Faculdade de Engenharia da Universidade do Porto


# Performance Evaluation - Single and Multi-Core Matrix Multiplication Algorithms

## (1st Lab Work)

Parallel and Distributed Computing 2024/25 – LEIC

L.EIC028-12

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# 1. Problem Description

In this project, we will analyse the performance differences between several matrix multiplication approaches. We will explore how building an algorithm with smarter cache usage affects execution time and how performance is increased by using parallel programming approaches through the use of the OpenMP C++ library.

# 2. Algorithms

Matrix multiplication performance is influenced by memory access patterns. To analyze this, we implemented and compared different approaches using **C++** and **Python**.

We started with a **basic row-by-column multiplication** in all two languages, followed by an **element-wise multiplication approach** in **C++** and **Python**. Finally, a **block-oriented optimization** was implemented in **C++**. Performance was measured across various matrix sizes, with larger tests conducted in C++ to evaluate scalability. The **Performance API (PAPI)** was used to gather execution metrics, and optimizations were applied where applicable.

The following sections describe each algorithm in detail.

## 2.1 Basic Matrix Multiplication

```
for ( i=0; i<m_ar; i++) {
      for ( j=0; j<m_br; j++) {
            temp = 0;
            for ( k=0; k<m_ar; k++) {
                  temp += pha[i*m_ar+k] * phb[k*m_br+j];
            }
            phc[i*m_ar+j] = temp;
      }
}
```

**Image 1** - Basic Matrix Multiplication (Row-by-Column) in C++

This implementation follows the traditional row-by-column approach for matrix multiplication. Each element of the resulting matrix is computed by iterating through the rows of matrix **A** and multiplying them with the corresponding columns of matrix **B**.

## 2.2 Line Matrix Multiplication

```
for (i = 0; i < m_ar; i++) {
      for (k = 0; k < m_ar; k++) {
            temp = pha[i * m_ar + k];
            for (j = 0; j < m_br; j++) {
                  phc[i * m_ar + j] += temp * phb[k * m_br + j];
            }
      }
}
```

**Image 2** - Line Matrix Multiplication in C++

This implementation optimizes the traditional row-by-column approach by reusing computed values from matrix **A** more efficiently, reducing redundant memory accesses. Instead of

computing each element of the result matrix individually by iterating over rows and columns, this approach processes **one row at a time**, multiplying it with all columns of matrix **B** before moving to the next row.

### 2.2.1 Parallel Nested Loops with OpenMP (*i-loop* parallelized)

```
#pragma omp parallel for
for (int i = 0; i < m_ar; i++) {
   for (int k = 0; k < m_ar; k++) {
      double temp = pha[i * m_ar + k];
      for (int j = 0; j < m_br; j++) {
         phc[i * m_ar + j] += temp * phb[k * m_br + j];
      }
   }
}
```

**Image 3** - Parallel Nested Loops with OpenMP in C++

This implementation applies OpenMP parallelization to the **outermost loop (`i-loop`)**, distributing the iterations across multiple threads. Each thread processes a different row of matrix **A**, performing all multiplications for that row in parallel. The num_threads parameter was also used to study the difference of the usage of different thread counts.

### 2.2.2 Parallel Nested Loops with OpenMP (*j-loop* parallelized)

```
#pragma omp parallel
for (int i = 0; i < m_ar; i++) {
   for (int k = 0; k < m_ar; k++) {
      double temp = pha[i * m_ar + k];

      #pragma omp for
      for (int j = 0; j < m_br; j++) {
         phc[i * m_ar + j] += temp * phb[k * m_br + j];
      }
   }
}
```

**Image 4** - Parallel Nested Loops with OpenMP in C++

This version applies **nested parallelization**, where the **outermost `i-loop`** is parallelized, and an additional `#pragma omp for` is applied to the **innermost `j-loop`**. This means that, within each thread, multiple threads are also distributing the work inside the row-wise multiplication process. Similarly to the algorithm above, num_threads was also used for similar effects.

## 2.3 Block Matrix Multiplication

```
for (int blockY = 0; blockY < blocksPerRow; ++blockY) {
  for (int blockX = 0; blockX < blocksPerRow; ++blockX) {
    for (int block = 0; block < blocksPerRow; ++block) {
      for (int i = 0; i < bkSize; ++i) {
        for (int n = 0; n < bkSize; ++n) {
          for (int j = 0; j < bkSize; ++j) {
            phc[blockIndex + (i*m_ar+j)] += pha[blockAindex + (i*m_ar+n)] * phb[blockBindex + (n*m_ar+j)];
```

**Image 5** - Block Matrix Multiplication in C++

This implementation optimizes matrix multiplication by **dividing the matrices into smaller sub-blocks**. Instead of processing entire rows or columns at once, computations are performed on blocks, which helps improve **cache efficiency** and **memory locality**. By keeping blocks small enough to fit in **cache memory**, we reduce memory access latency and enhance computational performance.

# 3. Performance Metrics

To assess the performance of the implemented matrix multiplication algorithms, we analyzed key metrics such as **execution time, cache misses, and floating-point operations per second (FLOP/s)**. These metrics provide insights into the efficiency of different approaches and programming languages, helping to evaluate the impact of memory hierarchy and parallelization.

### Performance Measurement Using PAPI

For the **C++ implementations**, we used the **Performance Application Programming Interface (PAPI)** to measure **hardware performance counters**, specifically:

- **L1 and L2 Data Cache Misses**: Indicates how well memory is being reused (**lower cache misses generally result in better performance** due to reduced memory access latency).
- **Execution Time (seconds)**: The total time taken to execute each algorithm.

## 3.1 Execution Time and Multi-Core Evaluation

The algorithms were implemented in **C++ and Python**, and execution time was measured for both. The **parallelized versions** of the algorithms using **OpenMP** were also evaluated to measure performance improvements due to **multi-core execution**.

To ensure accuracy, all measurements were conducted on the **same system**, running **Ubuntu 22.04**, with an **Intel i5 7200 processor (3.10 GHz)** featuring:

| Cache | Total Size | Associativity |
|---|---|---|
| L1 Data Cache | 64 KiB | 8-Way |
| L1 Information Cache | 64 KiB | 8-Way |
| L2 Cache | 512 KiB | 4-Way |
| L3 Cache | 3 MiB | 12-Way |

Each experiment was repeated **five times**, and the average execution time was reported to minimize variations due to system background processes.

### 3.1.1 FLOP Calculation and Performance Estimation

For performance evaluation, the **floating-point operations per second (FLOP/s)** metric was computed using:

$$FLOPS = \frac{2 \times (matrix\ size)^3}{execution\ time}$$

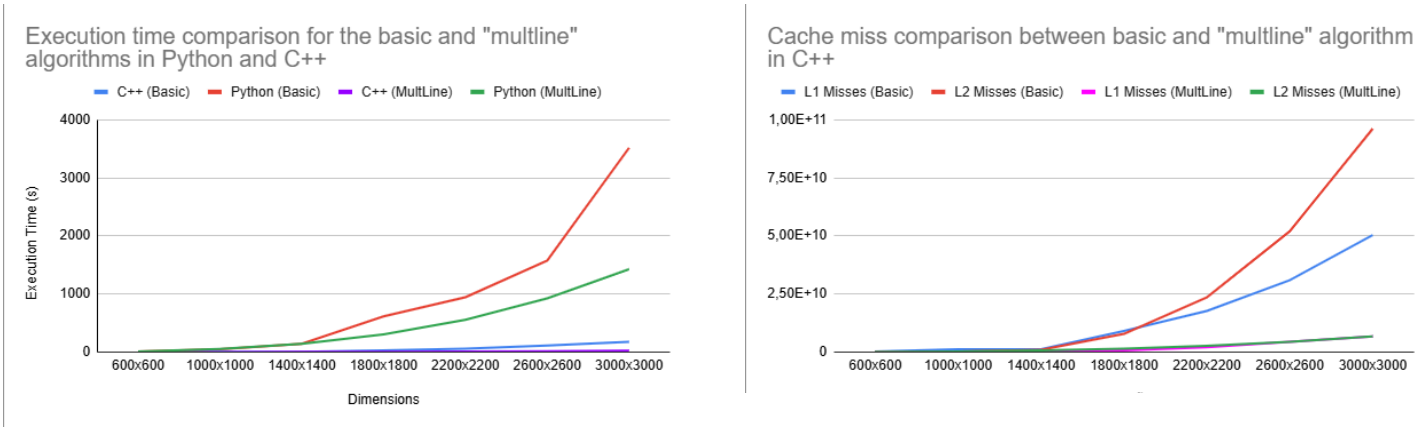
### 3.1.2 Speedup and Efficiency in Parallel Implementations

