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**ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ НЕСТАЦИОНАРНОГО ОДНОМЕРНОГО
ТЕЧЕНИЯ ГАЗА:
СХЕМА ДЛЯ $\log(\rho)$ С ЦЕНТРАЛЬНЫМИ РАЗНОСТЯМИ**

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1 Постановка задачи

Приведем систему уравнений, описывающую нестационарное двумерное движение вязкого баротропного газа:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_1}{\partial x_1} + \frac{\partial \rho u_2}{\partial x_2} = 0, \\ \frac{\partial \rho u_1}{\partial t} + \frac{\partial \rho u_1^2}{\partial x_1} + \frac{\partial \rho u_1 u_2}{\partial x_2} + \frac{\partial p}{\partial x_1} = \mu \left(\frac{4}{3} \frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2} + \frac{1}{3} \frac{\partial^2 u_2}{\partial x_1 \partial x_2} \right) + \rho f_1, \\ \frac{\partial \rho u_2}{\partial t} + \frac{\partial \rho u_1 u_2}{\partial x_1} + \frac{\partial \rho u_2^2}{\partial x_2} + \frac{\partial p}{\partial x_2} = \mu \left(\frac{1}{3} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_2}{\partial x_1^2} + \frac{4}{3} \frac{\partial^2 u_2}{\partial x_2^2} \right) + \rho f_2, \\ p = p(\rho). \end{cases} \quad (1.1)$$

Здесь μ – коэффициент вязкости газа (известная неотрицательная константа), p – давление газа (известная функция), f – вектор внешних сил (известная функция).

Неизвестные функции ρ и u , плотность и скорость соответственно, – функции от двух переменных t и x (переменные Эйлера), причем

$$(t, x) \in Q = [0, T] \times \Omega.$$

В качестве граничных условий берется следующее:

$$\rho|_{\Gamma^-} = \rho_\gamma = 1, \quad u_1|_{\Gamma^-} = \omega \in \{0, 1; 1\}, \quad \left. \frac{\partial u_1}{\partial x_1} \right|_{\Gamma^+} = 0. \quad (1.2)$$

На оставшейся границе компоненты скорости равны нулю, а функция плотности считается неизвестной.

Для решения задачи введем равномерную сетку ω_h с шагом h_x по оси x , с шагом h_y по оси y и с шагом τ по оси t . Введем константы M_x , M_y и N , такие что $X = M_x h_x$, $Y = M_y h_y$ и $T = N\tau$.

2 Схема для $\log(\rho)$ с центральными разностями

Для автоматического обеспечения условия положительности функции плотности систему дифференциальных уравнений можно преобразовать к виду

$$\begin{cases} \frac{\partial g}{\partial t} + \frac{1}{2} \sum_{k=1}^2 \left(u_k \frac{\partial g}{\partial x_k} + \frac{\partial u_k g}{\partial x_k} + (2 - g) \frac{\partial u_k}{\partial x_k} \right) = f_0, \\ \frac{\partial u_k}{\partial t} + \frac{1}{3} \left(u_k \frac{\partial u_k}{\partial x_k} + \frac{\partial u_k^2}{\partial x_k} \right) + \frac{1}{2} \sum_{m=1, m \neq k}^2 \left(u_m \frac{\partial u_k}{\partial x_m} + \frac{\partial u_m u_k}{\partial x_m} - u_k \frac{\partial u_m}{\partial x_m} \right) + p'_\rho \frac{\partial g}{\partial x_k} = \\ = \frac{\mu}{\rho} \left(\frac{4}{3} \frac{\partial^2 u_k}{\partial x_k^2} + \sum_{m=1, m \neq k}^2 \left(\frac{\partial^2 u_k}{\partial x_m^2} + \frac{1}{3} \frac{\partial^2 u_m}{\partial x_k \partial x_m} \right) \right) + f_k, \quad k = 1 \dots s, \\ p = p(\rho), \quad g = \ln \rho. \end{cases} \quad (2.1)$$

