Entanglement Criteria

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Abstract

Quantum entanglement is at the heart of quantum information processing, yet its detection is non-trivial [3]. This poster investigates the Positive Partial Transpose (PPT) criterion—a fundamental tool for proving entanglement in bipartite systems[5, 2].

Introduction

Quantum entanglement is a phenomenon where the quantum states of two or more particles become linked, such that the state of one cannot be described independently of the others, even when they are separated by large distances[3]. Entanglement is the foundation of quantum communication, ensuring ultra-secure encryption, and quantum computing, where entangled states enable powerful, parallel computations. It's also key in quantum teleportation: transferring the state of particles without moving the particles themselves.
Several entanglement detection criteria exist, each with its own strengths. Figure 1 visualizes the detection power overlap between three key approaches: PPT, RED, and RLN [3].

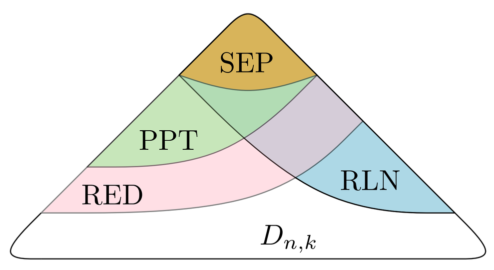

Figure 1. Detection power of entanglement criteria. PPT covers the largest region; RED and RLN detect distinct subsets beyond PPT. Adapted from [12].

Criterion	Conceptual Basis	Best Use Cases	Limitations
PPT	Partial transposition	Low-dimensional systems (2×2, 2×3), Gaussian states	Cannot detect bound entanglement in higher dimensions
RED	Reduction map	Mixed states, complements PPT	Misses some RLN-detectable states
RLN	Matrix realignment	Non-PPT entangled states	Less effective in some RED-detectable cases

None of the criteria alone is sufficient for detecting all entangled states. RED and RLN offer complementary perspectives beyond what PPT can reveal.

Positive Partial Transpose (PPT)

The PPT(Peres-Horodecki) criterion states:
If a bipartite quantum state $\rho \in S_A \otimes S_B$ is separable, then its partial transpose $\rho^{T_B} := (I_{d_A} \otimes T_B)[\rho] \geq 0$.
If **PPT quantum states=**$\{\rho : \rho^{\Gamma} \geq 0\}$, then PPT criterion has the following **practical form: if a state is not PPT, then it is entangled**.

This criterion is capable of:

- completely characterizing the set of separable quantum states only for 2×2 (qubit - qubit) and 2×3 (qubit - qutrit) bipartite systems [3]
- independent of transposed subsystem.
- in larger systems there exist entangled states which satisfy the PPT criterion, for example the so-called phenomenon of bound entanglement or entangled PPT states[4]. **Bound entanglement** refers to entangled states from which no entanglement can be distilled via local operations and classical communication (LOCC), making them particularly challenging to detect and utilize directly
- The PPT criterion is particularly effective for Gaussian states, where it acts as a complete entanglement detector in simple bipartite systems such as qubit × qubit or qubit × qutrit. This capability can be visualized the diagram below. It highlights entanglement via the interference patterns between the two Gaussians, providing a geometric perspective on the state's separability.

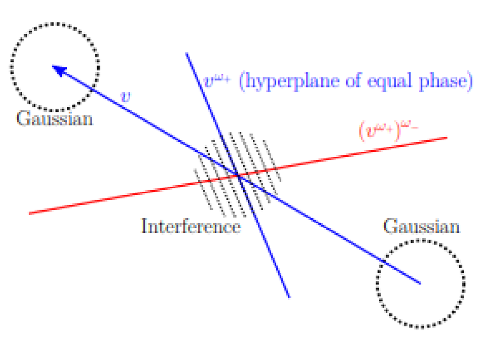

Figure 2. Schmetic diagram of the Wigner function of a bipartite cat state. Adapted from [11].

✍ The Wigner function is a quasi-probability distribution that represents a quantum state in phase space. For a bipartite cat state, as shown in Figure 2, the Wigner function can exhibit negative regions or interference patterns, which are key indicators of non-classicality and entanglement. The presence of such patterns here visually suggests entanglement.
✍ Gaussian states are quantum states whose Wigner function is a Gaussian function, typically describing bosonic systems (like light) in thermal or squeezed states. They are fully characterized by their first and second moments

Experiments

1.Quantum Metrology Activation: An optimal 3×3 PPT state, as described in [4], was subjected to a two-iteration sequential phase-imprinting protocol. The Quantum Fisher Information (QFI) was computed under depolarizing noise of strength p.The results showed:

- **No noise** ($p = 0$): QFI increased from the classical bound of 8 to **8.2762**.
- **Moderate noise** ($p = 0.1$): QFI remained **above 8** until approximately $p \approx 0.18$.

