March Mu Alpha Theta Problems

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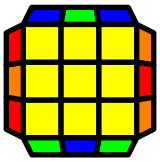


In this monthly problem set, we'll be exploring the mathematics of Rubik's cubes! I highly recommend watching this video and this video for some background information about the cube. Since there is a lot of information about the cube online, you are welcome to do more research to solve these puzzles, but please don't directly search up the problems themselves. Remember, I'm always available to provide hints!

1 Permuting the Last Layer

One method of solving the Rubik's cube is the CFOP method. At the last step of this method, all of the yellow pieces of the cube have their yellow stickers on the top face, but they are not in the correct positions. This final step uses one algorithm to permute the pieces so that they end up in the right spots.

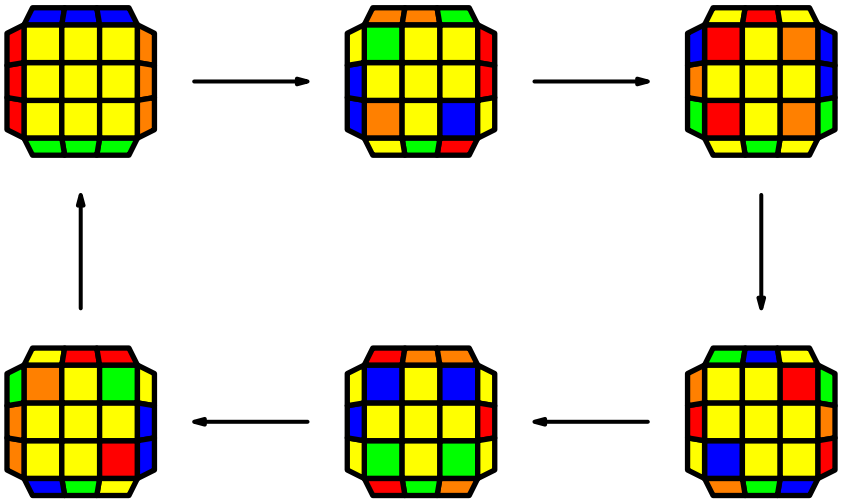
(a) Given that every possible permutation of the pieces is equally likely at this step, what is the probability that all corners are already solved (relative to each other)? (3 points)

(b) What is the probability that all the edges are already solved (relative to each other) at this step? (3 points)

(c) What is the probability of a PLL skip, where the cubies are already in the correct order by chance? (3 points)

2 Algorithm Orders

You may notice that applying an algorithm (any sequence of moves, possibly involving slices or cube rotations) a certain number of times will bring the cube back to its original state. The number of times the algorithm is applied is known as the order of the algorithm. Below is the Sune algorithm (R U R' U R U2 R'), which has an order of 6.



(a) Find an algorithm with an order of 4 and an algorithm with an order of 2. (1 point)

(b) Find an algorithm with an order of 3. (3 points)

(c) Find an algorithm that has an order of 7. (5 points)

(d) Find an algorithm with an order of 5. (5 points)

(e) Find an algorithm with an order of 11. (5 points)

3 Double Turns

In this section we explore a variant of the Rubik's cube where only double turns (U2, F2, R2, etc.) are allowed. Define a double-turn algorithm to be an algorithm that only uses these moves.

(a) Find any double-turn algorithm that swaps two pairs of edges while not moving any other pieces. (2 points)

(b) Find any double-turn algorithm that cycles three edges while not moving any other pieces. (5 points)

(c) Some turns are done on a Rubik's cube (not necessarily double turns) which brings it to a state where only opposite colors are on the same face. In other words, only yellow stickers are on the white-centered face, and so on. Prove or disprove that it is always possible to solve this cube only using double turns. (5 points)

(d) Find a method (which only uses double turns) to solve any Rubik's cube scrambled using double turns. (9 points)