Сеточную функцию, разностное приближение для плотности ρ , обозначим H . Аналогично, разностные аналоги g и u обозначим через G и V . Для поиска численного решения задачи используется следующая разностная схема:

$$G_t + \frac{1}{2} \sum_{k=1}^2 \left(V_k \hat{G}_{x_k^\circ} + (V_k \hat{G})_{x_k^\circ} + 2(\hat{V}_k)_{x_k^\circ} - G(V_k)_{x_k^\circ} \right) = f_0, \quad x \in \Omega_{\bar{h}}; \quad (2.2)$$

$$G_t + \frac{1}{2} \left((V_k \hat{G})_{x_k} + 2(\hat{V}_k)_{x_k} - G(V_k)_{x_k} \right) - \frac{1}{2} h_k \left((GV_k)_{x_k \bar{x}_k}^{+1_k} - \frac{1}{2} (GV_k)_{x_k \bar{x}_k}^{+2_k} + \right. \\ \left. + (2 - G) \left((V_k)_{x_k \bar{x}_k}^{+1_k} - \frac{1}{2} (V_k)_{x_k \bar{x}_k}^{+2_k} \right) \right) = f_0, \quad x \in \gamma_k^-, \quad k = 1, 2; \quad (2.3)$$

$$G_t + \frac{1}{2} \left((V_k \hat{G})_{\bar{x}_k} + 2(\hat{V}_k)_{\bar{x}_k} - G(V_k)_{\bar{x}_k} \right) + \frac{1}{2} h_k \left((GV_k)_{x_k \bar{x}_k}^{-1_k} - \frac{1}{2} (GV_k)_{x_k \bar{x}_k}^{-2_k} + \right. \\ \left. + (2 - G) \left((V_k)_{x_k \bar{x}_k}^{-1_k} - \frac{1}{2} (V_k)_{x_k \bar{x}_k}^{-2_k} \right) \right) = f_0, \quad x \in \gamma_k^+, \quad k = 1, 2; \quad (2.4)$$

$$(V_k)_t + \frac{1}{3} \left(V_k (\hat{V}_k)_{x_k^\circ} + (V_k \hat{V}_k)_{x_k^\circ} \right) + \\ + \frac{1}{2} \sum_{m=1, m \neq k}^2 \left(V_m (\hat{V}_k)_{x_m^\circ} + (V_m \hat{V}_k)_{x_m^\circ} - V_k (\hat{V}_m)_{x_m^\circ} \right) + p'_\rho (e^G) \hat{G}_{x_m^\circ} = \\ = \tilde{\mu} \left(\frac{4}{3} (\hat{V}_k)_{x_k \bar{x}_k} + \sum_{m=1, m \neq k}^2 (\hat{V}_k)_{x_m \bar{x}_m} \right) - (\tilde{\mu} - \mu e^{-G}) \times \\ \times \left(\frac{4}{3} (V_k)_{x_k \bar{x}_k} + \sum_{m=1, m \neq k}^2 (V_k)_{x_m \bar{x}_m} \right) + \frac{\mu e^{-G}}{3} \sum_{m=1, m \neq k}^2 (V_m)_{x_k^\circ x_m^\circ} + f_k, \quad \text{где } x \in \Omega_{\bar{h}}; \quad (2.5)$$

$$\hat{V}_k = 0, \quad x \in \gamma_{\bar{h}}, \quad k = 1, 2. \quad (2.6)$$

Распишем уравнения схемы в поточечном виде и преобразуем их, приведя подобные слагаемые при неизвестных значениях с верхнего слоя. Получим:

