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The AMEDD Uses Goal Programming to Optimize Workforce Planning Decisions

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The mission of the Army Medical Department (AMEDD) is to provide medical and healthcare delivery for the U.S. Army. Given the large number of medical specialties in the AMEDD, determining the appropriate number of hires and promotions for each medical specialty is a complex task. The AMEDD Personnel Proponency Directorate (APPD) previously used a manual approach to project the number of hires, promotions, and personnel inventory for each medical specialty across the AMEDD to support a 30-year life cycle. As a means of decision support to APPD, we proffer the objective force model (OFM) to optimize AMEDD workforce planning. We also employ a discrete-event simulation model to verify and validate the results.

In this paper, we describe the OFM applied to the Medical Specialist Corps, one of the six officer corps in the AMEDD. The OFM permits better transparency of personnel for senior AMEDD decision makers, whereas effectively projecting the optimal number of officers to meet the demands of the current workforce structure. The OFM provides tremendous value to APPD in terms of time, requiring only seconds to solve rather than months; this enables APPD to conduct quick what-if analyses for decision support, which was impossible to do manually.

Keywords: workforce planning; mixed-integer linear programming; stochastic optimization; goal programming; multiple-criteria decision making; military medicine.
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The Army Medical Department (AMEDD) is a special branch of the U.S. Army whose mission is to provide health services for the Army and, as directed, for other agencies, organizations, and military services. Since the establishment of the AMEDD in 1775, six officer corps (Medical Corps, Dental Corps, Nurse Corps, Veterinary Corps, Medical Specialist Corps, and Medical Service Corps) have been developed to provide the organizational leadership and professional and clinical expertise necessary to accomplish the broad soldier-support functions implicit to the mission (Department of the Army 2007). Each corps is made up of individually managed career fields and duty titles called areas of concentration (AOCs); the AMEDD includes 100 officer AOCs. The Medical Specialist Corps has the smallest number with four AOCs; the Medical Corps has the most with 41 AOCs.

The AMEDD manages medical officer personnel over a 30-year life cycle. Given the large number of AOCs in the AMEDD, determining the appropriate number of hires and promotions for each medical specialty is a complex task. The number of authorized medical personnel positions for each AOC varies significantly depending on the uniqueness of the career field and the needs of the Army. Some AMEDD officers enter the Army and remain in the same AOC throughout their careers. Others start their careers in one AOC but have the option of obtaining additional education to qualify for a more specialized AOC. Finally, some officers enter the Army in one AOC but must obtain additional education and move to a more specialized AOC to stay competitive for promotion.

The rank structure of the AMEDD includes officers in the ranks of second lieutenant through general, but the AMEDD Personnel Proponency Directorate (APPD) is only responsible for managing officers below the rank of general, namely second lieutenant (2LT), first lieutenant (1LT), captain (CPT), major (MAJ), lieutenant colonel (LTC), and colonel (COL). The pay grades for these ranks are O-1 to O-6, respectively. APPD is only responsible for managing these officers through the 30th year of commissioned federal service, because officers not selected for promotion to general are generally limited to a 30-year career in the military. The AMEDD's promotion policy is set forth by the Defense Officer Personnel Management Act (DOPMA) of 1980, which provides guidance on promotion selection target percentages (Rostker et al. 1993). For example, the targeted promotion rate from LTC to COL is 50 percent based on DOPMA, although this does not apply to physicians or dentists. In addition to the challenge of meeting federal mandates associated with promotion rates, uncertain officer continuation rates further complicate the workforce planning problem for the AMEDD, because an officer may decide to leave at any time (if that officer has no remaining active-duty service obligation). Thus, uncertainty caused by attrition makes the officer accession and promotion decisions even more complex.

Although the Army Surgeon General has authority over the entire AMEDD, each corps has a corps chief who is responsible for making many decisions impacting the officers in his (her) corps. Some of the key decisions include, how many new officers to recruit and hire into active duty each year within his (her) corps, how many officers to promote to the next higher rank (grade) each year, and how many officers to train in each career field (or clinical specialty). The APPD seeks to provide workforce planning decision support to the corps chiefs by projecting the number of hires, promotions, and personnel inventory needed to support a 30-year life cycle within a corps' authorized officer positions. A 30-year life cycle allows APPD to assess the availability of an officer throughout the anticipated lifespan. Although the model is rerun year after year to reassess each officer's availability, the number of hires must necessarily be based on the requirements forecast based on attrition and promotion data. This approach, which is used throughout the U.S. Army, was implemented initially by Gass (1991).

Literature Review

Military workforce planning models have been used for decades. Bres et al. (1980) developed a goal-programming model for planning officer hires to the U.S. Navy from various commissioning sources. They used transition rates to project the on-board expected flows between starts in successive periods in Markovian fashion to which they also added new hires into the system. Gass et al. (1988) developed the Army manpower long-range planning system, which integrated a Markov chain and linear goal-programming model to forecast the flow of an initial force (given by grade and years of service) to a future force over a 20-year planning horizon, and to determine the optimal transition rates (continuation, promotion, and skill migration) and accession values to obtain the desired end-state force structure or the rates required to minimize the deviation from the desired end-state force structure. Silverman et al. (1988) developed a multiperiod, multiple-criteria trajectory optimization system to help manage the enlisted force structure of the U.S. Navy. Their workforce accession planning model employs an interactive augmented weighted Tchebycheff method, while examining various recruitment and promotion strategies. Gass (1991) built network-flow goal-programming models to provide the U.S. Army with decision support tools to effectively manage its workforce. The transition-rate models describe people going from one state to another during their life cycle in the workforce system. Weigel and Wilcox (1993) developed the Army's enlisted personnel decision support system, which combines a variety of modeling approaches (goal programming, network models, linear programming, and Markov-type inventory projection) with a management information system to support the analysis of long-term personnel planning decisions.

In addition to these long-term workforce planning models, Corbett (1995) developed a workforce optimization model that assists personnel planners in determining yearly officer hires as well as transfers to functional areas as part of the branch detail program. The model employs a multiyear weighted goal program designed to maximize the Army's ability to meet forecasted authorization requirements. Yamada (2000) developed the infinite-horizon workforce planning model using convex quadratic programming for managing officer hires, promotions, and separations annually to best meet desired inventory targets. Henry and Ravindran (2005) presented both preemptive and nonpreemptive goal-programming models for determining the optimal-hires cohort—the number of new Army officers into each of 15 career branches. Shrimpton and Newman (2005) developed a network-optimization model to designate mid-career level officers into new career fields to meet end-strength requirements and maximize the overall utility of officers.

Cashbaugh et al. (2007) used network-based mathematical programming to model the assignment of U.S. Army enlisted personnel in a 96-month planning horizon. Kinstler et al. (2008) developed a Markov model using promotion and attrition rates to improve workforce management decisions in the U.S. Navy nurse corps. Hall (2009) used dynamic programming and linear programming techniques to model the optimal retirement behavior for an Army officer from any point in his (her) career. He addresses the optimal retirement policies for Army officers, incorporating the current retirement system, pay tables, and Army promotion opportunities. Coates et al. (2011) investigated the U.S. Army's captain retention program and used a chi-square and odds ratio analysis to determine whether the practice of providing financial bonuses to individuals agreeing to continue their service is an effective retention tool. Lesiński et al. (2011) used discrete-event simulation (DES) to model the current flow process that an officer negotiates from precommissioning to the first unit of assignment. This model assisted with synchronization of the officer accession and training with the Army force generation process.

Given that some of the officer continuation rates are uncertain parameters, we discuss several tools for addressing optimization problems in the presence of uncertainty. Different algorithms have been developed for stochastic optimization problems, and research has shown that they can be used successfully in many planning applications. The type of data available to the decision maker(s), the assumptions on risk, and the structure and properties of the stochastic optimization problem guide which method to use. Our workforce planning problem incorporates stochastic components in the constraints; therefore, we are concerned with methods for solving chance-constrained stochastic programming problems that Charnes and Cooper (1959) proposed originally. In general, chance-constrained stochastic programs have two difficulties (Ahmed and Shapiro 2008). One difficulty is accurately computing the probabilistic constraints. Without this difficulty, we could transform the stochastic optimization problems to their respective deterministic equivalents and then convert them to general nonlinear programs that are solvable with traditional nonlinear techniques. Cheon et al. (2006) and Ruszczyński (2002) proposed algorithms for these types of problems. However, such processes are usually difficult to solve in practice and are only successful for special cases. The second difficulty arises when the feasible region is not convex. In this case, which occurs frequently in workforce planning, the optimization problem becomes difficult to solve efficiently.

Most chance-constrained stochastic programs are solved using approximation methods. Numerous methods have been developed for problems in which both difficulties exist. Both Nemirovski and Shapiro (2006) and Calafiore and Campi (2005) proposed such solution methods. Kleywegt et al. (2001) introduce a Monte Carlo simulation-based approach to stochastic discrete optimization problems, in which a random sample is generated and the expected-value function is approximated by the corresponding sample average function. The obtained sample average approximation optimization problem is then solved.

A particular subclass within chance-constrained stochastic programs is chance-constrained stochastic goal programs, which can be used to solve multiple-criteria optimization problems under uncertainty. This subclass belongs to goal programming, where there are probabilistic rather than deterministic constraints. Because we cannot usually convert a chance-constrained stochastic goal program to a deterministic equivalent, we typically apply Monte Carlo simulation methods. The general concept is that we can approximate the uncertain constraint functions using stochastic simulation, and then solve the problem using the approximated function.

Motivation and Purpose

Effective long-term workforce planning and personnel management of all medical professionals within the AMEDD is a complex problem. Prior to 2010, APPD used a manual approach to project the appropriate hires and promotion goals for each medical specialty across the six separate corps. McMurry et al. (2010) developed a set of nonlinear mathematical programming models to provide the APPD with a multiple-criteria decision support mechanism for determining optimal hiring and promotion policies.

We extend the work of McMurry et al. (2010) by introducing the objective force model (OFM), a deterministic, mixed-integer linear weighted goal-programming model to optimize AMEDD workforce planning for the Medical Specialist Corps (SP). This linear, multicriteria optimization model is significantly more difficult in that the constraints are more varied as a result of substitutability. We also introduce two stochastic variants of the linear OFM, which incorporate probabilistic components associated with uncertain officer continuation rates (following the completion of grade-based active-duty service obligations); these rates may fluctuate significantly. We use discrete-event simulation to verify and validate the results of the deterministic OFM.