To quantify the performance gain from parallel execution, we computed:

$$Speedup = \frac{T_{sequential}}{T_{parallel}} \qquad Efficiency = \frac{Speedup}{\text{number of logical processors}}$$

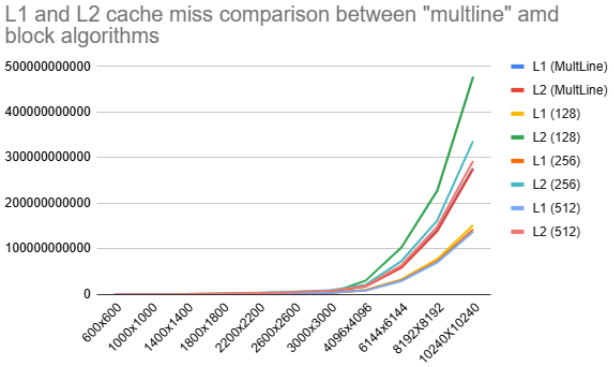
## 4. Results and Analysis

### 4.1 Basic vs Line Multiplication



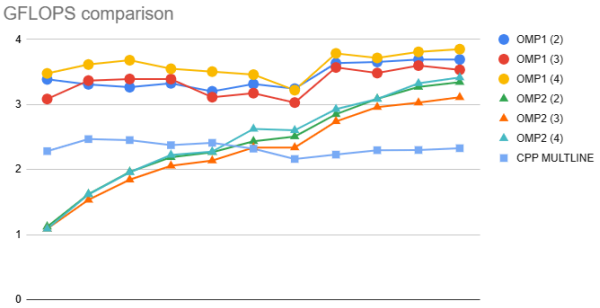

By looking at the first graph we can see that the execution time in Python is drastically higher than that of the one experienced when using C++. This was expected, as Python is an interpreted programming language while C++ is compiled: there is additional overhead due to code being “translated” at runtime. We can also tell that the line multiplication algorithm is significantly faster than the basic one originally provided by the teachers.
In relation to cache misses, expectedly the basic algorithm experiences more of them. It justifies the relative slowness in execution and shows the effects of poor cache management when building an algorithm.

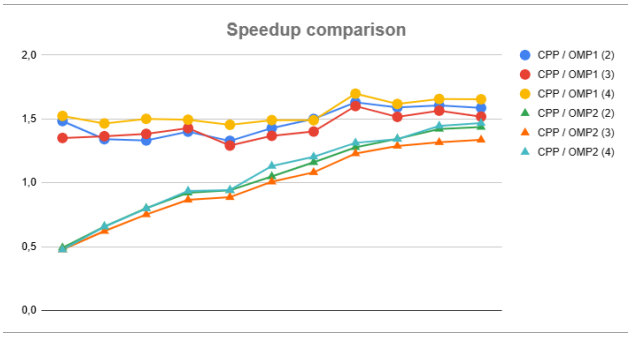
## 4.2 Line Multiplication vs Block Algorithm




When it comes to execution time, the conclusion was that both algorithms perform quite similarly when analysed in a line graph. The slight timing difference mainly lies in how these algorithms handle cache resources - as you can see, the block approach has slightly more L2 cache misses and slightly less L1 cache misses than the line multiplication approach, with the 512 block resulting in the lowest number of L1 cache misses overall.

## 4.3 Single vs Multi Core Line Multiplication




The computer used for testing only has two physical cores so a maximum of 4 threads was used. The number of FLOPS stayed stable for regular line multiplication and OMP1 and increased over time for OMP2 - this is explained by the need for a larger overhead of thread creation and synchronization, since we're parallelizing the innermost loop in this approach, which ends up being unsustainable especially in smaller matrices. Overall we can tell that the highest speedup was achieved by the 4 threaded OMP1 approach. We were also able to conclude that using 3 threads ends up being less efficient than using 4 or even 2 threads, since there is an workload imbalance between cores that calls for a significant context switching overhead for workload assignment.

## 5. Conclusions

In conclusion, this project allowed us to fully grasp the extent to which successful cache management and smartly built algorithms affect the performance of certain tasks. Furthermore we had the opportunity to try out some parallel programming approaches, which were foreign to us up until this point.
It will definitely serve as a reminder for the remainder of our academic and professional career that we should always take into consideration some of the lower level aspects of the programs we are running in order to optimize performance.

## References

freeCodeCamp. **Interpreted vs Compiled Programming Languages: What's the Difference?.** 2020. Accessed on 20 March 2025. <https://www.freecodecamp.org/news/compiled-versus-interpreted-languages/>

# Annexes

## A1. Basic Algorithm

### A1.1. C++ - execution time (s)

| Dimensions | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|
| 600x600 | 0,439693 | 0,419459 | 0,391727 | 0,42769 | 0,407302 | 0,4171742 |
| 1000x1000 | 2,352758 | 2,207778 | 2,310205 | 2,198498 | 2,214327 | 2,2567132 |
| 1400x1400 | 5,890811 | 6,01915 | 5,878399 | 6,22058 | 5,739422 | 5,9496724 |
| 1800x1800 | 29,268526 | 28,574936 | 29,469361 | 27,501497 | 27,289729 | 28,4208098 |
| 2200x2200 | 59,098902 | 58,8099 | 59,10075 | 58,822947 | 58,517598 | 58,8700194 |
| 2600x2600 | 111,589645 | 109,523708 | 107,390359 | 112,599718 | 108,725018 | 109,9656896 |
| 3000x3000 | 186,188844 | 171,556685 | 171,954471 | 179,419925 | 168,992275 | 175,62244 |

### A1.2. Python - execution time (s)

| Dimensions | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|
| 600x600 | 9,639553 | 9,604367 | 9,67931 | 9,973183 | 9,994624 | 9,7782074 |
| 1000x1000 | 47,732815 | 48,035698 | 47,761805 | 49,535921 | 48,063445 | 48,2259368 |
| 1400x1400 | 152,441989 | 141,919836 | 146,396799 | 138,014873 | 138,019214 | 143,3585422 |
| 1800x1800 | 635,329777 | 637,423009 | 587,58044 | 577,64713 | 624,95063 | 612,5861972 |
| 2200x2200 | 946,874424 | 967,744301 | 872,949756 | 951,401237 | 975,825248 | 942,9589932 |
| 2600x2600 | 1628,028695 | 1587,41334 | 1546,093184 | 1530,391618 | 1589,310286 | 1576,247425 |
| 3000x3000 | 3519,678407 | 3574,332479 | 3541,415252 | 3476,996507 | 3473,67521 | 3517,219571 |

### A1.3. C++ - cache misses L1

| Dimensions | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|
| 600x600 | 244170603 | 244545924 | 244690993 | 243957461 | 244298561 | 244332708,4 |
| 1000x1000 | 1227149425 | 1228541869 | 1229378949 | 1228506174 | 1228208050 | 1228356893 |
| 1400x1400 | 1227149425 | 1228541869 | 1229378949 | 1228506174 | 1228208050 | 1228356893 |
| 1800x1800 | 9068659941 | 9086327414 | 9087193201 | 9087330712 | 9069318795 | 9079766013 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2200x2200 | 17659290853 | 17660327097 | 17647178484 | 17636370466 | 17635257412 | 17647684862 |
| 2600x2600 | 30929545853 | 30882096670 | 30882829862 | 30884477028 | 30883622949 | 30892514472 |
| 3000x3000 | 50306908398 | 50321200773 | 50320309731 | 50309596231 | 50341456050 | 50319894237 |

### A1.4. C++ - cache misses L2

| Dimensions | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|
| 600x600 | 38375124 | 39428196 | 38763910 | 38692234 | 37894911 | 38630875 |
| 1000x1000 | 233482975 | 266676576 | 242436027 | 239857155 | 229334703 | 242357487,2 |
| 1400x1400 | 670316965 | 1329592129 | 1155520751 | 1272211041 | 641124775 | 1013753132 |
| 1800x1800 | 5592660964 | 8483799979 | 8503403610 | 8018560927 | 8427590392 | 7805203174 |
| 2200x2200 | 23488625800 | 24027791827 | 23718687713 | 23239861384 | 23304198911 | 23555833127 |
| 2600x2600 | 52428066623 | 52160409699 | 51483154059 | 51758056597 | 51987566146 | 51963450625 |
| 3000x3000 | 95370773532 | 98281602694 | 95864069723 | 95853165032 | 95099173586 | 96093756913 |

