CPSC 340 Assignment 2 (due Friday January 26th at 9:00pm)

Instructions

Rubric: {mechanics:5}

The above points are allocated for compliance with the CPSC 340 homework submission instructions: `https://github.ugrad.cs.ubc.ca/CPSC340-2017W-T2/home/blob/master/homework_instructions.md`

NOTE: for this assignment you'll need to separately download the data from the home repo. It can't be delivered in the normal way due to a limitation of the way we're using GitHub.

1 Naive Bayes

In this section we'll implement naive Bayes, a very fast classification method that is often surprisingly accurate for text data with simple representations like bag of words.

1.1 Naive Bayes by Hand

Rubric: {reasoning:3}

Consider the dataset below, which has 10 training examples and 2 features:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad y = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Suppose you believe that a naive Bayes model would be appropriate for this dataset, and you want to classify the following test example:

$$\hat{x} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

(a) Compute the estimates of the class prior probabilities (you don't need to show any work):

- $p(y = 1)$.
- $p(y = 0)$.

(b) Compute the estimates of the 4 conditional probabilities required by naive Bayes for this example (you don't need to show any work):

- $p(x_1 = 1|y = 1)$.
- $p(x_2 = 0|y = 1)$.
- $p(x_1 = 1|y = 0)$.
- $p(x_2 = 0|y = 0)$.

(c) Under the naive Bayes model and your estimates of the above probabilities, what is the most likely label for the test example? (Show your work.)

1.2 Bag of Words

Rubric: {reasoning:3}

If you run `python main.py -q 1.2`, it will load the following dataset:

1. X: A sparse binary matrix. Each row corresponds to a newsgroup post, and each column corresponds to whether a particular word was used in the post. A value of 1 means that the word occured in the post.
2. *wordlist*: The set of words that correspond to each column.
3. y: A vector with values 1 through 4, with the value corresponding to the newsgroup that the post came from.
4. *groupnames*: The names of the four newsgroups.
5. $Xvalidate$ and $yvalidate$: the word lists and newsgroup labels for additional newsgroup posts.

Answer the following:

1. Which word corresponds to column 50 of X?
2. Which words are present in training example 500?
3. Which newsgroup name does training example 500 come from?

1.3 Naive Bayes Implementation

Rubric: {code:5}

If you run `python main.py -q 1.3` it will load the newsgroups dataset and report the test error for a random forest, and also fit the basic naive Bayes model and report the test error.

The `predict()` function of the naive Bayes classifier is already implemented. However, in `fit()` the calculation of the variable `p_xy` is incorrect (right now, it just sets all values to 1/2). Modify this function so that `p_xy` correctly computes the conditional probabilities of these values based on the frequencies in the data set. Hand in your code and the validation error that you obtain. Also, briefly comment on the accuracy as compared to the random forest and scikit-learn's naive Bayes implementation.

1.4 Laplace smoothing

Rubric: {code:1}

Do the following:

1. Modify your code so that it uses Laplace smoothing, with β as a parameter taken in by the constructor.

2. Did you need to modify your fit function, predict function, or both?
3. Take a look at the documentation for the scikit-learn version of naive Bayes the code is using. How much Laplace smoothing does it use by default? Using the same amount of smoothing with your code, do you get the same results?

1.5 Runtime of Naive Bayes for Discrete Data

Rubric: {reasoning:3}

Assume you have the following setup:

- The training set has n objects each with d features.
- The test set has t objects with d features.
- Each feature can have up to c discrete values (you can assume $c \leq n$).
- There are k class labels (you can assume $k \leq n$)

You can implement the training phase of a naive Bayes classifier in this setup in $O(nd)$, since you only need to do a constant amount of work for each $X(i, j)$ value. (You do not have to actually implement it in this way for the previous question, but you should think about how this could be done). What is the cost of classifying t test examples with the model?

2 Random Forests

2.1 Implementation

Rubric: {reasoning:7}

The file *vowels.pkl* contains a supervised learning dataset where we are trying to predict which of the 11 "steady-state" English vowels that a speaker is trying to pronounce.

