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# Solitary Wave Families in Two Non-Integrable Models Using Reversible Systems Theory

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### Overview

- Definitions
- Background
- Literature Review
- Method of Solution
- The Generalized Pochammer-Chree Equations
- A Generalized Microstructure Equation

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# Reversible Dynamical System (looss & Adelmeyer)

Consider

$$\frac{dz}{dt} = F(z; \mu), z \in \mathbb{R}^n, \mu \in \mathbb{R}$$
 (1)

where

$$F\left( 0;0\right) =0$$

. If there exits a unitary map

$$S: \mathbb{R}^n \mapsto \mathbb{R}^n, S \neq I$$

such that

$$F(Sz; \mu) = -SF(z; \mu)$$

for all z and  $\mu$  then (1) is a reversible system.

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## Families of Solitary Waves

Here we will use the term solitary wave or "soliton" to mean a pulse-like solution to an evolution equation. For example, a two parameter family of solitary waves is

$$A(z) = \ell \operatorname{sech}^2 kz \tag{2}$$

where k and  $\ell$  are the parameters which determine the speed and the height of the wave.

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## Normal Form Theory

After a nonlinear change of variables [?] one may put the Center Manifold into Normal Form.

- Two-Dimenional Center Manifold
  - $\lambda_{1-4} = 0, 0, \pm \lambda, \lambda \in \mathbb{R}$
  - $Y = A\zeta_0 + B\zeta_1 + \Psi(\epsilon, A, B)$
- Four-Dimensional Center Manifold
  - $\lambda_{1-4} = 0, 0, \pm i\omega, \omega \in \mathbb{R}$
  - $Y = A\zeta_0 + B\zeta_1 + C\zeta_+ + \bar{C}\zeta_- + \Psi(\epsilon, A, B, C, \bar{C})$

where  $\zeta_0, \zeta_1, \zeta_+, \zeta_-$  are eigenvectors of the linearized operator.

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### Properties of Bilinear Functions

A function

 $B: \mathbb{C}x\mathbb{C} \mapsto \mathbb{C}$ 

satisfying the following axioms

$$B(x+y,z) = B(x,z) + B(y,z)$$
 (3a)

$$B(\lambda x, y) = \lambda B(x, y) \tag{3b}$$

$$B(x, y + z) = B(x, y) + B(x, z)$$
 (3c)

$$B(x, \lambda y) = \lambda B(x, y) \tag{3d}$$

where  $\lambda \in \mathbb{C}$ , is called **bilinear**.

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# The Generalized Pochammer-Chree Equations

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$$(u - u_{xx})_{tt} - (a_1u + a_2u^2 + a_3u^3)_{xx} = 0$$
 (4)

• GPC2

$$(u - u_{xx})_{tt} - (a_1u + a_3u^3 + a_5u^5)_{xx} = 0$$
 (5)

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## Travelling Wave ODE

Let z=x-ct and  $u(x,t)=\phi(z)$  to reduce (4) and (5) to the Travelling Wave ODE

$$\phi_{zzzz} - q\phi_{zz} + p\phi = \mathcal{N}_{1,2}[\phi] \tag{6}$$

where

$$\mathcal{N}_{1}[\phi] = -\frac{1}{c^{2}} \left[ 3a_{3} \left( 2\phi\phi_{z}^{2} + \phi^{2}\phi_{zz} \right) + 2a_{2} \left( \phi_{zz}\phi_{z} + \phi_{z}^{2} \right) \right] 
\mathcal{N}_{2}[\phi] = -\frac{1}{c^{2}} \left[ 3a_{3} \left( 2\phi\phi_{z}^{2} + \phi^{2}\phi_{zz} \right) + 5a_{5} \left( 4\phi^{3}\phi_{z}^{2} + \phi^{4}\phi_{zz} \right) \right]$$

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### A Generalized Microstructure PDE

$$v_{tt} - bv_{xx} - \frac{\mu}{2} \left( v^2 \right)_{xx} - \delta \left( \beta v_{tt} - \gamma v_{xx} \right)_{xx} = 0$$
 (8)

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## Travelling Wave ODE

Let z = x - ct and  $u(x, t) = \phi(z)$  to reduce to the Travelling Wave ODF

$$\phi_{zzzz} - q\phi_{zz} + p\phi = \mathcal{N}[\phi] \tag{9}$$

where

$$\mathcal{N}\left[\phi\right] = -\Delta_1 \phi_z^2 - b\Delta_1 \phi \phi_{zz} \tag{10}$$

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# Results: The Generalized Pochammer-Chree Equations

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## Results: A Generalized Microstructure PDE

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