Solitary Wave Families in Two Non-Integrable Models Using Reversible Systems Theory

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Overview

- Definitions
- Background
- Literature Review
- Method of Solution
- The Generalized Pochammer-Chree Equations
- A Generalized Microstructure Equation

Reversible Dynamical System (Iooss & Adelmeyer)

Consider

$$\frac{dz}{dt} = F(z;\mu), z \in \mathbb{R}^n, \mu \in \mathbb{R} \tag{1}$$

where

$$F(0;0) = 0$$

. If there exits a **unitary map**

$$S : \mathbb{R}^n \mapsto \mathbb{R}^n, S \neq I$$

such that

$$F(Sz;\mu) = -SF(z;\mu)$$

for all z and μ then (1) is a reversible system.

Families of Solitary Waves

Here we will use the term solitary wave or "solitons" in the broadest sense, as a solution to a nonlinear equation which ETC

Normal Form Theory

Properties of Bilinear Functions

Selected Literature Review

The Generalized Pochammer-Chree Equations

- GPC1

$$(u - u_{xx})_{tt} - \left(a_1 u + a_2 u^2 + a_3 u^3\right)_{xx} = 0 \tag{2}$$

- GPC2

$$(u - u_{xx})_{tt} - \left(a_1 u + a_3 u^3 + a_5 u^5\right)_{xx} = 0 \tag{3}$$

Travelling Wave ODE

Let $z = x - ct$ and $u(x,t) = \phi(z)$ to reduce (2) and (3) to the Travelling Wave ODE

$$\phi_{zzzz} - q\phi_{zz} + p\phi = \mathcal{N}_{1,2}[\phi] \tag{4}$$

where

$$\begin{aligned} \mathcal{N}_1[\phi] &= -\frac{1}{c^2}\left[3a_3\left(2\phi\phi_z^2 + \phi^2\phi_{zz}\right) + 2a_2\left(\phi_{zz}\phi_z + \phi_z^2\right)\right] \\ \mathcal{N}_2[\phi] &= -\frac{1}{c^2}\left[3a_3\left(2\phi\phi_z^2 + \phi^2\phi_{zz}\right) + 5a_5\left(4\phi^3\phi_z^2 + \phi^4\phi_{zz}\right)\right] \end{aligned}$$

A Generalized Microstructure PDE

$$v_{tt} - bv_{xx} - \frac{\mu}{2}\left(v^2\right)_{xx} - \delta\left(\beta v_{tt} - \gamma v_{xx}\right)_{xx} = 0 \tag{6}$$

Travelling Wave ODE

Let $z = x - ct$ and $u(x,t) = \phi(z)$ to reduce to the Travelling Wave ODE

$$\phi_{zzzz} - q\phi_{zz} + p\phi = \mathcal{N}[\phi] \tag{7}$$

where

$$\mathcal{N}[\phi] = -\Delta_1\phi_z^2 - b\Delta_1\phi\phi_{zz} \tag{8}$$

Results:The Generalized Pochammer-Chree Equations

Results: A Generalized Microstructure PDE