## A2. Line Multiplication (Single Core)

### A2.1. C++ - execution time (s)

| Dimensions | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|
| 600x600 | 0,166654 | 0,197872 | 0,167535 | 0,214487 | 0,19953 | 0,1892156 |
| 1000x1000 | 0,807852 | 0,812936 | 0,805502 | 0,815078 | 0,811128 | 0,8104992 |
| 1400x1400 | 2,218192 | 2,23107 | 2,219219 | 2,226751 | 2,292973 | 2,237641 |
| 1800x1800 | 4,728846 | 4,725345 | 4,711708 | 5,159114 | 5,229171 | 4,9108368 |
| 2200x2200 | 8,929431 | 8,779584 | 8,79437 | 8,797028 | 8,869663 | 8,8340152 |
| 2600x2600 | 14,801709 | 15,750524 | 14,95215 | 15,376528 | 14,856572 | 15,1474966 |
| 3000x3000 | 22,996702 | 25,769195 | 25,620781 | 26,036189 | 24,454995 | 24,9755724 |
| 4096x4096 | 59,10882 | 61,951388 | 61,754143 | 62,294359 | 63,088056 | 61,6393532 |
| 6144x6144 | 201,872146 | 201,034928 | 201,035997 | 201,76821 | 203,788372 | 201,8999306 |
| 8192x8192 | 472,392815 | 482,11446 | 478,736591 | 478,431713 | 478,953047 | 478,1257252 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 10240x10240 | 920,239415 | 930,133287 | 922,528146 | 919,882397 | 919,78131 | 922,512911 |

### A2.2. Python - execution time (s)

| Dimensions | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|
| 600x600 | 10,584092 | 10,650233 | 10,676091 | 10,502165 | 10,631115 | 10,6087392 |
| 1000x1000 | 52,336577 | 50,405668 | 50,983617 | 50,570455 | 50,601985 | 50,9796604 |
| 1400x1400 | 140,599531 | 141,502348 | 143,052117 | 141,578753 | 141,417281 | 141,630006 |
| 1800x1800 | 302,866577 | 302,840571 | 303,926355 | 303,256146 | 305,527085 | 303,6833468 |
| 2200x2200 | 555,43571 | 554,718959 | 555,463223 | 556,335064 | 559,351865 | 556,2609642 |
| 2600x2600 | 923,982903 | 921,010894 | 925,529727 | 925,723786 | 923,55403 | 923,960268 |
| 3000x3000 | 1417,615939 | 1416,803158 | 1418,985761 | 1429,85886 | 1446,47089 | 1425,946922 |

### A2.3. C++ - cache misses L1

| Dimensions | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|
| 600x600 | 27152125 | 27154161 | 27158034 | 27154541 | 27154725 | 27154717,2 |
| 1000x1000 | 125938814 | 125921227 | 125929517 | 126061555 | 125940697 | 125958362 |
| 1400x1400 | 347422556 | 347425491 | 347210798 | 348058431 | 348969448 | 347817344,8 |
| 1800x1800 | 752769186 | 750336376 | 748647236 | 770724158 | 779314230 | 760358237,2 |
| 2200x2200 | 2093795361 | 208379845 7 | 208425018 6 | 208387622 6 | 208722652 6 | 2086589351 |
| 2600x2600 | 4413336677 | 4411203820 | 4413165045 | 4412358366 | 4413051858 | 4412623153 |
| 3000x3000 | 6780399279 | 6777442980 | 6777215625 | 6776807529 | 6778009074 | 6777974897 |
| 4096x4096 | 17712072300 | 17715384631 | 17684276513 | 17684977896 | 17698601296 | 17699062527 |
| 6144x6144 | 59794964679 | 59809167132 | 59817491303 | 59767826502 | 59787858126 | 59795461548 |
| 8192X8192 | 141726928358 | 141679985501 | 141119100769 | 141152776476 | 141115214768 | 141358801174 |
| 10240X10240 | 276629570401 | 276393085500 | 276644828881 | 276767536358 | 276745646400 | 276636133508 |

### A2.4. C++ - cache misses L2

| Dimensions | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|
| 600x600 | 53631041 | 54441848 | 54529030 | 54739825 | 54566531 | 54381655 |
| 1000x1000 | 250624630 | 251865398 | 246026804 | 246581623 | 247044234 | 248428537,8 |
| 1400x1400 | 685642797 | 674535541 | 671529696 | 670000947 | 683349204 | 677011637 |
| 1800x1800 | 1474732612 | 1441819270 | 1469131525 | 1470405190 | 1444686339 | 1460154987 |
| 2200x2200 | 2687641360 | 2686607199 | 2696537044 | 2627910692 | 2617718259 | 2663282911 |
| 2600x2600 | 4374704052 | 4437473057 | 4374702515 | 4388727811 | 4502275520 | 4415576591 |
| 3000x3000 | 6737396997 | 6804616622 | 6790468182 | 6827335558 | 6701215835 | 6772206639 |
| 4096x4096 | 17582546234 | 17553782111 | 17558202402 | 17648275895 | 17373546872 | 17543270703 |
| 6144x6144 | 59236286867 | 58180901261 | 58204022218 | 59153483216 | 58595505263 | 58674039765 |
| 8192X8192 | 139353152647 | 140059648489 | 139567774690 | 137526789401 | 138342347536 | 138969942553 |
| 10240X10240 | 281551931080 | 273606965054 | 276029494217 | 271409619268 | 272823963343 | 275084394592 |

## A3. Block Multiplication

### A3.1. Execution time (s)

| Dimensions | Block SIze | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|---|
| 600x600 | 128 | 0,169692 | 0,165527 | 0,169001 | 0,167367 | 0,167091 | 0,1677356 |
| | 256 | 0,200018 | 0,195913 | 0,210652 | 0,209468 | 0,235808 | 0,2103718 |
| | 512 | 0,177651 | 0,191799 | 0,195314 | 0,267 | 0,177882 | 0,2019292 |
| 1000x1000 | 128 | 0,743163 | 0,817756 | 0,831334 | 0,800828 | 0,827871 | 0,8041904 |
| | 256 | 0,736131 | 0,736801 | 0,74509 | 0,734772 | 0,73613 | 0,7377848 |
| | 512 | 0,849685 | 0,83734 | 0,845516 | 0,817946 | 0,8363 | 0,837359 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | 8 | | | | |
| 1400x1400 | 128 | 2,04857 | 2,052335 | 2,049205 | 2,036978 | 2,071667 | 2,051751 |
| | 256 | 2,041896 | 2,036405 | 2,028108 | 2,039485 | 2,249262 | 2,0790312 |
| | 512 | 2,20949 | 2,198519 | 2,207738 | 2,201811 | 2,216112 | 2,206734 |
| 1800x1800 | 128 | 4,28972 | 4,832219 | 4,291887 | 4,717409 | 4,293226 | 4,4848922 |
| | 256 | 4,288478 | 4,314774 | 4,910938 | 4,821712 | 4,295664 | 4,5263132 |
| | 512 | 5,128629 | 4,757467 | 5,464643 | 5,060843 | 5,462048 | 5,174726 |
| 2200x2200 | 128 | 8,187262 | 8,057813 | 7,870871 | 7,947953 | 7,931159 | 7,9990116 |
| | 256 | 7,819376 | 8,614781 | 8,371213 | 8,439576 | 8,517552 | 8,3524996 |
| | 512 | 8,827248 | 9,696861 | 9,662971 | 8,709997 | 9,466025 | 9,2726204 |
| 2600x2600 | 128 | 12,931972 | 13,595905 | 13,92568 | 13,83276 | 13,787916 | 13,6148466 |
| | 256 | 12,875163 | 13,815314 | 13,948038 | 14,028577 | 12,915217 | 13,5164618 |
| | 512 | 14,604651 | 15,654414 | 14,984728 | 15,788725 | 15,184353 | 15,2433742 |
| 3000x3000 | 128 | 19,83103 | 19,819097 | 20,679933 | 19,880797 | 20,42467 | 20,1271054 |
| | 256 | 19,735704 | 19,741985 | 20,636479 | 19,732203 | 19,872044 | 19,943683 |
| | 512 | 22,505544 | 23,307896 | 23,268751 | 23,436277 | 22,147441 | 22,9331818 |
| 4096x4096 | 128 | 86,216835 | 83,943095 | 79,57701 | 80,398629 | 80,792582 | 82,1856302 |
| | 256 | 70,223737 | 69,80046 | 69,827641 | 69,93322 | 70,305864 | 70,0181844 |
| | 512 | 62,128087 | 61,013158 | 61,192701 | 61,521632 | 61,063067 | 61,383729 |
| 6144x6144 | 128 | 210,866738 | 220,762766 | 214,215706 | 220,183199 | 218,410988 | 216,8878794 |
| | 256 | 227,231946 | 227,546524 | 226,577129 | 227,033124 | 227,252658 | 227,1282762 |
| | 512 | 202,74466 | 204,958 | 204,9668 | 205,0070 | 204,954 | 204,526163 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 278 | 23 | 04 | 049 | |
| 8192x8192 | 128 | 810,612121 | 790,822141 | 803,088016 | 802,357899 | 803,597817 | 802,0955988 |
| | 256 | 594,163771 | 594,452017 | 615,07659 | 616,221892 | 620,752189 | 608,1332918 |
| | 512 | 518,898974 | 505,487237 | 513.088196 | 514,726568 | 511,543329 | 512,664027 |
| 10240x10240 | 128 | 1014,218588 | 1027,942598 | 1053,823548 | 1042,924869 | 1046,165015 | 1037,014924 |
| | 256 | 1036,154706 | 1030,992011 | 1038,562635 | 1038,079011 | 1048,243225 | 1038,406318 |
| | 512 | 952,945122 | 950,609856 | 944,101049 | 935,243307 | 943,157265 | 945,2113198 |

