

Data 605: Assignment 3

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Contents

```
# Install packages if needed and load library
#install.packages("Matrix")
library(Matrix)
```

Problem Set 1

(1) What is the rank of the matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

```
# Create matrix A
A <- matrix(c(1, 2, 3, 4, -1, 0, 1, 3, 0, 1, -2, 1, 5, 4, -2, -3), nrow = 4, byrow = TRUE)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```
# Rank of the matrix A
qr(A)$rank
```

```
## [1] 4
```

Solution: Given the matrix the rank is 4.

(2) Given an $m \times n$ matrix where $m > n$, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

Assuming that the matrix is a non-zero, the maximum and minimum rank is no greater than the smallest row or column dimension, in this case being n .

(3) What is the rank of matrix B?

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

```
# Create matrix B
B <- matrix(c(1, 2, 1, 3, 6, 3, 2, 4, 2), nrow = 3, byrow = TRUE)
qr(B)$rank
```

```
## [1] 1
```

Solution: The rank for matrix B is 1.

Problem Set 2

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

```
# Create matrix A
A_ps2 <- matrix(c(1, 2, 3, 0, 4, 5, 0, 0, 6), nrow = 3, byrow = T)

# Find Eigenvalues
eigen(A_ps2)$values
```

```
## [1] 6 4 1
```

```
# Find Eigenvectors
eigen(A_ps2)$vectors
```

```
##           [,1]      [,2] [,3]
## [1,] 0.5108407 0.5547002    1
## [2,] 0.7981886 0.8320503    0
## [3,] 0.3192754 0.0000000    0
```

Identity Matrix $Av = \lambda v$

$$Av = \lambda Iv$$

$$|A - \lambda I| = 0$$

Solving for λ

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

Which is:

$$\begin{vmatrix} (1 - \lambda) & 2 & 3 \\ 0 & (4 - \lambda) & 5 \\ 0 & 0 & (6 - \lambda) \end{vmatrix} = 0$$
$$(1 - \lambda) \begin{vmatrix} (4 - \lambda) & 5 \\ 0 & (6 - \lambda) \end{vmatrix} + 2 \begin{vmatrix} 0 & 5 \\ 0 & (6 - \lambda) \end{vmatrix} + 3 \begin{vmatrix} 0 & (4 - \lambda) \\ 0 & 0 \end{vmatrix}$$

Equation:

- $(1 - \lambda)[(4 - \lambda)(6 - \lambda) - (0 * 0)] - 2[(0)(6 - \lambda) - (0 * 0)] + 3[(0 * 0) - (0 * 4 - \lambda)]$
- $(1 - \lambda)[(4 - \lambda)(6 - \lambda) - 0] = 0$
- $(1 - \lambda)(\lambda^2 - 10\lambda + 24) = 0$

Characteristic polynomial:

- $\lambda^3 - 11\lambda^2 + 34\lambda - 24 = 0$

Therefore, the Eigenvalues are:

- $\lambda_1 = 1$
- $\lambda_2 = 4$
- $\lambda_3 = 6$

The Eigenvectors are:

$$\lambda_1 = 1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_2 = 4 = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}$$

$$\lambda_3 = 6 = \begin{bmatrix} 8 \\ 5 \\ 5 \\ 2 \\ 1 \end{bmatrix}$$

References:

- <https://www.mathsisfun.com/algebra/eigenvalue.html>
- <https://stackoverflow.com/questions/10881392/rank-of-a-matrix-in-r>
- <https://cran.r-project.org/web/packages/matlib/vignettes/eigen-ex1.html>