Data 605 - Assignment 7

Leticia Salazar

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Contents

Probability

1. Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y.

$P(X = 1) = \frac{k^n-(k-1)^n}{k^n}$

$P(X = 2) = \frac{(k-2+1)^n-(k-2)^n}{k^n}$

$P(X = y) = \frac{(k-y+1)^n-(k-y)^n}{k^n}$

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part)

```
# Fail each year
P_Fail <- 1 / 10

# Not Fail each year
P_NFail <- 1 - P_Fail

# Expected Value
ev <- 1 / P_Fail
ev
```

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years)

```
## [1] 10
```

```r
# Standard Deviation
sd <- sqrt(P_NFail / (P_Fail ^ 2))
round(sd, 2)
```

```
## [1] 9.49
```

```r
# Modeling as geometric
P <- 1 - pgeom(8 - 1, P_Fail)
round(P, 2)
```

```
## [1] 0.43
```

```r


# Probability of Failing
P_eFailing <- exp(-1 * (8/10))
round(P_eFailing, 2)
```

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

```
## [1] 0.45
```

```r
# Expected Value using lambda
ev <-  10
ev
```

```
## [1] 10
```

```r
# Standard Deviation
sd_expected <- sqrt(1 / (.10 ^ 2))
sd_expected
```

```
## [1] 10
```

```r


n <- 8
p <- 1 /10
q <- 1 - p
k <- 0

# Probability of Machine Failing
p_Bin <- dbinom(k, n, p)
round(p_Bin, 2)
```

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

```
## [1] 0.43
```

```r
# Expected Value
ev <- n * p
ev
```

```
## [1] 0.8
```

```r
# Standard Deviation
sd <- sqrt(n * p * q)
round(sd, 2)
```

```
## [1] 0.85
```

```r
lambda <- 8 / 10

# Probability of machine Failing
p_Poisson <- ppois(0, lambda = 0.8)
p_Poisson
```

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

```
## [1] 0.449329
```

```r
# Expected Value
ev_Poisson <- lambda
ev_Poisson
```

```
## [1] 0.8
```

```r
# Standard Deviation
sd_Poisson <- sqrt(lambda)
round(sd_Poisson, 2)
```

```
## [1] 0.89
```

References:

https://www.geeksforgeeks.org/poisson-functions-in-r-programming/#:~:text=ppois(),or%20less%20than%