## A3.2. Cache misses

| Dimensions | Block Size | Cache | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|---|---|
| 600x600 | 128 | L1 | 31444808 | 31401452 | 31403791 | 28536787 | 31407010 | 30838769,6 |
| | 128 | L2 | 28680424 | 24936550 | 31225214 | 28536787 | 27283187 | 28132432,4 |
| | 256 | L1 | 29769776 | 29775000 | 29770527 | 29774005 | 29775205 | 29772902,6 |
| | 256 | L2 | 68813422 | 67518394 | 65288460 | 66374134 | 66198475 | 66838577 |
| | 512 | L1 | 28867378 | 28869502 | 28867926 | 29094839 | 28866893 | 28913307,6 |
| | 512 | L2 | 61407484 | 61657570 | 61720179 | 63598132 | 62676612 | 62211995,4 |
| 1000x1000 | 128 | L1 | 143505157 | 143521431 | 143521038 | 143528247 | 143516208 | 143518416,2 |
| | 128 | L2 | 128072059 | 133781881 | 138882735 | 149511840 | 139913409 | 138032384,8 |
| | 256 | L1 | 135130227 | 135087338 | 135099852 | 135091788 | 135092353 | 135100311,6 |
| | 256 | L2 | 349669828 | 348734601 | 348475028 | 338943828 | 338899328 | 344944522,6 |
| | 512 | L1 | 130223901 | 130324774 | 130345297 | 130330506 | 130348169 | 130314529,4 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 512 | L2 | 288900151 | 287159804 | 293054043 | 287472680 | 286199963 | 288557328,2 |
| 1400x1400 | 128 | L1 | 392331618 | 391975441 | 391982476 | 391898979 | 392033401 | 392044383 |
| | 128 | L2 | 391840245 | 383950671 | 397629470 | 350353799 | 385208045 | 381796446 |
| | 256 | L1 | 372146021 | 372118474 | 372104313 | 372138257 | 37244857 | 372191128,4 |
| | 256 | L2 | 906640654 | 935338352 | 933954827 | 906232478 | 940069655 | 924447193,2 |
| | 512 | L1 | 358802894 | 358768340 | 358781094 | 358778883 | 358795565 | 358785355,2 |
| | 512 | L2 | 809215238 | 793306969 | 793968734 | 809133728 | 809670243 | 803058982,4 |
| 1800x1800 | 128 | L1 | 826848910 | 826855010 | 826255460 | 827564671 | 826342260 | 826773262,2 |
| | 128 | L2 | 162892946 | 475612693 | 176258794 | 281745221 | 262190633 | 271740057,4 |
| | 256 | L1 | 787474719 | 787487132 | 788218791 | 788186625 | 787361488 | 787745751 |
| | 256 | L2 | 2015066545 | 1948514619 | 2015448471 | 1992644160 | 1947127349 | 1983760229 |
| | 512 | L1 | 764449793 | 764114192 | 765285838 | 764381420 | 765254298 | 764697108,2 |
| | 512 | L2 | 1741196334 | 1733187895 | 1731282252 | 1738462698 | 1735852139 | 1735996264 |
| 2200x2200 | 128 | L1 | 1515233790 | 1514818320 | 1514574572 | 1514547717 | 1514750164 | 1514784913 |
| | 128 | L2 | 1017319652 | 961356285 | 863366504 | 825938345 | 888162339 | 911228625 |
| | 256 | L1 | 1445113232 | 1446177597 | 1445801933 | 1445957120 | 1446113309 | 1445832638 |
| | 256 | L2 | 3706792349 | 3636756644 | 3620036920 | 3634688811 | 3601307109 | 3639916367 |
| | 512 | L1 | 1399878668 | 1401459657 | 1401396476 | 1399743382 | 1400892269 | 1400674090 |
| | 512 | L2 | 3198717873 | 3156288535 | 3168276998 | 3141836321 | 3189348212 | 3170893588 |
| 2600x2600 | 128 | L1 | 2544591349 | 2545238814 | 2546046177 | 2546020251 | 2544583818 | 2545296082 |
| | 128 | L2 | 1545525348 | 1662937357 | 2009978035 | 2116460446 | 2075342897 | 1882048817 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 256 | L1 | 2388207092 | 2389428301 | 2389733472 | 2389484516 | 2388313823 | 2389033441 |
| | 256 | L2 | 5765525896 | 5948081676 | 5917271457 | 5865677974 | 5921656044 | 5883642609 |
| | 512 | L1 | 2311274983 | 2313078885 | 2311848160 | 2313533979 | 2312154810 | 2312378163 |
| | 512 | L2 | 5059467339 | 5115871723 | 5140701183 | 5088867062 | 5133359604 | 5107653382 |
| 3000x3000 | 128 | L1 | 3873571192 | 3873476962 | 3875726262 | 3873515220 | 3874356160 | 3874129159 |
| | 128 | L2 | 2740434242 | 2790803576 | 2874522104 | 2656938223 | 2872989333 | 2787137496 |
| | 256 | L1 | 3647906882 | 3647942066 | 3649419830 | 3647857161 | 3648330570 | 3648291302 |
| | 256 | L2 | 9424787125 | 9430000701 | 9127361124 | 9379230429 | 9357237376 | 9343723351 |
| | 512 | L1 | 3528785195 | 3530488617 | 3530958595 | 3530569818 | 3528237257 | 3529807896 |
| | 512 | L2 | 7781647836 | 7819509621 | 7802203291 | 7772971501 | 7776560576 | 7790578565 |
| 4096x4096 | 128 | L1 | 9720265732 | 9805629743 | 9828947101 | 9824304460 | 9823611613 | 9800551730 |
| | 128 | L2 | 30550639850 | 30396105260 | 30311366584 | 30746173163 | 30715040493 | 30543865070 |
| | 256 | L1 | 9117358007 | 9115646816 | 9116573636 | 9117286207 | 9117456975 | 9116864328 |
| | 256 | L2 | 21449495375 | 20734433131 | 21200499585 | 21326657980 | 21041272067 | 21150471628 |
| | 512 | L1 | 8795040074 | 8784904715 | 8784143871 | 8782520134 | 8779541156 | 8785229990 |
| | 512 | L2 | 18616164363 | 18536235666 | 18948935013 | 18611106022 | 18968174943 | 18736123201 |
| 6144x6144 | 128 | L1 | 32831082955 | 32813951502 | 32805006265 | 32811264674 | 32820621964 | 32816385472 |
| | 128 | L2 | 102323035163 | 103540169110 | 103581047041 | 103151420296 | 103129621290 | 103145058580 |
| | 256 | L1 | 30786020324 | 30785957678 | 30789904906 | 30788066187 | 30786760332 | 30787341885 |
| | 256 | L2 | 73422762323 | 73443550173 | 73356438168 | 73156410755 | 73157843185 | 73307400921 |
| | 512 | L1 | 29641605157 | 29697862231 | 29694482728 | 29696753573 | 29698412479 | 29685823234 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 512 | L2 | 64190256377 | 64094463800 | 63241338565 | 63356600374 | 63809781971 | 63738488217 |
| 8192x8192 | 128 | L1 | 77430805798 | 77977842306 | 77636704872 | 77652642798 | 77620541153 | 77663707385 |
| | 128 | L2 | 229020858001 | 225219601937 | 228412932908 | 228305304408 | 228185703496 | 227828880150 |
| | 256 | L1 | 731118739 45 | 731083931 48 | 730302847 47 | 730193782 29 | 729961209 52 | 73053210204 |
| | 256 | L2 | 161130337931 | 162089546167 | 160885116548 | 163916715151 | 164097720711 | 162423887302 |
| | 512 | L1 | 707682462 81 | 705285132 12 | 706552791 78 | 706894651 89 | 706285659 84 | 70654013969 |
| | 512 | L2 | 148995935900 | 147506902514 | 149409140046 | 149754341774 | 148058779033 | 148745019853 |
| 10240x10240 | 128 | L1 | 152030265097 | 152289066162 | 151889812347 | 152129883663 | 151998348810 | 152067475216 |
| | 128 | L2 | 476605171466 | 476679753965 | 480811351217 | 476284361463 | 478800435779 | 477836214778 |
| | 256 | L1 | 142301639236 | 142354637247 | 142490791914 | 142434195336 | 142514516633 | 142419156073 |
| | 256 | L2 | 332724139022 | 335295190652 | 338163831088 | 337190505131 | 338372009137 | 336349135006 |
| | 512 | L1 | 137336485620 | 137311900989 | 137129936497 | 137046745624 | 137080290444 | 137181071835 |
| | 512 | L2 | 291589769375 | 293810621151 | 293544779850 | 291469438725 | 292833349182 | 292649591657 |

