Two-Dimensional Dendritic Growth Using Phase-Field Model
Design Document

CS 294-73 Group H

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1 Discretization Methods and Numerical Schemes

Recall the governing equations:

$$
\begin{cases}
\frac{\partial u}{\partial t} = & D\nabla^2 u + \frac{1}{2}\frac{\partial \phi}{\partial t} \\
\tau\frac{\partial \phi}{\partial t} = & \phi(1-\phi)(\phi - \frac{1}{2} + \tilde{n}(u)) - \frac{\partial}{\partial x}(WW'\frac{\partial \phi}{\partial y}) \\
 & +\frac{\partial}{\partial y}(WW'\frac{\partial \phi}{\partial x}) + \nabla(W^2)\cdot\nabla\phi + W^2\nabla^2\phi \\
W = & W_0(1+\mu cos(a_0(\theta-\theta_0))) \\
\theta = & tan^{-1}(\frac{\partial \phi}{\partial y}/\frac{\partial \phi}{\partial x}) + \pi(1 - sign(\frac{\partial \phi}{\partial x}))
\end{cases}
\tag{1}
$$

A 2nd order central difference scheme will be used for spatial discretization while a 4th order Runge-Kutta scheme for time integration.

The final computational solution consists of time dependent phase field (ϕ) and dimensionless temperature field (u) in the form of vtk files.

2 Software Design

The following existing classes will be directly utilized:

Point, Box, RectMDarray

RK4

VisitWriter, WriteRectMDArray

CH_Timer

A new class `DendriticGrowth` is defined, along with a modified version of the original `RK4`. Inside `DendriticGrowth`, public member data and functions contain ϕ and u fields, as well as update and increment functions for both fields. As `DendriticGrowth` is the only input class for `RK4`, class setup in `RK4` is modified accordingly.

3 Algorithm and Flow Chart

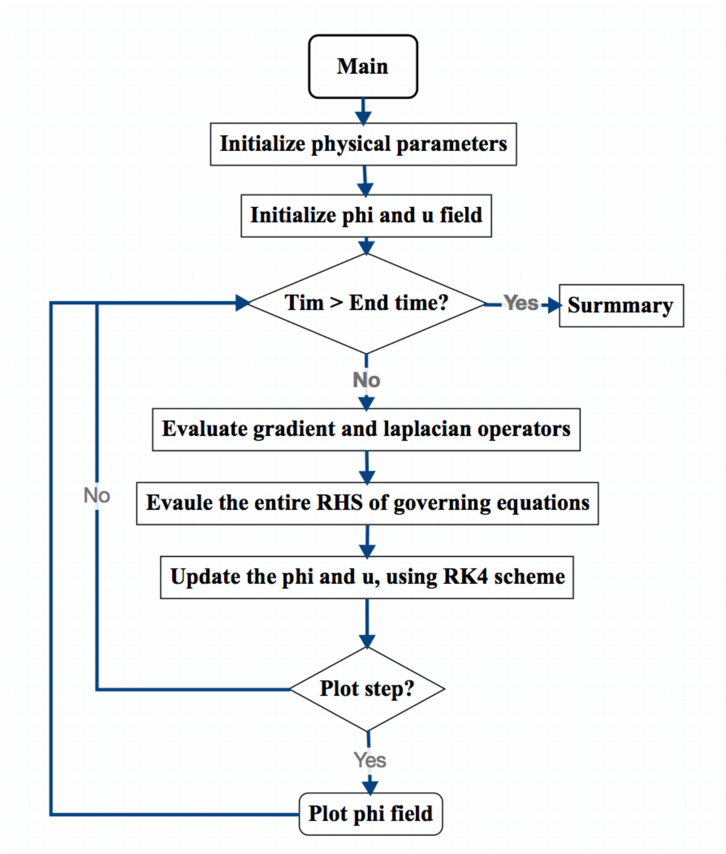

Figure 1: Pseudo code diagram for dendritic growth using phase-field model

1. Initialize the modeling parameters including timestep dt, end time t, grid dh, domain size L, etc.;
2. Initialize the ϕ and u field;
3. Evaluate the gradient and laplacian operators by 2nd order central difference scheme;
4. Evaluate the orientation angle θ and W;
5. Evaluate RHS of ϕ and u euqations, update ϕ and u using RK4;
6. Plot intermidiate time step contour of ϕ and u.