

Homework 08

1. Give a context – free grammar generating the language $\{a^{2n}b^{n+2k}c^k: n, k \geq 1\}$.
Give a derivation tree for the word $aaaabbbbbbbcc$

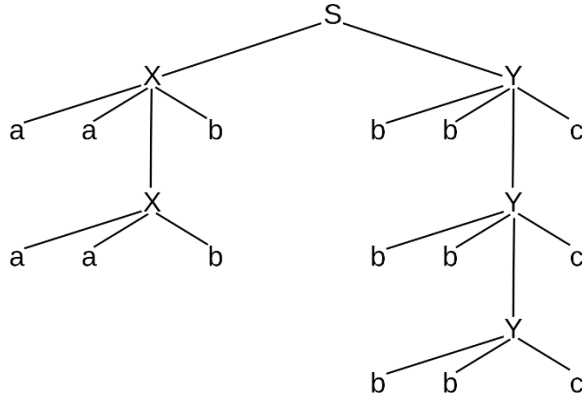
$a^{2n}b^{n+2k}c^k = a^{2n}b^n b^{2k}c^k$ so we can divide the language to two part: $X \rightarrow a^{2n}b^n$ and $Y \rightarrow b^{2k}c^k$

$S \rightarrow XY$

$X \rightarrow aab|aaXb$

$Y \rightarrow bbc|bbYc$

Derivation tree for word $aaaabbbbbbbcc$:



2. Give a linear grammar generating the language $a^+b(a^*b|ab^+)$

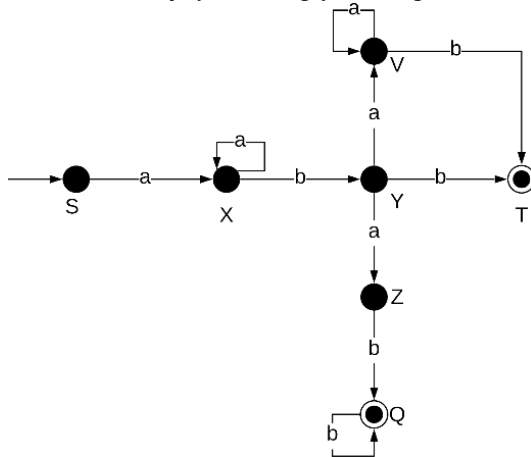
$S \rightarrow XbYb|XbaZ$

$X \rightarrow aX|a$

$Y \rightarrow aY|\varepsilon$

$Z \rightarrow bZ|b$

Attempt for strongly linear grammar



$S \rightarrow aX$

$X \rightarrow aX|bY$

$Y \rightarrow aV|bT|aZ$

$V \rightarrow aV|bT$

$T \rightarrow \varepsilon$

$Z \rightarrow bQ$

$Q \rightarrow bQ|\varepsilon$

Checking:

$Q \rightarrow bQ|\varepsilon \Rightarrow b^*$

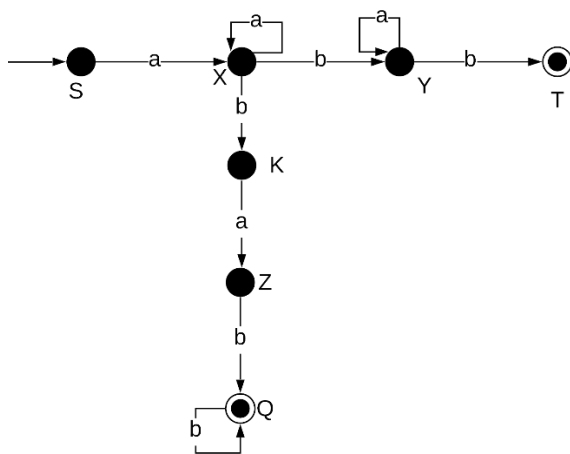
$Z \rightarrow bQ \Rightarrow bb^* = b^+$

$V \rightarrow aV|bT \Rightarrow a^+b|b \Rightarrow a^*b$

$Y \rightarrow aV|bT|aZ \Rightarrow aa^*b|b|ab^+ \Rightarrow a^+b|b|ab^+ \Rightarrow a^*b|ab^+$

$X \rightarrow aX|bY \Rightarrow a^+bY|bY \Rightarrow a^*bY \Rightarrow a^*ba^*b|a^*bab^+$

$S \rightarrow aX \Rightarrow aa^*ba^*b|aa^*bab^+ \Rightarrow a^+ba^*b|a^+bab^+ \Rightarrow a^+b(a^*b|ab^+)$



$S \rightarrow aX$

$X \rightarrow aX|bY|bK$

$Y \rightarrow aY|bT$

$T \rightarrow \varepsilon$

$K \rightarrow aZ$

$Z \rightarrow bQ$

$Q \rightarrow bQ|\varepsilon$

Checking:

$Q \rightarrow bQ|\varepsilon \Rightarrow b^*$

$Z \rightarrow bQ \Rightarrow bb^* = b^+$

$K \rightarrow aZ \Rightarrow ab^+$

$Y \rightarrow aY|bT \Rightarrow a^+b|b \Rightarrow a^*b$

$X \rightarrow aX|bY|bK \Rightarrow a^+bY|bY|a^+bK|bK \Rightarrow a^*bY|a^*bK \Rightarrow a^*ba^*b|a^*bab^+$

$S \rightarrow aX \Rightarrow aa^*ba^*b|aa^*bab^+ \Rightarrow a^+ba^*b|a^+bab^+ \Rightarrow a^+b(a^*b|ab^+)$