# A4. Multi-Core Line Multiplication

## A4.1. OMP1

### A4.1.1. Execution time (s)

| Dimensions | Threads | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|---|
| 600x600 | 2 | 0,133173 | 0,126825 | 0,127318 | 0,12552 | 0,125019 | 0,127571 |
| | 3 | 0,14065 | 0,14016 | 0,138799 | 0,136753 | 0,143946 | 0,1400616 |
| | 4 | 0,121696 | 0,120065 | 0,120519 | 0,121076 | 0,137687 | 0,1242086 |
| 1000x1000 | 2 | 0,605229 | 0,60219 | 0,607021 | 0,610244 | 0,59670 | 0,6042788 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | 9 | | | 1 | |
| | 3 | 0,59231 | 0,594017 | 0,590376 | 0,597523 | 0,595357 | 0,5939166 |
| | 4 | 0,548989 | 0,565182 | 0,545893 | 0,555952 | 0,550534 | 0,55331 |
| 1400x1400 | 2 | 1,72948 | 1,641767 | 1,716196 | 1,643082 | 1,672229 | 1,6805508 |
| | 3 | 1,593967 | 1,603807 | 1,637393 | 1,614101 | 1,640456 | 1,6179448 |
| | 4 | 1,493687 | 1,492441 | 1,488367 | 1,493782 | 1,487045 | 1,4910644 |
| 1800x1800 | 2 | 3,590526 | 3,489502 | 3,494568 | 3,483293 | 3,478633 | 3,5073044 |
| | 3 | 3,413368 | 3,40402 | 3,527402 | 3,398753 | 3,455244 | 3,4397574 |
| | 4 | 3,245397 | 3,236904 | 3,28394 | 3,30898 | 3,35934 | 3,2869122 |
| 2200x2200 | 2 | 6,789767 | 6,71304 | 6,581465 | 6,588806 | 6,576794 | 6,6499744 |
| | 3 | 7,4101 | 6,748105 | 6,684831 | 6,469265 | 6,896399 | 6,84174 |
| | 4 | 6,029528 | 6,145899 | 6,066905 | 6,062828 | 6,071029 | 6,0752378 |
| 2600x2600 | 2 | 10,496629 | 10,49743 | 10,589845 | 10,685093 | 10,752799 | 10,6043592 |
| | 3 | 11,130943 | 10,450068 | 10,598962 | 12,266784 | 10,922482 | 11,0738478 |
| | 4 | 10,108168 | 10,129539 | 10,133121 | 10,167169 | 10,271554 | 10,1619102 |
| 3000x3000 | 2 | 16,109718 | 16,143745 | 16,732097 | 16,950232 | 17,318446 | 16,6508476 |
| | 3 | 17,059422 | 17,784951 | 17,778587 | 17,738587 | 18,765854 | 17,8254802 |
| | 4 | 16,46765 | 16,636279 | 16,857979 | 16,866889 | 17,004635 | 16,7666864 |
| 4096x4096 | 2 | 38,982241 | 36,96382 | 37,182291 | 38,717788 | 37,208012 | 37,8108304 |
| | 3 | 39,117728 | 38,640117 | 37,045604 | 38,613794 | 39,122489 | 38,5079464 |
| | 4 | 36,172141 | 36,248772 | 36,394238 | 36,392645 | 36,329071 | 36,3073734 |
| 6144x6144 | 2 | 126,87527 | 126,665 | 127,1059 | 127,0127 | 127,115 | 126,9551096 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 9 | 833 | 58 | 28 | 75 | |
| | 3 | 133,887349 | 133,481616 | 132,745752 | 131,375752 | 134,310556 | 133,160205 |
| | 4 | 124,703687 | 125,201713 | 124,735206 | 124,849723 | 124,72487 | 124,8430398 |
| 8192x8192 | 2 | 298,841363 | 296,884687 | 297,55629 | 297,208629 | 298,74309 | 297,8468118 |
| | 3 | 311,094691 | 300,887257 | 303,097357 | 299,113801 | 313,728929 | 305,584407 |
| | 4 | 286,733685 | 288,988119 | 289,757055 | 289,607059 | 288,340651 | 288,6853138 |
| 10240x10240 | 2 | 582,119663 | 581,843125 | 579,897847 | 584,277291 | 579,793055 | 581,5861962 |
| | 3 | 614,496456 | 603,560405 | 605,630641 | 607,190975 | 608,10679 | 607,7970534 |
| | 4 | 559,749207 | 558,392613 | 548,760862 | 561,861382 | 560,172214 | 557,7872556 |