$$\begin{aligned}
& 4 \cdot G_{m_1, m_2}^{n+1} - \frac{\tau}{h_x} G_{m_1-1, m_2}^{n+1} (V_{1m_1, m_2}^n + V_{1m_1-1, m_2}^n) + \frac{\tau}{h_x} G_{m_1+1, m_2}^{n+1} (V_{1m_1, m_2}^n + V_{1m_1+1, m_2}^n) - \\
& - \frac{\tau}{h_y} G_{m_1, m_2-1}^{n+1} (V_{2m_1, m_2}^n + V_{2m_1, m_2-1}^n) + \frac{\tau}{h_y} G_{m_1, m_2+1}^{n+1} (V_{2m_1, m_2}^n + V_{2m_1, m_2+1}^n) - \\
& - \frac{2\tau}{h_x} V_{1m_1-1, m_2}^{n+1} + \frac{2\tau}{h_x} V_{1m_1+1, m_2}^{n+1} - \frac{2\tau}{h_y} V_{2m_1, m_2-1}^{n+1} + \frac{2\tau}{h_y} V_{2m_1, m_2+1}^{n+1} = \\
& = 4 \cdot G_{m_1, m_2}^n + \tau G_{m_1, m_2}^n \left(\frac{V_{1m_1+1, m_2}^n - V_{1m_1-1, m_2}^n}{h_x} + \frac{V_{2m_1, m_2+1}^n - V_{2m_1, m_2-1}^n}{h_y} \right) + 4\tau f_0,
\end{aligned}$$

где $x \in \Omega_h$

$$\begin{aligned}
& G_{0, m_2}^{n+1} \left(2 - \frac{\tau}{h_x} V_{10, m_2}^n \right) + G_{1, m_2}^{n+1} \frac{\tau}{h_x} V_{11, m_2}^n + \frac{2\tau}{h_x} V_{11, m_2}^{n+1} - \frac{2\tau}{h_x} V_{10, m_2}^{n+1} = \\
& = 2 \cdot G_{0, m_2}^n + \frac{\tau}{h_x} G_{0, m_2}^n (V_{11, m_2}^n - V_{10, m_2}^n) + 2\tau f_0 + \\
& + \frac{\tau}{h_x} \left(G_{0, m_2}^n V_{10, m_2}^n - \frac{5}{2} \cdot G_{1, m_2}^n V_{11, m_2}^n + 2 \cdot G_{2, m_2}^n V_{12, m_2}^n - \right. \\
& \left. - \frac{1}{2} \cdot G_{3, m_2}^n V_{13, m_2}^n + (2 - G_{0, m_2}^n) \cdot (V_{10, m_2}^n - 2.5 \cdot V_{11, m_2}^n + 2V_{12, m_2}^n - 0.5V_{13, m_2}^n) \right),
\end{aligned}$$

где $x \in \gamma_k^-$

$$\begin{aligned}
& G_{M, m_2}^{n+1} \left(2 + \frac{\tau}{h_x} V_{1M, m_2}^n \right) - G_{M-1, m_2}^{n+1} \frac{\tau}{h_x} V_{1M-1, m_2}^n + \frac{2\tau}{h_x} V_{1M, m_2}^{n+1} - \frac{2\tau}{h_x} V_{1M-1, m_2}^{n+1} = \\
& = 2 \cdot G_{M, m_2}^n + \frac{\tau}{h_x} G_{M, m_2}^n (V_{1M, m_2}^n - V_{1M-1, m_2}^n) + 2\tau f_0 - \frac{\tau}{h_x} \times \left[G_{M, m_2}^n V_{1M, m_2}^n - \right. \\
& \left. - \frac{5}{2} \cdot G_{M-1, m_2}^n V_{1M-1, m_2}^n + 2 \cdot G_{M-2, m_2}^n V_{1M-2, m_2}^n - \frac{1}{2} \cdot G_{M-3, m_2}^n V_{1M-3, m_2}^n + \right. \\
& \left. + (2 - G_{M, m_2}^n) \cdot \left(V_{1M, m_2}^n - \frac{5}{2} \cdot V_{1M-1, m_2}^n + 2 \cdot V_{1M-2, m_2}^n - \frac{1}{2} \cdot V_{1M-3, m_2}^n \right) \right],
\end{aligned}$$

