

Homework 09

1. Convert the following context – free grammar to Chomsky normal form:

$$S \rightarrow SS|SaP|P$$

$$P \rightarrow bbPa|\epsilon$$

Remove null productions $P \rightarrow \epsilon$.

$$S \rightarrow SS|SaP|P|Sa|\epsilon$$

$$P \rightarrow bbPa|bba$$

Remove null productions $S \rightarrow \epsilon$.

$$S \rightarrow SS|SaP|P|Sa|S|aP|a$$

$$P \rightarrow bbPa|bba$$

Remove unit productions $S \rightarrow S$.

$$S \rightarrow SS|SaP|P|Sa|aP|a$$

$$P \rightarrow bbPa|bba$$

Remove unit productions $S \rightarrow P$.

$$S \rightarrow SS|SaP|Sa|aP|a|bbPa|bba$$

$$P \rightarrow bbPa|bba$$

Substituting nonterminal A and B with existing terminal a and b .

$$S \rightarrow SS|SAP|SA|AP|a|BBPA|BBA$$

$$P \rightarrow BBPA|BBA$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Split the production $S \rightarrow SAP$

$$S \rightarrow SS|SX|SA|AP|a|BBPA|BBA$$

$$P \rightarrow BBPA|BBA$$

$$X \rightarrow AP$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Split the production $S \rightarrow BBA$ and $S \rightarrow BBPA$

$$S \rightarrow SS|SX|SA|AP|a|YZ|YA$$

$$P \rightarrow YZ|YA$$

$$Z \rightarrow PA$$

$$Y \rightarrow BB$$

$$X \rightarrow AP$$

$$A \rightarrow a$$

$$B \rightarrow b$$

2. For the following context – free grammar in Chomsky normal form and the word 'abaabb' simulate the CYK algorithm. Give the contents of the T array and the exemplary derivation tree.

$$S \rightarrow AB|BA$$

$$A \rightarrow AA|BB|a$$

$$B \rightarrow SB|b$$

A	S	—	—	S	A, B, S
	B	S	S	B	A, B, S
		A	A	S	A, B, S
			A	S	A, B
				B	A
					B

We consider the strings with length of 2.

$$ab \rightarrow S$$

$$ba \rightarrow S$$

$$aa \rightarrow A$$

$$ab \rightarrow S$$

$$bb \rightarrow A$$

We consider the strings with length of 3.

$$aba \rightarrow a \otimes ba \cup ab \otimes a \rightarrow A \otimes S \cup S \otimes A \rightarrow \begin{cases} AS \rightarrow - \\ SA \rightarrow - \end{cases}$$

$$baa \rightarrow b \otimes aa \cup ba \otimes a \rightarrow B \otimes A \cup S \otimes A \rightarrow \begin{cases} BA \rightarrow S \\ SA \rightarrow - \end{cases}$$

$$aab \rightarrow a \otimes ab \cup aa \otimes b \rightarrow A \otimes S \cup A \otimes B \rightarrow \begin{cases} AS \rightarrow - \\ AB \rightarrow S \end{cases}$$

$$abb \rightarrow a \otimes bb \cup ab \otimes b \rightarrow A \otimes A \cup S \otimes B \rightarrow \begin{cases} AA \rightarrow A \\ SB \rightarrow B \end{cases}$$

We consider the strings with length of 4.

$$\begin{aligned}
 abaa &\rightarrow a \otimes baa \cup ab \otimes aa \cup aba \otimes a \rightarrow A \otimes S \cup S \otimes A \cup - \otimes A \rightarrow \begin{cases} AS \rightarrow - \\ SA \rightarrow - \\ - \end{cases} \\
 baab &\rightarrow b \otimes aab \cup ba \otimes ab \cup baa \otimes b \rightarrow B \otimes S \cup S \otimes S \cup S \otimes B \rightarrow \begin{cases} BS \rightarrow - \\ SS \rightarrow - \\ SB \rightarrow B \end{cases} \\
 aabb &\rightarrow a \otimes abb \cup aa \otimes bb \cup aab \otimes b \rightarrow A \otimes \{A, B\} \cup A \otimes A \cup S \otimes B \rightarrow \begin{cases} AA \rightarrow A \\ AB \rightarrow S \\ AA \rightarrow A \\ SB \rightarrow B \end{cases}
 \end{aligned}$$

We consider the strings with length of 5.

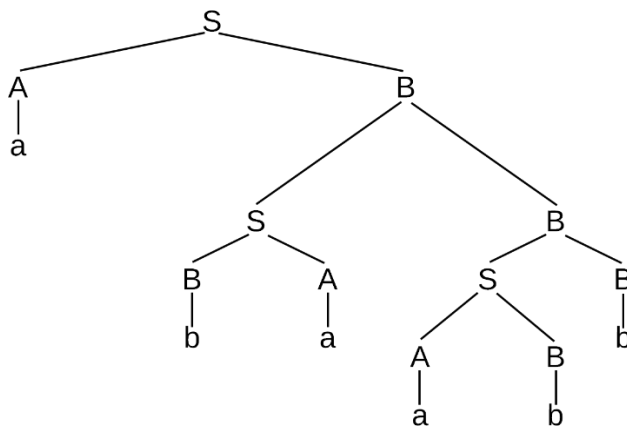
$$\begin{aligned}
 abaab &\rightarrow a \otimes baab \cup ab \otimes aab \cup aba \otimes ab \cup abaa \otimes b \rightarrow A \otimes B \cup S \otimes S \cup - \otimes S \cup - \otimes B \\
 &\rightarrow \begin{cases} AB \rightarrow S \\ SS \rightarrow - \\ - \end{cases} \\
 baabb &\rightarrow b \otimes aabb \cup ba \otimes abb \cup baa \otimes bb \cup baab \otimes b \rightarrow B \otimes \{A, B, S\} \cup S \otimes \{A, B\} \cup S \otimes A \cup B \otimes B \\
 &\rightarrow \begin{cases} BA \rightarrow S \\ BB \rightarrow A \\ BS \rightarrow - \\ SA \rightarrow - \\ SB \rightarrow B \\ SA \rightarrow - \\ BB \rightarrow A \end{cases}
 \end{aligned}$$

We consider the strings with length of 6.

$$\begin{aligned}
 abaabb &\rightarrow a \otimes baabb \cup ab \otimes aabb \cup aba \otimes abb \cup abaa \otimes bb \cup abaab \otimes b \\
 &\rightarrow A \otimes \{A, B, S\} \cup S \otimes \{A, B, S\} \cup - \otimes \{A, B\} \cup - \otimes A \cup S \otimes B \\
 &\rightarrow \begin{cases} AA \rightarrow A \\ AB \rightarrow S \\ AS \rightarrow - \\ SA \rightarrow - \\ SB \rightarrow B \\ SS \rightarrow - \\ - \\ SB \rightarrow B \end{cases}
 \end{aligned}$$

A	S	-	-	S	A, B, S
	B	S	S	B	A, B, S
		A	A	S	A, B, S
			A	S	A, B
				B	A
					B

The bolded font show the how derivation tree has been built



Testing: $S \rightarrow AB \rightarrow aB \rightarrow aSB \rightarrow aBAB \rightarrow abaB \rightarrow abaSB \rightarrow abaSb \rightarrow abaABb \rightarrow abaabb$