a set of directions `dirs` of a n-dimensional array.

### Parameters

**N** [`int`] Number of samples in model

**h** [numpy.ndarray] nd compact filter to be convolved to input signal  
**dims** [list] Number of samples for each dimension  
**offset** [tuple, optional] Indices of the center of the compact filter  
**dirs** [tuple, optional] Directions along which convolution is applied (set to None for filter of same dimension as input vector)  
**method** [str, optional] Method used to calculate the convolution (direct or fft).  
**dtype** [str, optional] Type of elements in input array.

## Notes

The ConvolveND operator applies n-dimensional convolution between the input signal  $d(x_1, x_2, \dots, x_N)$  and a compact filter kernel  $h(x_1, x_2, \dots, x_N)$  in forward model. This is a straightforward extension to multiple dimensions of `pylops.signalprocessing.Convolve2D` operator.

### Attributes

**shape** [tuple] Operator shape  
**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

---

<code>__init__(self, N, h, dims[, offset, dirs, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

---

## Examples using `pylops.signalprocessing.ConvolveND`

- sphx\_glr\_gallery\_plot\_smoothing2d.py
- sphx\_glr\_gallery\_plot\_causalintegration.py
- sphx\_glr\_gallery\_plot\_convolve.py
- 05. *Image deblurring*

## `pylops.signalprocessing.Interp`

`pylops.signalprocessing.Interp(M, iava, dims=None, dir=0, kind='linear', dtype='float64')`  
Interpolation operator.

Apply interpolation along direction `dir` from regularly sampled input vector into fractionary positions `iava`.

*Nearest neighbour* interpolation is a thin wrapper around `pylops.Restriction` at `np.round(iava)` locations.

*Linear interpolation* extracts values from input vector at locations `np.floor(iava)` and `np.floor(iava)+1` and linearly combines them in forward mode, places weighted versions of the interpolated values at locations `np.floor(iava)` and `np.floor(iava)+1` in an otherwise zero vector in adjoint mode.

---

**Note:** The vector `iava` should contain unique values. If the same index is repeated twice an error will be raised. This also applies when values beyond the last element of the input array for *linear interpolation* as those values are forced to be just before this element.

---

### Parameters

`M` [`int`] Number of samples in model.

`iava` [`list` or `numpy.ndarray`] Floating indices of locations of available samples for interpolation.

`dims` [`list`, optional] Number of samples for each dimension (`None` if only one dimension is available)

`dir` [`int`, optional] Direction along which restriction is applied.

`kind` [`str`, optional] Kind of interpolation (`nearest` and `linear` are currently supported)

`dtype` [`str`, optional] Type of elements in input array.

### Returns

`op` [`pylops.LinearOperator`] Linear interpolation operator

`iava` [`list` or `numpy.ndarray`] Corrected indices of locations of available samples (samples at `M-1` or beyond are forced to be at `M-1-eps`)

### Raises

`ValueError` If the vector `iava` contains repeated values.

`NotImplementedError` If `kind` is not `nearest` or `linear`

### See also:

`pylops.Restriction` Restriction operator

### Notes

Linear interpolation of a subset of  $N$  values at locations `iava` from an input (or model) vector `x` of size  $M$  can be expressed as:

$$y_i = (1 - w_i)x_{l_i^u} + w_i x_{l_i^d} \quad \forall i = 1, 2, \dots, N$$

where  $\mathbf{l}^l = [\lfloor l_1 \rfloor, \lfloor l_2 \rfloor, \dots, \lfloor l_N \rfloor]$  and  $\mathbf{l}^r = [\lfloor l_1 \rfloor + 1, \lfloor l_2 \rfloor + 1, \dots, \lfloor l_N \rfloor + 1]$  are vectors containing the indeces of the original array at which samples are taken, and  $\mathbf{w} = [l_1 - \lfloor l_1 \rfloor, l_2 - \lfloor l_2 \rfloor, \dots, l_N - \lfloor l_N \rfloor]$  are the linear interpolation weights.

This operator can be implemented by simply summing two `pylops.Restriction` operators which are weighted using `pylops.basicoperators.Diagonal` operators.

### Examples using `pylops.signalprocessing.Interp`

- `sphx_glr_gallery_plot_restriction.py`

## `pylops.signalprocessing.Bilinear`

**class** `pylops.signalprocessing.Bilinear(iava, dims, dtype='float64')`  
Bilinear interpolation operator.

Apply bilinear interpolation onto fractionary positions `iava` along the first two axes of a n-dimensional array.

---

**Note:** The vector `iava` should contain unique pairs. If the same pair is repeated twice an error will be raised.

---

### Parameters

**iava** [`list` or `numpy.ndarray`] Array of size  $[2 \times n_{ava}]$  containing pairs of floating indices of locations of available samples for interpolation.  
**dims** [`list`] Number of samples for each dimension  
**dtype** [`str`, optional] Type of elements in input array.

### Raises

**ValueError** If the vector `iava` contains repeated values.

### Notes

Bilnear interpolation of a subset of  $N$  values at locations `iava` from an input n-dimensional vector `x` of size  $[m_1 \times m_2 \times \dots \times m_{ndim}]$  can be expressed as:

$$y_i = (1 - w_i^0)(1 - w_i^1)x_{l_i^{l,0}, l_i^{l,1}} + w_i^0(1 - w_i^1)x_{l_i^{l,0}, l_i^{l,1}} + (1 - w_i^0)w_i^1x_{l_i^{l,0}, l_i^{l,1}} + w_i^0w_i^1x_{l_i^{l,0}, l_i^{l,1}} \quad \forall i = 1, 2, \dots, M$$

where  $\mathbf{l}^{l,0} = [\lfloor l_1^0 \rfloor, \lfloor l_2^0 \rfloor, \dots, \lfloor l_N^0 \rfloor]$ ,  $\mathbf{l}^{l,1} = [\lfloor l_1^1 \rfloor, \lfloor l_2^1 \rfloor, \dots, \lfloor l_N^1 \rfloor]$ ,  $\mathbf{l}^{r,0} = [\lfloor l_1^0 \rfloor + 1, \lfloor l_2^0 \rfloor + 1, \dots, \lfloor l_N^0 \rfloor + 1]$ ,  $\mathbf{l}^{r,1} = [\lfloor l_1^1 \rfloor + 1, \lfloor l_2^1 \rfloor + 1, \dots, \lfloor l_N^1 \rfloor + 1]$ , are vectors containing the indices of the original array at which samples are taken, and  $\mathbf{w}^j = [l_1^j - \lfloor l_1^j \rfloor, l_2^j - \lfloor l_2^j \rfloor, \dots, l_N^j - \lfloor l_N^j \rfloor]$  ( $\forall j = 0, 1$ ) are the bilinear interpolation weights.

### Attributes

**shape** [`tuple`] Operator shape  
**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

### Methods

---

<code>__init__(self, iava, dims[, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

---

## Examples using `pylops.signalprocessing.Bilinear`

- sphx\_glr\_gallery\_plot\_bilinear.py

## pylops.signalprocessing.FFT

`pylops.signalprocessing.FFT(dims, dir=0, nfft=None, sampling=1.0, real=False, fftshift=False, engine='numpy', dtype='complex128', **kwargs_fftw)`

One dimensional Fast-Fourier Transform.

Apply Fast-Fourier Transform (FFT) along a specific direction `dir` of a multi-dimensional array of size `dim`.

Note that the FFT operator is an overload to either the numpy `numpy.fft.fft` (or `numpy.fft.rfft` for real models) in forward mode and to the numpy `numpy.fft.ifft` (or `numpy.fft.irfft` for real models) in adjoint mode, or to the `pyfftw.FFTW` class.

In both cases, scaling is properly taken into account to guarantee that the operator is passing the dot-test.

---

**Note:** For a real valued input signal, it is possible to store the values of the Fourier transform at positive frequencies only as values at negative frequencies are simply their complex conjugates. However as the operation of removing the negative part of the frequency axis in forward mode and adding the complex conjugates in adjoint mode is nonlinear, the Linear Operator FTT with `real=True` is not expected to pass the dot-test. It is thus *only* advised to use this flag when a forward and adjoint FFT is used in the same chained operator (e.g., `FFT.H*Op*FFT`) such as in `pylops.waveeqprocessing.mdd.MDC`.

---

### Parameters

- dims** [`tuple`] Number of samples for each dimension
- dir** [`int`, optional] Direction along which FFT is applied.
- nfft** [`int`, optional] Number of samples in Fourier Transform (same as input if `nfft=None`)
- sampling** [`float`, optional] Sampling step `dt`.
- real** [`bool`, optional] Model to which fft is applied has real numbers (`True`) or not (`False`).  
Used to enforce that the output of adjoint of a real model is real.
- fftshift** [`bool`, optional] Apply fftshift/ifftshift (`True`) or not (`False`)

**engine** [`str`, optional] Engine used for fft computation (`numpy` or `fftw`)  
**dtype** [`str`, optional] Type of elements in input array.  
**\*\*kwargs\_fftw** Arbitrary keyword arguments for `pyfftw.FTTW`

**Raises**

**ValueError** If `dims` is not provided and if `dir` is bigger than `len(dims)`  
**NotImplementedError** If `engine` is neither `numpy` nor `numba`

**Notes**

The FFT operator applies the forward Fourier transform to a signal  $d(t)$  in forward mode:

$$D(f) = F(d) = \int d(t)e^{-j2\pi ft} dt$$

Similarly, the inverse Fourier transform is applied to the Fourier spectrum  $D(f)$  in adjoint mode:

$$d(t) = F^{-1}(D) = \int D(f)e^{j2\pi ft} df$$

Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform.

Note that the FFT operator is a special operator in that the adjoint is also the inverse of the forward mode. Moreover, in case of real signal in time domain, the Fourier transform is Hermitian.

**Attributes**

**shape** [`tuple`] Operator shape  
**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (True) or not (False)

**Examples using `pylops.signalprocessing.FFT`**

- sphx\_glr\_gallery\_plot\_fft.py
- [02. The Dot-Test](#)
- [03. Solvers](#)
- [04. Bayesian Inversion](#)

**pylops.signalprocessing.FFT2D**

**class** `pylops.signalprocessing.FFT2D` (`dims`, `dirs=(0, 1)`, `nffts=(None, None)`, `sampling=(1.0, 1.0)`, `dtype='complex128'`)

Two dimensional Fast-Fourier Transform.

Apply two dimensional Fast-Fourier Transform (FFT) to any pair of axes of a multi-dimensional array depending on the choice of `dirs`. Note that the FFT2D operator is a simple overload to the numpy `numpy.fft.fft2` in forward mode and to the numpy `numpy.fft.ifft2` in adjoint mode, however scaling is taken into account differently to guarantee that the operator is passing the dot-test.

**Parameters**

**dims** [`tuple`] Number of samples for each dimension

**dirs** [`tuple`, optional] Pair of directions along which FFT2D is applied  
**nffts** [`tuple`, optional] Number of samples in Fourier Transform for each direction (same as input if `nffts=(None, None)`)  
**sampling** [`tuple`, optional] Sampling steps `dy` and `dx`  
**dtype** [`str`, optional] Type of elements in input array

**Raises**

**ValueError** If `dims` has less than two elements, and if `dirs`, `nffts`, or `sampling` has more or less than two elements.

**Notes**

The FFT2D operator applies the two-dimensional forward Fourier transform to a signal  $d(y, x)$  in forward mode:

$$D(k_y, k_x) = F(d) = \int \int d(y, x) e^{-j2\pi k_y y} e^{-j2\pi k_x x} dy dx$$

Similarly, the two-dimensional inverse Fourier transform is applied to the Fourier spectrum  $D(k_y, k_x)$  in adjoint mode:

$$d(y, x) = F^{-1}(D) = \int \int D(k_y, k_x) e^{j2\pi k_y y} e^{j2\pi k_x x} dk_y dk_x$$

Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform.

**Attributes**

**shape** [`tuple`] Operator shape  
**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (True) or not (False)

**Methods**

<code>__init__(self, dims[, dirs, nffts, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\nkwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**Examples using `pylops.signalprocessing.FFT2D`**

- `sphx_glr_gallery_plot_fft.py`

- sphx\_glr\_gallery\_plot\_tvreg.py
- 12. Seismic regularization
- 14. Seismic wavefield decomposition

## pylops.signalprocessing.FFTND

```
class pylops.signalprocessing.FFTND(dims, dirs=(0, 1, 2), nffts=(None, None, None), sampling=(1.0, 1.0, 1.0), dtype='complex128')
```

N-dimensional Fast-Fourier Transform.

Apply n-dimensional Fast-Fourier Transform (FFT) to any n axes of a multi-dimensional array depending on the choice of `dirs`. Note that the FFTND operator is a simple overload to the numpy `numpy.fft.fftn` in forward mode and to the numpy `numpy.fft.ifftn` in adjoint mode, however scaling is taken into account differently to guarantee that the operator is passing the dot-test.

### Parameters

- `dims` [`tuple`] Number of samples for each dimension
- `dirs` [`tuple`, optional] Directions along which FFTND is applied
- `nffts` [`tuple`, optional] Number of samples in Fourier Transform for each direction (same as input if `nffts=(None, None, None, ..., None)`)
- `sampling` [`tuple`, optional] Sampling steps in each direction
- `dtype` [`str`, optional] Type of elements in input array

### Raises

- `ValueError` If `dims`, `dirs`, `nffts`, or `sampling` have less than three elements and if the dimension of `dirs`, `nffts`, and `sampling` is not the same

### Notes

The FFTND operator applies the n-dimensional forward Fourier transform to a multi-dimensional array. Without loss of generality we consider here a three-dimensional signal  $d(z, y, x)$ . The FFTND in forward mode is:

$$D(k_z, k_y, k_x) = F(d) = \int \int d(z, y, x) e^{-j2\pi k_z z} e^{-j2\pi k_y y} e^{-j2\pi k_x x} dz dy dx$$

Similarly, the three-dimensional inverse Fourier transform is applied to the Fourier spectrum  $D(k_z, k_y, k_x)$  in adjoint mode:

$$d(z, y, x) = F^{-1}(D) = \int \int D(k_z, k_y, k_x) e^{j2\pi k_z z} e^{j2\pi k_y y} e^{j2\pi k_x x} dk_z dk_y dk_x$$

Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform.

### Attributes

- `shape` [`tuple`] Operator shape
- `explicit` [`bool`] Operator contains a matrix that can be solved explicitly (True) or not (False)

### Methods

<code>__init__(self, dims[, dirs, nffts, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.signalprocessing.FFTND`

- sphx\_glr\_gallery\_plot\_fft.py

## pylops.signalprocessing.DWT

**class** `pylops.signalprocessing.DWT(dims, dir=0, wavelet='haar', level=1, dtype='float64')`  
One dimensional Wavelet operator.

Apply 1D-Wavelet Transform along a specific direction `dir` of a multi-dimensional array of size `dims`.

Note that the Wavelet operator is an overload of the `pywt` implementation of the wavelet transform. Refer to <https://pywavelets.readthedocs.io> for a detailed description of the input parameters.

### Parameters

**dims** [`int` or `tuple`] Number of samples for each dimension  
**dir** [`int`, optional] Direction along which FFT is applied.  
**wavelet** [`str`, optional] Name of wavelet type. Use `pywt.wavelist(kind='discrete')` for a list of available wavelets.  
**level** [`int`, optional] Number of scaling levels (must be  $\geq 0$ ).  
**dtype** [`str`, optional] Type of elements in input array.

### Raises

**ModuleNotFoundError** If `pywt` is not installed  
**ValueError** If `wavelet` does not belong to `pywt.families`

### Notes

The Wavelet operator applies the multilevel Discrete Wavelet Transform (DWT) in forward mode and the multilevel Inverse Discrete Wavelet Transform (IDWT) in adjoint mode.

Wavelet transforms can be used to compress signals and present a key advantage over Fourier transforms in that they captures both frequency and location information in time. Consider using this operator as sparsifying transform when using L1 solvers.

**Attributes**

**shape** [`tuple`] Operator shape  
**explicit** [`bool`] Operator contains a matrix that can be solved explicitly (True) or not (False)

**Methods**

<code>__init__(self, dims[, dir, wavelet, level, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, *\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

**Examples using `pylops.signalprocessing.DWT`**

- `sphx_glr_gallery_plot_wavelet.py`

**`pylops.signalprocessing.DWT2D`**

```
class pylops.signalprocessing.DWT2D(dims, dirs=(0, 1), wavelet='haar', level=1,
                                         dtype='float64')
```

Two dimensional Wavelet operator.

Apply 2D-Wavelet Transform along two directions `dirs` of a multi-dimensional array of size `dims`.

Note that the Wavelet operator is an overload of the `pywt` implementation of the wavelet transform. Refer to <https://pywavelets.readthedocs.io> for a detailed description of the input parameters.

**Parameters**

**dims** [`tuple`] Number of samples for each dimension  
**dirs** [`tuple`, optional] Direction along which FFT is applied.  
**wavelet** [`str`, optional] Name of wavelet type. Use `pywt.wavelist(kind='discrete')` for a list of available wavelets.  
**level** [`int`, optional] Number of scaling levels (must be  $\geq 0$ ).  
**dtype** [`str`, optional] Type of elements in input array.

**Raises**

**ModuleNotFoundError** If `pywt` is not installed  
**ValueError** If `wavelet` does not belong to `pywt.families`

## Notes

The Wavelet operator applies the 2-dimensional multilevel Discrete Wavelet Transform (DWT2) in forward mode and the 2-dimensional multilevel Inverse Discrete Wavelet Transform (IDWT2) in adjoint mode.

### Attributes

**shape** [tuple] Operator shape

**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, dims[, dirs, wavelet, level, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.signalprocessing.DWT2D`

- sphx\_glr\_gallery\_plot\_wavelet.py
- *05. Image deblurring*

## `pylops.signalprocessing.Radon2D`

`pylops.signalprocessing.Radon2D(taxis, haxis, pxaxis, kind='linear', centeredh=True, interp=True, onthefly=False, engine='numpy', dtype='float64')`

Two dimensional Radon transform.

Apply two dimensional Radon forward (and adjoint) transform to a 2-dimensional array of size  $[n_{px} \times n_t]$  (and  $[n_x \times n_t]$ ).

In forward mode this entails to spreading the model vector along parametric curves (lines, parabolas, or hyperbolas depending on the choice of `kind`), while stacking values in the data vector along the same parametric curves is performed in adjoint mode.

### Parameters

**taxis** [np.ndarray] Time axis

**haxis** [np.ndarray] Spatial axis

**pxaxis** [np.ndarray] Axis of scanning variable  $p_x$  of parametric curve

**kind** [`str`, optional] Curve to be used for stacking/spreading (linear, parabolic, and hyperbolic are currently supported) or a function that takes (`x`, `t0`, `px`) as input and returns `t` as output

**centeredh** [`bool`, optional] Assume centered spatial axis (`True`) or not (`False`)

**interp** [`bool`, optional] Apply linear interpolation (`True`) or nearest interpolation (`False`) during stacking/spreading along parametric curve

**onthefly** [`bool`, optional] Compute stacking parametric curves on-the-fly as part of forward and adjoint modelling (`True`) or at initialization and store them in look-up table (`False`). Using a look-up table is computationally more efficient but increases the memory burden

**engine** [`str`, optional] Engine used for computation (`numpy` or `numba`)

**dtype** [`str`, optional] Type of elements in input array.

## Returns

**r2op** [`pylops.LinearOperator`] Radon operator

## Raises

**KeyError** If `engine` is neither `numpy` nor `numba`

**NotImplementedError** If `kind` is not `linear`, `parabolic`, or `hyperbolic`

## See also:

`pylops.signalprocessing.Radon3D` Three dimensional Radon transform

`pylops.Spread` Spread operator

## Notes

The Radon2D operator applies the following linear transform in adjoint mode to the data after reshaping it into a 2-dimensional array of size  $[n_x \times n_t]$  in adjoint mode:

$$m(p_x, t_0) = \int d(x, t = f(p_x, x, t)) dx$$

where  $f(p_x, x, t) = t_0 + p_x * x$  where  $p_x = \sin(\theta)/v$  in linear mode,  $f(p_x, x, t) = t_0 + p_x * x^2$  in parabolic mode, and  $f(p_x, x, t) = \sqrt{t_0^2 + x^2/p_x^2}$  in hyperbolic mode.