где $x \in \gamma_k^+$

$$\begin{aligned}
& V_{1m_1,m_2}^{n+1} \left(6 + 4 \cdot \tau \tilde{\mu} \left(\frac{4}{h_x^2} + \frac{3}{h_y^2} \right) \right) + V_{1m_1-1,m_2}^{n+1} \left(-\frac{\tau}{h_x} (V_{1m_1,m_2}^n + V_{1m_1-1,m_2}^n) - \tilde{\mu} \frac{8\tau}{h_x^2} \right) + \\
& + V_{1m_1+1,m_2}^{n+1} \left(\frac{\tau}{h_x} (V_{1m_1,m_2}^n + V_{1m_1+1,m_2}^n) - \frac{8\tau}{h_x^2} \right) \cdot \tilde{\mu} + \\
& + V_{1m_1,m_2-1}^{n+1} \left(-\frac{3\tau}{2h_y} (V_{2m_1,m_2}^n + V_{2m_1,m_2-1}^n) - \frac{6\tau}{h_y^2} \right) \cdot \tilde{\mu} + \\
& + V_{1m_1,m_2+1}^{n+1} \left(\frac{3\tau}{2h_y} (V_{2m_1,m_2}^n + V_{2m_1,m_2+1}^n) - \frac{6\tau}{h_y^2} \right) \cdot \tilde{\mu} - \\
& - 3 \cdot \frac{\tau}{h_x} \cdot p'_\rho G_{m_1-1,m_2}^{n+1} + 3 \cdot \frac{\tau}{h_x} \cdot p'_\rho G_{m_1+1,m_2}^{n+1} = \\
= & 6 \cdot V_{1m_1,m_2}^n + 6 \cdot \tau f_1 + \frac{3}{2} \cdot \frac{\tau}{2h_y} V_{1m_1,m_2}^n (V_{2m_1,m_2+1}^n - V_{2m_1,m_2-1}^n) - \\
& - 6 \cdot \tau \cdot (\tilde{\mu} - \mu e^{-G}) \cdot \left(\frac{4}{3h_x^2} (V_{1m_1+1,m_2}^n - 2V_{1m_1,m_2}^n + V_{1m_1-1,m_2}^n) \right. \\
& \left. + \frac{1}{h_y^2} (V_{1m_1,m_2+1}^n - 2V_{1m_1,m_2}^n + V_{1m_1,m_2-1}^n) \right) + \\
& + \mu e^{-G} \frac{\tau}{2h_x h_y} (V_{2m_1+1,m_2+1}^n - V_{2m_1-1,m_2+1}^n - V_{2m_1+1,m_2-1}^n + V_{2m_1-1,m_2-1}^n),
\end{aligned}$$

где $\tilde{\mu} = \mu \|e^{-G}\|.$

3 Тестирование на известном гладком решении

Приведем результаты решения задач с помощью схемы на известном гладком решении:

$$\begin{cases} u_1 = \sin x \sin y e^t, \\ u_2 = \sin x \sin y e^{-1}, \\ \rho = e^{\cos x \sin y + t}. \end{cases} \quad (3.1)$$

Считаем, что $p_\rho = \rho$ и $p'_\rho = 1$.

3.1 $\mu = 0.1$

$\ g - G\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	3.191555e-01	2.247489e-01	2.243865e-01
0.250	3.043471e-01	1.629398e-01	1.491416e-01
0.125	2.876430e-01	1.187079e-01	9.785144e-02

$\ g - G\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	9.512504e-01	8.355700e-01	8.204384e-01
0.250	6.122799e-01	4.612271e-01	4.428729e-01
0.125	4.495678e-01	2.581024e-01	2.352407e-01

$\ g - G\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.385544e+00	1.114565e+00	9.792058e-01
0.250	1.210866e+00	8.926209e-01	7.084297e-01
0.125	1.155027e+00	8.174585e-01	6.073713e-01

$\ u_1 - V_1\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	3.979879e-01	3.622549e-01	3.583263e-01
0.250	2.643974e-01	2.274248e-01	2.187875e-01
0.125	2.065245e-01	1.377734e-01	1.260013e-01

$\ u_1 - V_1\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.149149e+00	1.132800e+00	1.131374e+00
0.250	6.822144e-01	6.370385e-01	6.334057e-01
0.125	4.361231e-01	3.451439e-01	3.385633e-01

$\ u_1 - V_1\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	4.621676e+00	3.323153e+00	2.462723e+00
0.250	4.690418e+00	3.298016e+00	2.352245e+00
0.125	4.759038e+00	3.321124e+00	2.336537e+00