Our improved optimization models allow for better transparency of AMEDD personnel for both the corps chiefs and the health services human resource planners at APPD, while effectively projecting the workforce skill levels (by grade) required to meet the demands of the current force. Note that we model a 30-year life cycle because of Title 10 U.S. code section 634, which states that each officer who holds the grade of colonel in the regular Army and is not on the selection list to brigadier general must retire the first day of the month after the month he (she) completes 30 years of active federal-commissioned service. Therefore, because we model officer ranks up to and including colonel, 30 years constitutes the maximum life cycle and represents the target steady-state inventory of officers within each specialty, rank, and years of service.

Methodology

We first present the formulation for the OFM, a mixed-integer linear weighted goal-programming model, to solve the workforce planning problem for AMEDD's Medical Specialist Corps, given deterministic continuation rates. We then briefly describe two solution methods for the stochastic goal programs used to solve the workforce planning problem under uncertainty. Finally, we discuss the discrete-event simulation model performed for OFM verification and validation.

Deterministic Variant of the Objective Force Model

AMEDD officers in the SP are hired into the Army at the grade (rank) of either O-1 (2LT), O-2 (1LT), or O-3 (CPT). Unlike the more specialized and diversified corps, these officers remain in the same AOC throughout their careers. SP consists of four career AOCs: occupational therapists (OTs), physical therapists (PTs), clinical dietitians (CDs), and physician assistants (PAs). We note that promotion decisions are made only for officers at the grade (rank) of O-4 (MAJ), O-5 (LTC), and O-6 (COL). According to APPD, a noninteger solution for promotions is an acceptable simplification. The noninteger structure is appropriate because of the concept of full-time equivalent employees, which may be fractional. Although we may not hire a fractional person, we can augment any fractional requirement along the entire 30-year timeline.

We now present a brief description of the deterministic OFM sets, parameters, variables, objective function, and goal and hard constraints (Appendix A provides a full description), and we provide some definitions and explanations of the military human resources terminology. The set G represents the grade of the SP officers that is indexed using $\{1, 2, 3, 4, 5, 6\}$. The set I represents the officer AOCs within the SP, which is indexed as $\{1 = \text{OT}, 2 = \text{PT}, 3 = \text{CD}, 4 = \text{PA}\}$.

The set K represents the year of service for an officer, which is $\{1, 2, \ldots, 30\}$ representing a full officer career. The set F represents the set of goals specified by APPD, which is $\{1, 2, 3, 4, 5\}$.

Authorizations are officer positions funded by the U.S. Congress to carry out the mission of the U.S. Army. A documented authorization is a funded position within an organization that identifies a specific specialty and rank required to meet a stated capability. There are two types of documented authorizations. The first is a career field specialty (or AOC) authorization that can be filled only by an officer specifically trained for that job (e.g., physical therapist). The second type of documented authorization is an immaterial authorization. Immaterial positions do not require an individual with a specific career specialty. Most immaterial authorizations are executive or leadership positions, such as commanders, directors, and administrators. The last type of authorization provides allowances for officers who are not assigned or contributing to the mission of an organization. This includes officers who are students, in transit between assignments, in long-term hospitalization or pending discharge (e.g., wounded warriors), or removed for disciplinary reasons (e.g., court martial).

In the OFM, the parameter c_{ig} reflects the documented authorizations for each AOC and grade, which reflect the requirements for SP officers to support both peace- and wartime healthcare delivery for the Army. The parameter $SPIMM_g$ represents the SP immaterial documented authorizations for each grade; these are SP authorizations that SP officers can fill regardless of AOC; $AMIMM_g$ represents the AMEDD immaterial documented authorizations for each grade that are AMEDD authorizations that SP officers can fill regardless of AOC; THS_g represent the transient, holdee, and student documented authorizations for each grade, which are authorizations that SP officers can fill regardless of AOC. Table 1 displays these data, which APPD provided.

Documented authorizations	Area of concentration				Total	*SPIMM*	*AMIMM*	*THS*
	OT	PT	CD	PA				
Total	75	255	122	780	1,505	13	44	216
COL	3	6	5	3	28	4	5	2
LTC	9	23	19	28	103	1	15	8
MAJ	20	46	38	149	325	2	15	55
CPT	31	111	28	534	798	6	7	81
1LT	12	19	10	66	179	0	2	70
2LT	0	50	22	0	72	0	0	0
Company grade	43	180	60	600	1,049	6	9	151

Table 1: This table shows the medical specialist corps documented number of authorizations (cells) by rank (rows) and area of concentration (columns).

Number of officers	Area of concentration			
	OT	PT	CD	PA
Total (Max)	96	295	154	
Total (Min)	93		149	
COL (Max)	4			
COL (Min)	4	8	7	
LTC (Max)	12			
LTC (Min)			20	

Table 2: This table details both the minimum and maximum number of medical specialist corps officers by rank (rows) and by AOC (columns).

The parameter Cap_i is the maximum allowable number of officers for each AOC (i.e., capacity). According to APPD, this upper bound applies only to OTs, PTs, and CDs. The parameter $Floor_i$ is the minimum acceptable number of officers for each AOC; this lower bound applies only to OTs and CDs. The parameter Cap_{ig} is the maximum allowable number of officers for each AOC and each grade; this upper bound applies only to COL and LTC who are OTs. The parameter $Floor_{ig}$ is the minimum acceptable number of officers for each AOC and each grade; this lower bound applies only to COL for OTs, PTs, and CDs and to LTC for CDs. Table 2 displays these data provided by APPD.

Promotion rate is the number of officers selected for promotion divided by the number of officers considered. The number of officers selected is a variable in the OFM bounded by promotion rates usually based on DOPMA objectives ± 10 percent when possible. In the OFM, the parameter pf_{ig} is the minimum promotion rate for each AOC and grade, which is not applicable to 2LT in each AOC. The parameter pc_{ig} is the maximum promotion rate for each AOC and grade, which is also not applicable to 2LT in each AOC. Table 3 displays these data, which APPD provided.

Note that although solving by hand might appear to be reasonable for some of our smaller models (such as the example of SP Corps OFM we discuss here),

DOPMA	OT					PT					CD					PA				
Promotion rate	1LT	CPT	MAJ	LTC	COL	1LT	CPT	MAJ	LTC	COL	1LT	CPT	MAJ	LTC	COL	1LT	CPT	MAJ	LTC	COL
Max	1	0.95	1	0.8	0.6	0.98	0.95	1	0.8	0.6	1	0.95	1	0.8	0.6	1	0.95	1	0.8	0.6
Min	1	0.95	0.7	0.6	0.4	0.98	0.95	0.7	0.55	0.3	1	0.95	0.7	0.6	0.3	1	0.95	0.6	0.4	0.2

Table 3: This table details the minimum and maximum DOPMA promotion rates for medical service corps officers by order statistic (rows), rank (columns), and area of concentration (table split).

	Year				
Promotion evaluation	2	4	11	17	22
2LT → 1LT	98%				
1LT → CPT		95%			
CPT → MAJ			80%		
MAJ → LTC				70%	
LTC → COL					50%

Table 4: This table details the DOPMA promotion standards by rank and year of service.

others are exceedingly large with significant range between the floor and ceiling constraints. Table 4 shows both the scheduled promotion evaluations by year and grade and the DOPMA standard promotion rates, which APPD again targets at ±10 percent.

The continuation rate is the percentage of officers that stay in the Army from one year to the next year, categorized by specialty, rank, and years of service. The rates are based on a five-year average of actual data collected on every officer on active duty for each specialty. In the OFM, the parameter r_{igk} reflects the deterministic continuation rate for each AOC, grade, and year of service, which reflects both those SP officers who are selected for promotion when considered and those SP officers who are considered for promotion but not selected. Note that officers not selected for promotion are limited to a set number of years that they may remain on active duty (rank dependent) before mandatory separation. These data provided by APPD come from the medical operational data system, which (again) is derived from multiyear averages. Finally, the parameter w_f reflects APPD's weight for each goal in the model.

The model decision variables p_{ig} represent the number of SP officers promoted in AOC i at grade g (for MAJ, LTC, and COL only). The model decision variables a_{ig} represent the number of SP officers hired for AOC i at grade g (for 2LT, 1LT, and CPT only). The model decision variables d_{ig} represent the actual number of SP officers in the system for each AOC and grade, whereas the model decision variables Inv_{igk} represent the projected inventory of SP officers in the system by AOC i, grade g, and year k. In terms of goal deviation variables, pos_f is the positive deviation for goal f and neg_f is the negative deviation for goal f.

The objective function of the deterministic OFM seeks to minimize the sum of the weighted goal deviations. The target for the first goal constraint is for the total number of officers (over each grade and AOC) to equal the total documented authorizations (over each grade and AOC as well as the SP immaterial, AMEDD immaterial, and THS). The target for the second goal constraint is for the number of COLs (over each AOC) to equal the COL documented authorizations (over each AOC as well as the SP immaterial, AMEDD immaterial, and THS). The target for the third goal constraint is for the number of LTCs (over each AOC) to equal the LTC documented authorizations (over each AOC as well as the SP immaterial, AMEDD immaterial, and THS). The target for the fourth goal constraint is for the number of MAJs (over each AOC) to equal the MAJ documented authorizations (over each AOC as well as the SP immaterial, AMEDD immaterial, and THS). The target for the last goal constraint is for the number of company grade (sum of 2LT, 1LT, and CPT) officers (over each AOC) to equal the company grade documented authorizations (over each AOC as well as the SP immaterial, AMEDD immaterial, and THS).

The hard constraints, as we define in Appendix A, force inventory controls, promotion controls (floors and ceilings by AOC), and transition controls. These constraints were developed based on known promotion restrictions, transition data, and (primarily) decision-maker input. Some constraints apply to all AOCs; however, others are AOC specific. All constraints were developed in conjunction with APPD. For example, multiple constraints provided promotion floors and ceilings by year, AOC, and grade. These constraints were necessary to achieve decision-maker personnel requirements. In addition, inventory constraints were necessary to ensure proper rollover from one period to another by AOC and grade. Additional constraints ensured that promotions were considered only during those years and by grade when feasible.