As the adjoint operator can be interpreted as a repeated summation of sets of elements of the model vector along chosen parametric curves, the forward is implemented as spreading of values in the data vector along the same parametric curves. This operator is actually a thin wrapper around the `pylops.Spread` operator.

## Examples using `pylops.signalprocessing.Radon2D`

- sphx\_glr\_gallery\_plot\_sliding.py
- sphx\_glr\_gallery\_plot\_radon.py
- 11. Radon filtering
- 12. Seismic regularization
- 16. CT Scan Imaging

**`pylops.signalprocessing.Radon3D`**

```
pylops.signalprocessing.Radon3D(taxis, hyaxis, hxaxis, pyaxis, pxaxis, kind='linear', centeredh=True, interp=True, onthefly=False, engine='numpy', dtype='float64')
```

Three dimensional Radon transform.

Apply three dimensional Radon forward (and adjoint) transform to a 3-dimensional array of size  $[n_{py} \times n_{px} \times n_t]$  (and  $[n_y \times n_x \times n_t]$ ).

In forward mode this entails to spreading the model vector along parametric curves (lines, parabolas, or hyperbolas depending on the choice of `kind`), while stacking values in the data vector along the same parametric curves is performed in adjoint mode.

**Parameters**

**taxis** [np.ndarray] Time axis  
**hxaxis** [np.ndarray] Fast patial axis  
**hyaxis** [np.ndarray] Slow spatial axis  
**pyaxis** [np.ndarray] Axis of scanning variable  $p_y$  of parametric curve  
**pxaxis** [np.ndarray] Axis of scanning variable  $p_x$  of parametric curve  
**kind** [str, optional] Curve to be used for stacking/spreading (linear, parabolic, and hyperbolic are currently supported)  
**centeredh** [bool, optional] Assume centered spatial axis (True) or not (False)  
**interp** [bool, optional] Apply linear interpolation (True) or nearest interpolation (False) during stacking/spreading along parametric curve  
**onthefly** [bool, optional] Compute stacking parametric curves on-the-fly as part of forward and adjoint modelling (True) or at initialization and store them in look-up table (False). Using a look-up table is computationally more efficient but increases the memory burden  
**engine** [str, optional] Engine used for computation (numpy or numba)  
**dtype** [str, optional] Type of elements in input array.

**Returns**

**r3op** [`pylops.LinearOperator`] Radon operator

**Raises**

**KeyError** If `engine` is neither numpy nor numba

**NotImplementedError** If `kind` is not linear, parabolic, or hyperbolic

**See also:**

`pylops.signalprocessing.Radon2D` Two dimensional Radon transform

`pylops.Spread` Spread operator

## Notes

The Radon3D operator applies the following linear transform in adjoint mode to the data after reshaping it into a 3-dimensional array of size  $[n_y \times n_x \times n_t]$  in adjoint mode:

$$m(p_y, p_x, t_0) = \int d(y, x, t = f(p_y, p_x, y, x, t)) dx dy$$

where  $f(p_y, p_x, y, x, t) = t_0 + p_y * y + p_x * x$  in linear mode,  $f(p_y, p_x, y, x, t) = t_0 + p_y * y^2 + p_x * x^2$  in parabolic mode, and  $f(p_y, p_x, y, x, t) = \sqrt{t_0^2 + y^2/p_y^2 + x^2/p_x^2}$  in hyperbolic mode.

As the adjoint operator can be interpreted as a repeated summation of sets of elements of the model vector along chosen parametric curves, the forward is implemented as spreading of values in the data vector along the same parametric curves. This operator is actually a thin wrapper around the `pylops.Spread` operator.

## Examples using `pylops.signalprocessing.Radon3D`

- sphx\_glr\_gallery\_plot\_sliding.py
- sphx\_glr\_gallery\_plot\_radon.py

## `pylops.signalprocessing.Sliding2D`

`pylops.signalprocessing.Sliding2D(Op, dims, dimsd, nwin, nover, tapertype='hanning', design=False)`

2D Sliding transform operator.

Apply a transform operator `Op` repeatedly to patches of the model vector in forward mode and patches of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into patches each patch is transformed, and patches are then recombined in a sliding window fashion. Both model and data should be 2-dimensional arrays in nature as they are internally reshaped and interpreted as 2-dimensional arrays. Each patch contains in fact a portion of the array in the first dimension (and the entire second dimension).

This operator can be used to perform local, overlapping transforms (e.g., `pylops.signalprocessing.FFT2` or `pylops.signalprocessing.Radon2D`) of 2-dimensional arrays.

---

**Note:** The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of `nwin` and `nover`, it is recommended to use `design=True` if unsure about the choice `dims` and use the number of windows printed on screen to define such input parameter.

---

## Parameters

- Op** [`pylops.LinearOperator`] Transform operator
- dims** [`tuple`] Shape of 2-dimensional model. Note that `dims[0]` should be multiple of the model size of the transform in the first dimension
- dimsd** [`tuple`] Shape of 2-dimensional data
- nwin** [`int`] Number of samples of window
- nover** [`int`] Number of samples of overlapping part of window
- tapertype** [`str`, optional] Type of taper (hanning, cosine, cosinesquare or None)

**design** [`bool`, optional] Print number of sliding window (`True`) or not (`False`)

#### Returns

**Sop** [`pylops.LinearOperator`] Sliding operator

#### Raises

**ValueError** Identified number of windows is not consistent with provided model shape (`dims`).

### Examples using `pylops.signalprocessing.Sliding2D`

- `sphx_glr_gallery_plot_sliding.py`
- *12. Seismic regularization*

### `pylops.signalprocessing.Sliding3D`

`pylops.signalprocessing.Sliding3D` (`Op, dims, dimsd, nwin, nover, nop, tapertype='hanning', design=False`)

3D Sliding transform operator.

Apply a transform operator `Op` repeatedly to patches of the model vector in forward mode and patches of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into patches each patch is transformed, and patches are then recombined in a sliding window fashion. Both model and data should be 3-dimensional arrays in nature as they are internally reshaped and interpreted as 3-dimensional arrays. Each patch contains in fact a portion of the array in the first and second dimensions (and the entire third dimension).

This operator can be used to perform local, overlapping transforms (e.g., `pylops.signalprocessing.FFTN` or `pylops.signalprocessing.Radon3D`) of 3-dimensional arrays.

---

**Note:** The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of `nwin` and `nover`, it is recommended to use `design=True` if unsure about the choice `dims` and use the number of windows printed on screen to define such input parameter.

---

#### Parameters

**Op** [`pylops.LinearOperator`] Transform operator

**dims** [`tuple`] Shape of 3-dimensional model. Note that `dims[0]` and `dims[1]` should be multiple of the model sizes of the transform in the first and second dimensions

**dimsd** [`tuple`] Shape of 3-dimensional data

**nwin** [`tuple`] Number of samples of window

**nover** [`tuple`] Number of samples of overlapping part of window

**nop** [`tuple`] Number of samples in axes of transformed domain associated to spatial axes in the data

**tapertype** [`str`, optional] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

**design** [`bool`, optional] Print number sliding window (`True`) or not (`False`)

#### Returns

**Sop** [`pylops.LinearOperator`] Sliding operator

**Raises**

**ValueError** Identified number of windows is not consistent with provided model shape (dims).

**Examples using `pylops.signalprocessing.Sliding3D`**

- sphx\_glr\_gallery\_plot\_sliding.py

**pylops.signalprocessing.Fredholm1**

```
class pylops.signalprocessing.Fredholm1(G, nz=1, saveGt=True, usematmul=True,
                                         dtype='float64')
```

Fredholm integral of first kind.

Implement a multi-dimensional Fredholm integral of first kind. Note that if the integral is two dimensional, this can be directly implemented using `pylops.basicoperators.MatrixMult`. A multi-dimensional Fredholm integral can be performed as a `pylops.basicoperators.BlockDiag` operator of a series of `pylops.basicoperators.MatrixMult`. However, here we take advantage of the structure of the kernel and perform it in a more efficient manner.

**Parameters**

**G** [numpy.ndarray] Multi-dimensional convolution kernel of size [ $n_{slice} \times n_x \times n_y$ ]

**nz** [numpy.ndarray, optional] Additional dimension of model

**saveGt** [bool, optional] Save G and  $G^H$  to speed up the computation of adjoint (True) or create  $G^H$  on-the-fly (False). Note that `saveGt=True` will double the amount of required memory

**usematmul** [bool, optional] Use `numpy.matmul` (True) or for-loop with `numpy.dot` (False). As it is not possible to define which approach is more performant (this is highly dependent on the size of G and input arrays as well as the hardware used in the computation), we advise users to time both methods for their specific problem prior to making a choice.

**dtype** [str, optional] Type of elements in input array.

**Notes**

A multi-dimensional Fredholm integral of first kind can be expressed as

$$d(sl, x, z) = \int G(sl, x, y)m(sl, y, z)dy \quad \forall sl = 1, n_{slice}$$

on the other hand its adjoint is expressed as

$$m(sl, y, z) = \int G^*(sl, y, x)d(sl, x, z)dx \quad \forall sl = 1, n_{slice}$$

In discrete form, this operator can be seen as a block-diagonal matrix multiplication:

$$\begin{bmatrix} G_{sl1} & 0 & \dots & 0 \\ 0 & G_{sl2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & G_{slN} \end{bmatrix} \begin{bmatrix} m_{sl1} \\ m_{sl2} \\ \dots \\ m_{slN} \end{bmatrix}$$

**Attributes**

**shape** [tuple] Operator shape

**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, G[, nz, saveGt, usematmul, dtype])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Wave-Equation processing

<code>PressureToVelocity(nt, nr, dt, dr, rho, vel)</code>	Pressure to Vertical velocity conversion.
<code>UpDownComposition2D(nt, nr, dt, dr, rho, vel)</code>	2D Up-down wavefield composition.
<code>UpDownComposition3D(nt, nr, dt, dr, rho, vel)</code>	3D Up-down wavefield composition.
<code>MDC(G, nt, nv[, dt, dr, twosided, fast, ...])</code>	Multi-dimensional convolution.
<code>PhaseShift(vel, dz, nt, freq, kx[, ky, dtype])</code>	Phase shift operator
<code>Demigration(z, x, t, srccs, recs, vel, wav, ...)</code>	Kirchoff Demigration operator.

### pylops.waveeqprocessing.PressureToVelocity

```
pylops.waveeqprocessing.PressureToVelocity(nt, nr, dt, dr, rho, vel, nffts=(None, None, None), critical=100.0, ntaper=10, topresure=False, dtype='complex128')
```

Pressure to Vertical velocity conversion.

Apply conversion from pressure to vertical velocity seismic wavefield (or vertical velocity to pressure). The input model and data required by the operator should be created by flattening the a wavefield of size  $([n_{r_y}] \times n_{r_x} \times n_t]$ .

#### Parameters

**nt** [int] Number of samples along the time axis

**nr** [int or tuple] Number of samples along the receiver axis (or axes)

**dt** [float] Sampling along the time axis

**dr** [float or tuple] Sampling(s) along the receiver array

**rho** [float] Density along the receiver array (must be constant)

**vel** [float] Velocity along the receiver array (must be constant)

**nffts** [`tuple`, optional] Number of samples along the wavenumber and frequency axes

**critical** [`float`, optional] Percentage of angles to retain in obliquity factor. For example, if `critical=100` only angles below the critical angle  $\sqrt{k_y^2 + k_x^2} < \frac{\omega}{vel}$  will be retained will be retained

**ntaper** [`float`, optional] Number of samples of taper applied to obliquity factor around critical angle

**topressure** [`bool`, optional] Perform conversion from particle velocity to pressure (True) or from pressure to particle velocity (False)

**dtype** [`str`, optional] Type of elements in input array.

#### Returns

**Cop** [`pylops.LinearOperator`] Pressure to particle velocity (or particle velocity to pressure) conversion operator

#### See also:

`UpDownComposition2D` 2D Wavefield composition

`UpDownComposition3D` 3D Wavefield composition

`WavefieldDecomposition` Wavefield decomposition

#### Notes

A pressure wavefield ( $p(x, t)$ ) can be converted into an equivalent vertical particle velocity wavefield ( $v_z(x, t)$ ) by applying the following frequency-wavenumber dependant scaling [1]:

$$v_z(k_x, \omega) = \frac{k_z}{\omega\rho} p(k_x, \omega)$$

where the vertical wavenumber  $k_z$  is defined as  $k_z = \sqrt{\omega^2/c^2 - k_x^2}$ .

Similarly a vertical particle velocity can be converted into an equivalent pressure wavefield by applying the following frequency-wavenumber dependant scaling [1]:

$$p(k_x, \omega) = \frac{\omega\rho}{k_z} v_z(k_x, \omega)$$

For 3-dimensional applications the only difference is represented by the vertical wavenumber  $k_z$ , which is defined as  $k_z = \sqrt{\omega^2/c^2 - k_x^2 - k_y^2}$ .

In both cases, this operator is implemented as a concatatanation of a 2 or 3-dimensional forward FFT (`pylops.signalprocessing.FFT2` or `pylops.signalprocessing.FFTN`), a weighting matrix implemented via `pylops.basicprocessing.Diagonal`, and 2 or 3-dimensional inverse FFT.

#### Examples using `pylops.waveeqprocessing.PressureToVelocity`

- 14. Seismic wavefield decomposition

## pylops.waveeqprocessing.UpDownComposition2D

```
pylops.waveeqprocessing.UpDownComposition2D(nt, nr, dt, dr, rho, vel, nffts=(None,
                                                               None), critical=100.0, ntaper=10, scal-
                                                               ing=1.0, dtype='complex128')
```

2D Up-down wavefield composition.

Apply multi-component seismic wavefield composition from its up- and down-going constituents. The input model required by the operator should be created by flattening the separated wavefields of size  $[n_r \times n_t]$  concatenated along the spatial axis.

Similarly, the data is also a flattened concatenation of pressure and vertical particle velocity wavefields.

### Parameters

**nt** [`int`] Number of samples along the time axis  
**nr** [`int`] Number of samples along the receiver axis  
**dt** [`float`] Sampling along the time axis  
**dr** [`float`] Sampling along the receiver array  
**rho** [`float`] Density along the receiver array (must be constant)  
**vel** [`float`] Velocity along the receiver array (must be constant)  
**nffts** [`tuple`, optional] Number of samples along the wavenumber and frequency axes  
**critical** [`float`, optional] Percentage of angles to retain in obliquity factor. For example, if  $\text{critical}=100$  only angles below the critical angle  $|k_x| < \frac{f(k_z)}{\text{vel}}$  will be retained will be retained  
**ntaper** [`float`, optional] Number of samples of taper applied to obliquity factor around critical angle  
**scaling** [`float`, optional] Scaling to apply to the operator (see Notes for more details)  
**dtype** [`str`, optional] Type of elements in input array.

### Returns

**UDop** [`pylops.LinearOperator`] Up-down wavefield composition operator

### See also:

**UpDownComposition3D** 3D Wavefield composition

**WavefieldDecomposition** Wavefield decomposition

### Notes

Multi-component seismic data ( $p(x, t)$  and  $v_z(x, t)$ ) can be synthesized in the frequency-wavenumber domain as the superposition of the up- and downgoing constituents of the pressure wavefield ( $p^-(x, t)$  and  $p^+(x, t)$ ) as follows [1]:

$$\begin{bmatrix} \mathbf{p}(k_x, \omega) \\ \mathbf{v}_z(k_x, \omega) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{k_z}{\omega\rho} & -\frac{k_z}{\omega\rho} \end{bmatrix} \begin{bmatrix} \mathbf{p}^+(k_x, \omega) \\ \mathbf{p}^-(k_x, \omega) \end{bmatrix}$$

where the vertical wavenumber  $k_z$  is defined as  $k_z = \sqrt{\omega^2/c^2 - k_x^2}$ .

We can write the entire composition process in a compact matrix-vector notation as follows:

$$\begin{bmatrix} \mathbf{p} \\ s * \mathbf{v_z} \end{bmatrix} = \begin{bmatrix} \mathbf{F} & 0 \\ 0 & s * \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{W}^+ & \mathbf{W}^- \end{bmatrix} \begin{bmatrix} \mathbf{F}^H & 0 \\ 0 & \mathbf{F}^H \end{bmatrix} \mathbf{p}^\pm$$

where  $\mathbf{F}$  is the 2-dimensional FFT (`pylops.signalprocessing.FFT2`),  $\mathbf{W}^\pm$  are weighting matrices which contain the scalings  $\pm \frac{k_z}{\omega \rho}$  implemented via `pylops.basicprocessing.Diagonal`, and  $s$  is a scaling factor that is applied to both the particle velocity data and to the operator has shown above. Such a scaling is required to balance out the different dynamic range of pressure and particle velocity when solving the wavefield separation problem as an inverse problem.

As the operator is effectively obtained by chaining basic PyLops operators the adjoint is automatically implemented for this operator.

### Examples using `pylops.waveeqprocessing.UpDownComposition2D`

- 14. Seismic wavefield decomposition

### `pylops.waveeqprocessing.UpDownComposition3D`

```
pylops.waveeqprocessing.UpDownComposition3D(nt, nr, dt, dr, rho, vel, nffts=(None, None,
                                                               None), critical=100.0, ntaper=10, scaling=1.0, dtype='complex128')
```

3D Up-down wavefield composition.

Apply multi-component seismic wavefield composition from its up- and down-going constituents. The input model required by the operator should be created by flattening the separated wavefields of size  $[n_{r_y} \times n_{r_x} \times n_t]$  concatenated along the first spatial axis.

Similarly, the data is also a flattened concatenation of pressure and vertical particle velocity wavefields.

#### Parameters

**nt** [`int`] Number of samples along the time axis  
**nr** [`tuple`] Number of samples along the receiver axes  
**dt** [`float`] Sampling along the time axis  
**dr** [`tuple`] Samplings along the receiver array  
**rho** [`float`] Density along the receiver array (must be constant)  
**vel** [`float`] Velocity along the receiver array (must be constant)  
**nffts** [`tuple`, optional] Number of samples along the wavenumbers and frequency axes (for the wavenumbers axes the same order as `nr` and `dr` must be followed)  
**critical** [`float`, optional] Percentage of angles to retain in obliquity factor. For example, if `critical=100` only angles below the critical angle  $\sqrt{k_y^2 + k_x^2} < \frac{\omega}{vel}$  will be retained  
**ntaper** [`float`, optional] Number of samples of taper applied to obliquity factor around critical angle  
**scaling** [`float`, optional] Scaling to apply to the operator (see Notes for more details)  
**dtype** [`str`, optional] Type of elements in input array.

#### Returns

**UDop** [`pylops.LinearOperator`] Up-down wavefield composition operator

**See also:**

[`UpDownComposition2D`](#) 2D Wavefield composition

[`WavefieldDecomposition`](#) Wavefield decomposition

**Notes**

Multi-component seismic data ( $p(y, x, t)$  and  $v_z(y, x, t)$ ) can be synthesized in the frequency-wavenumber domain as the superposition of the up- and downgoing constituents of the pressure wavefield ( $p^-(y, x, t)$  and  $p^+(y, x, t)$ ) as described [`pylops.waveeqprocessing.UpDownComposition2D`](#).

Here the vertical wavenumber  $k_z$  is defined as  $k_z = \sqrt{\omega^2/c^2 - k_y^2 - k_x^2}$ .

**`pylops.waveeqprocessing.MDC`**

```
pylops.waveeqprocessing.MDC(G, nt, nv, dt=1.0, dr=1.0, twosided=True, fast=None, dtype=None,
                            fftengine='numpy', transpose=True, saveGt=True, conj=False, use-
                            matmul=False, prescaled=False)
```

Multi-dimensional convolution.

Apply multi-dimensional convolution between two datasets. If `transpose=True`, model and data should be provided after flattening 2- or 3-dimensional arrays of size  $[n_r(\times n_{vs}) \times n_t]$  and  $[n_s(\times n_{vs}) \times n_t]$  (or  $2 * n_t - 1$  for `twosided=True`), respectively. If `transpose=False`, model and data should be provided after flattening 2- or 3-dimensional arrays of size  $[n_t \times n_r(\times n_{vs})]$  and  $[n_t \times n_s(\times n_{vs})]$  (or  $2 * n_t - 1$  for `twosided=True`), respectively.

**Warning:** A new implementation of MDC is provided in v1.5.0. This currently affects only the inner working of the operator and end-users can use the operator in the same way as they used to do with the previous one. Nevertheless, it is now recommended to use the operator with `transpose=False`, as this behaviour will become default in version v2.0.0 and the behaviour with `transpose=True` will be deprecated.

