Logistic Regression

Thu Hien Le

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1 Find the function

$$f'(x) = f(x)(1 - f(x)) \tag{1}$$

$$\Leftrightarrow \frac{d(f(x))}{dx} = f(x)(1 - f(x))$$

$$\Rightarrow \frac{d(f(x))}{f(x)(1 - f(x)} = dx$$

$$\Rightarrow \int \frac{1}{f(x)(1 - f(x))} d(f(x)) = \int dx$$

$$\Rightarrow \int \frac{1}{f(x)} + \frac{1}{1 - f(x)} d(f(x)) = \int dx$$

$$\Leftrightarrow \ln(f(x)) - \ln(1 - f(x)) = x \Leftrightarrow \frac{f(x)}{1 - f(x)} = e^x$$

$$\Rightarrow f(x) = e^x(1 - f(x))$$

$$\Leftrightarrow f(x) = e^x - e^x f(x) \Leftrightarrow f(x)(1 + e^x) = e^x$$

$$\Leftrightarrow f(x) = \frac{e^x}{1 + e^x} = \frac{1}{1 + e^{-x}}$$