Stochastic Variants of the Objective Force Model

In solution method #1, we use a scenario-based Monte Carlo simulation approach to approximate the objective value and the optimal solution of a stochastic goal program. We generate S scenarios, where each scenario corresponds to one realization of the deterministic optimization problem. We use the sample average across the S scenarios to approximate the optimal objective value and optimal solution. In solution method #2, we leverage the sample average approximation (SAA) method (Rubinstein and Shapiro 1990) because prior samples are easily generated. Appendix B provides additional details.

In the stochastic variant of the OFM, we model officer continuation rates as random variables from the normal distribution based on historical multiyear averages captured by APPD. We use both stochastic method #1 and stochastic method #2 to solve the stochastic variant of the mixed-integer linear weighted goal-programming model. For stochastic method #1, we solved the deterministic OFM for each scenario with stochastically generated continuation rates, where the final objective value and optimal solution is the average over the scenarios.

Appendix C contains the full description of the model formulation for stochastic method #2. The key differences between the deterministic and stochastic method #2 model formulations are as follows. First, we include an additional set S representing the scenario under consideration. Second, the data parameter for SP officer continuation (stochastic) rate r_{igks} is now computed over each AOC, grade, year, and scenario. Third, the decision variables d_{igs} and Inv_{igks} are now computed over each scenario. Fourth, there are now positive and negative goal deviations for each goal and scenario, pos_{fs} and neg_{fs}, respectively. Fifth, the objective function now seeks to minimize the sample average of the sum of the weighted goal deviations over the scenarios. Finally, the number of goal and hard constraints are increased as a result of execution over each scenario.

Discrete-Event Simulation Model

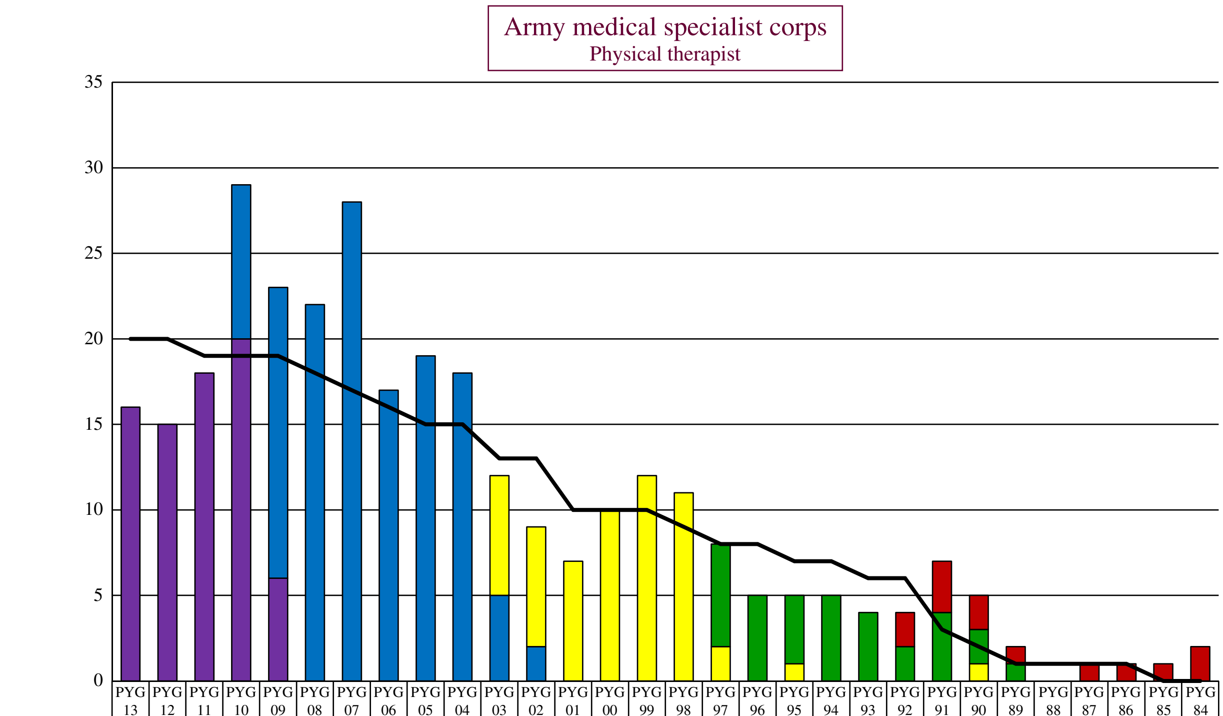
To verify and validate the deterministic OFM presented previously, we developed a DES model that processes SP officer hires (i.e., number of arrivals) for each AOC through the 30-year life cycle. At the beginning of each year, the DES model probabilistically determines if the arrival will be transferred to another AOC (not applicable for SP officers), promoted to a higher grade, or continue for another year of service. At the end of each year, for each AOC, the arrival's current grade (O1–O6) is incremented. At the end of each DES model replication, the model generates 30 years of data for each AOC, including replication number, life cycle year, and number of each grade. The DES model uses SP officer promotion rates and number of hires (entity arrival information) determined by the deterministic OFM. The deterministic SP officer continuation rates are also used as an empirical distribution in the simulation. The DES model is replicated 999 times using common random number streams for variance reduction.

Results and Discussion

We discuss the results of the deterministic and stochastic mixed-integer linear weighted goal-programming models to solve the SP workforce planning problem and provide decision support to APPD. We discuss how the results of the discrete-event simulation verify and validate the results of the deterministic variant of the OFM.

Deterministic Objective Force Model Results

Upon formulating and solving the deterministic OFM using Microsoft Excel with the OpenSolver, we obtained an objective function value of 0, indicating that all the specified goal constraints were satisfied. That is, the documented number of SP officer authorizations and the officer quantities determined by the model (for all the grades) were satisfied. Only two of



	PYG 13	PYG 12	PYG 11	PYG 10	PYG 09	PYG 08	PYG 07	PYG 06	PYG 05	PYG 04	PYG 03	PYG 02	PYG 01	PYG 00	PYG 99	PYG 98	PYG 97	PYG 96	PYG 95	PYG 94	PYG 93	PYG 92	PYG 91	PYG 90	PYG 89	PYG 88	PYG 87	PYG 86	PYG 85	PYG 84
COL																						2	3	2	1		1	1	1	2
LTC																	6	5	4	5	4	2	4	2	1					
MAJ											7	7	7	10	12	11	2		1					1						
CPT				9	17	22	28	17	19	18	5	2																		
LT	16	15	18	20	6																									
Obj force	20	20	19	19	19	18	17	16	15	15	13	13	10	10	10	9	8	8	7	7	6	6	3	2	1	1	1	1	0	0

Figure 1: (Color online) This chart is the typical OFM (what the AMEDD needs, depicted as a line by year) and the officer inventory by grade (depicted as stacked bars). The vertical bars of the histogram represent the actual inventory in terms of primary year group (cohort based on years of service) and rank. The black line represents the results of the OFM projections. Vertical bars that extend above the OFM line suggest that the AOC is over-strength for that particular year group, and vertical bars that fall below the OFM line suggest that the AOC is under-strength for that particular year group.

the goal deviation variables were positive. The model had 60 constraints and 27 variables, and solved in 0.53 seconds.

The optimal numbers of hires determined by the model were as follows: eight 1LT hires for OTs, 20 2LT hires for PTs, 10 2LT hires for CDs, 85 1LT hires for PAs, and five CPT hires for PAs. The goal for the average promotion rate (across all ranks) for each AOC was approximately 74 percent based on the DOPMA promotion standards (see Table 4); the optimal solution achieved an average promotion rate of 70 percent. On average, 95 percent of the CPTs, 76 percent of the MAJs, 55 percent of the LTCs, and 36 percent of the COLs were promoted across all AOCs. Table 5 displays the promotion percentage and the number of officers promoted by officer AOC and rank (CPT through COL).

Similarly, on average, 69 percent of CPTs, 75 percent of MAJs, and 81 percent of LTCs enter the rank (across all AOCs) and remain for consideration for promotion to the next higher rank. Table 6 shows officer continuation percentages for each AOC and rank (CPT, MAJ, LTC).

We first solved the OFM with equal decision-maker weights (i.e., the documented authorizations at all the grades are given equal weights). We then performed a sensitivity analysis to see if the adjustment of weights had any impact on the authorizations. We

	CPT	MAJ	LTC	COL
OT	95%	83%	60%	43%
	7.6	3.9	1.8	0.5
PT	95%	70%	55%	39%
	18.4	10.2	5.1	1.5
CD	95%	89%	65%	41%
	9.5	5.2	3	1.1
PA	95%	61%	40%	20%
	85.8	37.1	7	1.2

Table 5: This table provides both the percentage and number promoted (cells) by area of concentration (rows) and rank (columns).

	CPT (%)	MAJ (%)	LTC (%)
OT	62	75	71
PT	79	91	75
CD	62	88	91
PA	71	47	87

Table 6: This table details the percent of medical specialist corps officers remaining entering promotion consideration for the next-higher grade (cells) by area of concentration (rows) and rank (columns).

found that, irrespective of the weights, all the goals were achieved; however, the determined officer quantity for each grade depends on the weights that were chosen.

In addition to estimating the number of officer hires and promotions, APPD uses the OFM as a workforce management decision support tool for optimizing AOC grade structure and to assist in workforce-reduction-program decisions. In particular, APPD uses the OFM results to generate histograms that visually represent the current workforce inventory (partitioned by the fiscal year the officer entered active duty) plotted against the 30-year life cycle inventory as projected by OFM. For example, Figure 1 displays a generated histogram for the PTs.

In Figure 1, the vertical bars of the histogram represent the actual inventory in terms of primary year group (cohort based on years of service) and rank. The black line represents the results of the OFM projections. Vertical bars that extend above the OFM line suggest that the AOC is over-strength (i.e., above the targeted number of officers) for that particular year group, and vertical bars that fall below the OFM line suggest that the AOC is under-strength

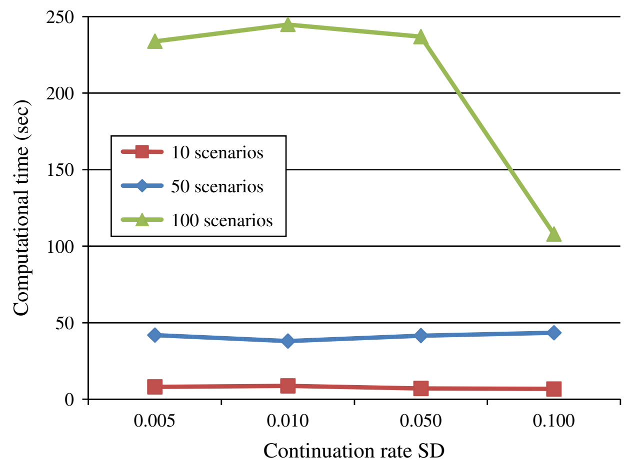


Figure 2: (Color online) This figure shows how computational time (y-axis) was affected by the continuation rate standard deviation (x-axis), given the number of scenarios run (separate lines).