You are provided with a `RandomStump` class that differs from `DecisionStump` in two ways: it uses the information gain splitting criterion (instead of classification accuracy), and it only considers $\lfloor \sqrt{d} \rfloor$ randomly-chosen features.[1] You are also provided with a `RandomTree` class that is exactly the same as `DecisionTree` except that it uses `RandomStump` instead of `DecisionStump` and it takes a bootstrap sample of the data before fitting. In other words, `RandomTree` is the entity we discussed in class, which makes up a random forest.

If you run `python main.py -q 2` it will fit a deep `DecisionTree` using the information gain splitting criterion. You will notice that the model overfits badly.

1. Why doesn't the random tree model have a training error of 0?
2. Create a class `RandomForest` in a file called `random_forest.py` that takes in hyperparameters `num_trees` and `max_depth` and fits `num_trees` random trees each with maximum depth `max_depth`. For prediction, have all trees predict and then take the mode.
3. Using 50 trees, and a max depth of ∞, report the training and testing error. Compare this to what we got with a single `DecisionTree` and with a single `RandomTree`. Are the results what you expected? Discuss.

[1] The notation $\lfloor x \rfloor$ means the "floor" of x, or "x rounded down". You can compute this with `np.floor(x)` or `math.floor(x)`.

4. Compare your implementation with scikit-learn's `RandomForestClassifier` for both speed and accuracy, and briefly discuss. You can use all default hyperparameters if you wish, or you can try changing them.

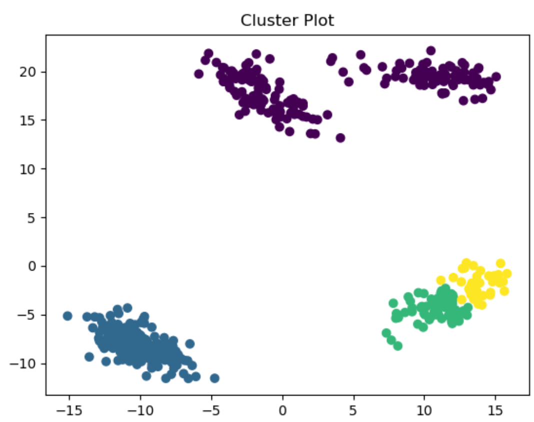
2.2 Very-Short Answer Questions

Rubric: {reasoning:3}

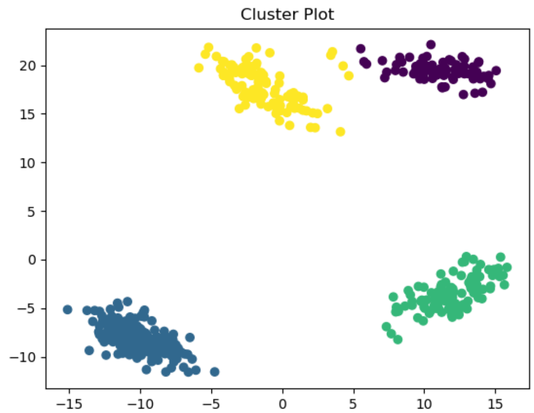
1. What is a a disadvantage of using a very-large number of trees in a random forest classifier?
2. Your random forest classifier has a training error of 0 and a very high test error. Which ones of the following could help performance?
   (a) Increase the maximum depth of the trees in your forest.
   (b) Decrease the maximum depth of the trees in your forest.
   (c) Increase the amout of data you consider for each tree (Collect more data and use $2n$ objects instead of n).
   (d) Decrease the amount of data you consider for each tree (Use $0.8n$ objects instead of n).
   (e) Increase the number of features you consider for each tree.
   (f) Decrease the number of features you consider for each tree.
3. Suppose that you were training on raw audio segments and trying to recognize vowel sounds. What could you do to encourage the final classifier to be invariant to translation?