**Parameters**

**G** [`numpy.ndarray`] Multi-dimensional convolution kernel in frequency domain of size  $[n_s \times n_r \times n_{fmax}]$  if `transpose=True` or size  $[n_{fmax} \times n_s \times n_r]$  if `transpose=False`

**nt** [`int`] Number of samples along time axis

**nv** [`int`] Number of samples along virtual source axis

**dt** [`float`, optional] Sampling of time integration axis

**dr** [`float`, optional] Sampling of receiver integration axis

**twosided** [`bool`, optional] MDC operator has both negative and positive time (`True`) or only positive (`False`)

**fast** [`bool`, optional] *Deprecated*, will be removed in v2.0.0

**dtype** [`str`, optional] *Deprecated*, will be removed in v2.0.0

**fftengine** [`str`, optional] Engine used for fft computation (`numpy` or `fftw`)

**transpose** [`bool`, optional] Transpose G and inputs such that time/frequency is placed in first dimension. This allows back-compatibility with v1.4.0 and older but will be removed in

v2.0.0 where time/frequency axis will be required to be in first dimension for efficiency reasons.

**saveGt** [`bool`, optional] Save G and  $G^H$  to speed up the computation of adjoint of `pylops.signalprocessing.Fredholm1` (`True`) or create  $G^H$  on-the-fly (`False`). Note that `saveGt=True` will be faster but double the amount of required memory

**conj** [`str`, optional] Perform Fredholm integral computation with complex conjugate of G

**usematmul** [`bool`, optional] Use `numpy.matmul` (`True`) or for-loop with `numpy.dot` (`False`) in `pylops.signalprocessing.Fredholm1` operator. Refer to `Fredholm1` documentation for details.

**prescaled** [`bool`, optional] Apply scaling to kernel (`False`) or not (`True`) when performing spatial and temporal summations. In case `prescaled=True`, the kernel is assumed to have been pre-scaled when passed to the MDC routine.

#### Raises

**ValueError** If `nt` is even and `twosided=True`

#### See also:

**MDD** Multi-dimensional deconvolution

#### Notes

The so-called multi-dimensional convolution (MDC) is a chained operator [1]. It is composed of a forward Fourier transform, a multi-dimensional integration, and an inverse Fourier transform:

$$y(t, s, v) = F^{-1} \left( \int_S G(f, s, r) F(x(t, r, v)) dr \right)$$

which is discretized as follows:

$$y(t, s, v) = F^{-1} \left( \sum_{i_r=0}^{n_r} (\sqrt{n_t} * d_t * d_r) G(f, s, i_r) F(x(t, i_r, v)) \right)$$

where  $(\sqrt{n_t} * d_t * d_r)$  is not applied if `prescaled=True`.

This operation can be discretized and performed by means of a linear operator

$$\mathbf{D} = \mathbf{F}^H \mathbf{G} \mathbf{F}$$

where **F** is the Fourier transform applied along the time axis and **G** is the multi-dimensional convolution kernel.

#### Examples using `pylops.waveeqprocessing.MDC`

- sphx\_glr\_gallery\_plot\_mdc.py
- 09. Multi-Dimensional Deconvolution

## pylops.waveeqprocessing.PhaseShift

`pylops.waveeqprocessing.PhaseShift(vel, dz, nt, freq, kx, ky=None, dtype='float64')`  
 Phase shift operator

Apply positive (forward) phase shift with constant velocity in forward mode, and negative (backward) phase shift with constant velocity in adjoint mode. Input model and data should be 2- or 3-dimensional arrays in time-space domain of size  $[n_t \times n_x (\times n_y)]$ .

### Parameters

**vel** [`float`, optional] Constant propagation velocity  
**dz** [`float`, optional] Depth step  
**nt** [`int`, optional] Number of time samples of model and data  
**freq** [`numpy.ndarray`] Positive frequency axis  
**kx** [`int`, optional] Horizontal wavenumber axis (centered around 0) of size  $[n_x \times 1]$ .  
**ky** [`int`, optional] Second horizontal wavenumber axis for 3d phase shift (centered around 0) of size  $[n_y \times 1]$ .  
**dtype** [`str`, optional] Type of elements in input array

### Returns

**Pop** [`pylops.LinearOperator`] Phase shift operator

### Notes

The phase shift operator implements a one-way wave equation forward propagation in frequency-wavenumber domain by applying the following transformation to the input model:

$$d(f, k_x, k_y) = m(f, k_x, k_y) * e^{-j\sqrt{\omega^2/v^2 - k_x^2 - k_y^2}\Delta z}$$

where  $v$  is the constant propagation velocity and  $\Delta z$  is the propagation depth. In adjoint mode, the data is propagated backward using the following transformation:

$$m(f, k_x, k_y) = d(f, k_x, k_y) * e^{j\sqrt{\omega^2/v^2 - k_x^2 - k_y^2}\Delta z}$$

Effectively, the input model and data are assumed to be in time-space domain and forward Fourier transform is applied to both dimensions, leading to the following operator:

$$\mathbf{d} = \mathbf{F}_t^H \mathbf{F}_x^H \mathbf{P} \mathbf{F}_x \mathbf{F}_t \mathbf{m}$$

where  $\mathbf{P}$  performs the phase-shift as discussed above.

## Examples using `pylops.waveeqprocessing.PhaseShift`

- sphx\_glr\_gallery\_plot\_phaseshift.py

**pylops.waveeqprocessing.Demigration**

```
pylops.waveeqprocessing.Demigration(z, x, t, sres, recs, vel, wav, wavcenter, y=None,
                                     trav=None, mode='eikonal')
```

Kirchoff Demigration operator.

Traveltime based seismic demigration/migration operator.

**Parameters**

**z** [numpy.ndarray] Depth axis  
**x** [numpy.ndarray] Spatial axis  
**t** [numpy.ndarray] Time axis for data  
**sres** [numpy.ndarray] Sources in array of size  $[2/3 \times n_s]$  The first axis should be ordered as (y,) x, z.  
**recs** [numpy.ndarray] Receivers in array of size  $[2/3 \times n_r]$  The first axis should be ordered as (y,) x, z.  
**vel** [numpy.ndarray or float] Velocity model of size  $[(n_y \times) n_x \times n_z]$  (or constant)  
**wav** [numpy.ndarray] Wavelet  
**wavcenter** [int] Index of wavelet center  
**y** [numpy.ndarray] Additional spatial axis (for 3-dimensional problems)  
**mode** [str, optional] Computation mode (analytic, eikonal or byot, see Notes for more details)  
**trav** [numpy.ndarray, optional] Traveltime table of size  $[(n_y \times) n_x \times n_z \times n_r]$  (to be provided if mode='byot')

**Returns**

**demop** [pylops.LinearOperator] Demigration/Migration operator

**Raises**

**NotImplementedError** If mode is neither analytic, eikonal, or byot

**Notes**

The demigration operator synthesizes seismic data given from a propagation velocity model  $v$  and a reflectivity model  $m$ . In forward mode:

$$d(\mathbf{x}_r, \mathbf{x}_s, t) = w(t) * \int_V G(\mathbf{x}, \mathbf{x}_s, t)G(\mathbf{x}_r, \mathbf{x}, t)m(\mathbf{x})d\mathbf{x}$$

where  $m(\mathbf{x})$  is the model and it represents the reflectivity at every location in the subsurface,  $G(\mathbf{x}, \mathbf{x}_s, t)$  and  $G(\mathbf{x}_r, \mathbf{x}, t)$  are the Green's functions from source-to-subsurface-to-receiver and finally  $w(t)$  is the wavelet. Depending on the choice of mode the Green's function will be computed and applied differently:

- mode=analytic or mode=eikonal: traveltimes curves between source to receiver pairs are computed for every subsurface point and Green's functions are implemented from traveltimes look-up tables, placing the reflectivity values at corresponding source-to-receiver time in the data.
- byot: bring your own table. Traveltimes table provided directly by user using trav input parameter. Green's functions are then implemented in the same way as previous options.

The adjoint of the demigration operator is a *migration* operator which projects data in the model domain creating an image of the subsurface reflectivity.

## Geophysical subsurface characterization

---

<code>avo.AVOLinearModelling(theta[, vsvp, nt0, AVO Linearized modelling. ...])</code>	AVO Linearized modelling.
<code>poststack.PoststackLinearModelling(wav, nt0)</code>	Post-stack linearized seismic modelling operator.
<code>prestack.PrestackLinearModelling(wav, theta)</code>	Pre-stack linearized seismic modelling operator.
<code>prestack.PrestackWaveletModelling(m, theta, nwav)</code>	Pre-stack linearized seismic modelling operator for wavelet.

---

### pylops.avo.avo.AVOLinearModelling

```
class pylops.avo.avo.AVOLinearModelling(theta, vsvp=0.5, nt0=1, spatdims=None, linearization='akirich', dtype='float64')
AVO Linearized modelling.

Create operator to be applied to a combination of elastic parameters for generation of seismic pre-stack reflectivity.
```

#### Parameters

**theta** [np.ndarray] Incident angles in degrees  
**vsvp** [np.ndarray or float] VS/VP ratio  
**nt0** [int, optional] number of samples (if vsvp is a scalar)  
**spatdims** [int or tuple, optional] Number of samples along spatial axis (or axes) (None if only one dimension is available)  
**linearization** [str, optional] choice of linearization, akirich: Aki-Richards, fatti: Fatti  
**dtype** [str, optional] Type of elements in input array.

#### Raises

**NotImplementedError** If linearization is not an implemented linearization

#### Notes

The AVO linearized operator performs a linear combination of three (or two) elastic parameters arranged in input vector **m** of size  $n_{t0} \times N$  to create the so-called seismic reflectivity:

$$r(t, \theta, x, y) = \sum_{i=1}^N G_i(t, \theta) m_i(t, x, y) \quad \forall t, \theta$$

where  $N = 2/3$ . Note that the reflectivity can be in 1d, 2d or 3d and **spatdims** contains the dimensions of the spatial axis (or axes)  $x$  and  $y$ .

#### Attributes

**shape** [tuple] Operator shape  
**explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

## Methods

<code>__init__(self, theta[, vsdp, nt0, spatdims, ...])</code>	Initialize this LinearOperator.
<code>adjoint(self)</code>	Hermitian adjoint.
<code>apply_columns(self, cols)</code>	Apply subset of columns of operator
<code>cond(self, \*\*kwargs_eig)</code>	Condition number of linear operator.
<code>conj(self)</code>	Complex conjugate operator
<code>div(self, y[, niter])</code>	Solve the linear problem $y = Ax$ .
<code>dot(self, x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>eigs(self[, neigs, symmetric, niter])</code>	Most significant eigenvalues of linear operator.
<code>matmat(self, X)</code>	Matrix-matrix multiplication.
<code>matvec(self, x)</code>	Matrix-vector multiplication.
<code>rmatmat(self, X)</code>	Adjoint matrix-matrix multiplication.
<code>rmatvec(self, x)</code>	Adjoint matrix-vector multiplication.
<code>todense(self)</code>	Return dense matrix.
<code>transpose(self)</code>	Transpose this linear operator.

## Examples using `pylops.avo.avo.AVOLinearModelling`

- sphx\_glr\_gallery\_plot\_avo.py

## `pylops.avo.poststack.PoststackLinearModelling`

`pylops.avo.poststack.PoststackLinearModelling(wav, nt0, spatdims=None, explicit=False, sparse=False)`

Post-stack linearized seismic modelling operator.

Create operator to be applied to an elastic parameter trace (or stack of traces) for generation of band-limited seismic post-stack data. The input model and data have shape  $[n_{t0}(\times n_x \times n_y)]$ .

### Parameters

**wav** [np.ndarray] Wavelet in time domain (must have odd number of elements and centered to zero). If 1d, assume stationary wavelet for the entire time axis. If 2d, use as non-stationary wavelet (user must provide one wavelet per time sample in an array of size  $[n_{t0} \times n_{wav}]$  where  $n_{wav}$  is the lenght of each wavelet)

**nt0** [int] Number of samples along time axis

**spatdims** [int or tuple, optional] Number of samples along spatial axis (or axes) (None if only one dimension is available)

**explicit** [bool, optional] Create a chained linear operator (False, preferred for large data) or a MatrixMult linear operator with dense matrix (True, preferred for small data)

**sparse** [bool, optional] Create a sparse matrix (True) or dense (False) when `explicit=True`

### Returns

**Pop** [LinearOperator] post-stack modelling operator.

### Raises

**ValueError** If `wav` is 2dimensional but does not contain `nt0` wavelets

## Notes

Post-stack seismic modelling is the process of constructing seismic post-stack data from a profile of an elastic parameter of choice in time (or depth) domain. This can be easily achieved using the following forward model:

$$d(t, \theta) = w(t) * \frac{d\ln(m(t))}{dt}$$

where  $m(t)$  is the elastic parameter profile and  $w(t)$  is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{WDm}$$

In the special case of acoustic impedance ( $m(t) = AI(t)$ ), the modelling operator can be used to create zero-offset data:

$$d(t, \theta = 0) = \frac{1}{2}w(t) * \frac{d\ln(m(t))}{dt}$$

where the scaling factor  $\frac{1}{2}$  can be easily included in the wavelet.

## Examples using `pylops.avo.poststack.PoststackLinearModelling`

- 07. Post-stack inversion

## `pylops.avo.prestack.PrestackLinearModelling`

```
pylops.avo.prestack.PrestackLinearModelling(wav, theta, vsvp=0.5, nt0=1, spat-
                                             dims=None, linearization='akirich', ex-
                                             plicit=False)
```

Pre-stack linearized seismic modelling operator.

Create operator to be applied to elastic property profiles for generation of band-limited seismic angle gathers from a linearized version of the Zoeppritz equation.

### Parameters

**wav** [np.ndarray] Wavelet in time domain (must had odd number of elements and centered to zero)

**theta** [np.ndarray] Incident angles in degrees

**vsvp** [float or np.ndarray] VS/VP ratio (constant or time/depth variant)

**nt0** [int, optional] number of samples (if vsvp is a scalar)

**spatdims** [int or tuple, optional] Number of samples along spatial axis (or axes) (None if only one dimension is available)

**linearization** [str, optional] choice of linearization, akirich: Aki-Richards, fatti: Fatti

**explicit** [bool, optional] Create a chained linear operator (False, preferred for large data) or a MatrixMult linear operator with dense matrix (True, preferred for small data)

### Returns

**Preop** [LinearOperator] pre-stack modelling operator.

### Raises

**NotImplementedError** If linearization is not an implemented linearization

## Notes

Pre-stack seismic modelling is the process of constructing seismic pre-stack data from three (or two) profiles of elastic parameters in time (or depth) domain arranged in an input vector  $\mathbf{m}$  of size  $n_{t0} \times n\theta$ . This can be easily achieved using the following forward model:

$$d(t, \theta) = w(t) * \sum_{i=1}^N G_i(t, \theta)m_i(t)$$

where  $w(t)$  is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

On the other hand, pre-stack inversion aims at recovering the different profiles of elastic properties from the band-limited seismic pre-stack data.

## Examples using `pylops.avo.prestack.PrestackLinearModelling`

- sphx\_glr\_gallery\_plot\_prestack.py
- [8. Pre-stack \(AVO\) inversion](#)

## `pylops.avo.prestack.PrestackWaveletModelling`

`pylops.avo.prestack.PrestackWaveletModelling`(*m*, *theta*, *nwav*, *wavc=None*, *vsvp=0.5*, *linearization='akirich'*)

Pre-stack linearized seismic modelling operator for wavelet.

Create operator to be applied to a wavelet for generation of band-limited seismic angle gathers using a linearized version of the Zoeppritz equation.

### Parameters

**m** [np.ndarray] elastic parameter profiles of size  $[n_{t0} \times N]$  where  $N = 3/2$   
**theta** [int] Incident angles in degrees  
**nwav** [np.ndarray] Number of samples of wavelet to be applied/estimated  
**wavc** [int, optional] Index of the center of the wavelet  
**vsvp** [np.ndarray or float, optional] VS/VP ratio  
**linearization** [str, optional] choice of linearization, akirich: Aki-Richards, fatti: Fatti

### Returns

**Mconv** [LinearOperator] pre-stack modelling operator for wavelet estimation.

### Raises

**NotImplementedError** If `linearization` is not an implemented linearization

## Notes

Pre-stack seismic modelling for wavelet estimate is the process of constructing seismic reflectivities using three (or two) profiles of elastic parameters in time (or depth) domain arranged in an input vector  $\mathbf{m}$  of size  $nt0 \times N$ :

$$d(t, \theta) = \sum_{i=1}^N G_i(t, \theta) m_i(t) * w(t)$$

where  $w(t)$  is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{Gw}$$

On the other hand, pre-stack wavelet estimation aims at recovering the wavelet given knowledge of the band-limited seismic pre-stack data and the elastic parameter profiles.

## Examples using `pylops.avo.prestack.PrestackWaveletModelling`

- sphx\_glr\_gallery\_plot\_wavest.py

## 3.6.2 Solvers

### Least-squares

---

```
leastsquares.NormalEquationsInversion(Op, inversion of normal equations.
...)
```

---

```
leastsquares.RegularizedInversion(Op, Regularized inversion.
Regs, data)
```

---

```
leastsquares.PreconditionedInversion(Op, Preconditioned inversion.
P, data)
```

---

### pylops.optimization.leastsquares.NormalEquationsInversion

```
pylops.optimization.leastsquares.NormalEquationsInversion(Op, Regs, data,
Weight=None,
dataregs=None,
epsI=0, epsRs=None,
x0=None, re-
turninfo=False,
**kwargs_cg)
```

Inversion of normal equations.

Solve the regularized normal equations for a system of equations given the operator  $Op$ , a data weighting operator  $Weight$  and a list of regularization terms  $Regs$

#### Parameters

**Op** [`pylops.LinearOperator`] Operator to invert

**Regs** [`list`] Regularization operators (None to avoid adding regularization)

**data** [`numpy.ndarray`] Data

**Weight** [`pylops.LinearOperator`, optional] Weight operator

**dataregs** [`list`, optional] Regularization data (must have the same number of elements as `Regs`)  
**epsI** [`float`, optional] Tikhonov damping  
**epsRs** [`list`, optional] Regularization dampings (must have the same number of elements as `Regs`)  
**x0** [`numpy.ndarray`, optional] Initial guess  
**returninfo** [`bool`, optional] Return info of CG solver  
**\*\*kwargs\_cg** Arbitrary keyword arguments for `scipy.sparse.linalg.cg` solver

**Returns**

**xinv** [`numpy.ndarray`] Inverted model.  
**istop** [`int`] Convergence information:  
    0: successful exit  
    >0: convergence to tolerance not achieved, number of iterations  
    <0: illegal input or breakdown

**See also:**

`RegularizedInversion` Regularized inversion  
`PreconditionedInversion` Preconditioned inversion

**Notes**

Solve the following normal equations for a system of regularized equations given the operator `Op`, a data weighting operator `W`, a list of regularization terms `Ri`, the data `d` and regularization damping factors  $\epsilon_I$  and  $\epsilon_{R_i}$ :

$$(\mathbf{Op}^T \mathbf{W} \mathbf{O} \mathbf{p} + \sum_i \epsilon_{R_i}^2 \mathbf{R}_i^T \mathbf{R}_i + \epsilon_I^2 \mathbf{I}) \mathbf{x} = \mathbf{O} \mathbf{p}^T \mathbf{W} \mathbf{d} + \sum_i \epsilon_{R_i}^2 \mathbf{R}_i^T \mathbf{d}_{R_i}$$

**Examples using `pylops.optimization.leastsquares.NormalEquationsInversion`**

- `sphx_glr_gallery_plot_bilinear.py`
- *03. Solvers*
- *05. Image deblurring*
- *06. 2D Interpolation*

**pylops.optimization.leastsquares.RegularizedInversion**

`pylops.optimization.leastsquares.RegularizedInversion`(`Op`, `Regs`, `data`, `Weight=None`,  
`dataregs=None`, `epsRs=None`,  
`x0=None`, `returninfo=False`,  
`**kwargs_lsqr`)

Regularized inversion.

Solve a system of regularized equations given the operator `Op`, a data weighting operator `Weight`, and a list of regularization terms `Regs`.