This demonstrates that bound entangled PPT states can be *activated* to surpass the shot-noise limit in practical metrological tasks.
*The Quantum Fisher Information (QFI) is a metric in quantum metrology, quantifying the maximum precision attainable in estimating a parameter using a given quantum state. A higher QFI indicates better precision. The results showed that even in the presence of moderate depolarizing noise (p=0.1), the QFI remained above 8 until approximately p=0.18 , demonstrating that bound entangled PPT states can be activated to surpass the shot-noise limit in practical metrological tasks.
The shot-noise limit represents the fundamental precision limit achievable with classical resources. Surpassing it indicates a quantum advantage.

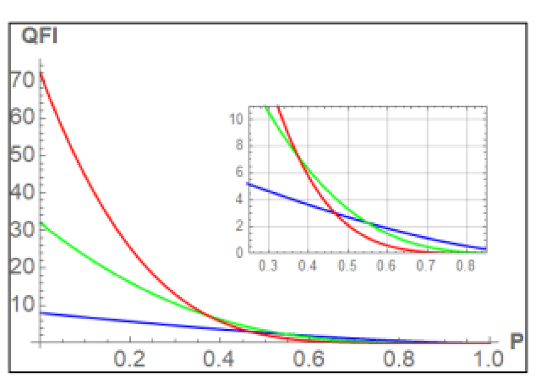

Figure 3. Quantum Fisher Information vs. noise p for the 3×3 PPT state. Adapted from [4].

Figure 3 illustrates the Quantum Fisher Information (QFI) as a function of depolarizing noise p for a 3×3 PPT state. The plot clearly shows that even with increasing noise, the QFI remains above the classical bound (typically 8 for this type of system), demonstrating the resilience and utility of these PPT-entangled states for enhancing measurement precision in quantum metrology—even when they exhibit bound entanglement.

2.Non-Gaussian Continuous-Variable Entanglement: A bipartite superposition of GQW states was considered [1]:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\big(|n,m\rangle + |m,n\rangle\big)$$

and it was computed the Wigner-moment matrix $M_{\tilde{f}}(\rho)$ via Airy-function integrals, and applied the non-Gaussian PPT criterion by partially transposing the matrix. For all levels $(n,m) \leq 50$, it was observed:

$$\det\left[M_{\tilde{f}}(\rho)\right]^{\Gamma} < 0$$

which unambiguously signals non-Gaussian entanglement in a continuous-variable system.

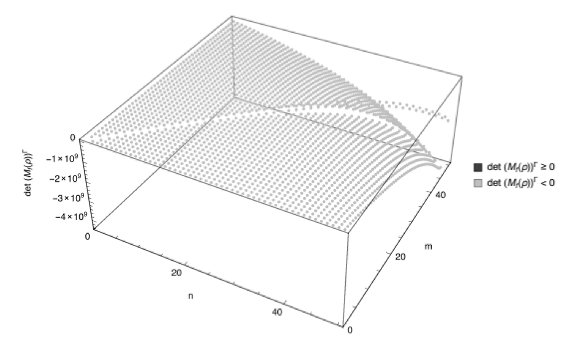

Figure 4. Determinant of the partially transposed moment matrix $M_{\tilde{f}}(\rho)$ as a function of levels n and m. Negative values indicate entanglement via the PPT criterion. Adapted from [1].

Wigner-moment matrix is constructed from the moments of the Wigner function and is used to apply entanglement criteria to continuous-variable systems. The non-Gaussian PPT criterion extends the standard PPT criterion to these systems. As shown, negative values of the determinant (indicated by blue regions from Figure2) unambiguously signal non-Gaussian entanglement according to the PPT criterion. This demonstrates how the PPT criterion, when applied to the Wigner-moment matrix, can effectively detect entanglement in continuous-variable systems, specifically those exhibiting non-Gaussian characteristics.