### A4.1.2. Cache misses

| Dimensions | Threads | Cache | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|---|---|
| 600x600 | 2 | L1 | 13612719 | 13594262 | 13598160 | 13589810 | 13597711 | 13598532,4 |
| | 2 | L2 | 27767061 | 27608734 | 27575887 | 27598695 | 27655033 | 27641082 |
| | 3 | L1 | 6692937 | 9223009 | 6354216 | 9212395 | 7583135 | 7813138,4 |
| | 3 | L2 | 13756862 | 18855231 | 12619679 | 18721672 | 15585448 | 15907778,4 |
| | 4 | L1 | 5582989 | 4890612 | 5600231 | 5247375 | 5840198 | 5432281 |
| | 4 | L2 | 11419465 | 9946891 | 11277501 | 10691339 | 11644050 | 10995849,2 |
| 1000x1000 | 2 | L1 | 63068327 | 62992640 | 62214554 | 63147670 | 62977378 | 62880113,8 |
| | 2 | L2 | 126856280 | 126537172 | 125390557 | 126502537 | 126538573 | 126365023,8 |
| | 3 | L1 | 38632096 | 36103898 | 30338111 | 45254113 | 32404723 | 36546588,2 |
| | 3 | L2 | 78339660 | 73058000 | 62971189 | 90254583 | 66805941 | 74285874,6 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 | L1 | 23232474 | 25244993 | 24374329 | 27256354 | 22781820 | 24577994 |
| | 4 | L2 | 47333124 | 51479693 | 49672622 | 55653149 | 46572839 | 50142285,4 |
| 1400x1400 | 2 | L1 | 178381731 | 173524213 | 176207422 | 173627759 | 173890081 | 175126241,2 |
| | 2 | L2 | 353663152 | 350738190 | 356165074 | 346912155 | 341610199 | 349817754 |
| | 3 | L1 | 115953689 | 130617825 | 131821208 | 130017582 | 118515650 | 125385190,8 |
| | 3 | L2 | 230861827 | 183937854 | 180322270 | 182601368 | 232712971 | 202087258 |
| | 4 | L1 | 114054805 | 152875072 | 149199273 | 143012906 | 148238998 | 141476210,8 |
| | 4 | L2 | 128826633 | 167428102 | 162890547 | 157075371 | 161137473 | 155471625,2 |
| 1800x1800 | 2 | L1 | 388486811 | 374533783 | 375999526 | 374896850 | 375740519 | 377931497,8 |
| | 2 | L2 | 739731495 | 733705130 | 735219566 | 741541306 | 744721158 | 738983731 |
| | 3 | L1 | 252043912 | 252327069 | 266028098 | 255121035 | 364119150 | 277927852,8 |
| | 3 | L2 | 491334790 | 490827595 | 500400555 | 506939610 | 450475736 | 487995657,2 |
| | 4 | L1 | 333417137 | 338646586 | 351762844 | 359927064 | 346113890 | 345973504,2 |
| | 4 | L2 | 325405277 | 335656823 | 368233082 | 383182607 | 359091909 | 354313939,6 |
| 2200x2200 | 2 | L1 | 1067934826 | 1064787815 | 1059835402 | 1058964979 | 1058982115 | 1062101027 |
| | 2 | L2 | 1367662891 | 1343901046 | 1336381061 | 1371056594 | 1374862507 | 1358772820 |
| | 3 | L1 | 846628481 | 774020057 | 819160204 | 791844719 | 763314151 | 798993522,4 |
| | 3 | L2 | 940022971 | 843740189 | 916034132 | 892039214 | 918103193 | 901987939,8 |
| | 4 | L1 | 631126469 | 654761529 | 643178927 | 659853466 | 650874415 | 647958961,2 |
| | 4 | L2 | 629613384 | 690325676 | 654932319 | 688328213 | 683030798 | 669246078 |
| 2600x2600 | 2 | L1 | 2209645563 | 2209721434 | 2209737100 | 2210022104 | 2209538386 | 2209732917 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 2 | L2 | 2205754212 | 2209140473 | 2207809742 | 2206747966 | 2219859031 | 2209862285 |
| | 3 | L1 | 1433146242 | 1302598335 | 1347636323 | 1446121791 | 1426414971 | 1391183532 |
| | 3 | L2 | 1404402501 | 1179532501 | 1277944137 | 1496671553 | 1457323014 | 1363174741 |
| | 4 | L1 | 1101344958 | 1089048820 | 1100225621 | 1088465611 | 1069338434 | 1089684689 |
| | 4 | L2 | 1194149457 | 1165329946 | 1191836541 | 1164961675 | 1104601368 | 1164175797 |
| 3000x3000 | 2 | L1 | 3393754204 | 3393671982 | 3393452051 | 3393751167 | 3393534011 | 3393632683 |
| | 2 | L2 | 3494474423 | 3503477654 | 3526852298 | 3443303479 | 3451127133 | 3483846997 |
| | 3 | L1 | 2262559887 | 2187433833 | 2255688832 | 2265179036 | 2231207065 | 2240413731 |
| | 3 | L2 | 2307461230 | 2217399728 | 2330782990 | 2406589341 | 2325362243 | 2317519106 |
| | 4 | L1 | 1666412031 | 1688481159 | 1672833316 | 1694529375 | 1677916906 | 1680034557 |
| | 4 | L2 | 1753155823 | 1806365926 | 1789231279 | 1839116356 | 1798096646 | 1797193206 |
| 4096x4096 | 2 | L1 | 8752719079 | 8768154559 | 8769713853 | 8784425038 | 8776317309 | 8770265968 |
| | 2 | L2 | 8065787720 | 7985128669 | 8097624550 | 8003566977 | 8064597294 | 8043341042 |
| | 3 | L1 | 5864456194 | 5837607850 | 5610102481 | 5844045886 | 5830224713 | 5797287425 |
| | 3 | L2 | 6003276443 | 6001615191 | 5313733334 | 5846532702 | 5842094263 | 5801450387 |
| | 4 | L1 | 4369807291 | 4341843783 | 4371657950 | 4378940633 | 4386892722 | 4369828476 |
| | 4 | L2 | 4580303128 | 4449281399 | 4551213656 | 4560205037 | 4608986183 | 4549997881 |
| 6144x6144 | 2 | L1 | 29566295259 | 29589991186 | 29644273581 | 29599184737 | 29598602222 | 29599669397 |
| | 2 | L2 | 27226769956 | 27499766011 | 27979598065 | 27911025756 | 28502486186 | 27823929195 |
| | 3 | L1 | 19702459915 | 19763197087 | 19767663643 | 19785591067 | 19817679810 | 19767318304 |
| | 3 | L2 | 19543297901 | 20379357359 | 20277496512 | 20757859700 | 21171178412 | 20425837977 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 | L1 | 148397 00420 | 148625 15590 | 148612 52477 | 148405 20523 | 148289 43116 | 14846586425 |
| | 4 | L2 | 164004 77384 | 163379 46733 | 164888 93332 | 164637 78859 | 165177 35010 | 16441766264 |
| 8192x8192 | 2 | L1 | 693853 32677 | 694061 70015 | 694085 25949 | 696049 55139 | 694026 87539 | 69441534264 |
| | 2 | L2 | 659029 50129 | 659067 24229 | 665947 81605 | 678253 42070 | 668631 40114 | 66618587629 |
| | 3 | L1 | 461436 29676 | 462262 51832 | 451232 19405 | 444326 67401 | 461078 18181 | 45606717299 |
| | 3 | L2 | 503219 74190 | 443548 82478 | 489989 00874 | 467026 22920 | 514170 07595 | 48359077611 |
| | 4 | L1 | 345409 53537 | 343795 53323 | 345442 54262 | 345176 94885 | 344409 23409 | 34484675883 |
| | 4 | L2 | 410662 94437 | 414374 45822 | 406718 03728 | 406965 62233 | 415482 03384 | 41084061921 |
| 10240x10240 | 2 | L1 | 137350 994016 | 137353 318168 | 137215 543092 | 137258 500420 | 137014 169688 | 137238505077 |
| | 2 | L2 | 132491 162753 | 130815 410155 | 131472 450010 | 133469 808979 | 130856 562904 | 131821078960 |
| | 3 | L1 | 908812 58986 | 909924 16828 | 908468 11640 | 913580 10505 | 911787 69296 | 91051453451 |
| | 3 | L2 | 126664 310820 | 131359 770690 | 131827 182425 | 108984 862723 | 125874 989655 | 124942223263 |
| | 4 | L1 | 684043 42201 | 680642 68458 | 681398 55366 | 685327 65003 | 683269 36632 | 68293633532 |
| | 4 | L2 | 117200 084760 | 115575 126225 | 115778 546318 | 117756 129817 | 115765 433829 | 116415064190 |

### A4.1.3. GFLOPS

| Dimensions | OMP1 ( 2 ) | OMP1 (3) | OMP1 (4) |
|---|---|---|---|
| 600 | 3,386350 | 3,084357 | 3,478020 |
| 1000 | 3,309731 | 3,367476 | 3,614610 |
| 1400 | 3,265596 | 3,391958 | 3,680592 |
| 1800 | 3,325631 | 3,390937 | 3,548619 |
| 2200 | 3,202418 | 3,112658 | 3,505377 |
| 2600 | 3,314863 | 3,174326 | 3,459192 |
| 3000 | 3,243078 | 3,029371 | 3,220672 |
| 4096 | 3,634910 | 3,569106 | 3,785428 |
| 6144 | 3,653705 | 3,483447 | 3,715517 |

| | | | |
|---|---|---|---|
| 8192 | 3,691534 | 3,598062 | 3,808686 |
| 10240 | 3,692460 | 3,533225 | 3,850005 |

A4.1.4. Speedup

| Speedup (CPP / OMP 1) | | |
|---|---|---|
| 1,483218 | 1,350946 | 1,523370 |
| 1,341267 | 1,364668 | 1,464819 |
| 1,331493 | 1,383014 | 1,500700 |
| 1,400174 | 1,427670 | 1,494058 |
| 1,328428 | 1,291194 | 1,454102 |
| 1,428422 | 1,367862 | 1,490615 |
| 1,499958 | 1,401116 | 1,489595 |
| 1,630204 | 1,600692 | 1,697709 |
| 1,590325 | 1,516218 | 1,617230 |
| 1,605274 | 1,564627 | 1,656218 |
| 1,586202 | 1,517798 | 1,653880 |