**Parameters**

**Op** [`pylops.LinearOperator`] Operator to invert  
**Regs** [`list`] Regularization operators (None to avoid adding regularization)  
**data** [`numpy.ndarray`] Data  
**Weight** [`pylops.LinearOperator`, optional] Weight operator  
**dataregs** [`list`, optional] Regularization data (if None a zero data will be used for every regularization operator in Regs)  
**epsRs** [`list`, optional] Regularization dampings  
**x0** [`numpy.ndarray`, optional] Initial guess  
**returninfo** [`bool`, optional] Return info of LSQR solver  
**\*\*kwargs\_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

**Returns**

**xinv** [`numpy.ndarray`] Inverted model Op  
**istop** [`int`] Gives the reason for termination  
 1 means x is an approximate solution to  $\mathbf{d} = \mathbf{Opx}$   
 2 means x approximately solves the least-squares problem  
**itn** [`int`] Iteration number upon termination  
**r1norm** [`float`]  $\|\mathbf{r}\|_2$ , where  $\mathbf{r} = \mathbf{d} - \mathbf{Opx}$   
**r2norm** [`float`]  $\sqrt{\mathbf{r}^T \mathbf{r} + \epsilon^2 \mathbf{x}^T \mathbf{x}}$ . Equal to r1norm if  $\epsilon = 0$

**See also:**

**RegularizedOperator** Regularized operator  
**NormalEquationsInversion** Normal equations inversion  
**PreconditionedInversion** Preconditioned inversion

**Notes**

Solve the following system of regularized equations given the operator **Op**, a data weighting operator  $\mathbf{W}^{1/2}$ , a list of regularization terms  $\mathbf{R}_i$ , the data  $\mathbf{d}$  and regularization damping factors  $\epsilon_I$ : and  $\epsilon_{R_i}$ :

$$\begin{bmatrix} \mathbf{W}^{1/2} \mathbf{Op} \\ \epsilon_{R_1} \mathbf{R}_1 \\ \dots \\ \epsilon_{R_N} \mathbf{R}_N \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{W}^{1/2} \mathbf{d} \\ \epsilon_{R_1} \mathbf{d}_{R_1} \\ \dots \\ \epsilon_{R_N} \mathbf{d}_{R_N} \end{bmatrix}$$

where the **Weight** provided here is equivalent to the square-root of the weight in `pylops.optimization.leastsquares.NormalEquationsInversion`. Note that this system is solved using the `scipy.sparse.linalg.lsqr` and an initial guess  $\mathbf{x0}$  can be provided to this solver, despite the original solver does not allow so.

**Examples using `pylops.optimization.leastsquares.RegularizedInversion`**

- sphx\_glr\_gallery\_plot\_ista.py
- sphx\_glr\_gallery\_plot\_tvreg.py
- sphx\_glr\_gallery\_plot\_wavest.py
- *03. Solvers*
- *06. 2D Interpolation*
- *16. CT Scan Imaging*

**`pylops.optimization.leastsquares.PreconditionedInversion`**

```
pylops.optimization.leastsquares.PreconditionedInversion(Op, P, data, x0=None,  
                                                       returninfo=False,  
                                                       **kwargs_lsqr)
```

Preconditioned inversion.

Solve a system of preconditioned equations given the operator  $\text{Op}$  and a preconditioner  $\text{P}$ .

**Parameters**

**Op** [`pylops.LinearOperator`] Operator to invert  
**P** [`pylops.LinearOperator`] Preconditioner  
**data** [`numpy.ndarray`] Data  
**x0** [`numpy.ndarray`] Initial guess  
**returninfo** [`bool`] Return info of LSQR solver  
**\*\*kwargs\_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

**Returns**

**xinv** [`numpy.ndarray`] Inverted model.  
**istop** [`int`] Gives the reason for termination

- 1 means  $\mathbf{x}$  is an approximate solution to  $\mathbf{d} = \text{Op}\mathbf{x}$
- 2 means  $\mathbf{x}$  approximately solves the least-squares problem

**itn** [`int`] Iteration number upon termination  
**r1norm** [`float`]  $\|\mathbf{r}\|_2$ , where  $\mathbf{r} = \mathbf{d} - \text{Op}\mathbf{x}$   
**r2norm** [`float`]  $\sqrt{\mathbf{r}^T \mathbf{r} + \epsilon^2 \mathbf{x}^T \mathbf{x}}$ . Equal to `r1norm` if  $\epsilon = 0$

**See also:**

**RegularizedInversion** Regularized inversion

**NormalEquationsInversion** Normal equations inversion

## Notes

Solve the following system of preconditioned equations given the operator  $\mathbf{Op}$ , a preconditioner  $\mathbf{P}$ , the data  $\mathbf{d}$

$$\mathbf{d} = \mathbf{Op}(\mathbf{P}\mathbf{p})$$

where  $\mathbf{p}$  is the solution in the preconditioned space and  $\mathbf{x} = \mathbf{P}\mathbf{p}$  is the solution in the original space.

## Examples using `pylops.optimization.leastsquares.PreconditionedInversion`

- `sphx_glr_gallery_plot_wavest.py`
- [03. Solvers](#)

## Sparsity

---

<code>sparsity.IRLS(Op, data, nouter[, threshR, ...])</code>	Iteratively reweighted least squares.
<code>sparsity.OMP(Op, data[, niter_outer, ...])</code>	Orthogonal Matching Pursuit (OMP).
<code>sparsityISTA(Op, data, niter[, eps, alpha, ...])</code>	Iterative Soft Thresholding Algorithm (ISTA).
<code>sparsity.FISTA(Op, data, niter[, eps, ...])</code>	Fast Iterative Soft Thresholding Algorithm (FISTA).
<code>sparsity.SPGL1(Op, data[, SOp, tau, sigma, x0])</code>	Spectral Projected-Gradient for L1 norm.
<code>sparsity.SplitBregman(Op, RegsL1, data[, ...])</code>	Split Bregman for mixed L2-L1 norms.

---

## pylops.optimization.sparsity.IRLS

```
pylops.optimization.sparsity.IRLS(Op, data, nouter, threshR=False, epsR=1e-10, epsI=1e-10, x0=None, tolIRLS=1e-10, returnhistory=False, **kwargs_cg)
```

Iteratively reweighted least squares.

Solve an optimization problem with  $L_1$  cost function given the operator  $\mathbf{Op}$  and data  $\mathbf{y}$ . The cost function is minimized by iteratively solving a weighted least squares problem with the weight at iteration  $i$  being based on the data residual at iteration  $i + 1$ .

The IRLS solver is robust to *outliers* since the  $L_1$  norm given less weight to large residuals than  $L_2$  norm does.

### Parameters

- Op** [`pylops.LinearOperator`] Operator to invert
- data** [`numpy.ndarray`] Data
- nouter** [`int`] Number of outer iterations
- threshR** [`bool`, optional] Apply thresholding in creation of weight (`True`) or damping (`False`)
- epsR** [`float`, optional] Damping to be applied to residuals for weighting term
- espI** [`float`, optional] Tikhonov damping
- x0** [`numpy.ndarray`, optional] Initial guess
- tolIRLS** [`float`, optional] Tolerance. Stop outer iterations if difference between inverted model at subsequent iterations is smaller than `tolIRLS`

**returnhistory** [`bool`, optional] Return history of inverted model for each outer iteration of IRLS

**\*\*kwargs\_cg** Arbitrary keyword arguments for `scipy.sparse.linalg.cg` solver

### Returns

**xinv** [`numpy.ndarray`] Inverted model

**nouter** [`int`] Number of effective outer iterations

**xinv\_hist** [`numpy.ndarray`, optional] History of inverted model

**rw\_hist** [`numpy.ndarray`, optional] History of weights

### Notes

Solves the following optimization problem for the operator **Op** and the data **d**:

$$J = \|\mathbf{d} - \mathbf{Opx}\|_1$$

by a set of outer iterations which require to repeatedly solve a weighted least squares problem of the form:

$$\mathbf{x}^{(i+1)} = \arg \min_{\mathbf{x}} \|\mathbf{d} - \mathbf{Opx}\|_{2,\mathbf{R}^{(i)}} + \epsilon_I^2 \|\mathbf{x}\|$$

where  $\mathbf{R}^{(i)}$  is a diagonal weight matrix whose diagonal elements at iteration  $i$  are equal to the absolute inverses of the residual vector  $\mathbf{r}^{(i)} = \mathbf{y} - \mathbf{Opx}^{(i)}$  at iteration  $i$ . More specifically the  $j$ -th element of the diagonal of  $\mathbf{R}^{(i)}$  is

$$R_{j,j}^{(i)} = \frac{1}{|r_j^{(i)}| + \epsilon_R}$$

or

$$R_{j,j}^{(i)} = \frac{1}{\max(|r_j^{(i)}|, \epsilon_R)}$$

depending on the choice `threshR`. In either case,  $\epsilon_R$  is the user-defined stabilization/thresholding factor [1].

### Examples using `pylops.optimization.sparsity.IRLS`

- `sphx_glr_gallery_plot_linearregr.py`
- `sphx_glr_gallery_plot_regr.py`

### `pylops.optimization.sparsity.OMP`

`pylops.optimization.sparsity.OMP(Op, data, niter_outer=10, niter_inner=40, sigma=0.0001, normalizecols=False, show=False)`

Orthogonal Matching Pursuit (OMP).

Solve an optimization problem with  $L_0$  regularization function given the operator **Op** and data **y**. The operator can be real or complex, and should ideally be either square  $N = M$  or underdetermined  $N < M$ .

### Parameters

**Op** [`pylops.LinearOperator`] Operator to invert

---

**data** [numpy.ndarray] Data  
**niter\_outer** [int] Number of iterations of outer loop  
**niter\_inner** [int] Number of iterations of inner loop  
**sigma** [list] Maximum L2 norm of residual. When smaller stop iterations.  
**normalizecols** [list] Normalize columns (True) or not (False). Note that this can be expensive as it requires applying the forward operator  $n_{cols}$  times to unit vectors (i.e., containing 1 at position j and zero otherwise); use only when the columns of the operator are expected to have highly varying norms.  
**show** [bool, optional] Display iterations log

**Returns**

**xinv** [numpy.ndarray] Inverted model  
**iiter** [int] Number of effective outer iterations  
**cost** [numpy.ndarray, optional] History of cost function

**See also:**

**ISTA** Iterative Soft Thresholding Algorithm (ISTA).  
**FISTA** Fast Iterative Soft Thresholding Algorithm (FISTA).  
**SPGL1** Spectral Projected-Gradient for L1 norm (SPGL1).  
**SplitBregman** Split Bregman for mixed L2-L1 norms.

**Notes**

Solves the following optimization problem for the operator **Op** and the data **d**:

$$\|\mathbf{x}\|_0 \text{ subj. to } \|\mathbf{Op}\mathbf{x} - \mathbf{b}\|_2 \leq \sigma,$$

using Orthogonal Matching Pursuit (OMP). This is a very simple iterative algorithm which applies the following step:

$$\begin{aligned} \Lambda_k &= \Lambda_{k-1} \cup \{\operatorname{argmax}_j |\mathbf{Op}_j^H \mathbf{r}_k|\} \\ \mathbf{x}_k &= \{\operatorname{argmin}_{\mathbf{x}} \|\mathbf{Op}_{\Lambda_k} \mathbf{x} - \mathbf{b}\|_2 \} \end{aligned}$$

**Examples using `pylops.optimization.sparsity.OMP`**

- sphx\_glr\_gallery\_plot\_ista.py

**pylops.optimization.sparsity.ISTA**

```
pylops.optimization.sparsity.ISTA(Op, data, niter, eps=0.1, alpha=None, eigsiter=None,
                                  eigstol=0, tol=1e-10, monitorres=False, returninfo=False,
                                  show=False)
```

Iterative Soft Thresholding Algorithm (ISTA).

Solve an optimization problem with  $L_1$  regularization function given the operator **Op** and data **y**. The operator can be real or complex, and should ideally be either square  $N = M$  or underdetermined  $N < M$ .

**Parameters**

**Op** [`pylops.LinearOperator`] Operator to invert  
**data** [`numpy.ndarray`] Data  
**niter** [`int`] Number of iterations  
**eps** [`float`, optional] Sparsity damping  
**alpha** [`float`, optional] Step size ( $\alpha \leq 1/\lambda_{\max}(\mathbf{O}^H \mathbf{O})$ ) guarantees convergence. If `None`, estimated to satisfy the condition, otherwise the condition will not be checked)  
**eigsiter** [`float`, optional] Number of iterations for eigenvalue estimation if `alpha=None`  
**eigstol** [`float`, optional] Tolerance for eigenvalue estimation if `alpha=None`  
**tol** [`float`, optional] Tolerance. Stop iterations if difference between inverted model at subsequent iterations is smaller than `tol`  
**monitorres** [`bool`, optional] Monitor that residual is decreasing  
**returninfo** [`bool`, optional] Return info of CG solver  
**show** [`bool`, optional] Display iterations log

**Returns**

**xinv** [`numpy.ndarray`] Inverted model  
**niter** [`int`] Number of effective iterations  
**cost** [`numpy.ndarray`, optional] History of cost function

**Raises**

**ValueError** If `monitorres=True` and residual increases

**See also:**

**OMP** Orthogonal Matching Pursuit (OMP).

**FISTA** Fast Iterative Soft Thresholding Algorithm (FISTA).

**SPGL1** Spectral Projected-Gradient for L1 norm (SPGL1).

**SplitBregman** Split Bregman for mixed L2-L1 norms.

**Notes**

Solves the following optimization problem for the operator **Op** and the data **d**:

$$J = \|\mathbf{d} - \mathbf{O}\mathbf{x}\|_2^2 + \epsilon\|\mathbf{x}\|_1$$

using the Iterative Soft Thresholding Algorithm (ISTA) [1]. This is a very simple iterative algorithm which applies the following step:

$$\mathbf{x}^{(i+1)} = \text{soft}(\mathbf{x}^{(i)} + \alpha \mathbf{O}^H (\mathbf{d} - \mathbf{O}\mathbf{x}^{(i)})), \epsilon\alpha/2)$$

where  $\epsilon\alpha/2$  is the threshold and `soft()` is the so-called soft-thresholding rule.

## Examples using `pylops.optimization.sparsityISTA`

- `sphx_glr_gallery_plot_ista.py`
- *03. Solvers*

## `pylops.optimization.sparsity.FISTA`

`pylops.optimization.sparsity.FISTA(Op, data, niter, eps=0.1, alpha=None, eigsiter=None, eigstol=0, tol=1e-10, returninfo=False, show=False)`

Fast Iterative Soft Thresholding Algorithm (FISTA).

Solve an optimization problem with  $L1$  regularization function given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square  $N = M$  or underdetermined  $N < M$ .

### Parameters

**Op** [`pylops.LinearOperator`] Operator to invert  
**data** [`numpy.ndarray`] Data  
**niter** [`int`] Number of iterations  
**eps** [`float`, optional] Sparsity damping  
**alpha** [`float`, optional] Step size ( $\alpha \leq 1/\lambda_{max}(\mathbf{Op}^H \mathbf{Op})$ ) guarantees convergence. If `None`, estimated to satisfy the condition, otherwise the condition will not be checked)  
**eigsiter** [`int`, optional] Number of iterations for eigenvalue estimation if `alpha=None`  
**eigstol** [`float`, optional] Tolerance for eigenvalue estimation if `alpha=None`  
**tol** [`float`, optional] Tolerance. Stop iterations if difference between inverted model at subsequent iterations is smaller than `tol`  
**returninfo** [`bool`, optional] Return info of FISTA solver  
**show** [`bool`, optional] Display iterations log

### Returns

**xinv** [`numpy.ndarray`] Inverted model  
**niter** [`int`] Number of effective iterations  
**cost** [`numpy.ndarray`, optional] History of cost function

### See also:

**OMP** Orthogonal Matching Pursuit (OMP).

**ISTA** Iterative Soft Thresholding Algorithm (FISTA).

**SPGL1** Spectral Projected-Gradient for L1 norm (SPGL1).

**SplitBregman** Split Bregman for mixed L2-L1 norms.

### Notes

Solves the following optimization problem for the operator `Op` and the data `d`:

$$J = \|\mathbf{d} - \mathbf{O}\mathbf{x}\|_2^2 + \epsilon\|\mathbf{x}\|_1$$

using the Fast Iterative Soft Thresholding Algorithm (FISTA) [1]. This is a modified version of ISTA solver with improved convergence properties and limited additional computational cost.

### Examples using `pylops.optimization.sparsity.FISTA`

- sphx\_glr\_gallery\_plot\_ista.py
- *03. Solvers*
- *05. Image deblurring*
- *11. Radon filtering*
- *15. Least-squares migration*

## `pylops.optimization.sparsity.SPGL1`

`pylops.optimization.sparsity.SPGL1(Op, data, SOp=None, tau=0, sigma=0, x0=None, **kwargs_spgl1)`

Spectral Projected-Gradient for L1 norm.

Solve a constrained system of equations given the operator `Op` and a sparsifying transform `SOp` aiming to retrieve a model that is sparse in the sparsifying domain.

This is a simple wrapper to `spgl1.spgl1` which is a porting of the well-known `SPGL1` MATLAB solver into Python. In order to be able to use this solver you need to have installed the `spgl1` library.

#### Parameters

- Op** [`pylops.LinearOperator`] Operator to invert
- data** [`numpy.ndarray`] Data
- SOp** [`pylops.LinearOperator`] Sparsifying transform
- tau** [`float`] Non-negative LASSO scalar. If different from 0, SPGL1 will solve LASSO problem
- sigma** [`list`] BPDN scalar. If different from 0, SPGL1 will solve BPDN problem
- x0** [`numpy.ndarray`] Initial guess
- \*\*kwargs\_spgl1** Arbitrary keyword arguments for `spgl1.spgl1` solver

#### Returns

- xinv** [`numpy.ndarray`] Inverted model in original domain.
- pinv** [`numpy.ndarray`] Inverted model in sparse domain.
- info** [`dict`] Dictionary with the following information:
  - `tau`, final value of tau (see sigma above)
  - `rnorm`, two-norm of the optimal residual
  - `rgap`, relative duality gap (an optimality measure)
  - `gnorm`, Lagrange multiplier of (LASSO)
  - `stat`, 1: found a BPDN solution, 2: found a BP solution; exit based on small gradient, 3: found a BP solution; exit based on small residual, 4: found a LASSO solution, 5: error, too many iterations, 6: error, linesearch failed, 7: error, found suboptimal BP solution, 8: error, too many matrix-vector products.

---

niters, number of iterations  
nProdA, number of multiplications with A  
nProdAt, number of multiplications with A'  
n\_newton, number of Newton steps  
time\_project, projection time (seconds)  
time\_matprod, matrix-vector multiplications time (seconds)  
time\_total, total solution time (seconds)  
niters\_lsqr, number of lsqr iterations (if subspace\_min=True)  
xnorm1, L1-norm model solution history through iterations  
rnorm2, L2-norm residual history through iterations  
lambdaaa, Lagrange multiplier history through iterations

**Raises**

**ModuleNotFoundError** If the `spgl1` library is not installed

**Notes**

Solve different variations of sparsity-promoting inverse problem by imposing sparsity in the retrieved model [1]. The first problem is called *basis pursuit denoise (BPDN)* and its cost function is

$$\|\mathbf{x}\|_1 \quad subj.to \quad \|\mathbf{OpS}^H \mathbf{x} - \mathbf{b}\|_2 \leq \sigma,$$

while the second problem is the *l1-regularized least-squares or LASSO* problem and its cost function is

$$\|\mathbf{OpS}^H \mathbf{x} - \mathbf{b}\|_2 \quad subj.to \quad \|\mathbf{x}\|_1 \leq \tau$$

**Examples using `pylops.optimization.sparsity.SPGL1`**

- *03. Solvers*

**`pylops.optimization.sparsity.SplitBregman`**

```
pylops.optimization.sparsity.SplitBregman(Op,      RegsL1,      data,      niter_outer=3,
                                              niter_inner=5,      RegsL2=None,
                                              dataregsL2=None,   mu=1.0,    epsRL1s=None,
                                              epsRL2s=None,    tol=1e-10, tau=1.0, x0=None,
                                              restart=False, show=False, **kwargs_lsqr)
```

Split Bregman for mixed L2-L1 norms.

Solve an unconstrained system of equations with mixed L2-L1 regularization terms given the operator `Op`, a list of L1 regularization terms `RegsL1`, and an optional list of L2 regularization terms `RegsL2`.