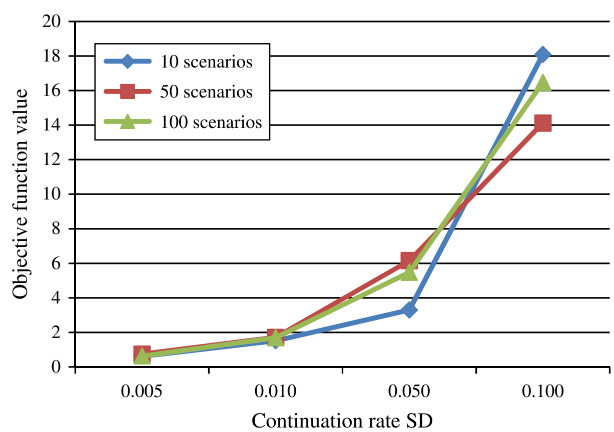


Figure 3: (Color online) This figure depicts the objective value function (y-axis) as a function of the continuation rate standard deviation (x-axis), given the number of scenario runs (separate lines).

(i.e., below the targeted number of officers) for that particular year group. APPD uses these results to provide the senior AMEDD leadership a quick reference to identify specific year groups that could require management focus and key personnel decisions. APPD periodically updates the histograms as personnel numbers change during the year and whenever a new OFM is produced (usually annually).

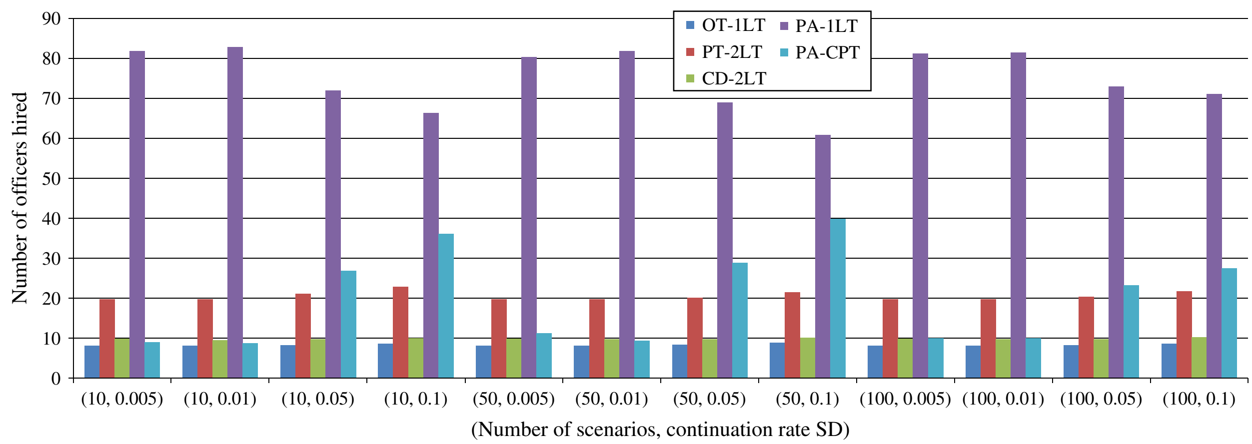


Figure 4: (Color online) This figure depicts the number of medical specialist corps officers hired (y-axis) by the number of scenarios and by the continuation rate standard deviation (x-axis) for various areas of concentration and ranks (separate bars).

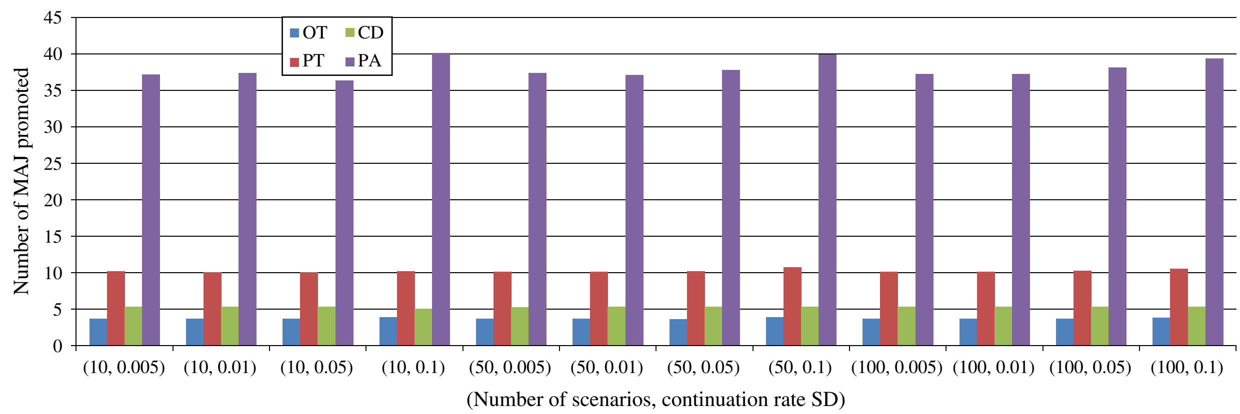


Figure 5: (Color online) This figure depicts the number of majors promoted (y-axis), given the number of scenarios and continuation rate standard deviation (x-axis) for various areas of concentration.

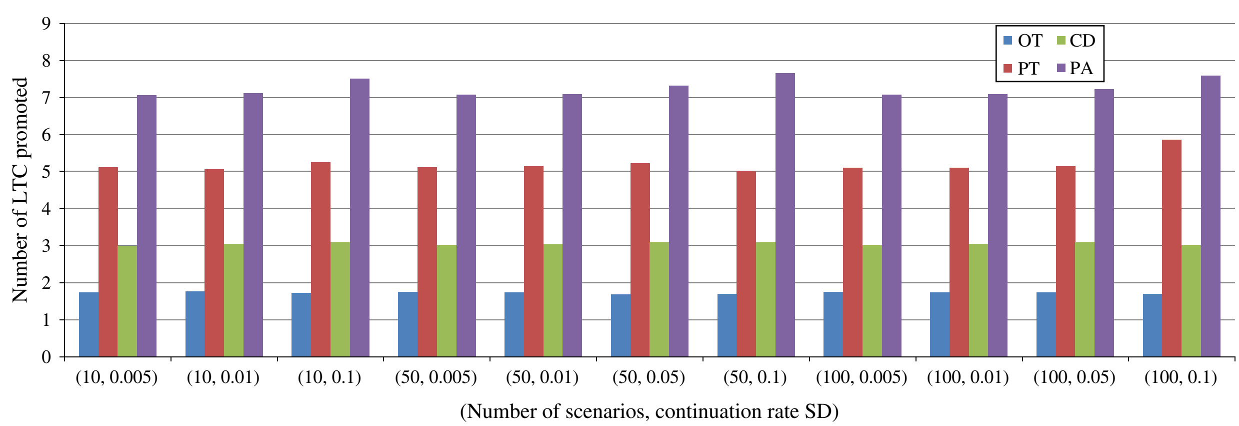


Figure 6: (Color online) This figure depicts the number of lieutenant colonels promoted (y-axis), given the number of scenarios and the continuation rate standard deviation (x-axis) for various areas of concentration.

Stochastic Method #1: Objective Force Model Results

We performed the analysis by varying the number of scenarios and the stochastic continuation rate standard deviation (SD). We varied the number of scenarios from 10 to 100 and the continuation rate SD from 0.005 to 0.1. For each setting (i.e., number of scenarios and continuation rate SD), we determined the number of SP officers hired, the number of SP officers promoted, and the objective function values. Table 7 contains the results of the stochastic method #1 that the OFM solved using Microsoft Excel with OpenSolver.

The average numbers of MAJs promoted were 3.77, 10.26, 5.33, and 37.93 in OT, PT, CD, and PA, respectively. The average numbers of LTCs promoted were 1.73, 5.17, 3.04, and 7.27 for OT, PT, CD, and PA, respectively. The average numbers of COLs promoted were 0.54, 1.52, 1.12, and 1.31 in OT, PT, CD, and PA respectively. In Figure 2, we see that the computational time increased with the number of scenarios run.

In most of the cases, we observed that the computational time was higher when the continuation rate SD was lower. The objective function value also appeared to depend on the continuation rate SD (see Figure 3). The value of the objective function increased as the continuation rate SD increased, irrespective of the scenarios. This might be because when the uncertainty level increases, achieving all the targets becomes difficult.

The number of SP officers hired remained relatively flat for all the settings (see Figure 3). On average, 8.36 officers of rank 1LT for OTs, 20.58 officers of rank 2LT for PTs, 9.90 officers of rank 2LT for CDs, 75.12 officers of rank 1LT for PAs, and 20.11 officers of rank CPT for PAs were hired. The number of SP officers promoted also remained the same for all the settings. We observed that on average, the number of officers for PAs promoted was higher than for any other AOC (see Figures 4–7).

Stochastic Method #2: Objective Force Model Results

Upon formulating and solving the stochastic method #2 OFM, we often did not achieve an integer solution because of the large number of additional constraints from each scenario. We therefore relaxed the integer constraints for number of SP officers hired. We performed 100 scenarios and 10 iterations, and then calculated the estimated expected values and variances for each decision variable. We assumed the probability density function of the officer continuation rates as a normal distribution with mean equal to the solution of the deterministic optimization model and standard deviation of 0.005.

