Solitary Wave Families in Two Non-Integrable Models Using Reversible Systems Theory

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Overview

- Definitions
- Background
- Literature Review
- Method of Solution
- The Generalized Pochammer-Chree Equations
- A Generalized Microstructure Equation

Reversible Dynamical System (Iooss & Adelmeyer)

Consider

$$\frac{dz}{dt} = F(z;\mu), z \in \mathbb{R}^n, \mu \in \mathbb{R} \tag{1}$$

where

$$F(0;0) = 0$$

. If there exits a **unitary map**

$$S : \mathbb{R}^n \mapsto \mathbb{R}^n, S \neq I$$

such that

$$F(Sz;\mu) = -SF(z;\mu)$$

for all z and μ then (1) is a reversible system.

Families of Solitary Waves

Here we will use the term solitary wave or "soliton" to mean a pulse-like solution to an evolution equation. For example, a two parameter family of solitary waves is

$$A(z) = \ell \operatorname{sech}^2 kz \tag{2}$$

where k and ℓ are the parameters which determine the speed and the height of the wave.

Normal Form Theory

After a nonlinear change of variables [?] one may put the Center Manifold into Normal Form.

- Two-Dimenional Center Manifold
  - $\lambda_{1-4} = 0, 0, \pm\lambda, \lambda \in \mathbb{R}$
  - $Y = A\zeta_0 + B\zeta_1 + \Psi(\epsilon, A, B)$
- Four-Dimenional Center Manifold
  - $\lambda_{1-4} = 0, 0, \pm i\omega, \omega \in \mathbb{R}$
  - $Y = A\zeta_0 + B\zeta_1 + C\zeta_+ + \bar{C}\zeta_- + \Psi(\epsilon, A, B, C, \bar{C})$

where $\zeta_0, \zeta_1, \zeta_+, \zeta_-$ are eigenvectors of the linearized operator.

Properties of Bilinear Functions

A function

$$B : \mathbb{C}x\mathbb{C} \mapsto \mathbb{C}$$

satisfying the following axioms

$$\begin{aligned} B(x+y,z) &= B(x,z)+B(y,z) && \text{(3a)} \\ B(\lambda x,y) &= \lambda B(x,y) && \text{(3b)} \\ B(x,y+z) &= B(x,y)+B(x,z) && \text{(3c)} \\ B(x,\lambda y) &= \lambda B(x,y) && \text{(3d)} \end{aligned}$$

where $\lambda \in \mathbb{C}$, is called **bilinear.**

Selected Literature Review

The Generalized Pochammer-Chree Equations

- GPC1

$$(u - u_{xx})_{tt} - \left(a_1 u + a_2 u^2 + a_3 u^3\right)_{xx} = 0 \tag{4}$$

- GPC2

$$(u - u_{xx})_{tt} - \left(a_1 u + a_3 u^3 + a_5 u^5\right)_{xx} = 0 \tag{5}$$

Travelling Wave ODE

Let $z = x - ct$ and $u(x,t) = \phi(z)$ to reduce (4) and (5) to the Travelling Wave ODE

$$\phi_{zzzz} - q\phi_{zz} + p\phi = \mathcal{N}_{1,2}[\phi] \tag{6}$$

where

$$\begin{aligned} \mathcal{N}_1[\phi] &= -\frac{1}{c^2}\left[3a_3\left(2\phi\phi_z^2 + \phi^2\phi_{zz}\right) + 2a_2\left(\phi_{zz}\phi_z + \phi_z^2\right)\right] \\ \mathcal{N}_2[\phi] &= -\frac{1}{c^2}\left[3a_3\left(2\phi\phi_z^2 + \phi^2\phi_{zz}\right) + 5a_5\left(4\phi^3\phi_z^2 + \phi^4\phi_{zz}\right)\right] \end{aligned}$$

A Generalized Microstructure PDE

$$v_{tt} - bv_{xx} - \frac{\mu}{2}\left(v^2\right)_{xx} - \delta\left(\beta v_{tt} - \gamma v_{xx}\right)_{xx} = 0 \tag{8}$$

Travelling Wave ODE

Let $z = x - ct$ and $u(x,t) = \phi(z)$ to reduce to the Travelling Wave ODE

$$\phi_{zzzz} - q\phi_{zz} + p\phi = \mathcal{N}[\phi] \tag{9}$$

where

$$\mathcal{N}[\phi] = -\Delta_1 \phi_z^2 - b\Delta_1 \phi\phi_{zz} \tag{10}$$

Results:The Generalized Pochammer-Chree Equations

(11)

Results: A Generalized Microstructure PDE

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