**Parameters**

**Op** [`pylops.LinearOperator`] Operator to invert

**RegsL1** [`list`] L1 regularization operators

**data** [`numpy.ndarray`] Data

**niter\_outer** [`int`] Number of iterations of outer loop

**niter\_inner** [`int`] Number of iterations of inner loop

**RegsL2** [`list`] Additional L2 regularization operators (if `None`, L2 regularization is not added to the problem)

**dataregsL2** [`list`, optional] L2 Regularization data (must have the same number of elements of `RegsL2` or equal to `None` to use a zero data for every regularization operator in `RegsL2`)

**mu** [`float`, optional] Data term damping

**epsRL1s** [`list`] L1 Regularization dampings (must have the same number of elements as `RegsL1`)

**epsRL2s** [`list`] L2 Regularization dampings (must have the same number of elements as `RegsL2`)

**tol** [`float`, optional] Tolerance. Stop outer iterations if difference between inverted model at subsequent iterations is smaller than `tol`

**tau** [`float`, optional] Scaling factor in the Bregman update (must be close to 1)

**x0** [`numpy.ndarray`, optional] Initial guess

**restart** [`bool`, optional] The unconstrained inverse problem in inner loop is initialized with the initial guess (`True`) or with the last estimate (`False`)

**show** [`bool`, optional] Display iterations log

**\*\*kwargs\_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

### Returns

**xinv** [`numpy.ndarray`] Inverted model

**itn\_out** [`int`] Iteration number of outer loop upon termination

### Notes

Solve the following system of unconstrained, regularized equations given the operator `Op` and a set of mixed norm (L2 and L1) regularization terms  $\mathbf{R}_{\mathbf{L2},i}$  and  $\mathbf{R}_{\mathbf{L1},i}$ , respectively:

$$J = \mu/2\|\mathbf{d} - \mathbf{Opx}\|_2 + \sum_i \epsilon_{R_{L2,i}} \|\mathbf{d}_{\mathbf{R}_{\mathbf{L2},i}} - \mathbf{R}_{\mathbf{L2},i}\mathbf{x}\|_2 + \sum_i \|\mathbf{R}_{\mathbf{L1},i}\mathbf{x}\|_1$$

where  $\mu$  and  $\epsilon_{R_{L2,i}}$  are the damping factors used to weight the different terms of the cost function.

The generalized Split Bergman algorithm is used to solve such cost function: the algorithm is composed of a sequence of unconstrained inverse problems and Bregman updates. Note that the L1 terms are not weighted in the original cost function but are first converted into constraints and then re-inserted in the cost function with Lagrange multipliers  $\epsilon_{R_{L1,i}}$ , which effectively act as damping factors for those terms. See [1] for detailed derivation.

The `scipy.sparse.linalg.lsqr` solver and a fast shrinkage algorithm are used within the inner loop to solve the unconstrained inverse problem, and the same procedure is repeated `niter_outer` times until convergence.

## Examples using `pylops.optimization.sparsity.SplitBregman`

- `sphx_glr_gallery_plot_tvreg.py`
- [05. Image deblurring](#)
- [16. CT Scan Imaging](#)

### 3.6.3 Applications

#### Wave-Equation processing

<code>SeismicInterpolation</code> (data, nrec, iava[, ...])	Seismic interpolation (or regularization).
<code>Deghosting</code> (p, nt, nr, dt, dr, vel, zrec[, ...])	Wavefield deghosting.
<code>WavefieldDecomposition</code> (p, vz, nt, nr, dt, ...)	Up-down wavefield decomposition.
<code>MDD</code> (G, d[, dt, dr, nfmax, wav, twosided, ...])	Multi-dimensional deconvolution.
<code>Marchenko</code> (R[, R1, dt, nt, dr, nfmax, wav, ...])	Marchenko redatuming
<code>LSM</code> (z, x, t, srccs, recs, vel, wav, wavcenter)	Least-squares Migration (LSM).

#### `pylops.waveeqprocessing.SeismicInterpolation`

```
pylops.waveeqprocessing.SeismicInterpolation(data, nrec, iava, iava1=None,
                                              kind='fk', nffts=None, sampling=None,
                                              spataxis=None, spat1axis=None,
                                              taxis=None, paxis=None, p1axis=None,
                                              centeredh=True, nwins=None,
                                              nwin=None, nover=None, design=False,
                                              engine='numba', dotest=False,
                                              **kwargs_solver)
```

Seismic interpolation (or regularization).

Interpolate seismic data from irregular to regular spatial grid. Depending on the size of the input `data`, interpolation is either 2- or 3-dimensional. In case of 3-dimensional interpolation, data can be irregularly sampled in either one or both spatial directions.

##### Parameters

`data` [np.ndarray] Irregularly sampled seismic data of size  $[n_{r_y} (\times n_{r_x} \times n_t)]$

`nrec` [int or tuple] Number of elements in the regularly sampled (reconstructed) spatial array,  $n_{R_y}$  for 2-dimensional data and  $(n_{R_y}, n_{R_x})$  for 3-dimensional data

`iava` [list or numpy.ndarray] Integer (or floating) indices of locations of available samples in first dimension of regularly sampled spatial grid of interpolated signal. The `pylops.basicoperators.Restriction` operator is used in case of integer indices, while the `pylops.signalprocessing.Iterp` operator is used in case of floating indices.

`iava1` [list or numpy.ndarray, optional] Integer (or floating) indices of locations of available samples in second dimension of regularly sampled spatial grid of interpolated signal. Can be used only in case of 3-dimensional data.

`kind` [str, optional] Type of inversion: `fk` (default), `spatial`, `radon-linear`, `radon-parabolic` or `radon-hyperbolic` and `sliding`

**nffts** [int or tuple, optional] nffts : tuple, optional Number of samples in Fourier Transform for each direction. Required if kind='fk'

**sampling** [tuple, optional] Sampling steps dy (, dx) and dt. Required if kind='fk' or kind='radon-lin'

**spataxis** [np.ndarray, optional] First spatial axis. Required for kind='radon-lin', can also be provided instead of sampling for kind='fk'

**spat1axis** [np.ndarray, optional] Second spatial axis. Required for kind='radon-lin', can also be provided instead of sampling for kind='fk'

**timeaxis** [np.ndarray, optional] Time axis. Required for kind='radon-lin', can also be provided instead of sampling for kind='fk'

**paxis** [np.ndarray, optional] First Radon axis. Required for kind='radon-linear', kind='radon-parabolic', kind='radon-hyperbolic' and kind='sliding'

**p1axis** [np.ndarray, optional] Second Radon axis. Required for kind='radon-linear', kind='radon-parabolic' and kind='radon-hyperbolic' and kind='sliding'

**centeredh** [bool, optional] Assume centered spatial axis (True) or not (False). Required for kind='radon-linear', kind='radon-parabolic' and kind='radon-hyperbolic'

**nwins** [int or tuple, optional] Number of windows. Required for kind='sliding'

**nwin** [int or tuple, optional] Number of samples of window. Required for kind='sliding'

**nover** [int or tuple, optional] Number of samples of overlapping part of window. Required for kind='sliding'

**design** [bool, optional] Print number of sliding window (True) or not (False) when using kind='sliding'

**engine** [str, optional] Engine used for Radon computations (numpy or numba)

**dottest** [bool, optional] Apply dot-test

**\*\*kwargs\_solver** Arbitrary keyword arguments for `pylops.optimization.leastsquares.RegularizedInversion` solver if kind='spatial' or `pylops.optimization.sparsity.FISTA` solver otherwise

### Returns

**recdata** [np.ndarray] Reconstructed data of size  $[n_{R_y} (\times n_{R_x} \times n_t)]$

**recprec** [np.ndarray] Reconstructed data in the sparse or preconditioned domain in case of kind='fk', kind='radon-linear', kind='radon-parabolic', kind='radon-hyperbolic' and kind='sliding'

**cost** [np.ndarray] Cost function norm

### Raises

**KeyError** If kind is neither spatial, fl, radon-linear, radon-parabolic, radon-hyperbolic nor sliding

## Notes

The problem of seismic data interpolation (or regularization) can be formally written as

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where a restriction or interpolation operator is applied along the spatial direction(s). Here  $\mathbf{y} = [\mathbf{y}_{R1}^T, \mathbf{y}_{R2}^T, \dots, \mathbf{y}_{RN}^T]^T$  where each vector  $\mathbf{y}_{Ri}$  contains all time samples recorded in the seismic data at the specific receiver  $R_i$ . Similarly,  $\mathbf{x} = [\mathbf{x}_{r1}^T, \mathbf{x}_{r2}^T, \dots, \mathbf{x}_{rM}^T]$ , contains all traces at the regularly and finely sampled receiver locations  $r_i$ .

Several alternative approaches can be taken to solve such a problem. They mostly differ in the choice of the regularization (or preconditioning) used to mitigate the ill-posedness of the problem:

- **spatial**: least-squares inversion in the original time-space domain with an additional spatial smoothing regularization term, corresponding to the cost function  $J = \|\mathbf{y} - \mathbf{R}\mathbf{x}\|_2 + \epsilon_{\nabla} \nabla \|\mathbf{x}\|_2$  where  $\nabla$  is a second order space derivative implemented via `pylops.basicoperators.SecondDerivative` in 2-dimensional case and `pylops.basicoperators.Laplacian` in 3-dimensional case
- **fk**: L1 inversion in frequency-wavenumber preconditioned domain corresponding to the cost function  $J = \|\mathbf{y} - \mathbf{R}\mathbf{F}\mathbf{x}\|_2$  where  $\mathbf{F}$  is frequency-wavenumber transform implemented via `pylops.signalprocessing.FFT2D` in 2-dimensional case and `pylops.signalprocessing.FFTND` in 3-dimensional case
- **radon-linear**: L1 inversion in linear Radon preconditioned domain using the same cost function as `fk` but with  $\mathbf{F}$  being a Radon transform implemented via `pylops.signalprocessing.Radon2D` in 2-dimensional case and `pylops.signalprocessing.Radon3D` in 3-dimensional case
- **radon-parabolic**: L1 inversion in parabolic Radon preconditioned domain
- **radon-hyperbolic**: L1 inversion in hyperbolic Radon preconditioned domain
- **sliding**: L1 inversion in sliding-linear Radon preconditioned domain using the same cost function as `fk` but with  $\mathbf{F}$  being a sliding Radon transform implemented via `pylops.signalprocessing.Sliding2D` in 2-dimensional case and `pylops.signalprocessing.Sliding3D` in 3-dimensional case

## Examples using `pylops.waveeqprocessing.SeismicInterpolation`

- 12. Seismic regularization

## `pylops.waveeqprocessing.Deghosting`

```
pylops.waveeqprocessing.Deghosting(p, nt, nr, dt, dr, vel, zrec, pd=None, win=None, npad=(11, 11), ntaper=(11, 11), restriction=None, sptransf=None, solver=<function lsqr at 0x7f1807f607b8>, dottest=False, dtype='complex128', **kwargs_solver)
```

Wavefield deghosting.

Apply seismic wavefield decomposition from single-component (pressure) data. This process is also generally referred to as model-based deghosting.

### Parameters

**p** [np.ndarray] Pressure data of size  $[n_{r_x} (\times n_{r_y}) \times n_t]$  (or  $[n_{r_x, sub} (\times n_{r_y, sub}) \times n_t]$ ) in case a restriction operator is provided. Note that  $n_{r_x, sub}$  (and  $n_{r_y, sub}$ ) must agree with the size of the output of this operator

**nt** [int] Number of samples along the time axis  
**nr** [int or tuple] Number of samples along the receiver axis (or axes)  
**dt** [float] Sampling along the time axis  
**dr** [float or tuple] Sampling along the receiver array of the separated pressure constituents  
**vel** [float] Velocity along the receiver array (must be constant)  
**zrec** [float] Depth of receiver array  
**pd** [np.ndarray, optional] Direct arrival to be subtracted from p  
**pd** [np.ndarray, optional] Time window to be applied to p to remove the direct arrival (if pd=None)  
**ntaper** [float or tuple, optional] Number of samples of taper applied to propagator to avoid edge effects  
**npad** [float or tuple, optional] Number of samples of padding applied to propagator to avoid edge effects angle  
**restriction** [pylops.LinearOperator, optional] Restriction operator  
**sptransf** [pylops.LinearOperator, optional] Sparsifying operator  
**solver** [float, optional] Function handle of solver to be used if kind='inverse'  
**dottest** [bool, optional] Apply dot-test  
**dtype** [str, optional] Type of elements in input array.  
\*\*kwargs\_solver Arbitrary keyword arguments for chosen solver

#### Returns

**pup** [np.ndarray] Up-going wavefield  
**pdown** [np.ndarray] Down-going wavefield

#### Notes

Up- and down-going components of seismic data ( $p^-(x, t)$  and  $p^+(x, t)$ ) can be estimated from single-component data ( $p(x, t)$ ) using a ghost model.

The basic idea is that of using a one-way propagator in the f-k domain (also referred to as ghost model) to predict the down-going field from the up-going one (excluded the direct arrival and its source ghost referred here to as  $p_d(x, t)$ ):

$$p^+ - p_d = e^{-jk_z 2z_{rec}} p^-$$

where  $k_z$  is the vertical wavenumber and  $z_{rec}$  is the depth of the array of receivers

In a matrix form we can thus write the total wavefield as:

$$\mathbf{p} - \mathbf{p}_d = (\mathbf{I} + \Phi)\mathbf{p}^-$$

where  $\Phi$  is one-way propagator implemented via the `pylops.waveeqprocessing.PhaseShift` operator.

#### Examples using `pylops.waveeqprocessing.Deghosting`

- 13. Deghosting

## pylops.waveeqprocessing.WavefieldDecomposition

```
pylops.waveeqprocessing.WavefieldDecomposition(p, vz, nt, nr, dt, dr, rho, vel,
                                                nffts=(None, None, None), criti-
                                                cal=100.0, ntaper=10, scaling=1.0,
                                                kind='inverse', restriction=None, sp-
                                                transf=None, solver=<function lsqr
                                                at 0x7f1807f607b8>, dottest=False,
                                                dtype='complex128', **kwargs_solver)
```

Up-down wavefield decomposition.

Apply seismic wavefield decomposition from multi-component (pressure and vertical particle velocity) data. This process is also generally referred to as data-based deghosting.

### Parameters

**p** [np.ndarray] Pressure data of size  $[n_{r_x} (\times n_{r_y}) \times n_t]$  (or  $[n_{r_x, \text{sub}} (\times n_{r_y, \text{sub}}) \times n_t]$  in case a restriction operator is provided. Note that  $n_{r_x, \text{sub}}$  (and  $n_{r_y, \text{sub}}$ ) must agree with the size of the output of this operator)

**vz** [np.ndarray] Vertical particle velocity data of same size as pressure data

**nt** [int] Number of samples along the time axis

**nr** [int or tuple] Number of samples along the receiver axis (or axes)

**dt** [float] Sampling along the time axis

**dr** [float or tuple] Sampling along the receiver array (or axes)

**rho** [float] Density along the receiver array (must be constant)

**vel** [float] Velocity along the receiver array (must be constant)

**nffts** [tuple, optional] Number of samples along the wavenumber and frequency axes

**critical** [float, optional] Percentage of angles to retain in obliquity factor. For example, if  $\text{critical}=100$  only angles below the critical angle  $\frac{f(k_x)}{v}$  will be retained

**ntaper** [float, optional] Number of samples of taper applied to obliquity factor around critical angle

**kind** [str, optional] Type of separation: `inverse` (default) or `analytical`

**scaling** [float, optional] Scaling to apply to the operator (see Notes of pylops.waveeqprocessing.wavedecomposition.UpDownComposition2D for more details)

**restriction** [pylops.LinearOperator, optional] Restriction operator

**sptransf** [pylops.LinearOperator, optional] Sparsifying operator

**solver** [float, optional] Function handle of solver to be used if `kind='inverse'`

**dottest** [bool, optional] Apply dot-test

**dtype** [str, optional] Type of elements in input array.

**\*\*kwargs\_solver** Arbitrary keyword arguments for chosen solver

### Returns

**pup** [np.ndarray] Up-going wavefield

**pdown** [np.ndarray] Down-going wavefield

**Raises**

**KeyError** If kind is neither analytical nor inverse

**Notes**

Up- and down-going components of seismic data ( $p^-(x, t)$  and  $p^+(x, t)$ ) can be estimated from multi-component data ( $p(x, t)$  and  $v_z(x, t)$ ) by computing the following expression [1]:

$$\begin{bmatrix} p^+(k_x, \omega) \\ p^-(k_x, \omega) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & \frac{\omega\rho}{k_z} \\ 1 & -\frac{\omega\rho}{k_z} \end{bmatrix} \begin{bmatrix} \mathbf{P}(k_x, \omega) \\ \mathbf{v}_z(k_x, \omega) \end{bmatrix}$$

if kind='analytical' or alternatively by solving the equation in `ptcpy.waveeqprocessingUpDownComposition2D` as an inverse problem, if kind='inverse'.

The latter approach has several advantages as data regularization can be included as part of the separation process allowing the input data to be aliased. This is obtained by solving the following problem:

$$\begin{bmatrix} \mathbf{P} \\ s * \mathbf{v}_z \end{bmatrix} = \begin{bmatrix} \mathbf{R}\mathbf{F} & 0 \\ 0 & s * \mathbf{R}\mathbf{F} \end{bmatrix} \mathbf{W} \begin{bmatrix} \mathbf{F}^H \mathbf{S} & 0 \\ 0 & \mathbf{F}^H \mathbf{S} \end{bmatrix} \mathbf{P}^\pm$$

where **R** is a `ptcpy.basicoperators.Restriction` operator and **S** is sparsifying transform operator (e.g., `ptcpy.signalprocessing.Radon2D`).

**Examples using `pylops.waveeqprocessing.WavefieldDecomposition`**

- 14. Seismic wavefield decomposition

**pylops.waveeqprocessing.MDD**

```
pylops.waveeqprocessing.MDD(G, d, dt=0.004, dr=1.0, nfmax=None, wav=None,
                             twosided=True, causality_precond=False, adjoint=False, psf=False,
                             dtype='float64', dottest=False, saveGt=True, add_negative=True,
                             smooth_precond=0, **kwargs_lsqr)
```

Multi-dimensional deconvolution.

Solve multi-dimensional deconvolution problem using `scipy.sparse.linalg.lsqr` iterative solver.

**Parameters**

**G** [`numpy.ndarray`] Multi-dimensional convolution kernel in time domain of size  $[n_s \times n_r \times n_t]$  for `twosided=False` or `twosided=True` and `add_negative=True` (with only positive times) or size  $[n_s \times n_r \times 2 * n_t - 1]$  for `twosided=True` and `add_negative=False` (with both positive and negative times)

**d** [`numpy.ndarray`] Data in time domain  $[n_s (\times n_v) \times n_t]$  if `twosided=False` or `twosided=True` and `add_negative=True` (with only positive times) or size  $[n_s (\times n_v) \times 2 * n_t - 1]$  if `twosided=True`

**dt** [`float`, optional] Sampling of time integration axis

**dr** [`float`, optional] Sampling of receiver integration axis

**nfmax** [`int`, optional] Index of max frequency to include in deconvolution process

**wav** [`numpy.ndarray`, optional] Wavelet to convolve to the inverted model and psf (must be centered around its index in the middle of the array). If None, the outputs of the inversion are returned directly.

**twosided** [`bool`, optional] MDC operator and data both negative and positive time (`True`) or only positive (`False`)

**add\_negative** [`bool`, optional] Add negative side to MDC operator and data (`True`) or not (`False`)- operator and data are already provided with both positive and negative sides. To be used only with `twosided=True`.

