Khuyen Le Thi Minh s5128 – ASD 1

Write down the algorithm which finds the value of the polynomial:

- using the standard form $P(x)=a_n\times x^n+a_{n-1}\times x^{n-1}+...+a_0$,
- using the Horner's schema $P(x)=(...((a_n\times x+a_{n-1})x+a_{n-2})x+...+a_1)x+a_0$.

Compare the complexity of both algorithms. Specify invariants which allow to demonstrate correctness of the algorithms being considered.

```
int Polynomial(int[ ]a, int x, int n){
    int result := a0; i:= 1; k:= 1;
    if(n > 0) then
        while (i ≤ n) do
            k := k * x
            result := result + (k * ai)
            i := i + 1
        od;
    fi
    return result;
}
```

Dominating operation: *comparison (n times), multiplication (2n times), addition (2n times).*

Data size: *n*

Assuming that the dominant operation in the FACT algorithm is the operations on natural numbers, then ***t(Polynomial, n)*** *= 5n=Θ (n)*

The FACT algorithm uses one auxiliary variables independently of the value of the argument n, therefore ***S (n)*** *= 1.*

invariant: *n>0 and i<=n*

Correctness:

The specification of the algorithm FACT becomes a pair, where WP = (n is a natural number) and WK=($x^n a_n$ *+…+* xa_1 *+*a_0*).*

The algorithms will terminate when i is equals to n which is natural number given from beginning (i is by beginning 1 and increase by 1 each time until it reach n). For the x=0 the result will be a_0 *and for the x>0 the result will be* $x^n a_n$ *+…+* xa_1 *+*a_0 *so the end condition is true so this algorithm is absolutely correct to the specification <WP, WK>*

$int\ Horner\ (int[\quad]a, int\ x, int\ n)\{$

$int\ result := a_n;\ i := n-1;$

$if\ (n > 0)\ \textbf{then}$

$while\ (i \geq 0)\ \textbf{do}$

$result := x * result + a_i$

$i := i - 1$

$\textbf{od};$

$\textbf{fi}$

$\textbf{return}\ result;$

$\}$

Dominating operation: *comparison (n times), multiplication (n times), addition (n times), subtraction (n times).*

Assuming that the dominant operation in the FACT algorithm is the operations on natural numbers, then ***t(Horner, n)*** *= 4n=Θ (n).*

The FACT algorithm uses one auxiliary variables independently of the value of the argument n, therefore ***S (n)*** *= 1.*

Data size: *n*

Invariant: *n>0 and i >0*

Correctness:

The specification of the algorithm FACT becomes a pair, where WP = (n is a natural number) and WK=((..((a_n*x+*a_{n-1}*)x+*a_{n-2}*)x+…+*a_1*)x +*a_0*).*

The algorithms will terminate when i is equals to 0 (i is by beginning n-1 which is natural number given from beginning and decrease by 1 each time until it reach 0). For the x=0 the result will be a_0 *and for the x>0 the result will be (..((*a_n*x+*a_{n-1}*)x+*a_{n-2}*)x+…+*a_1*)x +*a_0 *so the end condition is true so this algorithm is absolutely correct to the specification <WP, WK>*