Torsional Inertia Moment of Beam Element with Complex Section Analysis Based on FEM

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Abstract

Currently, for the analysis of complex bridge based on beam element, the calculation of section torsional inertia moment is still an unresolved technical problem. Mostly current calculation of section torsional inertia moment is an approximate analytic method for two-dimensional section, which is not fully consistent with the actual situation, not considering the effects of diaphragm in bridge. In order to analyze accurately cable-stayed bridge, suspension bridge and other complex bridge structures based on beam element, the calculation method of section torsional inertia moment based on finite element method (FEM) is invented in this paper. Firstly, setting up a local cantilever fine model with solid element or shell element and applied torsion on the end of the cantilever. Secondly, calculating the torsion angle of the cantilever by FEM method and then calculating the torsional moment through equivalent beam method. Finally, the examples of the section torsional moment calculation of concrete model based on solid element with diaphragm and steel girder box model based on shell element with diaphragm are used to verify the calculation method, which is applied in the suspension bridge design and construction control special software (Chinese name abbr: SBNA) developed by Research Institute of Highway Ministry of Transport, which has been applied in China Taizhou Yangtze River Bridge under construction.

Keyword: concrete beam; steal box girder; torsion inertia moment; FEM

1 Introduction

Combined with the scientific research of China Taizhou Yangtze River Bridge of multi tower continuous span suspension bridge, the suspension bridge design and construction control special software (Chinese name abbr: SBNA) was developed by RIOH (Research Institute of Highway, China). In the complex structure analysis of multi tower suspension bridge, for the engineers, it is hard to design and construction controlling analyze the whole bridge meticulous model based on solid or shell element. So the whole bridge model based on beam element is suitable. While, the calculation of section torsional inertia moment based on beam element is still an unresolved technical problem. The methods are not unified, and the calculation error is large, affecting the calculation results.

Most of the current theories of section torsional inertia moment calculation are as following:

(1) Saint-Venant formula. This method based on single-box single-chamber pure torsion theory is derived by membrane analogy, and is only applied in single-box structure.

(2) Bredt formula. This method based on single-box single-chamber pure torsion theory is derived from adjacent room shear flow equilibrium conditions, and is only applied in less wide single-box and multi-cell structure.

(3) E. G. Hambly formula. In this method, the box girder is analogized to the orthogonal homosexual board composed by upper and lower panels, not considering the impact of the web, calculating the unit length of the torsional constant according to the differential equations about physical and geometry relations. But this method is only applied in wide and low box girder which has the mechanical characteristics of plate.

From above, the current calculation methods of section torsional inertia moment are certainly limited, not considering the impact of diaphragm in beam segment of bridge. The analytical method can approximately calculate section torsional inertia moment of box girder with diaphragm and stiffener, but not match the actual situation and has many uncertainties. Therefore, the suspension bridge modeling assistant in SBNA sets a calculation method, through setting up local cantilever fine model and applying unit torque on the end of the cantilever, calculating the torsion angle, and then getting the torsional moment through equivalent beam method, in which the result is accurate and practical by practical certification.

2 Theoretical study on calculation of section torsional inertia moment

Firstly, according to the finite element method (FEM), it can easily calculate the nodal displacement of the three-dimensional cantilever model with diaphragm based on solid element or shell element. Secondly, applying unit torsion on the end of cantilever, the unit length torsion angle of cantilever can be got from the nodal displacement of the middle section in cantilever. Finally, the section torsional inertia moment is calculated through the torsion Hook Law. This calculation is called equivalent beam method.

The finite element analysis basic equations in three-dimensional elastic body V included:

1) Balance equation: $\boldsymbol{A}\cdot\boldsymbol{\sigma}+\overline{\boldsymbol{f}}=0$, where $\boldsymbol{A}$ is differential operator; $\overline{f}$ is body force vector; $\boldsymbol{\sigma}$ is nodal stress vector.

