Data 605 - Assignment 8

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Contents

Probability

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11. A company buys 100 lightbulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out? (See Exercise 10.) From exercise 10: $\frac{\mu}{n}$ where n is independent random variables with the exponential density and the mean μ.

Key: $\mu = 1000$ hours n = 100 lightbulbs

```
burnout <- 1000 / 100
burnout
```

```
## [1] 10
```

Solution: The expected time for the first of these bulbs to burn out is 10 hours.

14. Assume that X1 and X2 are independent random variables, each having an exponential density with parameter λ. Show that Z = X1 - X2 has density $fZ^{(z)} = (1/2)\lambda e^{-\lambda|z|}$

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Start with PDFs: $f(x_1) = \lambda e^{-\lambda x1}$

$f(x_2) = \lambda e^{-\lambda x2}$

$f(x_1) * f(x_2) = \lambda 2e^{-\lambda(x1+x2)}$

Known: $Z = x_1 - x_2$

$x_1 = Z + x_2$

Therefore: $\lambda 2e - \lambda((z + x2) + x2) = \lambda 2e = \lambda(z + 2x2)$

Z is negative: $\int \lambda 2e^{\lambda(z+2x_2)}dx = \frac{\lambda}{2}e^{\lambda z}$

Z is positive: $\int \lambda 2e^{-\lambda(z+2x2)}dx = \frac{\lambda}{2}e^{-\lambda|z|}$

Solution: $(1/2)\lambda e^{-\lambda|z|}$

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1. Let X be a continuous random variable with mean $\mu = 10$ and variance $\sigma^2 = 100/3$. Using Chebyshev's Inequality, find an upper bound for the following probabilities.

(a) P(|X - 10| $\geq$ 2).
(b) P(|X - 10| $\geq$ 5).
(c) P(|X - 10| $\geq$ 9).
(d) P(|X - 10| $\geq$ 20).

```
# (a)
k <- 2 / sqrt(100 / 3)
round((1 / k^2), 3)
```

```
## [1] 8.333
```

Solution: Since the highest value of probability is 1, $\therefore$ 8.333 $\approx$ 1

```
# (b)
k <- 5 / sqrt(100 / 3)
round((1 / k^2), 3)
```

```
## [1] 1.333
```

Solution: Since the highest value of probability is 1, $\therefore$ 1.333 $\approx$ 1

```
# (c)
k <- 9 / sqrt(100 / 3)
round((1 / k^2), 3)
```

```
## [1] 0.412
```

Solution: upper bound is 0.412

```r
# (d)
k <- 20 / sqrt(100 / 3)
round((1 / k^2), 3)
```

```
## [1] 0.083
```

Solution: upper bound is 0.083