Upon solving the model using the General Algebraic Modeling System (GAMS) 23.9.3 with IBM ILOG CPLEX 12.4.0.1, we obtained an average objective function value of 0.58. The total solution time was 160.3 seconds. There were 66,432 constraints and 75,449 variables in total per iteration. Table 8 shows the results for the number of estimated average SP officer hires and promotions. The optimal numbers of estimated average hires determined by the model were as follows: 8.44 1LT hires for OT, 19.81 2LT hires for PTs, 9.51 2LT hires for CDs, 89.44 1LT hires for PAs, and no CPT hires for PAs. Compared to the deterministic OFM results, the number of hires for the former 1LT-OT, 2LT-PT, and 2LT-CD was nearly identical. However, we see a clear trade-off for the latter 1LT-PA and CPT-PA. This stochastic model solution increased the number of 1LT hires for PAs by eliminating CPT hires for the same AOC. In terms of the estimated average number of SP officer promotions, these results were nearly identical to those determined by the deterministic variant of the optimization model.

Compared with the results of stochastic method #1 OFM (using the same condition—100 scenarios with standard deviation of 0.005), we see that the main differences are the number of 1LTs and CPTs hired for PAs. The model can induce infeasibility problems if the number of CPT hires for PAs is strictly positive, which is hidden in the stochastic method #1 because the number of constraints in stochastic method #1 is much less than in stochastic method #2. Thus, when we use the first method to solve a stochastic optimization problem, we must be aware of the possible infeasibility issue because of stochastic parameters. Aside from this difference, the other results were similar, providing verification for stochastic method #1.

From Table 9, we can see that the variances of each decision variable (number promoted, number hired) are not very large, which means that the actual

			Number of SP officers hired					Number of SP officers promoted												
Number of scenarios	Continuation SD	Comp time (sec)	OT 1LT	PT 2LT	CD 2LT	PA 1LT	PA CPT	OT MAJ	OT LTC	OT COL	PT MAJ	PT LTC	PT COL	CD MAJ	CD LTC	CD COL	PA MAJ	PA LTC	PA COL	Objective function
10	0.005	8.16	8.17	19.84	9.89	81.90	9.06	3.72	1.74	0.54	10.20	5.12	1.51	5.37	2.99	1.12	37.15	7.06	1.24	0.62
	0.01	8.78	8.11	19.83	9.60	82.78	8.80	3.75	1.76	0.54	10.09	5.06	1.51	5.36	3.05	1.22	37.36	7.11	1.24	1.52
	0.05	7.07	8.31	21.15	9.81	71.95	26.80	3.72	1.71	0.54	10.13	4.98	1.53	5.37	3.01	1.13	36.33	7.47	1.28	3.30
	0.1	6.76	8.67	22.89	10.04	66.35	36.15	3.89	1.72	0.55	10.24	5.24	1.54	5.09	3.08	1.16	40.10	7.50	1.47	18.08
50	0.005	41.92	8.13	19.80	9.89	80.31	11.29	3.76	1.75	0.54	10.16	5.12	1.51	5.31	3.01	1.14	37.34	7.07	1.24	0.74
	0.01	38.02	8.18	19.78	9.76	81.77	9.42	3.75	1.73	0.54	10.15	5.14	1.51	5.34	3.03	1.20	37.12	7.08	1.25	1.71
	0.05	41.49	8.48	20.23	9.75	68.90	28.93	3.66	1.67	0.55	10.25	5.22	1.51	5.36	3.08	1.14	37.84	7.32	1.31	6.15
	0.1	43.42	8.95	21.60	10.25	60.82	39.95	3.92	1.70	0.54	10.78	5.01	1.55	5.35	3.08	0.87	39.92	7.64	1.49	14.12
100	0.005	233.84	8.15	19.81	9.87	81.18	10.08	3.74	1.75	0.54	10.15	5.10	1.51	5.36	3.00	1.13	37.24	7.07	1.24	0.63
	0.01	244.80	8.19	19.81	9.71	81.44	10.05	3.71	1.74	0.54	10.15	5.09	1.51	5.38	3.05	1.22	37.26	7.08	1.24	1.69
	0.05	236.90	8.29	20.35	9.86	72.90	23.23	3.76	1.74	0.54	10.26	5.14	1.51	5.34	3.09	1.14	38.18	7.22	1.29	5.48
	0.1	107.93	8.71	21.82	10.34	71.12	27.54	3.85	1.69	0.55	10.52	5.86	1.57	5.34	3.00	0.95	39.35	7.58	1.40	16.47

Table 7: This table depicts the medical specialist corps officer continuation rates, computation time, hiring numbers, promotion numbers, and objective function of the optimization (separate cell values) by the number of scenarios (row values), area of concentration, and rank (column values).

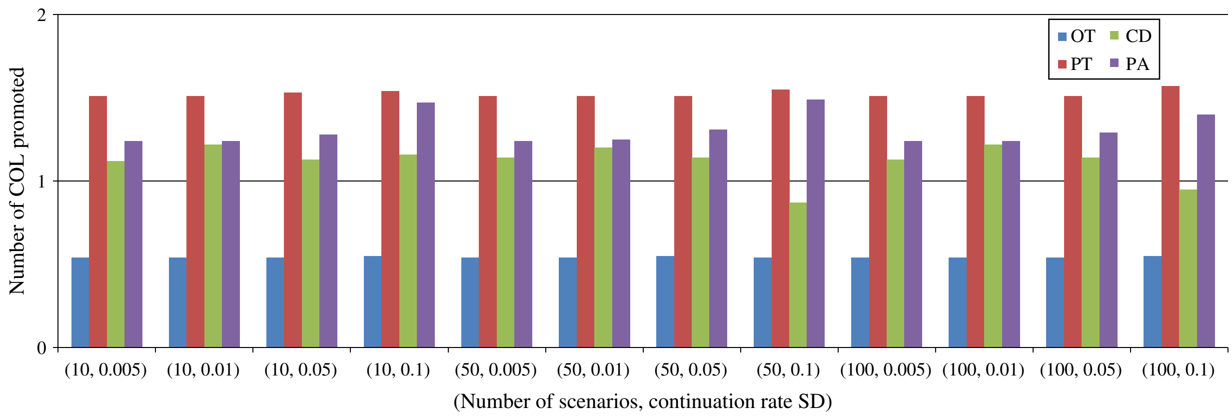


Figure 7: (Color online) This figure depicts the number of colonels promoted (y-axis), given the number of scenarios and the continuation rate standard deviation (x-axis) for various areas of concentration.

		2LT	1LT	CPT
OT	Mean	0	8.4435	0
	variance	0	0.0368	0
PT	Mean	19.8086	0	0
	variance	0.0498	0	0
CD	Mean	9.5065	0	0
	variance	0.2056	0	0
PA	Mean	0	89.4375	0
	variance	0	0	0

Table 8: This table shows the estimated mean and variance of medical specialist corps officer hires (cells) by area of concentration (rows) and rank (columns).

		MAJ	LTC	COL
OT	Mean	3.5991	1.6366	0.5282
	variance	0.0178	0.0085	0.0017
PT	Mean	10.3909	5.3402	1.5332
	variance	0.0468	0.0844	0.003
CD	Mean	4.7525	3.02	1.1294
	variance	0.5233	0.0022	0.0027
PA	Mean	36.932	7.2497	1.3927
	variance	0.3778	0.0972	0.2425

Table 9: This table shows the estimated mean and variance of medical specialist corps officer promotions (cells) by area of concentration (rows) and rank (columns).

AOC	2LT	1LT	CPT	MAJ	LTC	COL	Total
AOC average							
OT		8.6527	45.1211	23.4194	11.2202	4.002	92.4154
PT	20.3804	40.3904	122.926	76.4414	26.2923	8.1241	294.555
CD	10	20.2653	58.1712	38.8719	20.1291	6.7768	154.214
PA		177.867	543.512	186.128	44.4675	8.967	960.941
AOC standard deviation							
OT		1.0405	6.6646	9.6917	8.4417	5.3506	21.1086
PT	0.6084	2.4412	10.7675	19.7641	11.3142	6.9472	33.4218
CD	0	0.5072	6.6033	13.3928	10.4438	6.1126	25.9942
PA		3.715	22.3338	29.749	16.6983	8.0492	50.7138
AOC 95% confidence (half-width)							
OT	0	0.06452	0.41328	0.60099	0.52347	0.3318	1.30896
PT	0.03773	0.15138	0.6677	1.22559	0.7016	0.4308	2.07251
CD	0	0.03145	0.40947	0.83049	0.64762	0.37905	1.61191
PA	0	0.23037	1.38493	1.84475	1.03547	0.49914	3.14479
AOC inventory data from deterministic OFM							
OT	0	8.6667	45.2563	23.6091	11.4679	4	93
PT	20.3954	40.6908	123.328	75.7375	26.3997	8	294.551
CD	10	20.5	58.0988	38.4012	20	7	154
PA	0	177.846	544.234	187.124	45.2453	9	963.449

Table 10: This table shows the results of the discrete-event simulation and the deterministic OFM by area of concentration (rows) and rank (columns). The first three table splits are the average, standard deviation, and confidence interval for the simulation. The last split represents the deterministic model results.

expected values are not far from the estimated means. After comparing the stochastic model results with those of the deterministic OFM, we conclude that these results are both reasonable and useful to APPD.

We also implemented the stochastic method #2 optimization model in Microsoft Excel using VBA and the OpenSolver to provide APPD with a user-friendly decision support tool.

Discrete-Event Simulation Model Results

As a means to verify and validate the deterministic OFM, we used the output of the discrete-event simulation (coded in MedModel 2011) to create lower- and upper-bound limits to the SP officer total inventory values (summed over the 30-year officer life cycle by grade) for each AOC. We then checked whether the optimal values generated by the deterministic optimization model fell within these AOC total inventory limits. Based on the 999 DES model replications, we calculated the average and standard deviation of the total inventory values for each AOC and grade. Using these values, we computed the 95 percent confidence interval half widths to create the lower- and upper-bound AOC limits. We extracted the optimal values generated by the OFM. Table 10 shows these results.

We then compared the optimization values to the 95 percent confidence upper- and lower- 30-year AOC total inventory limits. From Table 11, we can see that all the optimal values computed from the deterministic OFM fell within the simulation-based upper and lower bounds, with the exception of two values (PT $\rightarrow$ 1LT and CD $\rightarrow$ 1LT); however, these two values (depicted in bold) were both just slightly greater than its respective upper-bound limit.