- Isotropic States: Are convex combinations of maximally entangled state and white noise: $\rho_\alpha = \alpha|\psi >< \psi| + (1-\alpha)\frac{I}{d^2}$ for $-\frac{1}{d^2-1} \leq \alpha \leq 1$
  - ρ_α is separable $\iff -\frac{1}{d^2-1} \leq \alpha \leq \frac{1}{d+1}$
  - if α is not in this range the state ρ_α is entangled

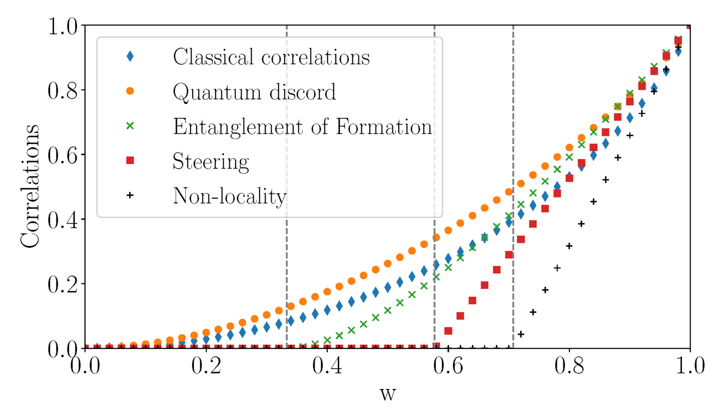

Figure 5. Visually compare growth of entanglement in Werner vs isotropic states. Those plots show how entanglement grows from zero at the separability threshold. Adapted from [11].

- Werner States: are a mixture of symmetric and antisymmetric subspace projections: $\sigma_\lambda = \lambda\pi_S + (1-\lambda)\pi_A$ for $0 \leq \lambda \leq 1$ where:
  - $\pi_S = \frac{2}{d(d+1)}(I+F)$
  - $\pi_A = \frac{2}{d(d-1)}(I-F)$ and F is called the flip operator $F(\phi_1 \otimes \phi_2) = \phi_2 \otimes \phi_1)$
    - σ_λ*isseparable* $\iff \lambda \geq \frac{1}{2}$
    - if $\lambda < \frac{1}{2}$ then the state is entangled

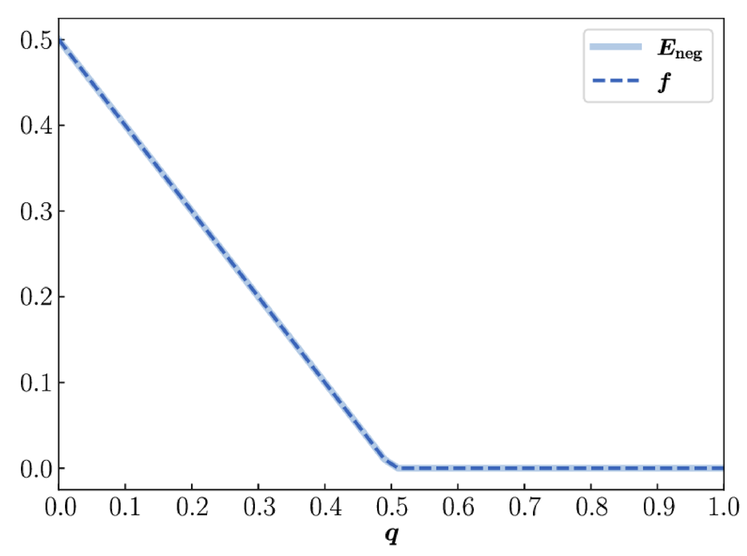

Figure 6. Negativity if Werner state $\rho(q)$. This plot shows negativity of two-qubit Werner states as a function of the mixing parameter q. It can be clearly seen that if $q \leq \frac{1}{2}$ then the state is separable, otherwise is entangled. Adapted from [11].

Negativity is an entanglement measure directly related to the partial transpose; a negative value of the eigenvalues of the partially transposed density matrix indicates entanglement. As shown, for q≤1/2, the Negativity is zero, indicating separability, while for q>1/2, the Negativity becomes positive, unambiguously signaling entanglement."

Conclusion

In this work, we investigated different entanglement detection criteria, with a focus on the PPT criterion. We showed its reliability in identifying entangled states in low-dimensional bipartite systems such as qubit–qubit and qubit–qutrit setups [5, 2]. Although PPT remains a widely used tool, we also pointed out its limitations—especially in higher-dimensional systems, where bound entangled states can satisfy the PPT condition yet still be inseparable [2, 3].
Through practical examples, we illustrated how PPT-entangled states can be activated for quantum metrology [4], and can also help detect non-Gaussian entanglement in continuous-variable systems [1]. These cases show that even a criterion with known limitations can still play a valuable role in quantum applications.
In summary, this poster provided insight into both the strengths and constraints of the PPT criterion, reinforcing its relevance while highlighting the need for complementary approaches in more complex scenarios.

References

Main references:

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