2) Physic equation: $\boldsymbol{\sigma}=\boldsymbol{D}\cdot\boldsymbol{\varepsilon}$, where $\boldsymbol{D}$ is elastic matrix decided by elastic module $\boldsymbol{E}$ of element materials and Passion ratio; $\boldsymbol{\varepsilon}$ is nodal strain vector.

3) Geometric equation: $\boldsymbol{\varepsilon}=L\cdot\boldsymbol{\mu}$, where $\boldsymbol{L}$ is differential operator, $\boldsymbol{\mu}$ is displacement vector.

According to those equations, the nodal displacement can be easily calculated. The unit length torsion angle can be got from Eq.(1)

$$\theta'=\arctan\frac{y_1-y_2}{a}/l \tag{1}$$

In Eq.(1), y_1 and y_2 are the displacements in Y direction of torsion angle calculation nodes; a is the distance in Z direction of torsion angle calculation nodes; l is the distance between torsion angle calculation side and constrained side.

And then the section torsional inertia moment is calculated through the torsion Hook Law, as in Eq.(2); where G is the shear modules; M_T is the applied torque.

$$J_d=\frac{M_T}{G\theta'} \tag{2}$$

3 Calculation of section torsional inertia moment with concrete box girder

In engineering practice, the finite element model of concrete box girder is simulated with solid element. Meanwhile, in order to consider the impact of diaphragm, the elastic module of diaphragm is 1000 times of that in beam girder. Therefore, the torsional deformation can be got through FEM calculation theory by setting up the three-dimensional cantilever model, and applying unit torsion on the end to reverse the model. And then the section torsional inertia moment of box girder can be calculated.

3.1 Sensitivity analysis of model length in torsional inertia moment calculation

In the progress of section torsional inertia moment calculation, we find the calculation result is related with the model length. In order to analyze this problem, this paper takes an example of concrete single-box five-chamber section, which is shown in Figure.1. Meanwhile, setting up a three-dimensional solid finite element cantilever model, the elastic module in model and diaphragm are 2.55×10^{7} N/m^{2} and 2.55×10^{10} N/m^{2}, and the Poisson ratio is 0.2, applying 1000N torsion on the end of the model.

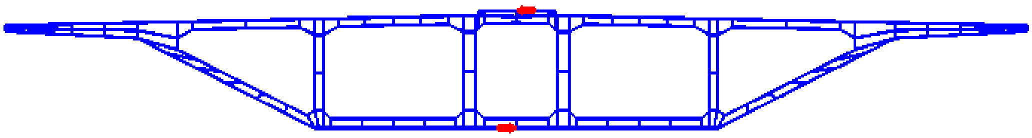


Figure 1. Concrete cross-section of torsional inertia moment calculation

This paper expands the length of the box girder model with 8.0m length, just as in Figure.2, in which the model contains diaphragm with 0.2m width to calculate the section torsional inertia moment.

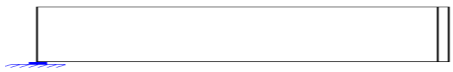


Figure 2. Elevation drawing of the three-dimensional cantilever model

To analyze accurately the sensitivity, this paper takes use of the results of ANSYS and SBNA for verification, and the results just as followed:

3.1.1 The section torsional inertia moment calculation with SBNA

The results of SBNA are just as Table 1, in which the unit of model length is “m”, and the unit of section torsional inertia moment is “m^{4}”. According to Saint Vinant principle, this paper takes the middle-section nodal displacement of the model to calculate the unit length torsion angle.

Table 1. Concrete cross-section torsional inertia moment of SBNA

L（m）	24.6	41	57.4	123	205
T（m^{4}）	129.3	113.9	107.2	98.9	96.3

3.1.2 The section torsional inertia moment calculation with ANSYS

The results of ANSYS are just as Table 2, with the same calculation process as SBNA.