These DES model results validate and verify the results of our deterministic, mixed-integer linear weighted goal-programming model for optimal workforce planning of the Medical Specialist Corps. In future work, we aim to apply this method to the stochastic variants of the optimization model by generating statistical quality control charts.

Concluding Remarks

In this paper, we described the objective force model currently in use by APPD for long-term workforce

Bounds	AOC	2LT	1LT	CPT	MAJ	LTC	COL	Total
Upper	OT		8.72	45.53	24.02	11.74	4.33	93.72
Opt value			8.67	45.26	23.61	11.47	4	93
Lower			8.59	44.71	22.82	10.7	3.67	91.11
Upper	PT	20.42	40.54	123.59	77.67	26.99	8.55	296.63
Opt value		20.4	**40.69**	123.33	75.74	26.4	8	294.55
Lower		20.34	40.24	122.26	75.22	25.59	7.69	292.48
Upper	CD	10	20.3	58.58	39.7	20.78	7.16	155.83
Opt value		10	**20.5**	58.1	38.4	20	7	154
Lower		10	20.23	57.76	38.04	19.48	6.4	152.6
Upper	PA		178.1	544.9	187.97	45.5	9.47	964.09
Opt value			177.85	544.23	187.12	45.25	9	963.45
Lower			177.64	542.13	184.28	43.43	8.47	957.8

Table 11: This table provides the mean estimate as well as the 95% confidence interval for the 30-year area of concentration total inventory limits (cells) by the parameter estimate (rows), area of concentration (table dividers), and rank.

planning of medical professionals in the AMEDD, and we specifically discussed the OFM and its stochastic variants applied to the Medical Specialist Corps. We investigated several methods of estimating the optimal number of hires and promotions for the AMEDD's Medical Specialist Corps and found high degrees of consistency among models. We described the deterministic optimization OFM and stochastic optimization OFM variants, and then further compared deterministic results with those from DES, finding a high level of congruency. The expert workforce management decision support for estimating the correct number of hires and promotions necessary to achieve desired personnel structure under uncertainty is of significance.

The OFM provides tremendous value to APPD; it requires only seconds to solve rather than months. This enables APPD to produce quick turnaround analysis in a transparent way and provides decision support that was almost impossible using a manual process. One of the greatest benefits of the OFM is that APPD uses the optimization results to provide senior AMEDD leaders with a quick reference to identify specific-year groups that could require specific management focus as well as key personnel decisions in terms of hiring, promoting, or firing. As the U.S. Army continues to reduce the size of its workforce structure, the use of the OFM will become vital for the AMEDD in its future medical professional reduction program. In future work, we plan to model the stochasticity of changing workforce structure authorization levels to provide decision support for downsizing decisions.

In terms of personnel strategy and policy, the impact from OFM-supported decisions includes recruiting goals for the Army's recruiting command, classroom capacity for specialty training, promotion requirements, and force-reduction objectives.

Appendix A. Deterministic Variant of the Objective Force Model

Objective Force Model Sets

G—Index for officer grade with $g \in G$.
I—Index for officer career specialty (AOC) with $i \in I$.
K—Index for an officer's year in service with $k \in K$.
F—Index for each goal with $f \in F$.

Objective Force Model Parameters

c_{ig}—Documented authorizations for AOC i in grade g.
r_{igk}—Continuation (deterministic) rate for AOC i in grade g in year k.
Cap_i—Maximum allowable officer quantity of AOC i.
$Floor_i$—Minimum acceptable officer quantity for AOC i.
Cap_{ig}—Maximum allowable officer quantity of AOC i at grade g.
$Floor_{ig}$—Minimum acceptable officer quantity for AOC i at grade g.
pf_{ig}—Minimum promotion rate for AOC i at grade g.
pc_{ig}—Maximum promotion rate for AOC i at grade g.
$SPIMM_g$—Medical Specialist Corps immaterial authorizations for grade g.
$AMIMM_g$—AMEDD immaterial authorizations for grade g.
THS_g—Transients, holdees, and students (THS) authorizations for grade g.
w_f—Decision-maker weight for goal f.

Objective Force Model Decision and Goal Deviation Variables

p_{ig}—Number of officers promoted in AOC i at grade g, $\forall g = 4, 5, 6$.
a_{ig}—Number of officers accessed (hired) for AOC i at grade g, $\forall g = 1, 2, 3$.
d_{ig}—Officer quantity for AOC i in grade g.
Inv_{igk}—Projected inventory of AOC i in grade g in year k.
pos_f—Positive deviation for goal f.
neg_f—Negative deviation for goal f.

Objective Force Model Formulation

$$\min \quad Z=\sum_f w_f(pos_f+neg_f). \tag{A1}$$

subject to

$$\sum_g\sum_i d_{ig}-\left(\sum_g\sum_i(c_{ig}+SPIMM_g+AMIMM_g+THS_g)\right)-pos_{f=1}+neg_{f=1}=0. \tag{A2}$$

$$\sum_i d_{ig}-\left(\sum_i c_{ig}+SPIMM_g+AMIMM_g+THS_g\right)-pos_{f=2}+neg_{f=2}=0 \quad \forall g=6. \tag{A3}$$

$$\sum_i d_{ig}-\left(\sum_i c_{ig}+SPIMM_g+AMIMM_g+THS_g\right)-pos_{f=3}+neg_{f=3}=0 \quad \forall g=5. \tag{A4}$$

$$\sum_i d_{ig}-\left(\sum_i c_{ig}+SPIMM_g+AMIMM_g+THS_g\right)-pos_{f=4}+neg_{f=4}=0 \quad \forall g=4. \tag{A5}$$

$$\sum_{g=1}^{3}\sum_i d_{ig}-\left(\sum_{g=1}^{3}\sum_i(c_{ig}+SPIMM_g+AMIMM_g+THS_g)\right)-pos_{f=5}+neg_{f=5}=0. \tag{A6}$$

The objective function in (A1) of the deterministic OFM seeks to minimize the sum of the weighted goal deviations. The target for the first goal constraint in constraint (A2) is for the total number of officers over each grade and AOC to equal the total documented authorizations (over each grade and AOC and the SP immaterial, AMEDD immaterial, and THS). The target for the second goal constraint in (A3) is for the total number of COLs over each AOC to equal the COL documented authorizations over each AOC and the SP immaterial, AMEDD immaterial, and THS. The target for the third goal constraint in (A4) is for the total number of LTCs over each AOC to equal the LTC documented authorizations over each AOC and the SP immaterial, AMEDD immaterial, and THS. The target for the fourth goal constraint in (A5) is for the total number of MAJs over each AOC to equal the MAJ documented authorizations over each AOC and the SP immaterial, AMEDD immaterial, and THS. The target for the last goal constraint in (A6) is for the total number of company grade (sum of 2LT, 1LT, and CPT) officers over each AOC to equal the company grade documented authorizations over each AOC and the SP immaterial, AMEDD immaterial, and THS.

$$\sum_g d_{i=1,g}\le Cap_{i=1}. \tag{A7}$$

$$\sum_g d_{i=3,g}\ge Floor_{i=3}. \tag{A8}$$

$$\sum_g d_{i=1,g}\ge Floor_{i=1}. \tag{A9}$$

$$\sum_g d_{i=3,g}\le Cap_{i=3}. \tag{A10}$$

$$\sum_g d_{i=2,g}\le Cap_{i=2}. \tag{A11}$$

$$\sum_g d_{ig}\ge \sum_g c_{ig} \quad \forall i\in I. \tag{A12}$$

$$\sum_{g=1}^{3} d_{ig}\ge \sum_{g=1}^{3} c_{ig} \quad \forall i\in I. \tag{A13}$$

$$d_{i=3,g=5}\ge Floor_{i=3,g=5}. \tag{A14}$$

$$d_{i=3,g=6}\ge Floor_{i=3,g=6}. \tag{A15}$$

$$d_{i=1,g=6}\le Cap_{i=1,g=6}. \tag{A16}$$

$$d_{i=2,g=6}\ge Floor_{i=2,g=6}. \tag{A17}$$

$$d_{i=1,g=5}\le Cap_{i=1,g=5}. \tag{A18}$$

$$d_{i=1,g=6}\ge Floor_{i=1,g=6}. \tag{A19}$$

$$d_{ig}\ge c_{ig} \quad \forall i\in I,\ g=4,5,6. \tag{A20}$$

Constraint (A7) is used to place a maximum allowable officer quantity for OTs, whereas constraint (A8) is used to place a minimum acceptable officer quantity for CDs. Constraint (A9) is used to place a minimum allowable officer quantity for OTs, whereas constraint (A10) is used to place a maximum allowable officer quantity for CDs. Constraint (A11) is used to place a maximum allowable officer quantity for PTs. Constraints (A12) ensure that the officer quantity must meet or exceed the total documented authorizations for each AOC i. Constraints (A13) ensure that the total officer quantity for company grade (sum of 2LT, 1LT, and CPT) officers must meet or exceed the total documented authorizations for company grade (sum of 2LT, 1LT, and CPT) officers for each AOC i. Constraint (A14) ensures that the LTC officer quantity for CDs must meet or exceed the minimum acceptable LTC officer quantity for CDs. Constraint (A15) ensures that the COL officer quantity for CDs must meet or exceed the minimum acceptable COL officer quantity for CDs. Constraint (A16) ensures that the COL officer quantity for OTs must be less than or equal to the maximum allowable COL officer quantity for OTs. Constraint (A17) ensures that the COL officer quantity for PTs must meet or exceed the minimum acceptable COL officer quantity for PTs. Constraint (A18) ensures that the LTC officer quantity for OTs must be less than or equal to the maximum allowable LTC officer quantity for OTs. Constraint (A19) ensures that the COL officer quantity for OTs must meet or exceed the minimum acceptable COL officer quantity for OTs. Constraints (A20) ensure that the MAJ, LTC, and COL officer quantity must meet or exceed MAJ, LTC, and COL documented authorizations for each AOC i.

