

February Mu Alpha Theta Problems

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February 2022

1 A Couple Combinatorics Conundrums

- (a) A cube's vertices are colored white, and its center is colored black. A point is uniformly picked from the three-dimensional region inside the cube. What is the probability that the point picked is closer to the black point than any white point? (3 points)
- (b) Pichu and 3 friends are celebrating Christmas together, and they have lots of gifts for each other! The four of them have each brought two gifts which they placed into a heap under the tree, which so happened to be wrapped with identical packaging and unlabeled. Due to this, no one can tell the gifts apart. They decided to resolve this by just picking two random gifts from the pile, one at a time. If they distribute their 8 gifts completely randomly among the 4 of them, what is the probability they get really lucky, and no one receives their own gift? (5 points)
- (c) Pichu eats chocolate in a very interesting manner. He starts with a 4 by 4 grid of chocolate squares. Then, he randomly picks a chocolate square, and eats all the chocolate squares in the same row or column as that square. After this, he will be left with one or more chocolate bars. He picks another square randomly among all of the bars, and eats all the squares in the same row or column in the same bar. He repeats this until all of the squares are gone. What is the expected value of the number of iterations it will take for the chocolate bar to be gone? (5 points)

2 Polynomial Peril II

- (a) Algebraic numbers are numbers which are the roots of polynomials with integer coefficients and finite length. Prove or disprove that the set of algebraic numbers is countable. (3 points)