$\ u_2 - V_2\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.544142e-01	1.391937e-01	1.379958e-01
0.250	1.151507e-01	8.370939e-02	8.120909e-02
0.125	9.038622e-02	4.830719e-02	4.470352e-02

$\ u_2 - V_2\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	5.098528e-01	4.514006e-01	4.401044e-01
0.250	3.377369e-01	2.619125e-01	2.484032e-01
0.125	2.432280e-01	1.504754e-01	1.349097e-01

$\ u_2 - V_2\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	9.951011e-01	7.361388e-01	5.979325e-01
0.250	8.503144e-01	5.894178e-01	4.431625e-01
0.125	7.775693e-01	5.189529e-01	3.699671e-01

3.2 $\mu = 0.01$

$\ g - G\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	3.332275e-01	2.317920e-01	2.308903e-01
0.250	3.227055e-01	1.738487e-01	1.583301e-01
0.125	3.101889e-01	1.297109e-01	1.059268e-01

$\ g - G\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	9.769827e-01	8.604924e-01	8.452480e-01
0.250	6.319185e-01	4.775693e-01	4.588406e-01
0.125	4.690277e-01	2.688623e-01	2.448913e-01

$\ g - G\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.404438e+00	1.133190e+00	9.999392e-01
0.250	1.224002e+00	9.019652e-01	7.190158e-01
0.125	1.168566e+00	8.230314e-01	6.124281e-01

$\ u_1 - V_1\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	4.081024e-01	3.697396e-01	3.661379e-01
0.250	2.740035e-01	2.351150e-01	2.260646e-01
0.125	2.252334e-01	1.456298e-01	1.317880e-01

$\ u_1 - V_1\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.200346e+00	1.181441e+00	1.179247e+00
0.250	7.210320e-01	6.706195e-01	6.656928e-01
0.125	4.662389e-01	3.667748e-01	3.583753e-01

$\ u_1 - V_1\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	4.620662e+00	3.331493e+00	2.479650e+00
0.250	4.682651e+00	3.296842e+00	2.356233e+00
0.125	4.750031e+00	3.317148e+00	2.335832e+00

$\ u_2 - V_2\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.651374e-01	1.502980e-01	1.491544e-01
0.250	1.226988e-01	9.135306e-02	8.934564e-02
0.125	9.669605e-02	5.290997e-02	4.973908e-02

$\ u_2 - V_2\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	5.296704e-01	4.727790e-01	4.619251e-01
0.250	3.525451e-01	2.763621e-01	2.631604e-01
0.125	2.552148e-01	1.592887e-01	1.438254e-01

$\ u_2 - V_2\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.007894e+00	7.515347e-01	6.155485e-01
0.250	8.573573e-01	5.973214e-01	4.526872e-01
0.125	7.812806e-01	5.219918e-01	3.738559e-01

3.3 $\mu = 0.001$

$\ g - G\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	3.346899e-01	2.325533e-01	2.315276e-01
0.250	3.246620e-01	1.750017e-01	1.592995e-01
0.125	3.126662e-01	1.309073e-01	1.068023e-01

$\ g - G\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	9.796632e-01	8.630954e-01	8.478420e-01
0.250	6.340402e-01	4.792887e-01	4.605218e-01
0.125	4.714032e-01	2.700253e-01	2.459161e-01

$\ g - G\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.406413e+00	1.135145e+00	1.002115e+00
0.250	1.225423e+00	9.029572e-01	7.201406e-01
0.125	1.170173e+00	8.236647e-01	6.129941e-01

$\ u_1 - V_1\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	4.091439e-01	3.705469e-01	3.669681e-01
0.250	2.750049e-01	2.359130e-01	2.268182e-01
0.125	2.273044e-01	1.464708e-01	1.323957e-01

$\ u_1 - V_1\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.205739e+00	1.186552e+00	1.184275e+00
0.250	7.252048e-01	6.741907e-01	6.691166e-01
0.125	4.695859e-01	3.691123e-01	3.604995e-01

