# Tree Implementations

Chapter 16

## Nodes in a Binary Tree

- Representing tree nodes
  - Must contain both data and "pointers" to node's children
  - Each node will be an object
- Array-based
  - Pointers will be array indices
- Link-based
  - Use C++ pointers

Class of array-based data members

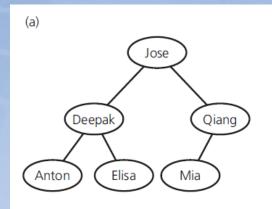
- Variable root is index to tree's root node within the array tree
- If tree is empty, root = -1

- As tree changes (additions, removals) ...
  - Nodes may not be in contiguous array elements
- Thus, need list of available nodes
  - Called a free list
- Node removed from tree
  - Placed in free list for later use

```
template<class ItemType>
class TreeNode
private:
  ItemType item;  // Data portion
           leftChild; // Index to left child
  int
            rightChild; // Index to right child
  int
public:
  TreeNode():
  TreeNode(const ItemType& nodeItem, int left, int right);
// Declarations of the methods setItem, getItem, setLeft, getLeft,
// setRight, and getRight are here.
}: // end TreeNode
```

LISTING 16-1 The class TreeNode for an array-based implementation of the ADT binary tree

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Qiang 5 Anton -1-1 Elisa -1 -1 Mia -1 7 -1 -1

The array tree leftChild

3

rightChild

root

0

free

6

Free list

FIGURE 16-1 (a) A binary tree of names; (b) its implementation using the array tree

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item

Jose

Deepak

#### Link-Based Representation

```
/** A class of nodes for a link-based binary tree.
     @file BinaryNode.h */
    #ifndef BINARY_NODE_
    #define BINARY NODE
    #include <memory>
 8
    template<class ItemType>
    class BinaryNode
10
    private:
11
                                                    // Data portion
12
       ItemType
                                        item:
       std::shared_ptr<BinaryNode<ItemType>> leftChildPtr; // Pointer to left child
13
       std::shared_ptr<BinaryNode<ItemType>> rightChildPtr; // Pointer to right child
14
15
    public:
16
       BinaryNode();
17
18
       BinaryNode(const ItemType& anItem);
       BinaryNode(const ItemType& anItem,
19
                 std::shared_ptr<BinaryNode<ItemType>> leftPtr,
20
                 std::shared ptr<BinaryNode<ItemType>> rightPtr);
```

LISTING 16-2 The header file containing the class BinaryNode for a link-based implementation of the ADT binary tree

## Link-Based Representation

```
void setItem(const ItemType& anItem);
23
24
      ItemType getItem() const;
25
      bool isLeaf() const;
26
27
       auto getLeftChildPtr() const;
28
       auto getRightChildPtr() const;
29
30
31
      void setLeftChildPtr(std::shared_ptr<BinaryNode<ItemType>> leftPtr);
      void setRightChildPtr(std::shared_ptr<BinaryNode<ItemType>> rightPtr);
32
   }; // end BinaryNode
33
34
   #include "BinaryNode.cpp"
35
   #endif
36
```

LISTING 16-2 The header file containing the class BinaryNode for a link-based implementation of the ADT binary tree