$$p_{i,g=4}\ge pf_{i,g=4}Inv_{i,g=3,k=10} \quad \forall i\in I. \tag{A21}$$

$$p_{i,g=5}\ge pf_{i,g=5}Inv_{i,g=4,k=16} \quad \forall i\in I. \tag{A22}$$

$$p_{i,\,g=6} \geq pf_{i,\,g=6} Inv_{i,\,g=5,\,k=21} \quad \forall\, i \in I. \tag{A23}$$

$$p_{i,\,g=4} \leq pc_{i,\,g=4} Inv_{i,\,g=3,\,k=10} \quad \forall\, i \in I. \tag{A24}$$

$$p_{i,\,g=5} \leq pc_{i,\,g=5} Inv_{i,\,g=4,\,k=16} \quad \forall\, i \in I. \tag{A25}$$

$$p_{i,\,g=6} \leq pc_{i,\,g=6} Inv_{i,\,g=5,\,k=21} \quad \forall\, i \in I. \tag{A26}$$

Constraints (A21), (A22), and (A23) ensure that the number of resultant field grade (MAJ, LTC, COL) promotions must be greater than or equal to the minimum number of promotions (i.e., the product of the minimum promotion rate and pool) for each respective field grade, for each AOC i. Constraints (A24), (A25), and (A26) ensure that the number of resultant field grade (MAJ, LTC, COL) promotions must be less than or equal to the maximum number of promotions (i.e., the product of the maximum promotion rate and pool) for each respective field grade for each AOC i.

$$d_{ig} = \sum_k Inv_{igk} \quad \forall\, i \in I,\, g \in G. \tag{A27}$$

$$Inv_{i,\,g=1,\,k=1} = a_{i,\,g=1} r_{i,\,g=1,\,k=1} \quad \forall\, i \in I. \tag{A28}$$

$$Inv_{i=1,\,g=1,\,k=3} = Inv_{i=1,\,g=1,\,k=2} r_{i=1,\,g=1,\,k=3}. \tag{A29}$$

$$Inv_{i=1,\,g=2,\,k=3} = (Inv_{i=1,\,g=2,\,k=2} + a_{i=1,\,g=2}) r_{i=1,\,g=2,\,k=3}. \tag{A30}$$

$$Inv_{ig,\,k=3} = Inv_{ig,\,k=2} r_{ig,\,k=3} \quad \forall\, g \in G,\, i = 2,3,4. \tag{A31}$$

$$Inv_{i=1,\,g=2,\,k=2} = Inv_{i,\,g=1,\,k=1} r_{i,\,g=2,\,k=2}. \tag{A32}$$

$$Inv_{i,\,g=2,\,k=2} = ((Inv_{i,\,g=1,\,k=1} pf_{i,\,g=2}) + a_{i,\,g=2}) r_{i,\,g=2,\,k=2} \quad \forall\, i = 2,3,4. \tag{A33}$$

$$Inv_{igk} = Inv_{igk-1} r_{igk} \quad \forall\, k = 5\text{–}10, 12\text{–}16, 18\text{–}21, 23\text{–}30,\; g \in G,\, i \in I. \tag{A34}$$

$$Inv_{i,\,g=1,\,k=2} = Inv_{i,\,g=1,\,k=1} (1 - pf_{i,\,g=2}) r_{i,\,g=1,\,k=2} \quad \forall\, i \in I. \tag{A35}$$

$$Inv_{i,\,g=3,\,k=4} = ((Inv_{i,\,g=2,\,k=3} pf_{i,\,g=3}) + a_{i,\,g=3}) r_{i,\,g=3,\,k=4} \quad \forall\, i \in I. \tag{A36}$$

$$Inv_{i,\,g=2,\,k=4} = Inv_{i,\,g=2,\,k=3} (1 - pf_{i,\,g=3}) r_{i,\,g=2,\,k=3} \quad \forall\, i \in I. \tag{A37}$$

$$Inv_{i,\,g=4,\,k=11} = p_{i,\,g=4} \quad \forall\, i \in I. \tag{A38}$$

$$Inv_{i,\,g=3,\,k=11} = (Inv_{i,\,g=3,\,k=10} - p_{i,\,g=4}) r_{i,\,g=3,\,k=11} \quad \forall\, i \in I. \tag{A39}$$

$$Inv_{i,\,g=5,\,k=17} = p_{i,\,g=5} \quad \forall\, i \in I. \tag{A40}$$

$$Inv_{i,\,g=4,\,k=17} = (Inv_{i,\,g=4,\,k=16} - p_{i,\,g=5}) r_{i,\,g=4,\,k=17} \quad \forall\, i \in I. \tag{A41}$$

$$Inv_{i,\,g=6,\,k=22} = p_{i,\,g=6} \quad \forall\, i \in I. \tag{A42}$$

$$Inv_{i,\,g=5,\,k=22} = (Inv_{i,\,g=5,\,k=21} - p_{i,\,g=6}) r_{i,\,g=5,\,k=22} \quad \forall\, i \in I. \tag{A43}$$

$$p_{ig} \geq 0 \quad \forall\, i \in I,\, g = 4,5,6; \quad a_{ig} \geq 0 \text{ and integer} \quad \forall\, i \in I,\, g = 1,2,3; \quad pos_f,\, neg_f \geq 0 \quad \forall\, f \in F. \tag{A44}$$

Constraints (A27) assign the officer quantity as the total projected inventory (over all years) for each AOC i and grade g. Constraints (A28) assign the inventory for year $k = 1$ and grade $g = 1$ for each AOC i. Constraint (A29) assigns the inventory for year $k = 3$, grade $g = 1$, and $i = 1$, whereas constraint (A30) assigns the inventory for year $k = 3$, grade $g = 2$, and $i = 1$. Constraints (A31) assign the inventory for $k = 3$ for all grades and $i = 2, 3, 4$. Constraint (A32) assigns the inventory for $i = 1$, $g = 2$, and $k = 2$, whereas constraints (A33) assign the inventory for $g = 2$, $k = 2$, and $i = 2, 3, 4$. Constraints (A34) assign the inventory for all years that are not year $k = 1, 3$ or a promotion year, for each grade g and each AOC i. Constraints (A35) assign the inventory for year $k = 2$ and grade $g = 1$ for each AOC i. Constraints (A36) assign the inventory for year $k = 4$ and grade $g = 3$ for each AOC i. Constraints (A37) assign the inventory for year $k = 4$ and grade $g = 2$ for each AOC i. Constraints (A38) assign the inventory for year $k = 11$ and grade $g = 4$ for each AOC i. Constraints (A39) assign the inventory for year $k = 11$ and grade $g = 3$ for each AOC i. Constraints (A40) assign the inventory for year $k = 17$ and grade $g = 5$ for each AOC i. Constraints (A41) assign the inventory for year $k = 17$ and grade $g = 4$ for each AOC i. Constraints (A42) assign the inventory for year $k = 22$ and grade $g = 6$ for each AOC i. Constraints (A43) assign the inventory for year $k = 22$ and grade $g = 5$ for each AOC i. Constraint (A44) represents the nonnegativity and integer constraints for the decision and deviational variables.

Appendix B. Technical Details for Stochastic Optimization Methods

In stochastic method #1, we use a scenario-based Monte Carlo simulation approach to approximate the optimal objective value and the optimal solution of a stochastic goal program:

$$\min_{x \in X} E[f(x, \xi)].$$

We generate S scenarios and each scenario corresponds to one realization of the random vector ξ. We have S deterministic optimization problems with forms of

$$\min_{x \in X} f(x, \xi_i) \quad \text{where } i = 1, \ldots, S.$$

Let $z_i = \min_{x \in X} f(x, \xi_i)$ and $x_i = \arg\min_{x \in X} f(x, \xi_i)$. We use $\bar{z} = (1/S) \sum_i^S z_i$ and $\bar{x} = (1/S) \sum_i^S x_i$ to be the approximated optimal objective value and the approximated optimal solution, respectively.

In stochastic method #2, we formulate the stochastic optimization model using the following form:

$$\min_{x \in X} E[f(x, \xi)].$$

We leverage the sample average approximation (SAA) method (Rubinstein and Shapiro 1990) because prior samples are easily generated. Suppose $\xi_1, \ldots, \xi_N$ are N independent samples from a probability distribution; we then obtain an estimator:

$$\hat{f}_N(x) = \frac{1}{N}\sum_{j=1}^{N} f(x, \xi_j).$$

Because $\hat{f}_N(x)$ is an unbiased estimator of $f(x)$, we instead minimize the SAA to obtain an estimate of the optimal value:

$$\min_{x\in X} \hat{f}_N(x) = \frac{1}{N}\sum_{j=1}^{N} f(x, \xi_j).$$

Let $z^* = \min_{x\in X} f(x)$ and $\hat{z}_N = \min_{x\in X} \hat{f}_N(x)$. We then construct statistical lower and upper bounds suggested by Mak et al. (1999), who show that $E[\hat{z}_N] \le z^*$. To estimate $E[\hat{z}_N]$, we iterate the SAA program M times to obtain M optimal solutions and compute the average of the solutions:

$$v_{N,M} = \frac{1}{M}\sum_{j=1}^{M} \hat{z}_N^j,$$

which is an unbiased estimate of $E[\hat{z}_N]$. This is a lower bound on z^*. We compute the estimate of variance of the previous estimator as follows:

$$S^2_{vN,M} = \frac{1}{M(M-1)}\sum_{j=1}^{M} (\hat{z}_N^j - v_{N,M})^2.$$

For the upper bound, we estimate $f(\hat{x})$ at each of the M solutions $(x^1, \ldots, x^M)$ obtained before by N' (which is independent of N and usually very large) independent samples. Because z^* is the optimal value,

$$\hat{f}_{N'}(\hat{x}) = \frac{1}{N'}\sum_{j=1}^{N'} f^j_{N'}(\hat{x}) \ge z^*,$$

and $\hat{f}_{N'}(x)$ is an upper bound. An estimate of the variance of the previous estimator is given by:

$$S^2_{fN'\hat{x}} = \frac{1}{N'(N'-1)}\sum_{j=1}^{N'} (f^j_{N'}(\hat{x}) - \hat{f}_{N'}(\hat{x}))^2.$$

Further, we set the stopping rule according to the value of the estimate of $f(\hat{x}) - z^*$ and the variance. If either gap is not small enough, we increase the sample size and repeat the process.

Appendix C. Stochastic Method #2: Variant of the Objective Force Model

Stochastic Method #2 OFM Sets

G—Index for officer grade with $g \in G$.
I—Index for officer career specialty (AOC) with $i \in I$.
K—Index for an officer's year in service with $k \in K$.
F—Index for each goal with $f \in F$.
S—Index for the scenario under consideration with $s \in S$.