$\ u_1 - V_1\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	4.620623e+00	3.332436e+00	2.481492e+00
0.250	4.681850e+00	3.296748e+00	2.356691e+00
0.125	4.749051e+00	3.316714e+00	2.335765e+00

$\ u_2 - V_2\ _C$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.662841e-01	1.514788e-01	1.503662e-01
0.250	1.234986e-01	9.217657e-02	9.022623e-02
0.125	9.738230e-02	5.341048e-02	5.029295e-02

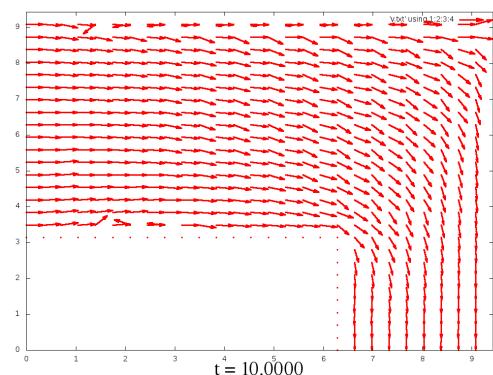
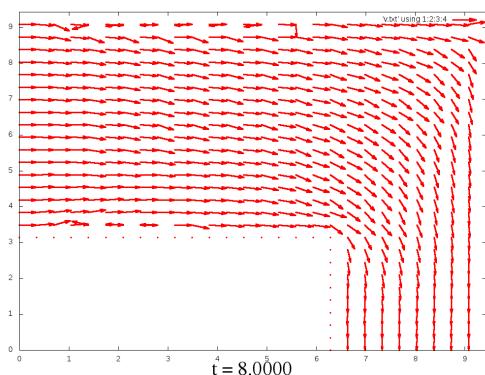
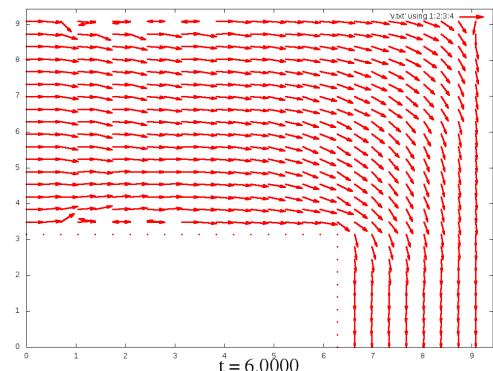
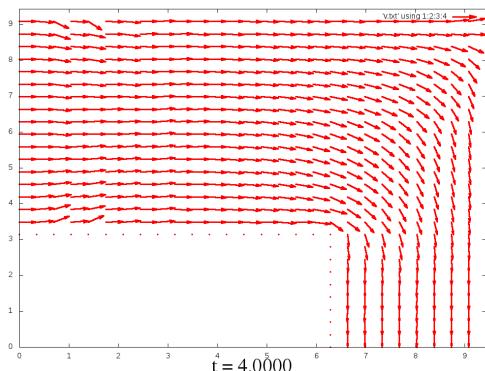
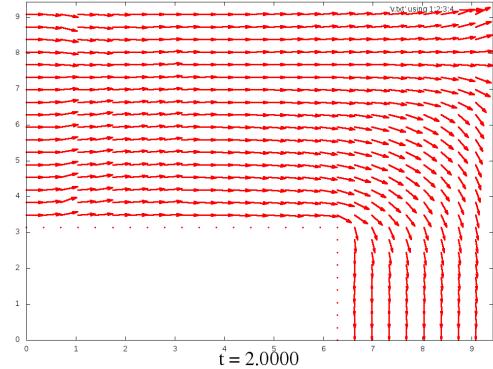
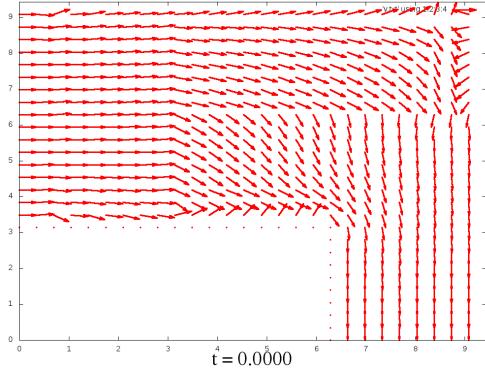
$\ u_2 - V_2\ _{L_2}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	5.318001e-01	4.750835e-01	4.642791e-01
0.250	3.541905e-01	2.779583e-01	2.647925e-01
0.125	2.566181e-01	1.602784e-01	1.448298e-01