Table 2. Concrete cross-section torsional inertia moment of ANSYS

L（m）	24.6	41	57.4	123	205
T（m^{4}）	122.1	111.7	108.3	106.4	104.8

From the data in the above tables, it is obtained that the section torsional inertia moment is smaller with the length of the calculation model increasing, which is stabled in a certain length of the cantilever model. As for the reason, it is that, according to the Saint Vinant principle, the middle-section nodal displacement of cantilever solid model is affected larger by the torsion in the end, when the length of model is much shorter. Therefore, when the length of the calculation model is longer, the affection of the constraint and the concentrated force can be ignored, and the result tends to be stable, which is as Table 2.

3.2 Precision analysis of the calculation of section torsional inertia moment

The Precision of section torsional inertia moment is also related with the unit division of the calculation model. To get the reasonable precision of the unit division, this paper takes the single-box five-chamber section as the research object, setting up a 24.6m cantilever model with diaphragms. In detail, the control parameters of mesh generation are as following: the most dimension of unite transverse is 1m, and the unit length ratio is 4:1. With the unit division in one time, two times, and quadruple grid in the calculation model, the section torsional inertia moments are followed:

With the one-time grid division, the node number and the cell number are 2768 and 1440, which are shown in Figure.3, and the result is 105.88 m^4 calculated by ANSYS.

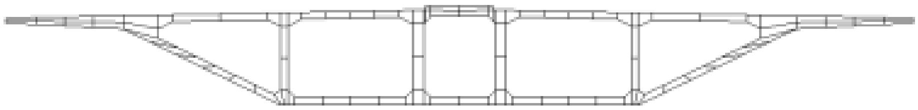


Figure 3. Cross-section drawing of one-time grid division

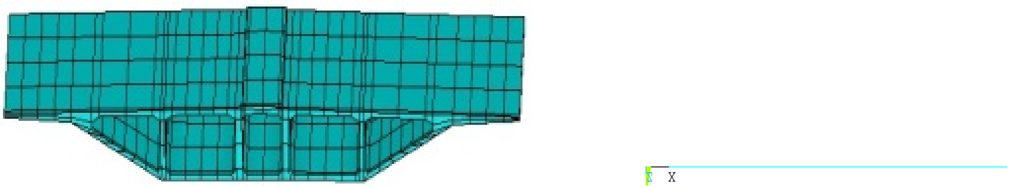
With the two-time grid division, the node number and the cell number are 13850 and 9408, and the result is 101.8 m^4 calculated by ANSYS.

With the quadruple grid division, the node number and the cell number are 87216 and 70560, and the result is 101.6 m^4 calculated by ANSYS.

From the data in above, with the unit division of the calculation model finer, the section torsional inertia moment tends to be stable. In view of this, it is suggested that the unit division of calculation model should be enough fine.

3.3 Verification and Comparison of calculation model results based on solid element and beam element

For further verifying the accuracy of this calculation method, the section torsional inertia moment calculated with FEM is applied in the beam element model, in the same working and loading conditions, comparing the displacement result of these two different types' models. Taking an example of the same section in above, the basic length of the calculation cantilever model is 8.2m; and the solid element model is just as Figure.4 (a), while the beam element model is just as Figure.4 (b).



(a) Solid element model (b) Beam element model

Figure 4 Calculation cantilever model

There are results of three different length calculation models, which are shown in Table 3, in which θ represents the unit length torsion angle of the solid element model, and θ' represents the unit length torsion angle of the beam element model.

Table 3 Results of solid and beam model

L（m）	8.2	24.6	41
θ（rad/m）	0.130e-5	0.343e-5	0.609e-5
θ'（rad/m）	0.135e-5	0.337e-5	0.608e-5
ERROR	3.7%	1.75%	0.16%

From the data in Table 3, it is summarized that the displacement result of beam element model applying the section torsional inertia moment of solid element model based on FEM is consistent with that of solid element model. And the error is smaller with the length increasing. Therefore, it is feasible to calculate the section torsional inertia moment of solid element model with this method.