#### Link-Based Representation

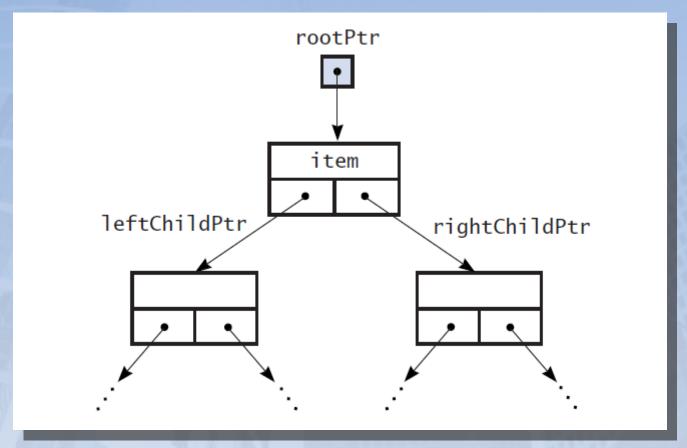


FIGURE 16-2 A link-based implementation of a binary tree

```
ADT binary tree: Link-based implementation.
     @file BinaryNodeTree.h */
    #ifndef BINARY NODE TREE
    #define BINARY NODE TREE
    #include "BinaryTreeInterface.h"
    #include "BinaryNode.h"
    #include "PrecondViolatedExcept.h"
    #include "NotFoundException.h"
10
    #include <memory>
11
12
    template<class ItemType>
13
    class BinaryNodeTree : public BinaryTreeInterface<ItemType>
14
15
    private:
16
       std::shared ptr<BinaryNode<ItemType>> rootPtr;
17
```

```
protected:
20
          Protected Utility Methods Section:
21
          Recursive helper methods for the public methods.
22
23
      int getHeightHelper(std::shared ptr<BinaryNode<ItemType>> subTreePtr) const;
24
      int getNumberOfNodesHelper(std::shared ptr<BinaryNode<ItemType>> subTreePtr) const;
25
26
      // Recursively adds a new node to the tree in a left/right fashion to keep tree balanced.
27
      auto balancedAdd(std::shared_ptr<BinaryNode<ItemType>> subTreePtr,
28
                     std::shared ptr<BinarvNode<ItemTvpe>> newNodePtr):
29
```

```
// Removes the target value from the tree.
31
      virtual auto removeValue(std::shared ptr<BinaryNode<ItemType>> subTreePtr,
32
                                             const ItemType target, bool& isSuccessful);
33
34
      // Copies values up the tree to overwrite value in current node until
35
      // a leaf is reached; the leaf is then removed, since its value is stored in the parent.
36
      auto moveValuesUpTree(std::shared ptr<BinaryNode<ItemType>> subTreePtr);
37
38
      // Recursively searches for target value.
39
      virtual auto findNode(std::shared ptr<BinaryNode<ItemType>> treePtr.
40
                                          const ItemType& target, bool& isSuccessful) const;
41
42
      // Copies the tree rooted at treePtr and returns a pointer to the root of the copy.
43
      auto copyTree(const std::shared ptr<BinaryNode<ItemType>> oldTreeRootPtr) const;
44
      // Recursively deletes all nodes from the tree.
46
      void destroyTree(std::shared_ptr<BinaryNode<ItemType>> subTreePtr);
```

```
// Recursive traversal helper methods:
       void preorder(void visit(ItemType&), std::shared ptr<BinaryNode<ItemType>> treePtr) const;
50
       void inorder(void visit(ItemType&), std::shared ptr<BinaryNode<ItemType>> treePtr) const;
51
       void postorder(void visit(ItemType&), std::shared ptr<BinaryNode<ItemType>> treePtr) const;
52
53
    public:
54
            Constructor and Destructor Section.
56
57
       BinaryNodeTree();
58
       BinaryNodeTree(const ItemType& rootItem);
59
       BinaryNodeTree(const ItemType& rootItem,
60
                     const std::shared ptr<BinaryNodeTree<ItemType>> leftTreePtr.
61
                     const std::shared ptr<BinaryNodeTree<ItemType>> rightTreePtr);
62
       BinaryNodeTree(const std::shared ptr<BinaryNodeTree<ItemType>>& tree);
63
       virtual ~BinaryNodeTree();
64
```

```
Public BinaryTreeInterface Methods Section.
67
      bool isEmpty() const;
69
      int getHeight() const;
70
      int getNumberOfNodes() const;
71
      ItemType getRootData() const throw(PrecondViolatedExcept);
72
      void setRootData(const ItemType& newData);
73
      bool add(const ItemType& newData); // Adds an item to the tree
74
      bool remove(const ItemType& data); // Removes specified item from the tree
75
      void clear();
76
      ItemType getEntry(const ItemType& anEntry) const throw(NotFoundException);
77
      bool contains(const ItemType& anEntry) const;
78
```

```
80
           Public Traversals Section.
81
82
      void preorderTraverse(void visit(ItemType&)) const;
83
      void inorderTraverse(void visit(ItemType&)) const;
84
      void postorderTraverse(void visit(ItemType&)) const;
85
86
87
      Overloaded Operator Section.
88
89
      BinaryNodeTree& operator = (const BinaryNodeTree& rightHandSide);
90
   }; // end BinaryNodeTree
91
92
   #include "BinaryNodeTree.cpp"
93
   #endif
94
```

#### Constructors

#### Constructors

```
template < class ItemType >
std::shared ptr<BinarvNode<ItemType>> BinarvNodeTree<ItemType>::copyTree(
     const std::shared_ptr<BinaryNode<ItemType>> oldTreeRootPtr) const
   std::shared ptr<BinaryNode<ItemType>> newTreePtr;
   // Copy tree nodes during a preorder traversal
   if (oldTreeRootPtr != nullptr)
      // Copy node
      newTreePtr = std::make_shared<BinaryNode<ItemType>>(oldTreeRootPtr->getItem(),
                                                        nullptr, nullptr);
      newTreePtr->setLeftChildPtr(copyTree(oldTreeRootPtr->getLeftChildPtr()));
      newTreePtr->setRightChildPtr(copyTree(oldTreeRootPtr->getRightChildPtr()));
     // end if
   // Else tree is empty (newTreePtr is nullptr)
   return newTreePtr:
  // end copyTree
```

Protected method copyTree called by copy constructor

#### Copy constructor

```
template < class ItemType >
void BinaryNodeTree < ItemType > ::
    destroyTree(std::shared_ptr < BinaryNode < ItemType >> subTreePtr)
{
    if (subTreePtr != nullptr)
    {
        destroyTree(subTreePtr -> getLeftChildPtr());
        destroyTree(subTreePtr -> getRightChildPtr());
        subTreePtr.reset(); // Decrement reference count to node
    } // end if
} // end destroyTree
```

destroyTree used by destructor which simply calls this method

Protected method getHeightHelper

```
template < class ItemType >
bool BinaryNodeTree < ItemType > :: add(const ItemType & newData)
{
    auto newNodePtr = std::make_shared < BinaryNode < ItemType >> (newData);
    rootPtr = balancedAdd(rootPtr, newNodePtr);

    return true;
} // end add
```

#### Method add

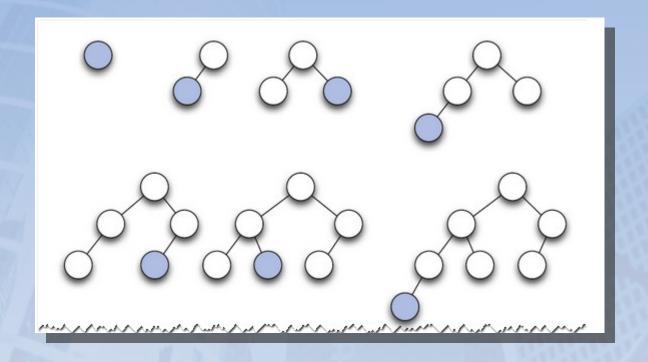


FIGURE 16-3 Adding nodes to an initially empty binary tree

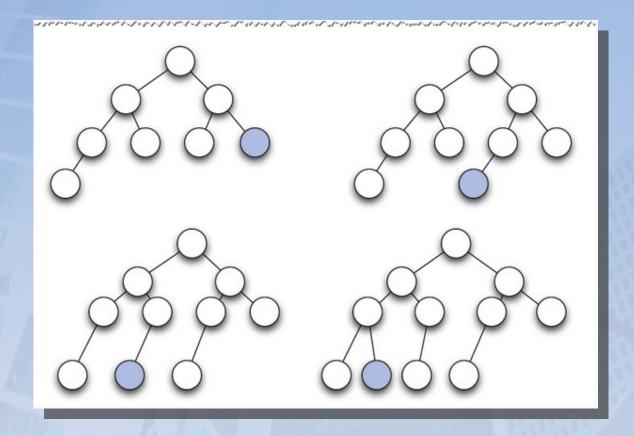


FIGURE 16-3 Adding nodes to an initially empty binary tree

Protected method that enables recursive traversals.