A4.1.5. Efficiency

| Efficiency OMP 1 | Logical Processors | | |
|---|---|---|---|
| Dimensions | 2 | 3 | 4 |
| 600 | 0,741609 | 0,450315 | 0,380842 |
| 1000 | 0,670633 | 0,454889 | 0,366205 |
| 1400 | 0,665746 | 0,461005 | 0,375175 |
| 1800 | 0,700087 | 0,475890 | 0,373514 |
| 2200 | 0,664214 | 0,430398 | 0,363525 |
| 2600 | 0,714211 | 0,455954 | 0,372654 |
| 3000 | 0,749979 | 0,467039 | 0,372399 |
| 4096 | 0,815102 | 0,533564 | 0,424427 |
| 6144 | 0,795163 | 0,505406 | 0,404308 |
| 8192 | 0,802637 | 0,521542 | 0,414054 |
| 10240 | 0,793101 | 0,505933 | 0,413470 |

## A4.2. OMP2

### A4.2.1. Execution time (s)

| Dimensions | Threads | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|---|
| 600x600 | 2 | 0,38163 | 0,392733 | 0,379815 | 0,379744 | 0,390338 | 0,384852 |
| | 3 | 0,397561 | 0,394668 | 0,400662 | 0,400201 | 0,392408 | 0,3971 |
| | 4 | 0,397735 | 0,399575 | 0,39723 | 0,397645 | 0,394297 | 0,3972964 |
| 1000x1000 | 2 | 1,248541 | 1,228382 | 1,233716 | 1,22875 | 1,225606 | 1,232999 |
| | 3 | 1,300675 | 1,31213 | 1,302337 | 1,308675 | 1,297564 | 1,3042762 |
| | 4 | 1,218565 | 1,220448 | 1,227221 | 1,259262 | 1,253017 | 1,2357026 |
| 1400x1400 | 2 | 2,792621 | 2,804032 | 2,792309 | 2,791545 | 2,793258 | 2,794753 |
| | 3 | 2,934621 | 3,051819 | 2,927747 | 2,973609 | 2,998605 | 2,9772802 |
| | 4 | 2,755612 | 2,773105 | 2,803406 | 2,856783 | 2,830516 | 2,8038844 |
| 1800x1800 | 2 | 5,312119 | 5,333321 | 5,321964 | 5,352766 | 5,32603 | 5,32924 |
| | 3 | 5,666931 | 5,745775 | 5,67773 | 5,727682 | 5,527767 | 5,669177 |
| | 4 | 5,273973 | 5,228035 | 5,198614 | 5,212823 | 5,306235 | 5,243936 |
| 2200x2200 | 2 | 10,030336 | 9,174896 | 9,179749 | 9,19493 | 9,408216 | 9,3976254 |
| | 3 | 11,010673 | 9,775009 | 9,712424 | 9,613366 | 9,708169 | 9,9639282 |
| | 4 | 9,298742 | 9,303683 | 9,205017 | 9,461139 | 9,556161 | 9,3649484 |
| 2600x2600 | 2 | 14,488955 | 14,308279 | 14,58885 | 14,35973 | 14,425365 | 14,4342358 |
| | 3 | 14,71801 | 14,920742 | 14,915261 | 15,533724 | 15,053726 | 15,0282926 |
| | 4 | 13,529372 | 13,560038 | 13,591233 | 13,149842 | 13,135386 | 13,3931742 |
| 3000x3000 | 2 | 22,581963 | 21,175399 | 21,35833 | 21,272864 | 21,266345 | 21,5309802 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 3 | 23,267658 | 22,833769 | 22,930497 | 23,673099 | 22,753468 | 23,0916982 |
| | 4 | 20,035861 | 20,585245 | 20,94112 | 21,007768 | 21,170657 | 20,7481302 |
| 4096x4096 | 2 | 47,491802 | 47,321194 | 50,962338 | 47,709228 | 47,666066 | 48,2301256 |
| | 3 | 49,563646 | 50,576377 | 49,803411 | 50,081797 | 50,695248 | 50,1440958 |
| | 4 | 46,703251 | 47,654319 | 46,879156 | 46,74238 | 46,748251 | 46,9454714 |
| 6144x6144 | 2 | 149,702326 | 150,156668 | 150,273697 | 150,388595 | 150,552286 | 150,2147144 |
| | 3 | 157,139126 | 157,093076 | 156,945781 | 155,628854 | 156,713785 | 156,7041244 |
| | 4 | 150,539076 | 150,728752 | 149,802398 | 150,969813 | 150,219092 | 150,4518262 |
| 8192x8192 | 2 | 336,053894 | 335,618405 | 336,593993 | 337,412598 | 336,230105 | 336,381799 |
| | 3 | 363,256032 | 362,968359 | 361,744941 | 364,851638 | 362,889184 | 363,1420308 |
| | 4 | 331,77254 | 331,339003 | 329,286161 | 332,159449 | 329,241117 | 330,759654 |
| 10240x10240 | 2 | 641,751855 | 641,748247 | 642,251178 | 642,817221 | 642,834293 | 642,2805588 |
| | 3 | 689,663426 | 694,615393 | 690,261908 | 688,60193 | 689,272338 | 690,482999 |
| | 4 | 625,508442 | 625,992264 | 629,18624 | 631,184514 | 632,690873 | 628,9124666 |

