Bandit Machine 2 Play Posterior Probability Calculation

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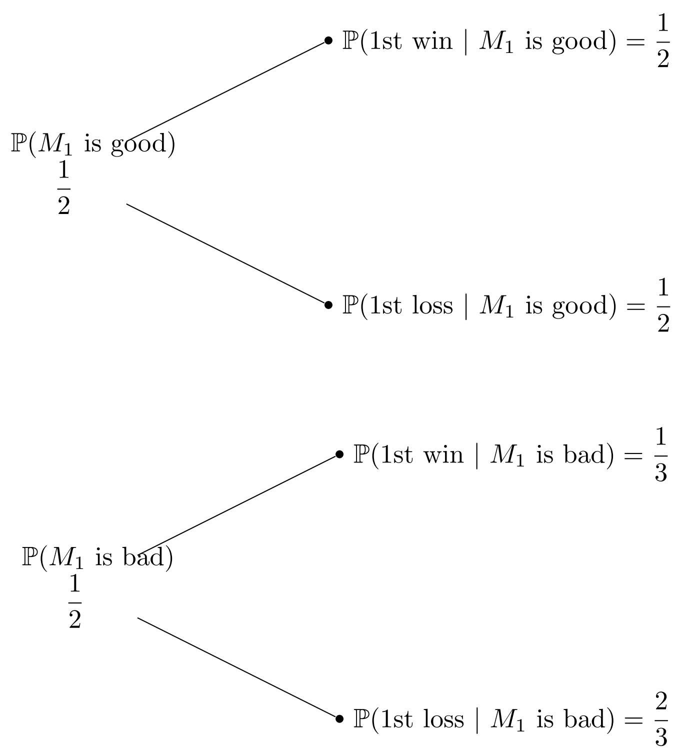
Play Machine 1 Twice and Get 1 Win then 1 Loss

In this example, we have `data = data.frame(machine = c(1L, 1L), outcome = c("W", "L"))`. We can calculate the posterior probabilities **step-by-step** as in the video mentioned.

Step 1: Machine 1 gets the 1st win

We build the probability tree as follows:



From the tree, we can calculate the following **posterior probabilities**:

$$\mathbb{P}(M_1 \text{ is good} \mid \text{1st win}) = \frac{\mathbb{P}(\text{1st win} \mid M_1 \text{ is good})\mathbb{P}(M_1 \text{ is good})}{\mathbb{P}(\text{1st win} \mid M_1 \text{ is good})\mathbb{P}(M_1 \text{ is good}) + \mathbb{P}(\text{1st win} \mid M_1 \text{ is bad})\mathbb{P}(M_1 \text{ is bad})}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2}}$$

$$= \frac{3}{5} = 0.6$$

Since M_1 is bad after we have observed the 1st win is the complement of M_1 is good after we have observed the 1st win, we can simply get

$$\mathbb{P}(M_1 \text{ is bad} \mid \text{1st win}) = 1 - \mathbb{P}(M_1 \text{ is good} \mid \text{1st win}) = \frac{2}{5} = 0.4$$

The above resuults match what we can get from running the following codes in RStudio Console:

```
data = data.frame(machine = 1L, outcome = "W")
bandit_posterior(data)
```

The results are shown in the `.Rmd` file:

```
## m1_good m2_good
## 0.6 0.4
```

Step 2: Machine 1 gets the 2nd loss

Now we need to use the posterior probabilities from the previous step as our new prior, and follow the probability tree:

$\mathbb{P}(M_1 \text{ is good} \mid \text{1st win})$ $\dfrac{3}{5} = 0.6$

- $\mathbb{P}(\text{2nd win} \mid M_1 \text{ is good, 1st win}) = \dfrac{1}{2}$
- $\mathbb{P}(\text{2nd loss} \mid M_1 \text{ is good, 1st win}) = \dfrac{1}{2}$

$\mathbb{P}(M_1 \text{ is bad} \mid \text{1st win})$ $\dfrac{2}{5} = 0.4$

- $\mathbb{P}(\text{2nd win} \mid M_1 \text{ is bad, 1st win}) = \dfrac{1}{3}$
- $\mathbb{P}(\text{2nd loss} \mid M_1 \text{ is bad, 1st win}) = \dfrac{2}{3}$

As you can see, we can just ignore the branches indicating the 2nd play is win and just focus on those that gives win for the 2nd play. To update the new posterior probability, we use Bayes' Rule again:

$$
\begin{aligned}
&\mathbb{P}(M_1 \text{ is good} \mid \text{1st win, 2nd loss}) \\
&= \frac{\mathbb{P}(\text{2nd loss} \mid M_1 \text{ is good, 1st win})\mathbb{P}(M_1 \text{ is good} \mid \text{1st win})}{\mathbb{P}(\text{2nd loss} \mid M_1 \text{ is good, 1st win})\mathbb{P}(M_1 \text{ is good} \mid \text{1st win}) + \mathbb{P}(\text{2nd loss} \mid M_1 \text{ is bad, 1st win})\mathbb{P}(M_1 \text{ is bad} \mid \text{1st win})} \\
&= \frac{\frac{1}{2} \times \frac{3}{5}}{\frac{1}{2} \times \frac{3}{5} + \frac{2}{3} \times \frac{2}{5}} \\
&= \frac{9}{17} \approx 0.5294118
\end{aligned}
$$

And of course, the posterior probability that Machine 1 is bad after 2 plays with 1st win then 2nd play a loss is the complement of the above posterior probability, which is

$$\mathbb{P}(M_1 \text{ is bad} \mid \text{1st win, 2nd loss}) = 1 - \mathbb{P}(M_1 \text{ is good} \mid \text{1st win, 2nd loss}) = 1 - 0.5294118 = 0.4705882$$