Stochastic Method #2 OFM Parameters

c_{ig}—Documented authorizations for AOC i in grade g.
r_{igks}—Continuation (stochastic) rate for AOC i in grade g in year k in scenario s.
Cap_i—Maximum allowable officer quantity of AOC i.
$Floor_i$—Minimum acceptable officer quantity for AOC i.
Cap_{ig}—Maximum allowable officer quantity of AOC i at grade g.
$Floor_{ig}$—Minimum acceptable officer quantity for AOC i at grade g.
pf_{ig}—Minimum promotion rate for AOC i at grade g.
pc_{ig}—Maximum promotion rate for AOC i at grade g.
$SPIMM_g$—Medical Specialist Corps immaterial authorizations for grade g.
$AMIMM_g$—AMEDD immaterial authorizations for grade g.
THS_g—Transient, holdee, and student (THS) authorizations for grade g.
w_f—Decision-maker weight for goal f.

Stochastic Method #2 OFM Decision and Goal Deviation Variables

p_{ig}—Number of officers promoted in AOC i at grade g, $\forall g = 4, 5, 6$.
a_{ig}—Number of officers accessed (hired) for AOC i at grade g, $\forall g = 1, 2, 3$.
d_{igs}—Number of officers for AOC i in grade g in scenario s.
Inv_{igks}—Projected inventory of AOC i in grade g in year k in scenario s.
pos_{fs}—Positive deviation for goal f in scenario s.
neg_{fs}—Negative deviation for goal f in scenario s.

Stochastic Method #2 OFM Formulation

$$\min \quad Z = \sum_s \frac{1}{S}\sum_f w_f(pos_f + neg_f). \tag{B1}$$

subject to

$$\sum_g\sum_i d_{igs} - \left(\sum_g\sum_i (c_{ig} + SPIMM_g + AMIMM_g + THS_g)\right) - pos_{f=1,s} + neg_{f=1,s} = 0 \quad \forall s \in S. \tag{B2}$$

$$\sum_i d_{igs} - \left(\sum_i c_{ig} + SPIMM_g + AMIMM_g + THS_g\right) - pos_{f=2,s} + neg_{f=2,s} = 0 \quad \forall g = 6, s \in S. \tag{B3}$$

$$\sum_i d_{igs} - \left(\sum_i c_{ig} + SPIMM_g + AMIMM_g + THS_g\right) - pos_{f=3,s} + neg_{f=3,s} = 0 \quad \forall g = 5, s \in S. \tag{B4}$$

$$\sum_i d_{igs} - \left(\sum_i c_{ig} + SPIMM_g + AMIMM_g + THS_g \right) - pos_{f=4,s} + neg_{f=4,s} = 0 \quad \forall g=4,\, s\in S. \tag{B5}$$

$$\sum_{g=1}^{3}\sum_i d_{igs} - \left(\sum_{g=1}^{3}\sum_i (c_{ig} + SPIMM_g + AMIMM_g + THS_g) \right) - pos_{f=5,s} + neg_{f=5,s} = 0, \quad \forall s\in S. \tag{B6}$$

$$\sum_g d_{i=1,gs} \le Cap_{i=1} \quad \forall s\in S. \tag{B7}$$

$$\sum_g d_{i=3,gs} \ge Floor_{i=3} \quad \forall s\in S. \tag{B8}$$

$$\sum_g d_{i=1,gs} \ge Floor_{i=1} \quad \forall s\in S. \tag{B9}$$

$$\sum_g d_{i=3,gs} \le Cap_{i=3} \quad \forall s\in S. \tag{B10}$$

$$\sum_g d_{i=2,gs} \le Cap_{i=2} \quad \forall s\in S. \tag{B11}$$

$$\sum_g d_{igs} \ge \sum_g c_{ig} \quad \forall i\in I,\, s\in S. \tag{B12}$$

$$\sum_{g=1}^{3} d_{igs} \ge \sum_{g=1}^{3} c_{ig} \quad \forall i\in I,\, s\in S. \tag{B13}$$

$$d_{i=3,g=5,s} \ge Floor_{i=3,g=5} \quad \forall s\in S. \tag{B14}$$

$$d_{i=3,g=6,s} \ge Floor_{i=3,g=6} \quad \forall s\in S. \tag{B15}$$

$$d_{i=1,g=6,s} \le Cap_{i=1,g=6} \quad \forall s\in S. \tag{B16}$$

$$d_{i=2,g=6,s} \ge Floor_{i=2,g=6} \quad \forall s\in S. \tag{B17}$$

$$d_{i=1,g=5,s} \le Cap_{i=1,g=5} \quad \forall s\in S. \tag{B18}$$

$$d_{i=1,g=6,s} \ge Floor_{i=1,g=6} \quad \forall s\in S. \tag{B19}$$

$$d_{igs} \ge c_{ig} \quad \forall i\in I, g=4,5,6, s\in S \tag{B20}$$

$$p_{i,g=4} \ge pf_{i,g=4} Inv_{i,g=3,k=10,s} \quad \forall i\in I,\, s\in S. \tag{B21}$$

$$p_{i,g=5} \ge pf_{i,g=5} Inv_{i,g=4,k=16,s} \quad \forall i\in I,\, s\in S. \tag{B22}$$

$$p_{i,g=6} \ge pf_{i,g=6} Inv_{i,g=5,k=21,s} \quad \forall i\in I,\, s\in S. \tag{B23}$$

$$p_{i,g=4} \le pc_{i,g=4} Inv_{i,g=3,k=10,s} \quad \forall i\in I,\, s\in S. \tag{B24}$$

$$p_{i,g=5} \le pc_{i,g=5} Inv_{i,g=4,k=16,s} \quad \forall i\in I,\, s\in S. \tag{B25}$$

$$p_{i,g=6} \le pc_{i,g=6} Inv_{i,g=5,k=21,s} \quad \forall i\in I,\, s\in S. \tag{B26}$$

$$d_{igs} = \sum_k Inv_{igks} \quad \forall i\in I,\, g\in G,\, s\in S. \tag{B27}$$

$$Inv_{i,g=1,k=1,s} = a_{i,g=1} r_{i,g=1,k=1,s} \quad \forall i\in I,\, s\in S. \tag{B28}$$

$$Inv_{i=1,g=1,k=3,s} = Inv_{i=1,g=1,k=2,s} r_{i=1,g=1,k=3,s} \quad \forall s\in S. \tag{B29}$$

$$Inv_{i=1,g=2,k=3,s} = (Inv_{i=1,g=2,k=2,s} + a_{i=1,g=2}) \cdot r_{i=1,g=2,k=3,s} \quad \forall s\in S. \tag{B30}$$

$$Inv_{ig,k=3,s} = Inv_{ig,k=2,s} r_{ig,k=3,s} \quad \forall g\in G,\, i=2,3,4,\, s\in S. \tag{B31}$$

$$Inv_{i=1,g=2,k=2,s} = Inv_{i,g=1,k=1,s} r_{i,g=2,k=2,s} \quad \forall s\in S. \tag{B32}$$

$$Inv_{i,g=2,k=2,s} = ((Inv_{i,g=1,k=1,s} pf_{i,g=2}) + a_{i,g=2}) \cdot r_{i,g=2,k=2,s} \quad \forall i=2,3,4,\, s\in S. \tag{B33}$$

$$Inv_{igks} = Inv_{igk-1,s} r_{igks} \forall k=5\text{–}10, 12\text{–}16, 18\text{–}21, 23\text{–}30,\; g\in G,\, i\in I,\, s\in S. \tag{B34}$$

$$Inv_{i,g=1,k=2,s} = Inv_{i,g=1,k=1,s}(1-pf_{i,g=2}) \cdot r_{i,g=1,k=2,s} \quad \forall i\in I,\, s\in S. \tag{B35}$$

$$Inv_{i,g=3,k=4,s} = ((Inv_{i,g=2,k=3,s} pf_{i,g=3}) + a_{i,g=3}) \cdot r_{i,g=3,k=4,s} \quad \forall i\in I,\, s\in S. \tag{B36}$$

$$Inv_{i,g=2,k=4,s} = Inv_{i,g=2,k=3,s}(1-pf_{i,g=3}) \cdot r_{i,g=2,k=3,s} \quad \forall i\in I,\, s\in S. \tag{B37}$$

$$Inv_{i,g=4,k=11,s} = p_{i,g=4} \quad \forall i\in I,\, s\in S. \tag{B38}$$

$$Inv_{i,g=3,k=11,s} = (Inv_{i,g=3,k=10,s} - p_{i,g=4}) \cdot r_{i,g=3,k=11,s} \quad \forall i\in I,\, s\in S. \tag{B39}$$

$$Inv_{i,g=5,k=17,s} = p_{i,g=5} \quad \forall i\in I,\, s\in S. \tag{B40}$$

$$Inv_{i,g=4,k=17,s} = (Inv_{i,g=4,k=16,s} - p_{i,g=5}) \cdot r_{i,g=4,k=17,s} \quad \forall i\in I,\, s\in S. \tag{B41}$$

$$Inv_{i,g=6,k=22,s} = p_{i,g=6} \quad \forall i\in I,\, s\in S. \tag{B42}$$

$$Inv_{i,g=5,k=22,s} = (Inv_{i,g=5,k=21,s} - p_{i,g=6}) \cdot r_{i,g=5,k=22,s} \quad \forall i\in I,\, s\in S. \tag{B43}$$

$$p_{ig} \ge 0 \quad \forall i\in I,\, g=4,5,6; \quad a_{ig} \ge 0 \text{ and integer } \forall i\in I,\, g=1,2,3; \quad pos_{fs}, neg_{fs} \ge 0 \quad \forall f\in F,\, s\in S. \tag{B44}$$

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Verification Letter

Paul J. Goymerac, Colonel, U.S. Army, Director, AMEDD Personnel Proponency Directorate, U.S. Army Medical Department Center & School, Fort Sam Houston, Texas 78234-6100, writes:

"This is to verify that the Army Medical Department Personnel Proponent Directorate has adopted and continues to use the modeling process described by Nathan Bastian in his manuscript "The AMEDD Uses Goal Programming to Optimize Workforce Planning Decisions." The Objective Force Models are a key factor in the officer personnel recommendations my directorate provides to the Army Medical Department leadership to support decisions impacting the number of officers brought on to active duty, the number trained in the various specialties, and the number promoted to a higher rank each year."

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