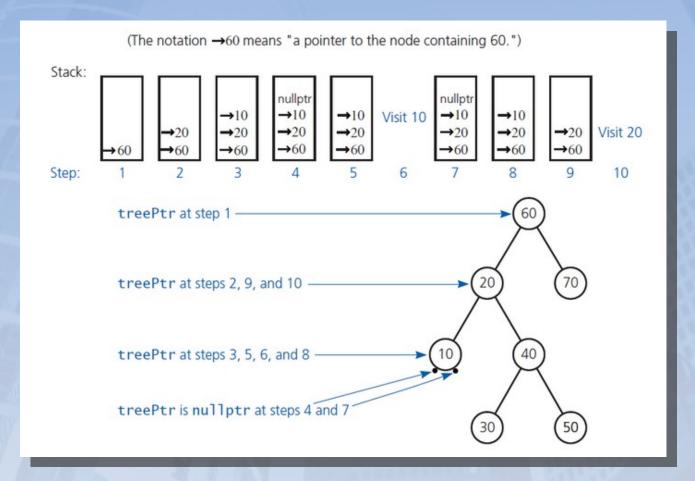


FIGURE 16-4 Contents of the implicit stack as treePtr progresses through a given tree during a recursive inorder traversal

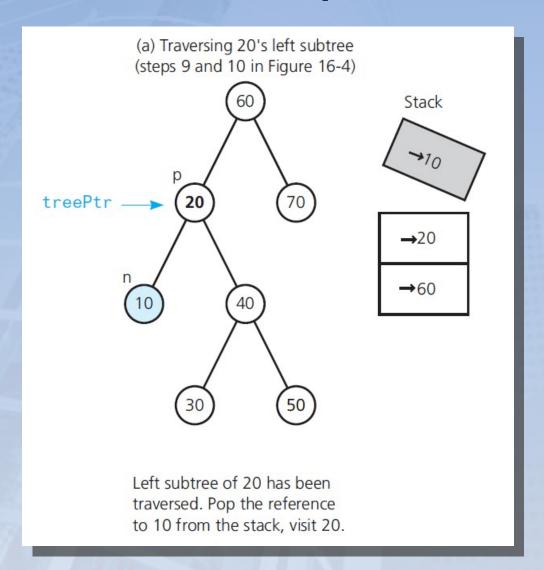


Figure 16-5
Steps during an inorder traversal of the subtrees of 20

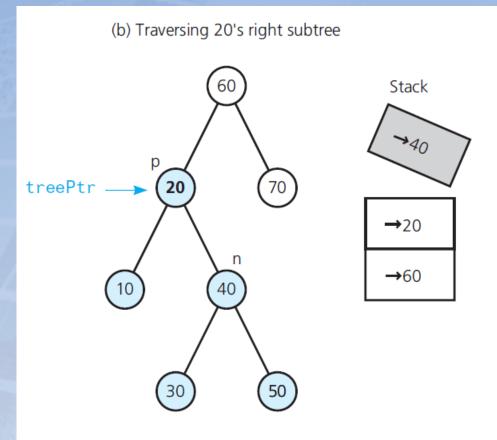


Figure 16-5
Steps during an inorder traversal of the subtrees of 20

Right subtree of 20 has been traversed. Pop the reference to 40 from stack.

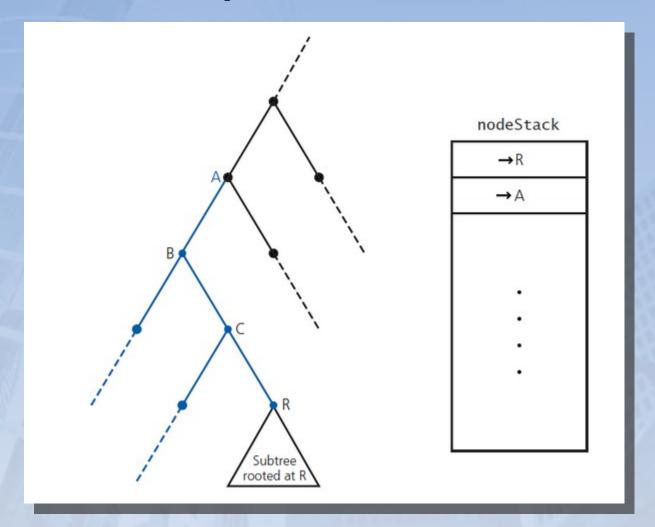


FIGURE 16-6 Avoiding returns to nodes B and C

```
11 Nonrecursively traverses a binary tree in inorder.
traverse(visit(item: ItemType): void): void
   11 Initialize
   nodeStack = A new, empty stack
   curPtr = rootPtr // Start at root
   done = false
   while (!done)
      if (curPtr != nullptr)
          11 Place pointer to node on stack before traversing the node's left subtree
          nodeStack.push(curPtr)
          11 Traverse the left subtree
          curPtr = curPtr->getLeftChildPtr()
        lse [1] Backtrack from the empty subtree and visit the node at the top o
```

#### Nonrecursive inorder traversal

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#### Nonrecursive inorder traversal

- Uses same node objects as for binary-tree implementation.
- Class BinaryNode from Listing16-2 will be used
- Recursive search algorithm from Section15.3.2 is basis for operations