3 Clustering

If you run `python main.py -q 3`, it will load a dataset with two features and a very obvious clustering structure. It will then apply the k-means algorithm with a random initialization. The result of applying the algorithm will thus depend on the randomization, but a typical run might look like this:



(Note that the colours are arbitrary – this is the label switching issue.) But the 'correct' clustering (that was used to make the data) is this:



3.1 Selecting among k-means Initializations

Rubric: {reasoning:5}

If you run the demo several times, it will find different clusterings. To select among clusterings for a *fixed* value of k, one strategy is to minimize the sum of squared distances between examples x_i and their means w_{y_i},

$$f(w_1, w_2, \ldots, w_k, y_1, y_2, \ldots, y_n) = \sum_{i=1}^{n} \|x_i - w_{y_i}\|_2^2 = \sum_{i=1}^{n} \sum_{j=1}^{d} (x_{ij} - w_{y_i j})^2.$$

where y_i is the index of the closest mean to x_i. This is a natural criterion because the steps of k-means alternately optimize this objective function in terms of the w_c and the y_i values.

1. In the *kmeans.py* file, add a new function called *error* that takes the same input as the *predict* function but that returns the value of this above objective function. Hand in your code.
2. What trend do you observe if you print the value of *error* after each iteration of the k-means algorithm?
3. Using `plot_2dclustering`, output the clustering obtained by running k-means 50 times (with $k = 4$) and taking the one with the lowest error.
4. Looking at the hyperparameters of scikit-learn's `KMeans`, explain the first four (`n_clusters`, `init`, `n_init`, `max_iter`) very briefly.

3.2 Selecting k in k-means

Rubric: {reasoning:5}

We now turn to the task of choosing the number of clusters k.

1. Explain why the *error* function should not be used to choose k.
2. Explain why even evaluating the *error* function on test data still wouldn't be a suitable approach to choosing k.
3. Hand in a plot of the minimum error found across 50 random initializations, as a function of k, taking k from 1 to 10.

4. The *elbow method* for choosing k consists of looking at the above plot and visually trying to choose the k that makes the sharpest "elbow" (the biggest change in slope). What values of k might be reasonable according to this method? Note: there is not a single correct answer here; it is somewhat open to interpretation and there is a range of reasonable answers.

3.3 k-medians

Rubric: {reasoning:5}

The data in *clusterData2.pkl* is the exact same as the above data, except it has 4 outliers that are very far away from the data.

1. Using the `plot_2dclustering` function, output the clustering obtained by running k-means 50 times (with $k = 4$) and taking the one with the lowest error. Are you satisfied with the result?
2. What values of k might be chosen by the elbow method for this dataset?
3. Implement the k-*medians* algorithm, which assigns examples to the nearest w_c in the L1-norm and to updates the w_c by setting them to the "median" of the points assigned to the cluster (we define the d-dimensional median as the concatenation of the median of the points along each dimension). Hand in your code and plot obtained with 50 random initializations for $k = 4$.
4. Using the L1-norm version of the error (where y_i now represents the closest median in the L1-norm),

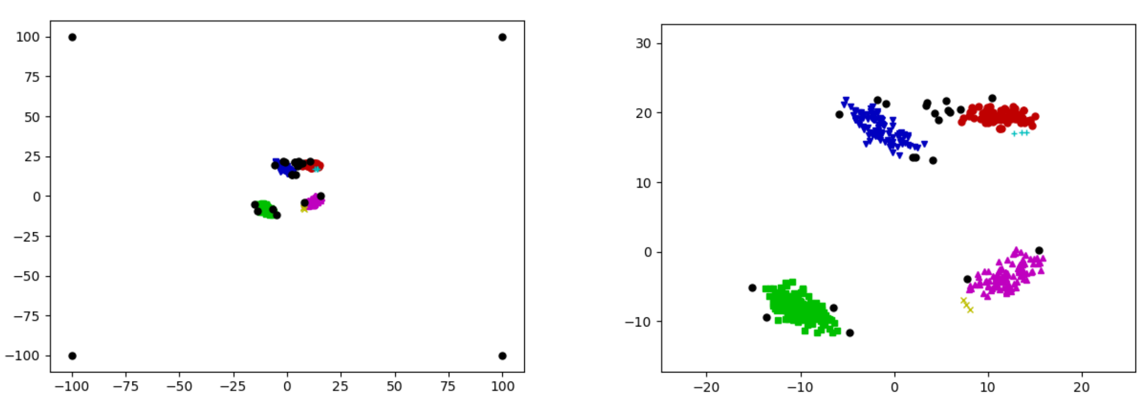
$$f(w_1, w_2, \dots, w_k, y_1, y_2, \dots, y_n) = \sum_{i=1}^{n} \|x_i - w_{y_i}\|_1 = \sum_{i=1}^{n}\sum_{j=1}^{d} |x_{ij} - w_{y_i j}|,$$