4 Calculation of section torsional inertia moment with steel box girder

In engineering practice, the finite element model of steel box girder is simulated with shell element. And then the three-dimensional cantilever model can be built based on the same conditions as that concrete box girder.

4.1 Sensitivity analysis of model length in torsional inertia moment calculation

Because of the same reason, this paper also analyzes the sensitivity of the steel box girder model length of section torsional inertia moment calculation. Taking an example of steel single-box five-chamber section, which is shown in Figure.5, the elastic module in model and diaphragm are $2.06\times10^{8}N/m^{2}$ and 2.06×10^{11} N/m^{2}, and the Poisson ratio is 0.3. The calculation process is like as concrete box girder.

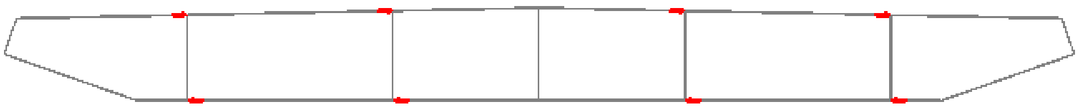


Figure 5. Steel cross-section of torsional inertia moment calculation

The results of SNBA and ANSYS are followed in Table 4, in which T represents the result calculated by SBNA, while T' represents the result calculated by ANSYS.

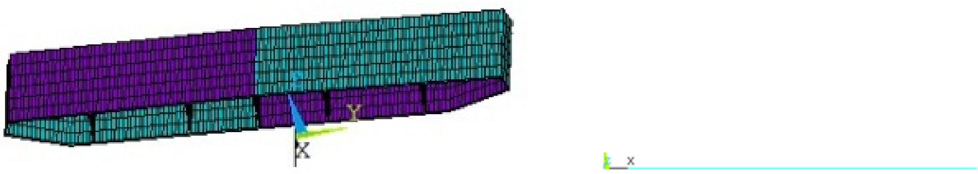
Table 4. Steel cross-section torsional inertia moment of SBNA and ANSYS

L（m）	8.2	24.6	57.4	123	205	287	328
T（m4）	3.91	3.93	5.56	13.2	16.96	17.51	17.86
T'（m4）	3.87	3.62	4.29	10.68	15.85	16.22	16.17

For the data in Table 4, it can be summarized that when the length of the calculation model is longer, the affection of the constraint and the concentrated force can be ignored, and the result tends to be stable.

4.2 Verification and Comparison of calculation model results based on shell element and beam element

For further verifying the calculation method, in the same calculation process as concrete box girder, the shell element model and beam element model are built, which are just as Figure.6.



(a) Shell element model (b) Beam element model

Figure 6. Calculation cantilever model

There are results of three different length calculation models, which are just as Table 5, in which θ represents the unit length torsion angle of the shell element model, and θ' represents the unit length torsion angle of the beam element model.

Table 5. Results of shell and beam model

L（m）	8.2	24.6	41
θ（rad/m）	0.425e-5	0.139e-4	0.229e-4
θ'（rad/m）	0.415e-5	0.134e-4	0.225e-4
ERROR	2.4%	3.6%	1.7%

From the data in Table 5, the same conclusion as that of concrete box girder can be obtained. Therefore, it is also feasible to calculate the section torsional inertia moment of shell element model with this method.

5 Conclusion

In this paper, when calculating the section torsional inertia moment with FEM, it is necessary to consider the length of calculation model. In detail, when the model length is shorter enough, the nodal displacement is affected larger by the constraint and the concentrated force, so the result is larger than true value. According to Saint Vinant principle, when the node is enough far from the constrained side and concentration side of calculation model, the affection can be ignored. In view of this, it is very necessary to increase the length of the calculation model to the appropriate value.

6 Acknowledgement

This work was supported by scientific research item 50908211 from National Natural Science Foundation of China and scientific research item 200831822343 from Ministry of Transport.

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