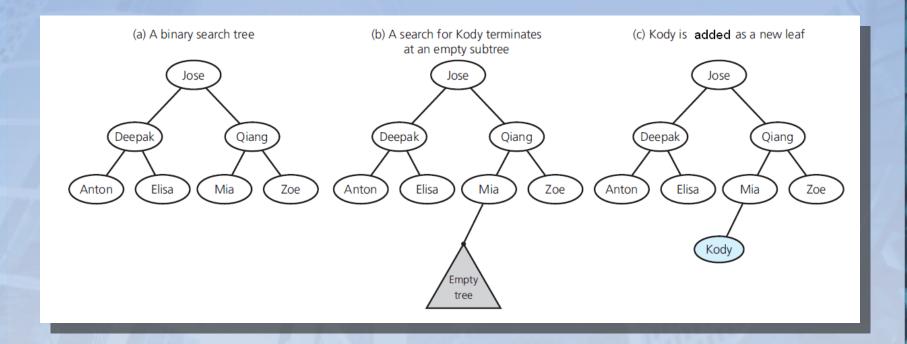


FIGURE 16-7 Adding Kody to a binary search tree

```
template < class ItemType >
bool BinarySearchTree < ItemType > ::add(const ItemType & newData)
{
    auto newNodePtr = std::make_shared < BinaryNode < ItemType >> (newData);
    rootPtr = placeNode(rootPtr, newNodePtr);

    return true;
} // end add
```

#### Method add

```
11 Recursively places a given new node at its proper position in a binary search tree.
placeNode(subTreePtr: BinaryNodePointer,
          newNodePtr: BinaryNodePointer): BinaryNodePointer
   if (subTreePtr is nullptr)
      return newNodePtr
   else if (subTreePtr->getItem() > newNodePtr->getItem())
      tempPtr = placeNode(subTreePtr->getLeftChildPtr(), newNodePtr)
      subTreePtr->setLeftChildPtr(tempPtr)
   else
     tempPtr = placeNode(subTreePtr->getRightChildPtr(), newNodePtr)
     subTreePtr->setRightChildPtr(tempPtr)
  return subTreePtr
```

#### Refinement of addition algorithm

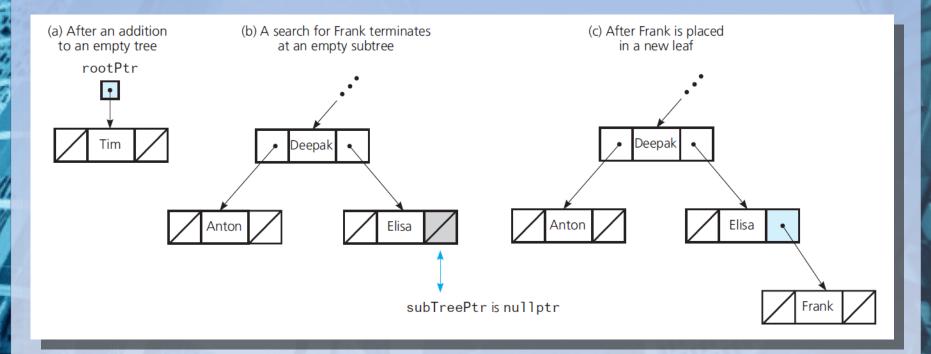


Figure 16-8 Adding new data to a binary search tree

```
// Removes the given target from a binary search tree.
// Returns true if the removal is successful or false otherwise.
removeValue(target: ItemType): boolean
{
    Locate the target by using the search algorithm
    if (target is found)
    {
        Remove target from the tree
        return true
    }
    else
        return false
}
```

Tree
 Cases for node N containing item to be removed

#### 1.N is a leaf

- Remove leaf containing target
- Set pointer in parent to nullptr

• Cases for node N containing item to be removed

- 2.N has only left (or right) child cases are symmetrical
  - After N removed, all data items rooted at L (or R) are adopted by root of N
  - All items adopted are in correct order, binary search tree property preserved

Tree
 Cases for node N containing item to be removed

#### 3.N has two children

- Locate another node M easier to remove from tree than N
- Copy item that is in M to N
- Remove M from tree

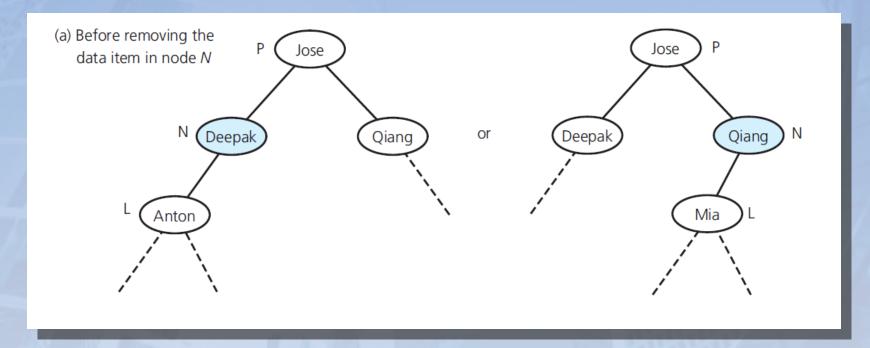


Figure 16-9 Case 2 for removeValue: The data item to remove is in a node N that has only a left child and whose parent is node P

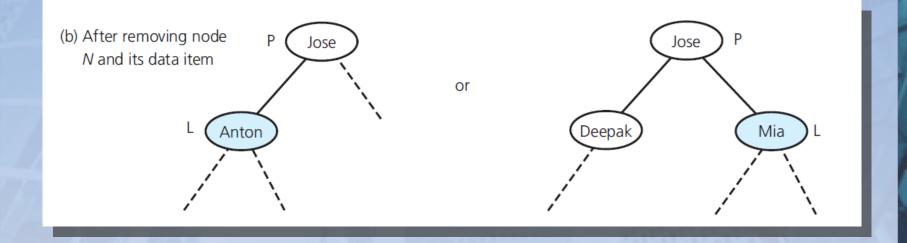


Figure 16-9 Case 2 for removeValue: The data item to remove is in a node N that has only a left child and whose parent is node P