A4.2.2. Cache misses

| Dimensions | Threads | Cache | Exp 1 | Exp 2 | Exp 3 | Exp 4 | Exp 5 | Average |
|---|---|---|---|---|---|---|---|---|
| 600x600 | 2 | L1 | 17934777 | 17990196 | 17900486 | 17943142 | 17954300 | 17944580,2 |
| | 2 | L2 | 46667725 | 46963873 | 46556308 | 46801777 | 46772998 | 46752536,2 |
| | 3 | L1 | 13354228 | 10612887 | 13275614 | 12527945 | 10469946 | 12048124 |
| | 3 | L2 | 40717795 | 26220832 | 40446769 | 27240282 | 24366039 | 31798343,4 |
| | 4 | L1 | 7988640 | 7979790 | 8000542 | 7992331 | 8026984 | 7997657,4 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 | L2 | 18417481 | 18253828 | 18473855 | 18479500 | 18986992 | 18522331,2 |
| 1000x1000 | 2 | L1 | 75620721 | 75615765 | 75675222 | 75772758 | 75765014 | 75689896 |
| | 2 | L2 | 175488089 | 175910957 | 176082169 | 176441778 | 176479449 | 176080488,4 |
| | 3 | L1 | 45934412 | 51282643 | 46697259 | 47167571 | 46127937 | 47441964,4 |
| | 3 | L2 | 95126360 | 117834253 | 99255200 | 102680147 | 98794273 | 102738046,6 |
| | 4 | L1 | 34867026 | 34904169 | 34838203 | 34804528 | 34896031 | 34861991,4 |
| | 4 | L2 | 64782999 | 65307426 | 64718851 | 64542914 | 65477510 | 64965940 |
| 1400x1400 | 2 | L1 | 197165579 | 197089662 | 197049613 | 197073725 | 197042426 | 197084201 |
| | 2 | L2 | 437889701 | 437176310 | 437131334 | 429076416 | 428636912 | 433982134,6 |
| | 3 | L1 | 133430609 | 119705311 | 133318412 | 140324243 | 124707916 | 130297298,2 |
| | 3 | L2 | 291572877 | 238725054 | 290314618 | 323902777 | 259952659 | 280893597 |
| | 4 | L1 | 92320346 | 92235093 | 10838000 | 10831653 | 10840884 | 101932035,2 |
| | 4 | L2 | 164944815 | 165744047 | 265453110 | 265713767 | 265037305 | 225378608,8 |
| 1800x1800 | 2 | L1 | 410290155 | 410459461 | 410256888 | 410647841 | 410083332 | 410347535,4 |
| | 2 | L2 | 871124028 | 868904082 | 864935621 | 869710770 | 867057534 | 868346407 |
| | 3 | L1 | 287315969 | 259377686 | 287363443 | 258850158 | 282109812 | 275003413,6 |
| | 3 | L2 | 628088738 | 472472239 | 634826124 | 460355101 | 615472539 | 562242948,2 |
| | 4 | L1 | 223387623 | 223403313 | 223243367 | 223303513 | 223192873 | 223306137,8 |
| | 4 | L2 | 541247805 | 540991914 | 540831391 | 541874961 | 541876751 | 541364564,4 |
| 2200x2200 | 2 | L1 | 762956383 | 738031047 | 737129410 | 738077614 | 740513518 | 743341594,4 |
| | 2 | L2 | 1535857146 | 1520649298 | 1525994357 | 1535439984 | 1539730307 | 1531534218 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 3 | L1 | 505941971 | 517489590 | 496706987 | 537252075 | 496265777 | 510731280 |
| | 3 | L2 | 981508963 | 1051066833 | 892976296 | 103906585 | 876441020 | 968211739,4 |
| | 4 | L1 | 384869944 | 369468030 | 369513621 | 385022148 | 375378813 | 376850511,2 |
| | 4 | L2 | 907303114 | 786623826 | 784762936 | 901754919 | 837752940 | 843639547 |
| 2600x2600 | 2 | L1 | 1211421660 | 1203120500 | 1208861290 | 1204814707 | 1208580693 | 1207359770 |
| | 2 | L2 | 2427999705 | 2433526698 | 2430402276 | 2428151569 | 2423068742 | 2428629798 |
| | 3 | L1 | 948954466 | 915981370 | 940603080 | 863186446 | 827918654 | 899328803,2 |
| | 3 | L2 | 1747087599 | 1563264682 | 1695303061 | 1674656223 | 1743861565 | 1684834626 |
| | 4 | L1 | 644737327 | 644435304 | 645480522 | 634107761 | 632855253 | 640323233,4 |
| | 4 | L2 | 1504724964 | 1505435764 | 1506357732 | 1348038392 | 1345181651 | 1441947701 |
| 3000x3000 | 2 | L1 | 1930120184 | 1859812952 | 1858569484 | 1848272391 | 1846852159 | 1868725434 |
| | 2 | L2 | 3705189645 | 3705673249 | 3656428785 | 3639458406 | 3640081243 | 3669366266 |
| | 3 | L1 | 1736530697 | 1254527857 | 1253922995 | 1708391329 | 1761754727 | 1543025521 |
| | 3 | L2 | 2462842165 | 2621504586 | 2703924176 | 2315774839 | 2631788106 | 2547166774 |
| | 4 | L1 | 905846775 | 957632429 | 1046399505 | 1047636463 | 1047008788 | 1000904792 |
| | 4 | L2 | 1668152493 | 2029079223 | 2252678161 | 2225345878 | 2226998237 | 2080450798 |
| 4096x4096 | 2 | L1 | 5204574060 | 5136114436 | 5455944146 | 5151830367 | 5145957473 | 5218884096 |
| | 2 | L2 | 9181310336 | 8907427120 | 9181747211 | 8975390300 | 8973244651 | 9043823924 |
| | 3 | L1 | 5988239676 | 3251776458 | 6046026871 | 5767509820 | 3965259880 | 5003762541 |
| | 3 | L2 | 6064302670 | 6334522496 | 5801353321 | 5903714678 | 6295252574 | 6079829148 |
| | 4 | L1 | 2811095098 | 2711734365 | 2735351785 | 2772040053 | 2821107156 | 2770265691 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 | L2 | 5003717207 | 5213006300 | 5125317866 | 4970852607 | 4930451387 | 5048669073 |
| 6144x6144 | 2 | L1 | 30197331804 | 30192350349 | 30198544785 | 30198089847 | 30201676794 | 30197598716 |
| | 2 | L2 | 29152037769 | 28899008807 | 29436596548 | 29148687533 | 29353581122 | 29197982356 |
| | 3 | L1 | 18044223484 | 19066021533 | 19054507387 | 17160402653 | 17924054987 | 18249842009 |
| | 3 | L2 | 20651737983 | 20596117234 | 20578774820 | 20500655136 | 20718678637 | 20609192762 |
| | 4 | L1 | 15385243634 | 15394215465 | 15406704798 | 15309434747 | 15315723686 | 15362264466 |
| | 4 | L2 | 16465581201 | 16499746578 | 16505463109 | 15756135478 | 15813654994 | 16208116272 |
| 8192x8192 | 2 | L1 | 70778434314 | 70729280637 | 70783479056 | 70878024480 | 71156449771 | 70865133652 |
| | 2 | L2 | 70415755889 | 70808735314 | 70135212063 | 69534770754 | 69127113918 | 70004317588 |
| | 3 | L1 | 47471001831 | 47440562169 | 47402358968 | 47461414059 | 47387517226 | 47432570851 |
| | 3 | L2 | 50614530874 | 50142160867 | 48906960082 | 49148547212 | 48239804742 | 49410400755 |
| | 4 | L1 | 35909914770 | 35763074521 | 35762685589 | 35766881422 | 35988972708 | 35838305802 |
| | 4 | L2 | 38339847757 | 36743041951 | 37708375167 | 36863581233 | 36979930693 | 37326955360 |
| 10240x10240 | 2 | L1 | 139260755119 | 139043162331 | 139288618587 | 139303043929 | 139025062021 | 139184128397 |
| | 2 | L2 | 131678043326 | 134153828495 | 130474122403 | 133045539070 | 132506920917 | 132371690842 |
| | 3 | L1 | 93640583500 | 93484566928 | 93752079215 | 93660032919 | 93650833122 | 93637619137 |
| | 3 | L2 | 95621427104 | 96673664445 | 95562395142 | 97035062093 | 93923750828 | 95763259922 |
| | 4 | L1 | 70479437459 | 70449068230 | 70227340798 | 70247376418 | 70365247048 | 70353693991 |
| | 4 | L2 | 74823954937 | 74115311990 | 74222311104 | 73798808896 | 72204918511 | 73833061088 |

### A4.2.3. GFLOPS

| Dimensions | OMP2 ( 2 ) | OMP2 (3) | OMP2 (4) |
|---|---|---|---|

| | | | |
|---|---|---|---|
| 600 | 1,122509 | 1,087887 | 1,087349 |
| 1000 | 1,622061 | 1,533418 | 1,618512 |
| 1400 | 1,963680 | 1,843293 | 1,957285 |
| 1800 | 2,188680 | 2,057441 | 2,224283 |
| 2200 | 2,266104 | 2,137310 | 2,274011 |
| 2600 | 2,435321 | 2,339055 | 2,624621 |
| 3000 | 2,508014 | 2,338503 | 2,602644 |
| 4096 | 2,849650 | 2,740880 | 2,927630 |
| 6144 | 3,087956 | 2,960078 | 3,083090 |
| 8192 | 3,268642 | 3,027773 | 3,324201 |
| 10240 | 3,343529 | 3,110118 | 3,414599 |

A4.2.4. Speedup

| Speedup (CPP / OMP 2) | | |
|---|---|---|
| 0,491658 | 0,476494 | 0,476258 |
| 0,657340 | 0,621417 | 0,655902 |
| 0,800658 | 0,751572 | 0,798050 |
| 0,921489 | 0,866235 | 0,936479 |
| 0,940026 | 0,886600 | 0,943306 |
| 1,049415 | 1,007932 | 1,130986 |
| 1,159983 | 1,081582 | 1,203751 |
| 1,278026 | 1,229244 | 1,312999 |
| 1,344076 | 1,288415 | 1,341957 |
| 1,421378 | 1,316636 | 1,445538 |
| 1,436308 | 1,336040 | 1,466838 |

A4.2.5. Efficiency

| Efficiency OMP 2 | Logical Processors | | |
|---|---|---|---|
| Dimensions | 2 | 3 | 4 |
| 600 | 0,245829 | 0,158831 | 0,119065 |
| 1000 | 0,328670 | 0,207139 | 0,163975 |
| 1400 | 0,400329 | 0,250524 | 0,199513 |
| 1800 | 0,460745 | 0,288745 | 0,234120 |
| 2200 | 0,470013 | 0,295533 | 0,235827 |
| 2600 | 0,524707 | 0,335977 | 0,282747 |
| 3000 | 0,579992 | 0,360527 | 0,300938 |

| | | | |
|---|---|---|---|
| 4096 | 0,639013 | 0,409748 | 0,328250 |
| 6144 | 0,672038 | 0,429472 | 0,335489 |
| 8192 | 0,710689 | 0,438879 | 0,361385 |
| 10240 | 0,718154 | 0,445347 | 0,366710 |