   what value of k would be chosen by the elbow method under this strategy? Are you satisfied with this result?

3.4 Density-Based Clustering

Rubric: {reasoning:2}

If you run `python main.py -q 3.4`, it will apply the basic density-based clustering algorithm to the dataset from the previous part. The final output should look somewhat like this:



(The right plot is zoomed in to show the non-outlier part of the data.) Even though we know that each object was generated from one of four clusters (and we have 4 outliers), the algorithm finds 6 clusters and does not assign some of the original non-outlier objects to any cluster. However, the clusters will change if we change the parameters of the algorithm. Find and report values for the two parameters, `eps` (which we called the "radius" in class) and `minPts`, such that the density-based clustering method finds:

1. The 4 "true" clusters.
2. 3 clusters (merging the top two, which also seems like a reasonable interpretaition).
3. 2 clusters.
4. 1 cluster (consisting of the non-outlier points).

3.5 Very-Short Answer Questions

Rubric: {reasoning:3}

1. Does the standard k-means clustering algorithm always yield the optimal clustering solution for a given k?
2. If your set out to minimize the distance between each point and its mean in a k-means clustering, what value of k minimizes this cost? Is this value useful?
3. Describe a dataset with k clusters where k-means would not be able to find the true clusters.
4. Suppose that you had only two features and that they have very-different scales (like kilograms vs. milligrams). How would this affect the result of density-based clustering?
5. Name a key advantage and drawback of using a supervised outlier detection method rather than an unsupervised method?

4 Vector Quantization

Rubric: {reasoning:6}

Discovering object groups is one motivation for clustering. Another motivation is *vector quantization*, where we find a prototype point for each cluster and replace points in the cluster by their prototype. If our inputs are images, vector quantization gives us a rudimentary image compression algorithm.

Your task is to implement image quantization in *quantize_image.py* with `quantize` and `dequantize` functions. The `quantize` function should take in an image and, using the pixels as examples and the 3 colour channels as features, run k-means clustering on the data with 2^b clusters for some hyperparameter b. The code should store the cluster means and return the cluster assignments. The `dequantize` function should return a version of the image (the same size as the original) where each pixel's orignal colour is replaced with the nearest prototype colour.

To understand why this is compression, consider the original image space. Say the image can take on the values $0, 1, \ldots, 254, 255$ in each colour channel. Since $2^8 = 256$ this means we need 8 bits to represent each colour channel, for a total of 24 bits per pixel. Using our method, we are restricting each pixel to only take on one of 2^b colour values. In other words, we are compressing each pixel from a 24-bit colour representation to a b-bit colour representation by picking the 2^b prototype colours that are "most representative" given the content of the image. So, for example, if $b = 6$ then we have 4x compression.

The loaded image contains a 3D-array representing the RGB values of a picture. Implement the *quantize* and *dequantize* functions and show the image obtained if you encode the colours using 1, 2, 4, and 6 bits with the provided image. You are welcome to use the provided implementation of k-means or the scikit-learn version.

1. Hand in your *quantizeImage* and *deQuantizeImage* functions.
2. Show the image obtained if you encode the colours using 1, 2, 4, and 6 bits per pixel (instead of the original 24-bits).
3. Briefly comment on the prototype colours learned in case each, which are saved by the code.