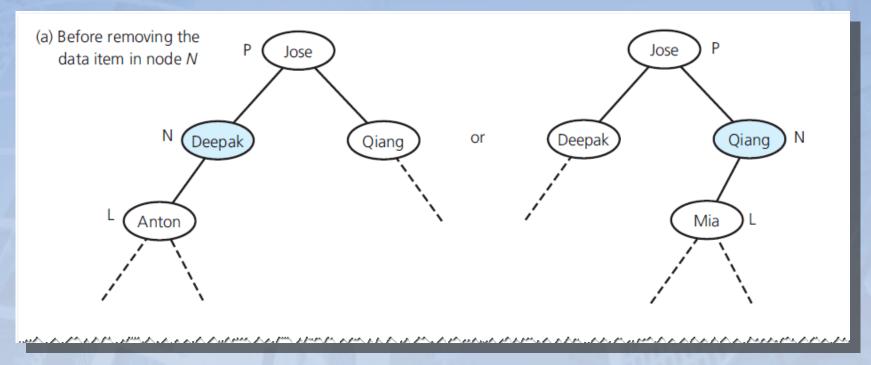


FIGURE 16-9 Case 2 for removeValue: The data item to remove is in a node N that has only a left child and whose parent is node P

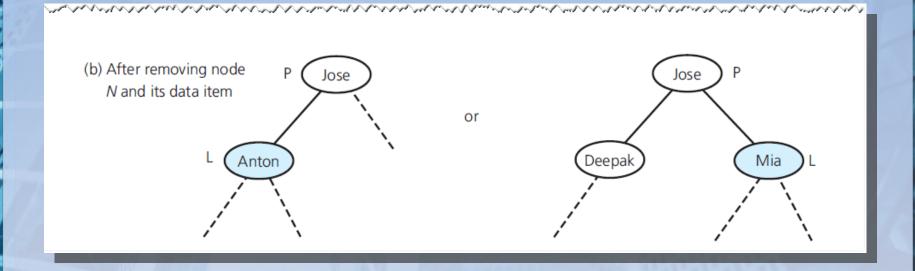


FIGURE 16-9 Case 2 for removeValue: The data item to remove is in a node N that has only a left child and whose parent is node P

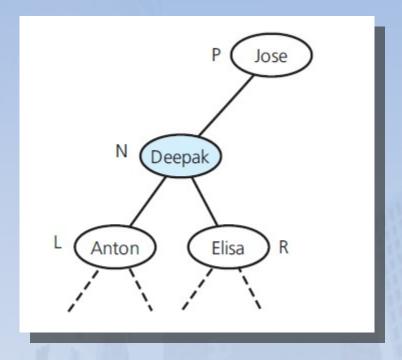
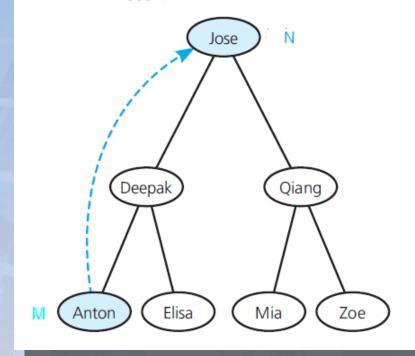


FIGURE 16-10 Case 3: The data item to remove is in a node N that has two children

(a) Removing the data item in node *N* by replacing it with data from an arbitrary node *M* 



(b) The result is no longer a binary search tree

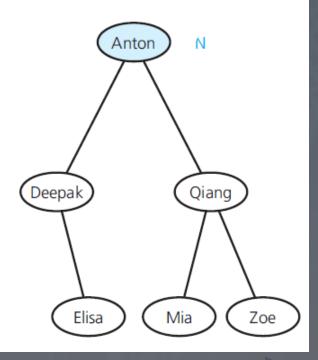
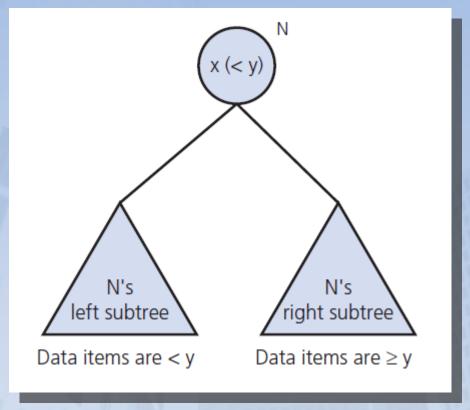