**causality\_precond** [`bool`, optional] Apply causality mask (`True`) or not (`False`)

**adjoint** [`bool`, optional] Compute and return adjoint(s)

**psf** [`bool`, optional] Compute and return Point Spread Function (PSF) and its inverse

**dtype** [`bool`, optional] Type of elements in input array.

**dottest** [`bool`, optional] Apply dot-test

**saveGt** [`bool`, optional] Save  $G$  and  $G^H$  to speed up the computation of adjoint of `pylops.signalprocessing.Fredholm1` (`True`) or create  $G^H$  on-the-fly (`False`) Note that `saveGt=True` will be faster but double the amount of required memory

**\*\*kwargs\_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

### Returns

**minv** [`numpy.ndarray`] Inverted model of size  $[n_r(\times n_{vs}) \times n_t]$  for `twosided=False` or  $[n_r(\times n_{vs}) \times 2 * n_t - 1]$  for `twosided=True`

**madj** [`numpy.ndarray`] Adjoint model of size  $[n_r(\times n_{vs}) \times n_t]$  for `twosided=False` or  $[n_r(\times n_r) \times 2 * n_t - 1]$  for `twosided=True`

**psfinv** [`numpy.ndarray`] Inverted psf of size  $[n_r \times n_r \times n_t]$  for `twosided=False` or  $[n_r \times n_r \times 2 * n_t - 1]$  for `twosided=True`

**psfadj** [`numpy.ndarray`] Adjoint psf of size  $[n_r \times n_r \times n_t]$  for `twosided=False` or  $[n_r \times n_r \times 2 * n_t - 1]$  for `twosided=True`

### See also:

**MDC** Multi-dimensional convolution

### Notes

Multi-dimensional deconvolution (MDD) is a mathematical ill-solved problem, well-known in the image processing and geophysical community [1].

MDD aims at removing the effects of a Multi-dimensional Convolution (MDC) kernel or the so-called blurring operator or point-spread function (PSF) from a given data. It can be written as

$$\mathbf{d} = \mathbf{D}\mathbf{m}$$

or, equivalently, by means of its normal equation

$$\mathbf{m} = (\mathbf{D}^H \mathbf{D})^{-1} \mathbf{D}^H \mathbf{d}$$

where  $\mathbf{D}^H \mathbf{D}$  is the PSF.

### Examples using `pylops.waveeqprocessing.MDD`

- 09. Multi-Dimensional Deconvolution

**pylops.waveeqprocessing.Marchenko**

```
class pylops.waveeqprocessing.Marchenko(R, R1=None, dt=0.004, nt=None, dr=1.0, nfmax=None, wav=None, toff=0.0, nsmooth=10, dtype='float64', saveRt=True, prescaled=False)
```

Marchenko redatuming

Solve multi-dimensional Marchenko redatuming problem using `scipy.sparse.linalg.lsqr` iterative solver.

**Parameters**

**R** [numpy.ndarray] Multi-dimensional reflection response in time or frequency domain of size  $[n_s \times n_r \times n_t / n_{f\max}]$ . If provided in time, *R* should not be of complex type. Note that the reflection response should have already been multiplied by 2.

**R1** [bool, optional] *Deprecated*, will be removed in v2.0.0. Simply kept for back-compatibility with previous implementation

**dt** [float, optional] Sampling of time integration axis

**nt** [float, optional] Number of samples in time (not required if *R* is in time)

**dr** [float, optional] Sampling of receiver integration axis

**nfmax** [int, optional] Index of max frequency to include in deconvolution process

**wav** [numpy.ndarray, optional] Wavelet to apply to direct arrival when created using `trav`

**toff** [float, optional] Time-offset to apply to traveltime

**nsmooth** [int, optional] Number of samples of smoothing operator to apply to window

**dtype** [bool, optional] Type of elements in input array.

**saveRt** [bool, optional] Save *R* and  $R^H$  to speed up the computation of adjoint of `pylops.signalprocessing.Fredholm1` (True) or create  $R^H$  on-the-fly (False). Note that `saveRt=True` will be faster but double the amount of required memory

**prescaled** [bool, optional] Apply scaling to *R* (False) or not (True) when performing spatial and temporal summations within the `pylops.waveeqprocessing.MDC` operator. In case `prescaled=True`, the *R* is assumed to have been pre-scaled by the user.

**Raises**

**TypeError** If *t* is not `numpy.ndarray`.

**See also:**

**MDC** Multi-dimensional convolution

**MDD** Multi-dimensional deconvolution

**Notes**

Marchenko redatuming is a method that allows to produce correct subsurface-to-surface responses given the availability of a reflection data and a macro-velocity model [R1c4066d82e61-1].

The Marchenko equations can be written in a compact matrix form [R1c4066d82e61-2] and solved by means of iterative solvers such as LSQR:

$$\begin{bmatrix} \Theta R f_d^+ \\ 0 \end{bmatrix} = I - \begin{bmatrix} 0 & \Theta R \\ \Theta R^* & 0 \end{bmatrix} \begin{bmatrix} f^- \\ f_m^+ \end{bmatrix}$$

Finally the subsurface Green's functions can be obtained applying the following operator to the retrieved focusing functions

$$\begin{bmatrix} -\mathbf{g}^- \\ \mathbf{g}^{+*} \end{bmatrix} = \mathbf{I} - \begin{bmatrix} \mathbf{0} & \mathbf{R} \\ \mathbf{R}^* & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{f}^- \\ \mathbf{f}^+ \end{bmatrix}$$

Here  $\mathbf{R}$  is the monopole-to-particle velocity seismic response (already multiplied by 2).

### Attributes

- ns** [int] Number of samples along source axis
- nr** [int] Number of samples along receiver axis
- shape** [tuple] Operator shape
- explicit** [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

### Methods

---

<code>__init__(self, R[, R1, dt, nt, dr, nfmax, ...])</code>	Initialize self.
<code>apply_multiplepoints(self, trav[, G0, nfft, ...])</code>	Marchenko redatuming for multiple points
<code>apply_onepoint(self, trav[, G0, nfft, rtm, ...])</code>	Marchenko redatuming for one point

---

**apply\_onepoint** (*self*, *trav*, *G0=None*, *nfft=None*, *rtm=False*, *greens=False*, *dottest=False*, *fast=None*, *\*\*kwargs\_lsqr*)  
Marchenko redatuming for one point

Solve the Marchenko redatuming inverse problem for a single point given its direct arrival traveltime curve (*trav*) and waveform (*G0*).

### Parameters

- trav** [numpy.ndarray] Travelttime of first arrival from subsurface point to surface receivers of size  $[n_r \times 1]$
- G0** [numpy.ndarray, optional] Direct arrival in time domain of size  $[n_r \times n_t]$  (if None, create arrival using *trav*)
- nfft** [int, optional] Number of samples in fft when creating the analytical direct wave
- rtm** [bool, optional] Compute and return rtm redatuming
- greens** [bool, optional] Compute and return Green's functions
- dottest** [bool, optional] Apply dot-test
- fast** [bool] *Deprecated*, will be removed in v2.0.0
- \*\*kwargs\_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

### Returns

- f1\_inv\_minus** [numpy.ndarray] Inverted upgoing focusing function of size  $[n_r \times n_t]$
- f1\_inv\_plus** [numpy.ndarray] Inverted downgoing focusing function of size  $[n_r \times n_t]$
- p0\_minus** [numpy.ndarray] Single-scattering standard redatuming upgoing Green's function of size  $[n_r \times n_t]$
- g\_inv\_minus** [numpy.ndarray] Inverted upgoing Green's function of size  $[n_r \times n_t]$

**g\_inv\_plus** [numpy.ndarray] Inverted downgoing Green's function of size  $[n_r \times n_t]$   
**apply\_multiplepoints** (self, trav, G0=None, nfft=None, rtm=False, greens=False, dottest=False,  
                          \*\*kwargs\_lsqr)  
Marchenko redatuming for multiple points  
Solve the Marchenko redatuming inverse problem for multiple points given their direct arrival traveltimes (trav) and waveforms (G0).

#### Parameters

**trav** [numpy.ndarray] Traveltimes of first arrival from subsurface points to surface receivers of size  $[n_r \times n_{vs}]$   
**G0** [numpy.ndarray, optional] Direct arrival in time domain of size  $[n_r \times n_{vs} \times n_t]$  (if None, create arrival using trav)  
**nfft** [int, optional] Number of samples in fft when creating the analytical direct wave  
**rtm** [bool, optional] Compute and return rtm redatuming  
**greens** [bool, optional] Compute and return Green's functions  
**dottest** [bool, optional] Apply dot-test  
**\*\*kwargs\_lsqr** Arbitrary keyword arguments for scipy.sparse.linalg.lsqr solver

#### Returns

**f1\_inv\_minus** [numpy.ndarray] Inverted upgoing focusing function of size  $[n_r \times n_{vs} \times n_t]$   
**f1\_inv\_plus** [numpy.ndarray] Inverted downgoing focusing function of size  $[n_r \times n_{vs} \times n_t]$   
**p0\_minus** [numpy.ndarray] Single-scattering standard redatuming upgoing Green's function of size  $[n_r \times n_{vs} \times n_t]$   
**g\_inv\_minus** [numpy.ndarray] Inverted upgoing Green's function of size  $[n_r \times n_{vs} \times n_t]$   
**g\_inv\_plus** [numpy.ndarray] Inverted downgoing Green's function of size  $[n_r \times n_{vs} \times n_t]$

## Examples using `pylops.waveeqprocessing.Marchenko`

- 10. Marchenko redatuming by inversion

## pylops.waveeqprocessing.LSM

**class** pylops.waveeqprocessing.LSM(z, x, t, srcs, recs, vel, wav, wavcenter, y=None, mode='eikonal', dottest=False)  
Least-squares Migration (LSM).  
Solve seismic migration as inverse problem given smooth velocity model vel and an acquisition setup identified by sources (src) and receivers (recs)

#### Parameters

**z** [numpy.ndarray] Depth axis  
**x** [numpy.ndarray] Spatial axis

---

**t** [numpy.ndarray] Time axis for data  
**srcs** [numpy.ndarray] Sources in array of size  $[2/3 \times n_s]$   
**recs** [numpy.ndarray] Receivers in array of size  $[2/3 \times n_r]$   
**vel** [numpy.ndarray or float] Velocity model of size  $[(n_y \times) n_x \times n_z]$  (or constant)  
**wav** [numpy.ndarray] Wavelet  
**wavcenter** [int] Index of wavelet center  
**y** [numpy.ndarray] Additional spatial axis (for 3-dimensional problems)  
**mode** [numpy.ndarray, optional] Computation mode (eikonal, analytic - only for constant velocity)  
**dottest** [bool, optional] Apply dot-test

See also:

[pylops.waveeqprocessing.Demigration](#) Demigration operator

## Notes

Inverting a demigration operator is generally referred in the literature as least-squares migration (LSM) as historically a least-squares cost function has been used for this purpose. In practice any other cost function could be used, for examples if `solver='pylops.optimization.sparsity.FISTA'` a sparse representation of reflectivity is produced as result of the inversion.

Finally, it is worth noting that in the first iteration of an iterative scheme aimed at inverting the demigration operator, a projection of the recorded data in the model domain is performed and an approximate (band-limited) image of the subsurface is created. This process is referred to in the literature as *migration*.

## Attributes

**Demop** [[pylops.LinearOperator](#)] Demigration operator

## Methods

---

<code>__init__(self, z, x, t, srcs, recs, vel, ...)</code>	Initialize self.
<code>solve(self, d[, solver])</code>	Solve least-squares migration equations with chosen solver

---

**solve** (`self, d, solver=<function lsqr at 0x7f1807f607b8>, **kwargs_solver`)  
Solve least-squares migration equations with chosen `solver`

### Parameters

**d** [numpy.ndarray] Input data of size  $[n_s \times n_r \times n_t]$   
**solver** [func, optional] Solver to be used for inversion  
**\*\*kwargs\_solver** Arbitrary keyword arguments for chosen `solver`

### Returns

**minv** [np.ndarray] Inverted reflectivity model of size  $[(n_y \times) n_x \times n_z]$

**Examples using `pylops.waveeqprocessing.LSM`**

- 15. Least-squares migration

**Geophysical subsurface characterization**

---

```
poststack.PoststackInversion(data, wav[, Post-stack linearized seismic inversion.
```

```
...])
```

---

```
prestack.PrestackInversion(data, theta, Pre-stack linearized seismic inversion.
```

```
wav)
```

---

**pylops.avo.poststack.PoststackInversion**

```
pylops.avo.poststack.PoststackInversion(data, wav, m0=None, explicit=False, simultaneous=False, epsI=None, epsR=None, dottest=False, epsRL1=None, **kwargs_solver)
```

Post-stack linearized seismic inversion.

Invert post-stack seismic operator to retrieve an elastic parameter of choice from band-limited seismic post-stack data. Depending on the choice of input parameters, inversion can be trace-by-trace with explicit operator or global with either explicit or linear operator.

**Parameters**

**data** [np.ndarray] Band-limited seismic post-stack data of size  $[n_{t0}(\times n_x \times n_y)]$

**wav** [np.ndarray] Wavelet in time domain (must have odd number of elements and centered to zero). If 1d, assume stationary wavelet for the entire time axis. If 2d of size  $[n_{t0} \times n_h]$  use as non-stationary wavelet

**m0** [np.ndarray, optional] Background model of size  $[n_{t0}(\times n_x \times n_y)]$

**explicit** [bool, optional] Create a chained linear operator (False, preferred for large data) or a MatrixMult linear operator with dense matrix (True, preferred for small data)

**simultaneous** [bool, optional] Simultaneously invert entire data (True) or invert trace-by-trace (False) when using explicit operator (note that the entire data is always inverted when working with linear operator)

**epsI** [float, optional] Damping factor for Tikhonov regularization term

**epsR** [float, optional] Damping factor for additional Laplacian regularization term

**dottest** [bool, optional] Apply dot-test

**epsRL1** [float, optional] Damping factor for additional blockiness regularization term

**\*\*kwargs\_solver** Arbitrary keyword arguments for `scipy.linalg.lstsq` solver (if `explicit=True` and `epsR=None`) or `scipy.sparse.linalg.lsqr` solver (if `explicit=False` and/or `epsR` is not None)

**Returns**

**minv** [np.ndarray] Inverted model of size  $[n_{t0}(\times n_x \times n_y)]$

**datar** [np.ndarray] Residual data (i.e., data - background data) of size  $[n_{t0}(\times n_x \times n_y)]$

## Notes

The cost function and solver used in the seismic post-stack inversion module depends on the choice of explicit, simultaneous, epsI, and epsR parameters:

- explicit=True, epsI=None and epsR=None: the explicit solver `scipy.linalg.lstsq` is used if simultaneous=False (or the iterative solver `scipy.sparse.linalg.lsqr` is used if simultaneous=True)
- explicit=True with epsI and epsR=None: the regularized normal equations  $\mathbf{W}^T \mathbf{d} = (\mathbf{W}^T \mathbf{W} + \epsilon_I^2 \mathbf{I}) \mathbf{A} \mathbf{I}$  are instead fed into the `scipy.linalg.lstsq` solver if simultaneous=False (or the iterative solver `scipy.sparse.linalg.lsqr` if simultaneous=True)
- explicit=False and epsR=None: the iterative solver `scipy.sparse.linalg.lsqr` is used
- explicit=False with epsR and epsRL1=None: the iterative solver `pylops.optimization.leastsquares.RegularizedInversion` is used to solve the spatially regularized problem.
- explicit=False with epsR and epsRL1: the iterative solver `pylops.optimization.sparsity.SplitBregman` is used to solve the blockiness-promoting (in vertical direction) and spatially regularized (in additional horizontal directions) problem.

Note that the convergence of iterative solvers such as `scipy.sparse.linalg.lsqr` can be very slow for this type of operator. It is suggested to take a two steps approach with first a trace-by-trace inversion using the explicit operator, followed by a regularized global inversion using the outcome of the previous inversion as initial guess.

## Examples using `pylops.avo.poststack.PoststackInversion`

- 07. Post-stack inversion

## `pylops.avo.prestack.PrestackInversion`

```
pylops.avo.prestack.PrestackInversion(data, theta, wav, m0=None, linearization='akirich',
                                       explicit=False, simultaneous=False, epsI=None,
                                       epsR=None, dottest=False, returnres=False, ep-
                                       sRL1=None, **kwargs_solver)
```

Pre-stack linearized seismic inversion.

Invert pre-stack seismic operator to retrieve a set of elastic property profiles from band-limited seismic pre-stack data (i.e., angle gathers). Depending on the choice of input parameters, inversion can be trace-by-trace with explicit operator or global with either explicit or linear operator.

### Parameters

**data** [np.ndarray] Band-limited seismic post-stack data of size  $[n_{t0} \times n_\theta (\times n_x \times n_y)]$

**theta** [np.ndarray] Incident angles in degrees

**wav** [np.ndarray] Wavelet in time domain (must had odd number of elements and centered to zero)

**m0** [np.ndarray, optional] Background model of size  $[n_{t0} \times n_m (\times n_x \times n_y)]$

**linearization** [str, optional] choice of linearization, `akirich`: Aki-Richards, `fatti`: Fatti (required only when `m0` is None)

**explicit** [bool, optional] Create a chained linear operator (False, preferred for large data) or a `MatrixMult` linear operator with dense matrix (True, preferred for small data)

**simultaneous** [`bool`, optional] Simultaneously invert entire data (`True`) or invert trace-by-trace (`False`) when using `explicit` operator (note that the entire data is always inverted when working with linear operator)

**epsI** [`float` or `list`, optional] Damping factor(s) for Tikhonov regularization term. If a list of  $n_m$  elements is provided, the regularization term will have different strength for each elastic property

**epsR** [`float`, optional] Damping factor for additional Laplacian regularization term

**dottest** [`bool`, optional] Apply dot-test

**returnres** [`bool`, optional] Return residuals

**epsRL1** [`float`, optional] Damping factor for additional blockiness regularization term

**\*\*kwargs\_solver** Arbitrary keyword arguments for `scipy.linalg.lstsq` solver (if `explicit=True` and `epsR=None`) or `scipy.sparse.linalg.lsqr` solver (if `explicit=False` and/or `epsR` is not `None`))

### Returns

**minv** [`np.ndarray`] Inverted model of size  $[n_{t0} \times n_m (\times n_x \times n_y)]$

**datar** [`np.ndarray`] Residual data (i.e., data - background data) of size  $[n_{t0} \times n_\theta (\times n_x \times n_y)]$

### Notes

The different choices of cost functions and solvers used in the seismic pre-stack inversion module follow the same convention of the seismic post-stack inversion module.

Refer to `pylops.avo.poststack.PoststackInversion` for more details.

## Examples using `pylops.avo.prestack.PrestackInversion`

- 08. Pre-stack (AVO) inversion

## 3.7 PyLops Utilities

Alongside with its *Linear Operators* and *Solvers*, PyLops contains also a number of auxiliary routines performing universal tasks that are used by several operators or simply within one or more *Tutorials* for the preparation of input data and subsequent visualization of results.

### 3.7.1 Shared

#### Dot-test

---

<code>dottest(Op, nr, nc[, tol, complexflag, ...])</code>	Dot test.
---	-----------

---

#### `pylops.utils.dottest`

`pylops.utils.dottest(Op, nr, nc, tol=1e-06, complexflag=0, raiseerror=True, verb=False)`

Dot test.

Generate random vectors **u** and **v** and perform dot-test to verify the validity of forward and adjoint operators. This test can help to detect errors in the operator implementation.

### Parameters

**Op** [`pylops.LinearOperator`] Linear operator to test.  
**nr** [`int`] Number of rows of operator (i.e., elements in data)  
**nc** [`int`] Number of columns of operator (i.e., elements in model)  
**tol** [`float`, optional] Dottest tolerance  
**complexflag** [`bool`, optional] generate random vectors with real (0) or complex numbers (1: only model, 2: only data, 3:both)  
**raiseerror** [`bool`, optional] Raise error or simply return `False` when dottest fails  
**verb** [`bool`, optional] Verbosity

### Raises

**ValueError** If dot-test is not verified within chosen tolerance.

### Notes

A dot-test is mathematical tool used in the development of numerical linear operators.