FIGURE16-11 Not any node will do



**FIGURE 16-12** 

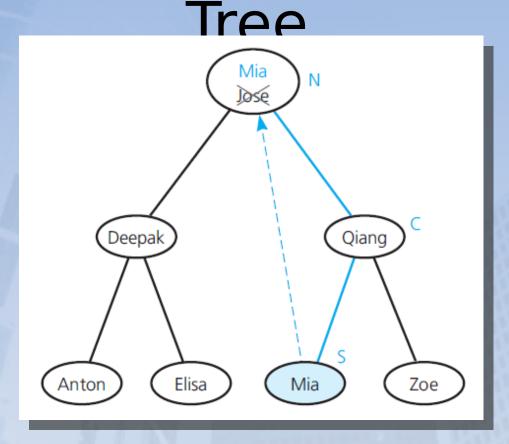


FIGURE 16-13 Replacing the data item in node N with its inorder successor

```
// Removes the given target from the binary search tree to which subTreePtr points.
// Returns a pointer to the node at this tree location after the value is removed.
// Sets isSuccessful to true if the removal is successful, or false otherwise.
removeValue(subTreePtr: BinaryNodePointer, target: ItemType,
             isSuccessful: boolean&): BinaryNodePointer
   if (subTreePtr == nullptr)
      isSuccessful = false
   else if (subTreePtr->getItem() == target)
      11 Item is in the root of some subtree
      subTreePtr = removeNode(subTreePtr) // Remove the item
      isSuccessful = true
   else if (subTreePtr->getItem() > target)
      11 Search the left subtree
      tempPtr = removeValue(subTreePtr->getLeftChildPtr(), target, isSuccessful)
      subTreePtr->setLeftChildPtr(tempPtr)
```

```
else
      11 Search the right subtree
      tempPtr = removeValue(subTreePtr->getRightChildPtr(), target, isSuccessful)
      subTreePtr->setRightChildPtr(tempPtr)
   return subTreePtr
11 Removes the data item in the node, N, to which nodePtr points.
11 Returns a pointer to the node at this tree location after the removal.
removeNode(nodePtr: BinaryNodePointer): BinaryNodePointer
   if (N is a leaf)
      11 Remove leaf from the tree
      Delete the node to which nodePtr points (done for us if nodePtr is a smart pointer)
      return nodePtr
   else if (N \text{ has only one child } C)
```

```
else if (N has only one child C)
{
    // C replaces N as the child of N's parent
    if (C is a left child)
        nodeToConnectPtr = nodePtr->getLeftChildPtr()
    else
        nodeToConnectPtr = nodePtr->getRightChildPtr()

    Delete the node to which nodePtr points (done for us if nodePtr is a smart pointer)
    return nodeToConnectPtr
}
```

```
else // N has two children
{
    // Find the inorder successor of the entry in N: it is in the left subtree rooted
    // at N's right child
    tempPtr = removeLeftmostNode(nodePtr->getRightChildPtr(), newNodeValue)
    nodePtr->setRightChildPtr(tempPtr)
    nodePtr->setItem(newNodeValue) // Put replacement value in node N
    return nodePtr
}
}

// Removes the leftmost node in the left subtree of the node pointed to by nodePtr.
// Sets inorderSuccessor to the value in this node.
// Returns a pointer to the revised subtree.
removeLeftmostNode(nodePtr: BinaryNodePointer,
    inorderSuccessor: ItemType&): BinaryNodePointer
{
```

```
inorderSuccessor: ItemType&): BinaryNodePointer

if (nodePtr->getLeftChildPtr() == nullptr)

{
    // This is the node you want; it has no left child, but it might have a right subtree
    inorderSuccessor = nodePtr->getItem()
    return removeNode(nodePtr)
}
else
{
    tempPtr = removeLeftmostNode(nodePtr->getLeftChildPtr(), inorderSuccessor)
    nodePtr->setLeftChildPtr(tempPtr)
    return nodePtr
}
```

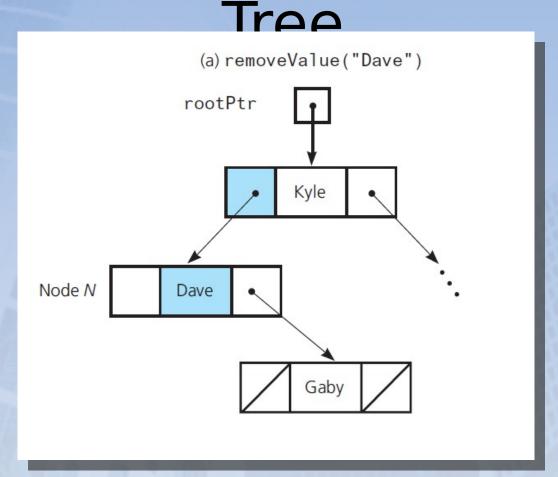


FIGURE 16-14 Recursive removal of node N

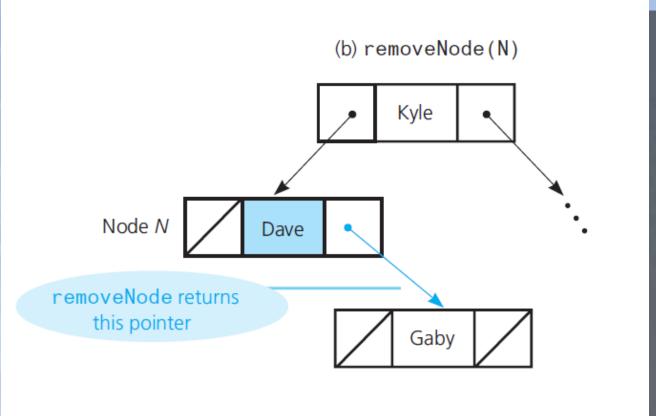


FIGURE 16-14 Recursive removal of node N

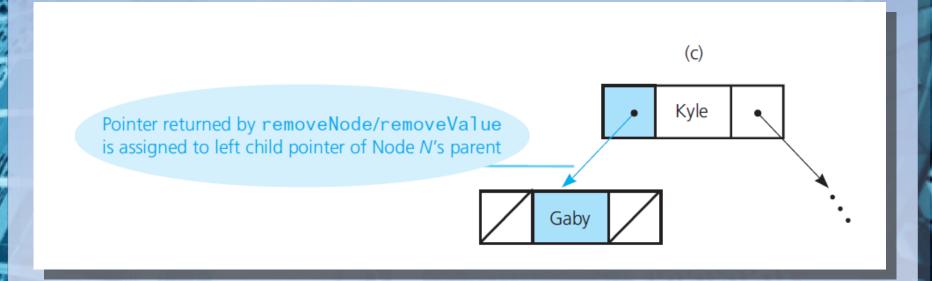


FIGURE 16-14 Recursive removal of node N

```
11 Locates the node in the binary search tree to which subTreePtr points and that contains
II the value target. Returns either a pointer to the located node or nullptr if such a
11 node is not found.
findNode(subTreePtr: BinaryNodePointer, target: ItemType): BinaryNodePointer
   if (subTreePtr == nullptr)
                                                    11 Not found
      return nullptr
    else if (subTreePtr->getItem() == target)
                                               11 Found
       return subTreePtr;
    else if (subTreePtr->getItem() > target)
       11 Search left subtree
       return findNode(subTreePtr->getLeftChildPtr(), target)
    else
       11 Search right subtree
       return findNode(subTreePtr->getRightChildPtr(), target)
```

#### Algorithm for findNode

```
/** Link-based implementation of the ADT binary search tree.
     @file BinarySearchTree.h */
    #ifndef BINARY_SEARCH_TREE_
    #define BINARY SEARCH TREE
6
    #include "BinaryTreeInterface.h"
    #include "BinaryNode.h"
    #include "BinaryNodeTree.h"
    #include "NotFoundException.h"
10
   #include "PrecondViolatedExcept.h"
11
    #include <memory>
12
13
    template < class ItemType >
14
    class BinarySearchTree : public BinaryNodeTree<ItemType>
15
16
    private:
17
       std::shared_ptr<BinaryNode<ItemType>> rootPtr;
```

```
protected:
20
           Protected Utility Methods Section:
21
           Recursive helper methods for the public methods.
22
23
24
      // Places a given new node at its proper position in this binary
      // search tree
25
      auto placeNode(std::shared_ptr<BinaryNode<ItemType>> subTreePtr.
26
                   std::shared ptr<BinaryNode<ItemType>> newNode);
27
28
      // Removes the given target value from the tree while maintaining a
29
30
      // binary search tree.
      auto removeValue(std::shared ptr<BinaryNode<ItemType>> subTreePtr,
31
                                  const ItemType target,
32
                                  bool& isSuccessful) override;
33
34
35
      // Removes a given node from a tree while maintaining a binary search tree.
      auto removeNode(std::shared ptr<BinaryNode<ItemType>> nodePtr);
36
```

```
""" / Tremoves a given node from a tree while manntaining a binary search tree.
       auto removeNode(std::shared ptr<BinaryNode<ItemType>> nodePtr);
36
37
       // Removes the leftmost node in the left subtree of the node
38
       // pointed to by nodePtr.
39
       // Sets inorderSuccessor to the value in this node.
40
       // Returns a pointer to the revised subtree.
41
42
       auto removeLeftmostNode(std::shared_ptr<BinaryNode<ItemType>>subTreePtr,
                                                 ItemType& inorderSuccessor):
43
44
       // Returns a pointer to the node containing the given value,
45
       // or nullptr if not found.
46
       auto findNode(std::shared_ptr<BinaryNode<ItemType>> treePtr,
47
                                      const ItemType& target) const;
48
49
    public:
50
51
            Constructor and Destructor Section.
52
53
       BinarySearchTree();
54
       BinarySearchTree(const ItemType& rootItem);
55
       BinarySearchTree(const BinarySearchTree<ItemType>& tree);
56
       virtual ~BinarySearchTree();
```

```
BinarySearchTree(const BinarySearchTree<ItemType>& tree);
56
      virtual ~BinarySearchTree();
57
58
59
          Public Methods Section.
60
61
      bool isEmpty() const;
62
      int getHeight() const;
63
      int getNumberOfNodes() const;
64
      ItemType getRootData() const throw(PrecondViolatedExcept);
65
      void setRootData(const ItemType& newData);
66
      bool add(const ItemType& newEntry);
67
      bool remove(const ItemType& target);
68
      void clear():
69
      ItemType getEntry(const ItemType& anEntry) const throw(NotFoundException);
```

```
bool contains(const ItemType& anEntry) const;
71
72
73
       // Public Traversals Section.
74
75
       void preorderTraverse(void visit(ItemType&)) const;
76
       void inorderTraverse(void visit(ItemType&)) const;
77
       void postorderTraverse(void visit(ItemType&)) const;
78
79
80
           Overloaded Operator Section.
81
82
       BinarySearchTree<ItemType>&
83
                    operator=(const BinarySearchTree<ItemType>& rightHandSide);
84
    }; // end BinarySearchTree
85
    #include "BinarySearchTree.cpp"
    #endif
87
```

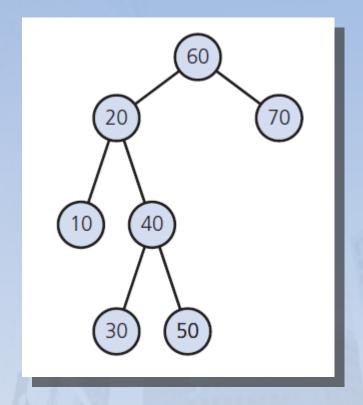


FIGURE 16-15 An initially empty binary search tree after the addition of 60, 20, 10, 40, 30, 50, and 70

- Use preorder traversal to save binary search tree in a file
  - Restore to original shape by using method add
- Balanced binary search tree increases efficiency of ADT operations