More specifically, a correct implementation of forward and adjoint for a linear operator should verify the following *equality* within a numerical tolerance:

$$(\mathbf{Op} * \mathbf{u})^H * \mathbf{v} = \mathbf{u}^H * (\mathbf{Op}^H * \mathbf{v})$$

### Examples using `pylops.utils.dottest`

- 02. The Dot-Test

## 3.7.2 Others

### Synthetics

<code>seismicevents.makeaxis(par)</code>	Create axes t, x, and y axes
<code>seismicevents.linear2d(x, t, v, t0, theta, ...)</code>	Linear 2D events
<code>seismicevents.parabolic2d(x, t, t0, px, pxx, ...)</code>	Parabolic 2D events
<code>seismicevents.hyperbolic2d(x, t, t0, vrms, ...)</code>	Hyperbolic 2D events
<code>seismicevents.linear3d(x, y, t, v, t0, ...)</code>	Linear 3D events
<code>seismicevents.hyperbolic3d(x, y, t, t0, ...)</code>	Hyperbolic 3D events

### `pylops.utils.seismicevents.makeaxis`

`pylops.utils.seismicevents.makeaxis (par)`  
Create axes t, x, and y axes

Create space and time axes from dictionary containing initial values ( $ot, ox, oy$ ), sampling steps ( $dt, dx, dy$ ) and number of elements ( $nt, nx, ny$ ) for each axis

**Parameters**

**par** [`dict`] Dictionary containing initial values, sampling steps, and number of elements

**Returns**

**t** [`numpy.ndarray`] time axis

**t2** [`numpy.ndarray`] double time axis (symmetric to zero)

**x** [`numpy.ndarray`] x axis

**y** [`numpy.ndarray`] y axis (None, if  $oy, dy, ny$  are not provided)

**Examples**

```
>>> par = {'ox':0, 'dx':2, 'nx':60,
>>>      'oy':0, 'dy':2, 'ny':100,
>>>      'ot':0, 'dt':4, 'nt':400}
>>> # Create axis
>>> t, t2, x, y = makeaxis(par)
```

**Examples using `pylops.utils.seismicevents.makeaxis`**

- sphx\_glr\_gallery\_plot\_sliding.py
- sphx\_glr\_gallery\_plot\_mdc.py
- sphx\_glr\_gallery\_plot\_phaseshift.py
- sphx\_glr\_gallery\_plot\_seismicevents.py
- [09. Multi-Dimensional Deconvolution](#)
- [11. Radon filtering](#)
- [12. Seismic regularization](#)
- [14. Seismic wavefield decomposition](#)

**pylops.utils.seismicevents.linear2d**

`pylops.utils.seismicevents.linear2d(x, t, v, t0, theta, amp, wav)`

Linear 2D events

Create 2d linear events given propagation velocity, intercept time, angle, and amplitude of each event

**Parameters**

**x** [`numpy.ndarray`] space axis

**t** [`numpy.ndarray`] time axis

**v** [`float`] propagation velocity

**t0** [`tuple` or `float`] intercept time at  $x = 0$  of each linear event

**theta** [`tuple` or `float`] angle (in degrees) of each linear event

**amp** [tuple or float] amplitude of each linear event

**wav** [numpy.ndarray] wavelet to be applied to data

#### Returns

**d** [numpy.ndarray] data without wavelet of size  $[n_x \times n_t]$

**dwav** [numpy.ndarray] data with wavelet of size  $[n_x \times n_t]$

#### Notes

Each event is created using the following relation:

$$t_i(x) = t_{0,i} + p_{x,i}x$$

where  $p_{x,i} = \sin(\theta_i)/v$

#### Examples using `pylops.utils.seismicevents.linear2d`

- sphx\_glr\_gallery\_plot\_seismicevents.py
- 11. Radon filtering
- 12. Seismic regularization

#### pylops.utils.seismicevents.parabolic2d

`pylops.utils.seismicevents.parabolic2d(x, t, t0, px, pxx, amp, wav)`

Parabolic 2D events

Create 2d parabolic events given intercept time, slowness, curvature, and amplitude of each event

#### Parameters

**x** [numpy.ndarray] space axis

**t** [numpy.ndarray] time axis

**t0** [tuple or float] intercept time at  $x = 0$  of each parabolic event

**px** [tuple or float] slowness of each parabolic event

**pxx** [tuple or float] curvature of each parabolic event

**amp** [tuple or float] amplitude of each parabolic event

**wav** [numpy.ndarray] wavelet to be applied to data

#### Returns

**d** [numpy.ndarray] data without wavelet of size  $[n_x \times n_t]$

**dwav** [numpy.ndarray] data with wavelet of size  $[n_x \times n_t]$

#### Notes

Each event is created using the following relation:

$$t_i(x) = t_{0,i} + p_{x,i}x + p_{xx,i}x^2$$

### Examples using `pylops.utils.seismicevents.parabolic2d`

- `sphx_glr_gallery_plot_sliding.py`
- `sphx_glr_gallery_plot_seismicevents.py`
- *11. Radon filtering*

### `pylops.utils.seismicevents.hyperbolic2d`

`pylops.utils.seismicevents.hyperbolic2d(x, t, t0, vrms, amp, wav)`

Hyperbolic 2D events

Create 2d hyperbolic events given intercept time, root-mean-square velocity, and amplitude of each event

#### Parameters

- `x` [numpy.ndarray] space axis
- `t` [numpy.ndarray] time axis
- `t0` [tuple or float] intercept time at  $x = 0$  of each of hyperbolic event
- `vrms` [tuple or float] root-mean-square velocity of each hyperbolic event
- `amp` [tuple or float] amplitude of each hyperbolic event
- `wav` [numpy.ndarray] wavelet to be applied to data

#### Returns

- `d` [numpy.ndarray] data without wavelet of size  $[n_x \times n_t]$
- `dwav` [numpy.ndarray] data with wavelet of size  $[n_x \times n_t]$

#### Notes

Each event is created using the following relation:

$$t_i(x) = \sqrt{t_{0,i}^2 + x^2/v_{rms,i}^2}$$

### Examples using `pylops.utils.seismicevents.hyperbolic2d`

- `sphx_glr_gallery_plot_mdc.py`
- `sphx_glr_gallery_plot_phaseshift.py`
- `sphx_glr_gallery_plot_seismicevents.py`
- *09. Multi-Dimensional Deconvolution*
- *14. Seismic wavefield decomposition*

### `pylops.utils.seismicevents.linear3d`

`pylops.utils.seismicevents.linear3d(x, y, t, v, t0, theta, phi, amp, wav)`

Linear 3D events

Create 3d linear events given propagation velocity, intercept time, angles, and amplitude of each event.

**Parameters**

**x** [numpy.ndarray] space axis in x direction  
**y** [numpy.ndarray] space axis in y direction  
**t** [numpy.ndarray] time axis  
**v** [float] propagation velocity  
**t0** [tuple or float] intercept time at  $x = 0$  of each linear event  
**theta** [tuple or float] angle in x direction (in degrees) of each linear event  
**phi** [tuple or float] angle in y direction (in degrees) of each linear event  
**amp** [tuple or float] amplitude of each linear event  
**wav** [numpy.ndarray] wavelet to be applied to data

**Returns**

**d** [numpy.ndarray] data without wavelet of size  $[n_y \times n_x \times n_t]$   
**dwav** [numpy.ndarray] data with wavelet of size  $[n_y \times n_x \times n_t]$

**Notes**

Each event is created using the following relation:

$$t_i(x, y) = t_{0,i} + p_{x,i}x + p_{y,i}y$$

where  $p_{x,i} = \sin(\theta_i)\cos(\phi_i)/v$  and  $p_{y,i} = \sin(\theta_i)\sin(\phi_i)/v$ .

**Examples using `pylops.utils.seismicevents.linear3d`**

- sphx\_glr\_gallery\_plot\_seismicevents.py

**pylops.utils.seismicevents.hyperbolic3d**

`pylops.utils.seismicevents.hyperbolic3d(x, y, t, t0, vrms_x, vrms_y, amp, wav)`  
Hyperbolic 3D events

Create 3d hyperbolic events given intercept time, root-mean-square velocities, and amplitude of each event

**Parameters**

**x** [numpy.ndarray] space axis in x direction  
**y** [numpy.ndarray] space axis in y direction  
**t** [numpy.ndarray] time axis  
**t0** [tuple or float] intercept time at  $x = 0$  of each of hyperbolic event  
**vrms\_x** [tuple or float] root-mean-square velocity in x direction for each hyperbolic event  
**vrms\_y** [tuple or float] root-mean-square velocity in y direction for each hyperbolic event  
**amp** [tuple or float] amplitude of each hyperbolic event  
**wav** [numpy.ndarray] wavelet to be applied to data

## Returns

- d** [numpy.ndarray] data without wavelet of size  $[n_y \times n_x \times n_t]$
- dwav** [numpy.ndarray] data with wavelet of size  $[n_y \times n_x \times n_t]$

## Notes

Each event is created using the following relation:

$$t_i(x, y) = \sqrt{t_{0,i}^2 + x^2/v_{rms_x,i}^2 + y^2/v_{rms_y,i}^2}$$

Note that velocities do not have a physical meaning here (compared to the corresponding `pylops.utils.seismicevents.hyperbolic2d`), they rather simply control the curvature of the hyperboloid along the spatial axes.

## Examples using `pylops.utils.seismicevents.hyperbolic3d`

- sphx\_glr\_gallery\_plot\_sliding.py
- sphx\_glr\_gallery\_plot\_phaseshift.py
- sphx\_glr\_gallery\_plot\_seismicevents.py

## Signal-processing

<code>signalprocessing.convmtx(h, n)</code>	Convolution matrix
<code>signalprocessing.</code>	Convolution matrix from a bank of filters
<code>nonstationary_convmtx(H, n)</code>	

## `pylops.utils.signalprocessing.convmtx`

`pylops.utils.signalprocessing.convmtx(h, n)`  
Convolution matrix

Equivalent of MATLAB's `convmtx` function . Makes a dense convolution matrix **C** such that the dot product `np.dot(C, x)` is the convolution of the filter *h* and the input signal *x*.

### Parameters

- h** [np.ndarray] Convolution filter (1D array)
- n** [int] Number of columns (if  $\text{len}(h) < n$ ) or rows (if  $\text{len}(h) \geq n$ ) of convolution matrix

### Returns

- C** [np.ndarray] Convolution matrix of size  $\text{len}(h) + n - 1 \times n$  (if  $\text{len}(h) < n$ ) or  $n \times \text{len}(h) + n - 1$  (if  $\text{len}(h) \geq n$ )

## `pylops.utils.signalprocessing.nonstationary_convmtx`

`pylops.utils.signalprocessing.nonstationary_convmtx(H, n, hc=0, pad=(0, 0))`  
Convolution matrix from a bank of filters

Makes a dense convolution matrix  $\mathbf{C}$  such that the dot product  $\text{np}.\text{dot}(\mathbf{C}, \mathbf{x})$  is the nonstationary convolution of the bank of filters  $H = [h_1, h_2, h_n]$  and the input signal  $x$ .

### Parameters

- H** [`np.ndarray`] Convolution filters (2D array of shape  $[n_{filters} \times n_h]$ )
- n** [`int`] Number of columns of convolution matrix
- hc** [`np.ndarray`, optional] Index of center of first filter
- pad** [`np.ndarray`] Zero-padding to apply to the bank of filters before and after the provided values (use it to avoid wrap-around or pass filters with enough padding)

### Returns

- C** [`np.ndarray`] Convolution matrix

## Tapers

---

<code>tapers.taper2d(nt, nmask, ntap[, tapertype])</code>	2D taper
<code>tapers.taper3d(nt, nmask, ntap[, tapertype])</code>	3D taper

---

### `pylops.utils.tapers.taper2d`

`pylops.utils.tapers.taper2d(nt, nmask, ntap, tapertype='hanning')`  
2D taper

Create 2d mask of size  $[n_{mask} \times n_t]$  with tapering of size  $ntap$  along the first dimension

### Parameters

- nt** [`int`] Number of time samples of mask along second dimension
- nmask** [`int`] Number of space samples of mask along first dimension
- ntap** [`int`] Number of samples of tapering at edges of first dimension
- tapertype** [`str`, optional] Type of taper (hanning, cosine, cosinesquare or None)

### Returns

- taper** [`numpy.ndarray`] 2d mask with tapering along first dimension of size  $[n_{mask} \times n_t]$

### Examples using `pylops.utils.tapers.taper2d`

- sphx\_glr\_gallery\_plot\_phaseshift.py
- sphx\_glr\_gallery\_plot\_tapers.py

### `pylops.utils.tapers.taper3d`

`pylops.utils.tapers.taper3d(nt, nmask, ntap, tapertype='hanning')`  
3D taper

Create 2d mask of size  $[n_{mask}[0] \times n_{mask}[1] \times n_t]$  with tapering of size  $ntap$  along the first and second dimension

### Parameters

**nt** [int] Number of time samples of mask along third dimension  
**nmask** [tuple] Number of space samples of mask along first dimension  
**ntap** [tuple] Number of samples of tapering at edges of first dimension  
**tapertype** [int] Type of taper (hanning, cosine, cosinesquare or None)

### Returns

**taper** [numpy.ndarray] 2d mask with tapering along first dimension of size  $[n_{mask,0} \times n_{mask,1} \times n_t]$

## Examples using `pylops.utils.tapers.taper3d`

- sphx\_glr\_gallery\_plot\_mdc.py
- sphx\_glr\_gallery\_plot\_phaseshift.py
- sphx\_glr\_gallery\_plot\_tapers.py
- [09. Multi-Dimensional Deconvolution](#)

## Wavelets

<code>wavelets.ricker(t[, f0])</code>	Ricker wavelet
<code>wavelets.gaussian(t[, std])</code>	Gaussian wavelet

## pylops.utils.wavelets.ricker

```
pylops.utils.wavelets.ricker(t,f0=10)
Ricker wavelet
```

Create a Ricker wavelet given time axis `t` and central frequency `f_0`

### Parameters

**t** [numpy.ndarray] Time axis (positive part including zero sample)  
**f0** [float, optional] Central frequency

### Returns

**w** [numpy.ndarray] Wavelet  
**t** [numpy.ndarray] Symmetric time axis  
**wcenter** [int] Index of center of wavelet

## Examples using `pylops.utils.wavelets.ricker`

- sphx\_glr\_gallery\_plot\_sliding.py
- sphx\_glr\_gallery\_plot\_avo.py
- sphx\_glr\_gallery\_plot\_convolve.py
- sphx\_glr\_gallery\_plot\_mdc.py
- sphx\_glr\_gallery\_plot\_ista.py

- sphx\_glr\_gallery\_plot\_phaseshift.py
- sphx\_glr\_gallery\_plot\_prestack.py
- sphx\_glr\_gallery\_plot\_seismicevents.py
- sphx\_glr\_gallery\_plot\_wavest.py
- 07. Post-stack inversion
- 08. Pre-stack (AVO) inversion
- 09. Multi-Dimensional Deconvolution
- 11. Radon filtering
- 12. Seismic regularization
- 14. Seismic wavefield decomposition
- 15. Least-squares migration

## pylops.utils.wavelets.gaussian

`pylops.utils.wavelets.gaussian(t, std=1)`  
Gaussian wavelet

Create a Gaussian wavelet given time axis `t` and standard deviation `std` using `scipy.signal.windows.gaussian`.

### Parameters

`t` [`numpy.ndarray`] Time axis (positive part including zero sample)  
`std` [`float`, optional] Standard deviation of gaussian

### Returns

`w` [`numpy.ndarray`] Wavelet  
`t` [`numpy.ndarray`] Symmetric time axis  
`wcenter` [`int`] Index of center of wavelet

## Geophysical Reservoir characterization

---

<code>avo.zoeppritz_scattering(vp1, vs1, rho1, ...)</code>	Zoeppritz solution.
<code>avo.zoeppritz_element(vp1, vs1, rho1, vp0, ...)</code>	Single element of Zoeppritz solution.
<code>avo.zoeppritz_pp(vp1, vs1, rho1, vp0, ...)</code>	PP reflection coefficient from the Zoeppritz scattering matrix.
<code>avo.approx_zoeppritz_pp(vp1, vs1, rho1, vp0, ...)</code>	PP reflection coefficient from the approximate Zoeppritz equation.
<code>avo.akirichards(theta, vsvp[, n])</code>	Three terms Aki-Richards approximation.
<code>avo.fatti(theta, vsvp[, n])</code>	Three terms Fatti approximation.

---

**pylops.avo.avo.zoeppritz\_scattering**

`pylops.avo.avo.zoeppritz_scattering(vp1, vs1, rho1, vp0, vs0, rho0, theta1)`  
Zoeppritz solution.

Calculates the angle dependent p-wave reflectivity of an interface between two media for a set of incident angles.

**Parameters**

**vp1** [`float`] P-wave velocity of the upper medium  
**vs1** [`float`] S-wave velocity of the upper medium  
**rho1** [`float`] Density of the upper medium  
**vp0** [`float`] P-wave velocity of the lower medium  
**vs0** [`float`] S-wave velocity of the lower medium  
**rho0** [`float`] Density of the lower medium  
**theta1** [np.ndarray or `float`] Incident angles in degrees

**Returns**

**zoep** [np.ndarray]  $4 \times 4$  matrix representing the scattering matrix for the incident angle theta1

**See also:**

`zoeppritz_element` Single reflectivity element of Zoeppritz solution

`zoeppritz_PP` PP reflectivity element of Zoeppritz solution

**pylops.avo.avo.zoeppritz\_element**

`pylops.avo.avo.zoeppritz_element(vp1, vs1, rho1, vp0, vs0, rho0, theta1, element='PdPu')`  
Single element of Zoeppritz solution.

Simple wrapper to `pylops.avo.avo.scattering_matrix`, returning any mode reflection coefficient from the Zoeppritz scattering matrix for specific combination of incident and reflected wave and a set of incident angles

**Parameters**

**vp1** [`float`] P-wave velocity of the upper medium  
**vs1** [`float`] S-wave velocity of the upper medium  
**rho1** [`float`] Density of the upper medium  
**vp0** [`float`] P-wave velocity of the lower medium  
**vs0** [`float`] S-wave velocity of the lower medium  
**rho0** [`float`] Density of the lower medium  
**theta1** [np.ndarray or `float`] Incident angles in degrees  
**element** [`str`, optional] specific choice of incident and reflected wave combining any two of the following strings: Pd P-wave downgoing, Sd S-wave downgoing, Pu P-wave upgoing, Su S-wave upgoing (e.g., PdPu)

**Returns**

---

**refl** [np.ndarray] reflectivity values for all input angles for specific combination of incident and reflected wave.

See also:

[zoepritz\\_scattering](#) Zoeppritz solution

[zoepritz\\_PP](#) PP reflectivity element of Zoeppritz solution

## pylops.avo.avo.zoepritz\_pp

pylops.avo.avo.**zoepritz\_pp** (*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*)

PP reflection coefficient from the Zoeppritz scattering matrix.

Simple wrapper to pylops.avo.avo.scattering\_matrix, returning the PP reflection coefficient from the Zoeppritz scattering matrix for a set of incident angles

### Parameters

**vp1** [float] P-wave velocity of the upper medium

**vs1** [float] S-wave velocity of the upper medium

**rho1** [float] Density of the upper medium

**vp0** [float] P-wave velocity of the lower medium

**vs0** [float] S-wave velocity of the lower medium

**rho0** [float] Density of the lower medium

**theta1** [np.ndarray or float] Incident angles in degrees

### Returns

**PPrefl** [np.ndarray] PP reflectivity values for all input angles.

See also:

[zoepritz\\_scattering](#) Zoeppritz solution

[zoepritz\\_element](#) Single reflectivity element of Zoeppritz solution

## pylops.avo.avo.approx\_zoepritz\_pp

pylops.avo.avo.**approx\_zoepritz\_pp** (*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*)

PP reflection coefficient from the approximate Zoeppritz equation.

Approximate calculation of PP reflection from the Zoeppritz scattering matrix for a set of incident angles [1].

### Parameters

**vp1** [np.ndarray or list or tuple] P-wave velocity of the upper medium

**vs1** [np.ndarray or list or tuple] S-wave velocity of the upper medium

**rho1** [np.ndarray or list or tuple] Density of the upper medium

**vp0** [np.ndarray or list or tuple] P-wave velocity of the lower medium

**vs0** [np.ndarray or list or tuple] S-wave velocity of the lower medium

**rho0** [np.ndarray or list or tuple] Density of the lower medium

**theta1** [np.ndarray or float] Incident angles in degrees

**Returns**

**PPref1** [np.ndarray] PP reflectivity values for all input angles.

**See also:**

[zoepritz\\_scattering](#) Zoeppritz solution

[zoepritz\\_element](#) Single reflectivity element of Zoeppritz solution

[zoepritz\\_PP](#) PP reflectivity element of Zoeppritz solution

## pylops.avo.avo.akirichards

`pylops.avo.avo.akirichards(theta, vsvp, n=1)`

Three terms Aki-Richards approximation.

Computes the coefficients of the of three terms Aki-Richards approximation for a set of angles and a constant or variable VS/VP ratio.