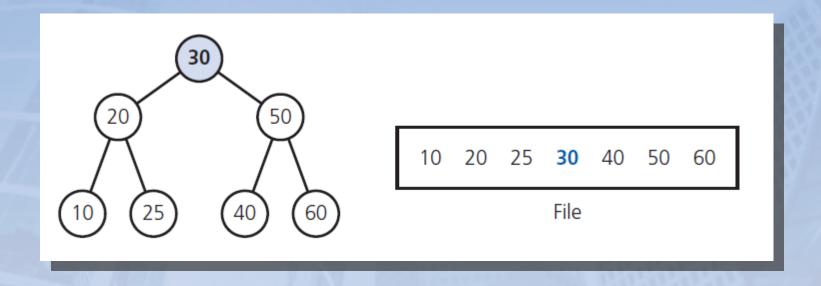


FIGURE 16-16 A full tree saved in a file by using inorder traversal

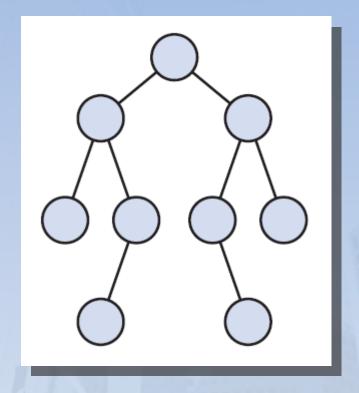


FIGURE 16-17 A tree of minimum height that is not complete

```
// Builds a minimum-height binary search tree from n sorted values in a file.
// Returns a pointer to the tree's root.
readTree(treePtr: BinaryNodePointer, n: integer): BinaryNodePointer
{
    if (n > 0)
    {
        treePtr = pointer to new node with nullptr as its child pointers

        // Construct the left subtree
        leftPtr = readTree(treePtr->getLeftChildPtr(), n / 2)
        treePtr->setLeftChildPtr(leftPtr)
```

Building a minimum-height binary search tree

```
// Get the data item for this node
rootItem = next data item from file
treePtr->setItem(rootItem)

// Construct the right subtree
rightPtr = readTree(treePtr->getRightChildPtr(), (n - 1) / 2)
treePtr->setRightChildPtr(rightPtr)

return treePtr
}
else
return nullptr
}
```

Building a minimum-height binary search tree

#### Tree Sort

```
// Sorts the integers in an array into ascending order.
treeSort(anArray: array, n: integer)
{
    Add anArray's entries to a binary search tree bst
    Traverse bst in inorder. As you visit bst's nodes, copy their data items into successive
    locations of anArray
}
```

Tree sort uses a binary search tree.

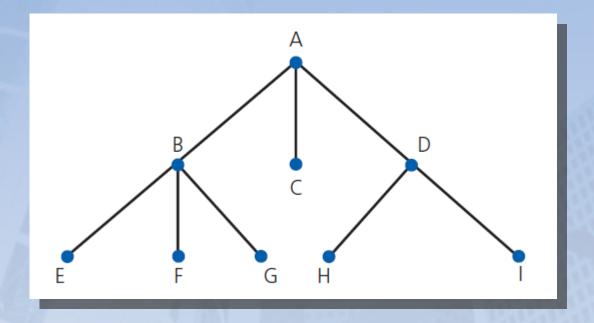


FIGURE 16-18 A general tree or an n-ary tree with n = 3

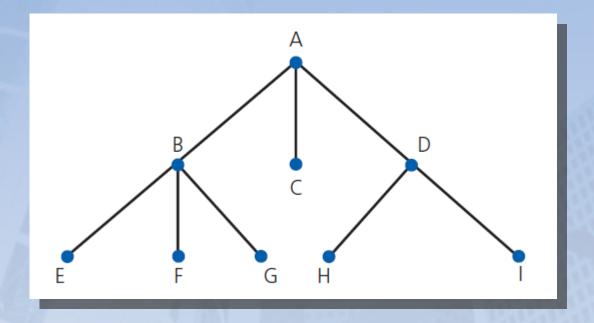


FIGURE 16-18 A general tree or an n-ary tree with n = 3

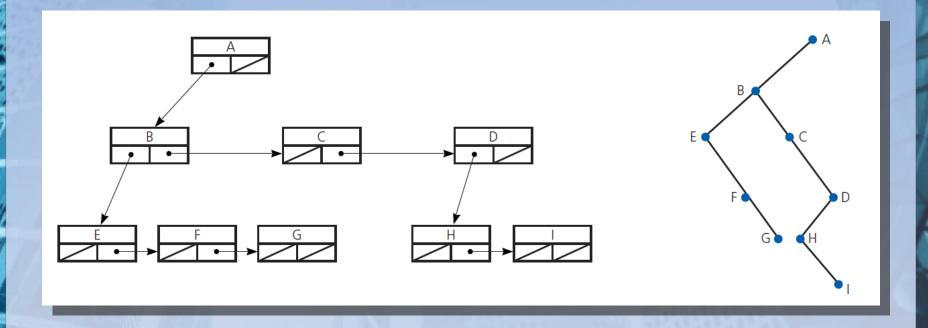


FIGURE 16-19 An implementation of a general tree and its equivalent binary tree

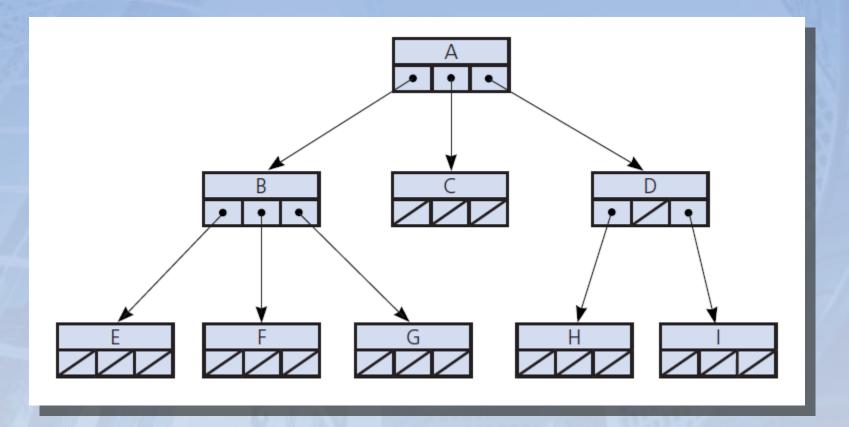


FIGURE 16-20 An implementation of the *n*-ary tree in Figure 16-18

#### End

Chapter 16