**Parameters**

**theta** [np.ndarray] Incident angles in degrees

**vs vp** [np.ndarray or float] VS/VP ratio

**n** [int, optional] number of samples (if vs vp is a scalare)

**Returns**

**G1** [np.ndarray] first coefficient of three terms Aki-Richards approximation [ $n_{theta} \times n_{vs vp}$ ]

**G2** [np.ndarray] second coefficient of three terms Aki-Richards approximation [ $n_{theta} \times n_{vs vp}$ ]

**G3** [np.ndarray] third coefficient of three terms Aki-Richards approximation [ $n_{theta} \times n_{vs vp}$ ]

## Notes

The three terms Aki-Richards approximation is used to compute the reflection coefficient as linear combination of contrasts in  $V_P$ ,  $V_S$ , and  $\rho$ . More specifically:

$$R(\theta) = G_1(\theta) \frac{\Delta V_P}{\bar{V}_P} + G_2(\theta) \frac{\Delta V_S}{\bar{V}_S} + G_3(\theta) \frac{\Delta \rho}{\bar{\rho}}$$

where  $G_1(\theta) = \frac{1}{2\cos^2\theta}$ ,  $G_2(\theta) = -4(V_S/V_P)^2 \sin^2\theta$ ,  $G_3(\theta) = 0.5 - 2(V_S/V_P)^2 \sin^2\theta$ ,  $\frac{\Delta V_P}{V_P} = 2 \frac{V_{P,2} - V_{P,1}}{V_{P,2} + V_{P,1}}$ ,  $\frac{\Delta V_S}{V_S} = 2 \frac{V_{S,2} - V_{S,1}}{V_{S,2} + V_{S,1}}$ , and  $\frac{\Delta \rho}{\rho} = 2 \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ .

## pylops.avo.avo.fatti

`pylops.avo.avo.fatti(theta, vsvp, n=1)`

Three terms Fatti approximation.

Computes the coefficients of the of three terms Fatti approximation for a set of angles and a constant or variable VS/VP ratio.

**Parameters**

**theta** [np.ndarray] Incident angles in degrees  
**vsvp** [np.ndarray or float] VS/VP ratio  
**n** [int, optional] number of samples (if vsvp is a scalare)

**Returns**

**G1** [np.ndarray] first coefficient of three terms Smith-Gidlow approximation [ $n_{theta} \times n_{vsvp}$ ]  
**G2** [np.ndarray] second coefficient of three terms Smith-Gidlow approximation [ $n_{theta} \times n_{vsvp}$ ]  
**G3** [np.ndarray] third coefficient of three terms Smith-Gidlow approximation [ $n_{theta} \times n_{vsvp}$ ]

**Notes**

The three terms Fatti approximation is used to compute the reflection coefficient as linear combination of contrasts in  $AI$ ,  $SI$ , and  $\rho$ . More specifically:

$$R(\theta) = G_1(\theta) \frac{\Delta AI}{AI} + G_2(\theta) \frac{\Delta SI}{SI} + G_3(\theta) \frac{\Delta \rho}{\bar{\rho}}$$

where  $G_1(\theta) = 0.5(1 + \tan^2 \theta)$ ,  $G_2(\theta) = -4(V_S/V_P)^2 \sin^2 \theta$ ,  $G_3(\theta) = 0.5(4(V_S/V_P)^2 \sin^2 \theta - \tan^2 \theta)$ ,  $\frac{\Delta AI}{AI} = 2 \frac{AI_2 - AI_1}{AI_2 + AI_1}$ ,  $\frac{\Delta SI}{SI} = 2 \frac{SI_2 - SI_1}{SI_2 + SI_1}$ ,  $\frac{\Delta \rho}{\bar{\rho}} = 2 \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ .

## 3.8 Implementing new operators

Users are welcome to create new operators and add them to the PyLops library.

In this tutorial, we will go through the key steps in the definition of an operator, using the `pylops.Diagonal` as an example. This is a very simple operator that applies a diagonal matrix to the model in forward mode and to the data in adjoint mode.

### 3.8.1 Creating the operator

The first thing we need to do is to create a new file with the name of the operator we would like to implement. Note that as the operator will be a class, we need to follow the UpperCaseCamelCase convention both for the class itself and for the filename.

Once we have created the file, we will start by importing the modules that will be needed by the operator. While this varies from operator to operator, you will always need to import the `pylops.LinearOperator` class, which will be used as *parent* class for any of our operators:

```
from pylops import LinearOperator
```

This class is a child of the `scipy.sparse.linalg.LinearOperator` class itself which implements the same methods of its parent class as well as an additional method for quick inversion: such method can be easily accessed by using `\` between the operator and the data (e.g., `A\y`).

After that we define our new object:

```
class Diagonal(LinearOperator):
```

followed by a `numpydoc` docstring (starting with `r"""` and ending with `"""`) containing the documentation of the operator. Such docstring should contain at least a short description of the operator, a `Parameters` section with a detailed description of the input parameters and a `Notes` section providing a mathematical explanation of the operator. Take a look at some of the core operators of PyLops to get a feeling of the level of details of the mathematical explanation.

We then need to create the `__init__` where the input parameters are passed and saved as members of our class. While the input parameters change from operator to operator, it is always required to create three members, the first called `shape` with a tuple containing the dimensions of the operator in the data and model space, the second called `dtype` with the data type object (`np.dtype`) of the model and data, and the third called `explicit` with a boolean (`True` or `False`) identifying if the operator can be inverted by a direct solver or requires an iterative solver. This member is `True` if the operator has also a member `A` that contains the matrix to be inverted like for example in the `pylops.MatrixMult` operator, and it will be `False` otherwise. In this case we have another member called `d` which is equal to the input vector containing the diagonal elements of the matrix we want to multiply to the model and data.

```
def __init__(self, d, dtype=None):
    self.d = d.flatten()
    self.shape = (len(self.d), len(self.d))
    self.dtype = np.dtype(dtype)
    self.explicit = False
```

We can then move onto writing the *forward mode* in the method `_matvec`. In other words, we will need to write the piece of code that will implement the following operation  $\mathbf{y} = \mathbf{A}\mathbf{x}$ . Such method is always composed of two inputs (the object itself `self` and the input model `x`). In our case the code to be added to the forward is very simple, we will just need to apply element-wise multiplication between the model `x` and the elements along the diagonal contained in the array `d`. We will finally need to `return` the result of this operation:

```
def _matvec(self, x):
    return self.d*x
```

Finally we need to implement the *adjoint mode* in the method `_rmatvec`. In other words, we will need to write the piece of code that will implement the following operation  $\mathbf{x} = \mathbf{A}^H\mathbf{y}$ . Such method is also composed of two inputs (the object itself `self` and the input data `y`). In our case the code to be added to the forward is the same as the one from the forward (but this will obviously be different from operator to operator):

```
def _rmatvec(self, x):
    return self.d*x
```

And that's it, we have implemented our first linear operator!

### 3.8.2 Testing the operator

Being able to write an operator is not yet a guarantee of the fact that the operator is correct, or in other words that the adjoint code is actually the *adjoint* of the forward code. Luckily for us, a simple test can be performed to check the validity of forward and adjoint operators, the so called *dot-test*.

We can generate random vectors `u` and `v` and verify the the following *equality* within a numerical tolerance:

$$(\mathbf{A} * \mathbf{u})^H * \mathbf{v} = \mathbf{u}^H * (\mathbf{A}^H * \mathbf{v})$$

The method `pylops.utils.dottest` implements such a test for you, all you need to do is create a new test within an existing `test_*.py` file in the `pytests` folder (or in a new file).

Generally a test file will start with a number of dictionaries containing different parameters we would like to use in the testing of one or more operators. The test itself starts with a *decorator* that contains a list of all (or some) of dictionaries that will be used for our specific operator, followed by the definition of the test

```
@pytest.mark.parametrize("par", [(par1), (par2)])
def test_Diagonal(par):
```

At this point we can first of all create the operator and run the `pylops.utils.dottest` preceded by the `assert` command. Moreover, the forward and adjoint methods should be tested towards expected outputs or even better, when the operator allows it (i.e., operator is invertible), a small inversion should be run and the inverted model tested towards the input model.

```
"""Dot-test and inversion for diagonal operator
"""
d = np.arange(par['nx']) + 1.

Dop = Diagonal(d)
assert dottest(Dop, par['nx'], par['nx'],
               complexflag=0 if par['imag'] == 1 else 3)

x = np.ones(par['nx'])
xlsqr = lsqr(Dop, Dop * x, damp=1e-20, iter_lim=300, show=0)[0]
assert_array_almost_equal(x, xlsqr, decimal=4)
```

### 3.8.3 Documenting the operator

Once the operator has been created, we can add it to the documentation of PyLops. To do so, simply add the name of the operator within the `index.rst` file in `docs/source/api` directory.

Moreover, in order to facilitate the user of your operator by other users, a simple example should be provided as part of the Sphinx-gallery of the documentation of the PyLops library. The directory `examples` contains several scripts that can be used as template.

### 3.8.4 Final checklist

Before submitting your new operator for review, use the following **checklist** to ensure that your code adheres to the guidelines of PyLops:

- you have created a new file containing a single class (or a function when the new operator is a simple combination of existing operators - see `pylops.Laplacian` for an example of such operator) and added to a new or existing directory within the `pylops` package.
- the new class contains at least `__init__`, `_matvec` and `_rmatvec` methods.
- the new class (or function) has a `numpydoc docstring` documenting at least the `input Parameters` and with a `Notes` section providing a mathematical explanation of the operator
- a new test has been added to an existing `test_*.py` file within the `pytests` folder. The test should verify that the new operator passes the `pylops.utils.dottest`. Moreover it is advisable to create a small toy example where the operator is applied in forward mode and the resulting data is inverted using \ from `pylops.LinearOperator`.
- the new operator is used within at least one `example` (in `examples` directory) or one `tutorial` (in `tutorials` directory).

## 3.9 Contributing

Contributions are welcome and greatly appreciated!

The best way to get in touch with the core developers and maintainers is to join the PyLops slack channel as well as open new *Issues* directly from the [github repo](#).

Moreover, take a look at the [Roadmap](#) page for a list of current ideas for improvements and additions to the PyLops library.

### 3.9.1 Types of Contributions

#### Report Bugs

Report bugs at <https://github.com/equinor/pylops/issues>.

If you are playing with the PyLops library and find a bug, please report it including:

- Your operating system name and version.
- Any details about your Python environment.
- Detailed steps to reproduce the bug.

#### Propose New Operators or Features

Open an issue at <https://github.com/equinor/pylops/issues> with tag *enhancement*.

If you are proposing a new operator or a new feature:

- Explain in detail how it should work.
- Keep the scope as narrow as possible, to make it easier to implement.

#### Implement Operators or Features

Look through the Git issues for operator or feature requests. Anything tagged with *enhancement* is open to whoever wants to implement it.

#### Add Examples or improve Documentation

Writing new operators is not the only way to get involved and contribute. Create examples with existing operators as well as improving the documentation of existing operators is as important as making new operators and very much encouraged.

### 3.9.2 Getting Started to contribute

Ready to contribute?

1. Fork the *PyLops* repo.
2. Clone your fork locally:

```
>> git clone https://github.com/your_name_here/pylops.git
```

3. Follow the installation instructions for *developers* that you find in [Installation](#) page. Ensure that you are able to *pass all the tests before moving forward*.
4. Add the main repository to the list of your remotes (this will be important to ensure you pull the latest changes before trying to merge your local changes):

```
>> git remote add upstream https://github.com/equinor/pylops
```

5. Create a branch for local development:

```
>> git checkout -b name-of-your-branch
```

Now you can make your changes locally.

6. When you're done making changes, check that your code follows the guidelines for [Implementing new operators](#) and that the both old and new tests pass successfully:

```
>> make tests
```

7. Commit your changes and push your branch to GitLab:

```
>> git add .
>> git commit -m "Your detailed description of your changes."
>> git push origin name-of-your-branch
```

Remember to add `-u` when pushing the branch for the first time.

8. Submit a pull request through the GitHub website.

### 3.9.3 Pull Request Guidelines

Before you submit a pull request, check that it meets these guidelines:

1. The pull request should include new tests for all the core routines that have been developed.
2. If the pull request adds functionality, the docs should be updated accordingly.
3. Ensure that the updated code passes all tests.

## 3.10 Changelog

### 3.10.1 Version 1.8.0

*Released on: 12/01/2020*

- Added `pylops.LinearOperator.todense` method to `pylops.LinearOperator`
- Added `pylops.signalprocessing.Bilinear`, `pylops.signalprocessing.DWT`, and `pylops.signalprocessing.DWT2` operators
- Added `pylops.waveeqprocessing.PressureToVelocity`, `pylops.waveeqprocessing.UpDownComposition3Doperator`, and `pylops.waveeqprocessing.PhaseShift` operators
- Fix bug in `pylops.basicoperators.Kronecker` (see Issue #125)

### 3.10.2 Version 1.7.0

*Released on: 10/11/2019*

- Added `pylops.Gradient`, `pylops.Sum`, `pylops.FirstDirectionalDerivative`, and `pylops.SecondDirectionalDerivative` operators
- Added `pylops.LinearOperator._ColumnLinearOperator` private operator
- Added possibility to directly mix Linear operators and numpy/scipy 2d arrays in `pylops.VStack` and `pylops.HStack` and `pylops.BlockDiag` operators
- Added `pylops.optimization.sparsity.OMP` solver

### 3.10.3 Version 1.6.0

*Released on: 10/08/2019*

- Added `pylops.signalprocessing.ConvolveND` operator
- Added `pylops.utils.signalprocessing.nonstationary_convmtx` to create matrix for non-stationary convolution
- Added possibility to perform seismic modelling (and inversion) with non-stationary wavelet in `pylops.avo.poststack.PoststackLinearModelling`
- Create private methods for `pylops.Block`, `pylops.avo.poststack.PoststackLinearModelling`, `pylops.waveeqprocessing.MDC` to allow calling different operators (e.g., from pylops-distributed or pylops-gpu) within the method

### 3.10.4 Version 1.5.0

*Released on: 30/06/2019*

- Added `conj` method to `pylops.LinearOperator`
- Added `pylops.Kronecker`, `pylops.Roll`, and `pylops.Transpose` operators
- Added `pylops.signalprocessing.Fredholm1` operator
- Added `pylops.optimization.sparsity.SPGL1` and `pylops.optimization.sparsity.SplitBregman` solvers
- Sped up `pylops.signalprocessing.Convolve1D` using `scipy.signal.fftconvolve` for multi-dimensional signals
- Changes in implementation of `pylops.waveeqprocessing.MDC` and `pylops.waveeqprocessing.Marchenko` to take advantage of primitives operators
- Added `epsRL1` option to `pylops.avo.poststack.PoststackInversion` and `pylops.avo.prestack.PrestackInversion` to include TV-regularization terms by means of `pylops.optimization.sparsity.SplitBregman` solver

### 3.10.5 Version 1.4.0

*Released on: 01/05/2019*

- Added numba engine to `pylops.Spread` and `pylops.signalprocessing.Radon2D` operators
- Added `pylops.signalprocessing.Radon3D` operator

- Added `pylops.signalprocessing.Sliding2D` and `pylops.signalprocessing.Sliding3D` operators
- Added `pylops.signalprocessing.FFTND` operator
- Added `pylops.signalprocessing.Radon3D` operator
- Added `niter` option to `pylops.LinearOperator.eigs` method
- Added `show` option to `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` solvers
- Added `pylops.waveeqprocessing.seismicinterpolation`, `pylops.waveeqprocessing.waveeqdecomposition` and `pylops.waveeqprocessing.lsm` submodules
- Added tests for `engine` in various operators
- Added documentation regarding usage of PyLops Docker container

### 3.10.6 Version 1.3.0

*Released on: 24/02/2019*

- Added `fftw` engine to `pylops.signalprocessing.FFT` operator
- Added `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` sparse solvers
- Added possibility to broadcast (handle multi-dimensional arrays) to `pylops.Diagonal` and `pylops..Restriction` operators
- Added `pylops.signalprocessing.Interp` operator
- Added `pylops.Spread` operator
- Added `pylops.signalprocessing.Radon2D` operator

### 3.10.7 Version 1.2.0

*Released on: 13/01/2019*

- Added `pylops.LinearOperator.eigs` and `pylops.LinearOperator.cond` methods to estimate eigenvalues and conditioning number using `scipy` wrapping of `ARPACK`
- Modified default `dtype` for all operators to be `float64` (or `complex128`) to be consistent with default `dtypes` used by `numpy` (and `scipy`) for real and complex floating point numbers.
- Added `pylops.Flip` operator
- Added `pylops.Symmetrize` operator
- Added `pylops.Block` operator
- Added `pylops.Regression` operator performing polynomial regression and modified `pylops.LinearRegression` to be a simple wrapper of `pylops.Regression` when `order=1`
- Modified `pylops.MatrixMult` operator to work with both `numpy` ndarrays and `scipy` sparse matrices
- Added `pylops.avo.prestack.PrestackInversion` routine
- Added possibility to have a data weight via `Weight` input parameter to `pylops.optimization.leastsquares.NormalEquationsInversion` and `pylops.optimization.leastsquares.RegularizedInversion` solvers

- Added `pylops.optimization.sparsity.IRLS` solver

### 3.10.8 Version 1.1.0

*Released on: 13/12/2018*

- Added `pylops.CausalIntegration` operator

### 3.10.9 Version 1.0.1

*Released on: 09/12/2018*

- Changed module from `lops` to `pylops` for consistency with library name (and pip install).
- Removed quickplots from utilities and `matplotlib` from requirements of *PyLops*.

### 3.10.10 Version 1.0.0

*Released on: 04/12/2018*

- First official release.

## 3.11 Roadmap

This roadmap is aimed at providing an high-level overview on the bug fixes, improvements and new functionality that are planned for the PyLops library.

Any of the fixes/additions mentioned in the roadmap are directly linked to a *Github Issue* that provides more details onto the reason and initial thoughts for the implementation of such a fix/addition.

Striked tasks have been completed and related github issue closed with more details regarding how this task has been carried out.

### 3.11.1 Library structure

- Create a child repository and python library called `geolops` (just a suggestion) where geoscience-related operators and examples are moved across, keeping the core `pylops` library very generic and multi-purpose - [Issue #22](#).

### 3.11.2 Code cleaning

- Change all `np.flatten()` into `np.ravel()` - [Issue #24](#).
- Fix all `if: return ... else: ...` statements to enforce a single return with the same number of outputs - [Issue #26](#).
- Protected attributes and `@property` attributes in linear operator classes? - [Issue #27](#).

### 3.11.3 Code optimization

- Investigate speed-up given by decorating `_matvec` and `_rmatvec` methods with `numba @jit` and `@stencil` decorators - [Issue #23](#).
- Replace `np.fft.*` routines used in several submodules with double engine, numpy and `pyFFTW` - [Issue #20](#).

### 3.11.4 Modules

#### avo

- Add possibility to choose different damping factors for each elastic parameter to invert for in `pylops.avo.prestack.PrestackInversion` - [Issue #25](#).

#### basicoperators

- Create Kronecker operator - [Issue #28](#).
- Deal with edges in FirstDerivative and SecondDerivative operators - [Issue #34](#).

#### optimization

- Sparse solvers - [Issue #44](#).

#### signalprocessing

- Compare performance in FTT operator of performing `np.swap+np.fft.fft(..., axis=-1)` versus `np.fft.fft(..., axis=chosen)` - [Issue #33](#).
- Add Wavelet operator performing the wavelet transform. `pywavelets` can be used as back-end - [Issue #21](#).
- Fredholm1 operator applying Fredholm integrals of first kind - [Issue #31](#).
- Fredholm2 operators applying Fredholm integrals of second kind - [Issue #31](#).

#### utils

Nothing so far

#### waveeqprocessing

- `numpy.matmul` as a way to speed up integral computation (i.e., inner for loop) in ‘‘MDC‘ operator’ - [Issue #32](#).
- NMO operator performing NMO modelling - [Issue #29](#).
- WavefieldDecomposition operator performing acoustic wavefield separation by inversion - [Issue #30](#).

## 3.12 Papers

This section lists various conference abstracts and papers using `pylops`:

- Ruan, J., and Vasconcelos I., *Data-and prior-driven sampling and wavefield reconstruction for sparse, irregularly-sampled, higher-order gradient data*, SEG Technical Program Expanded Abstracts, [link](#) (2019).

## 3.13 Citing

When using `pylops` in scientific publications, please cite the following paper:

- Ravasi, M., and Vasconcelos I., *PyLops—A Linear-Operator Python Library for large scale optimization*, Software X, (2019). [link](#).

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