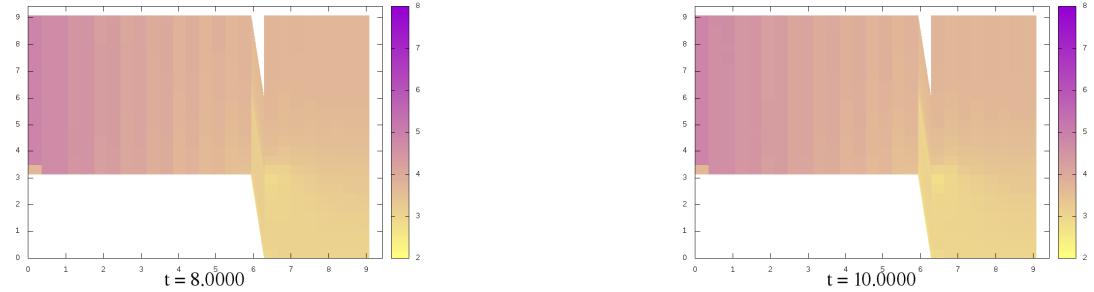
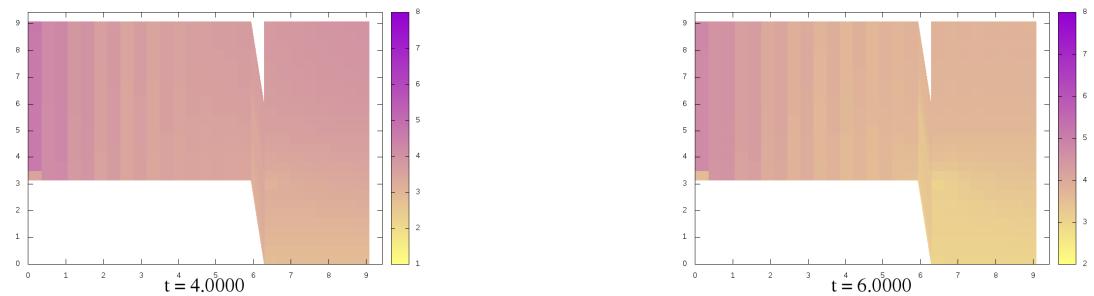
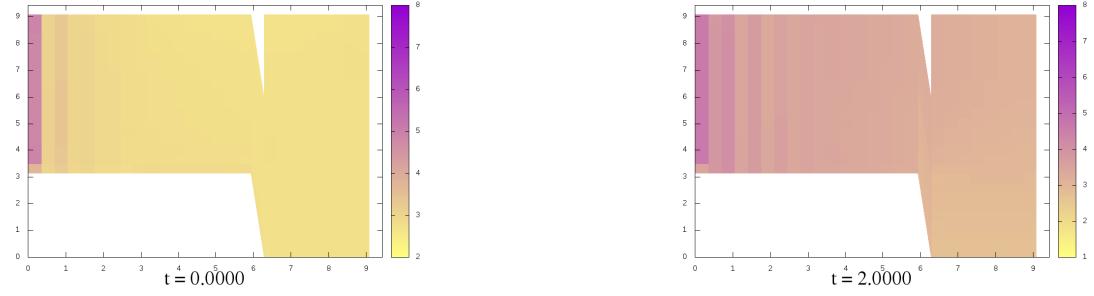
$\ u_2 - V_2\ _{v_2^1}$			
$\tau \setminus h$	1.00000	0.50000	0.25000
0.500	1.009266e+00	7.531993e-01	6.174574e-01
0.250	8.581391e-01	5.981973e-01	4.537501e-01
0.125	7.817225e-01	5.223316e-01	3.742988e-01

4 Рассчет для неизвестного решения

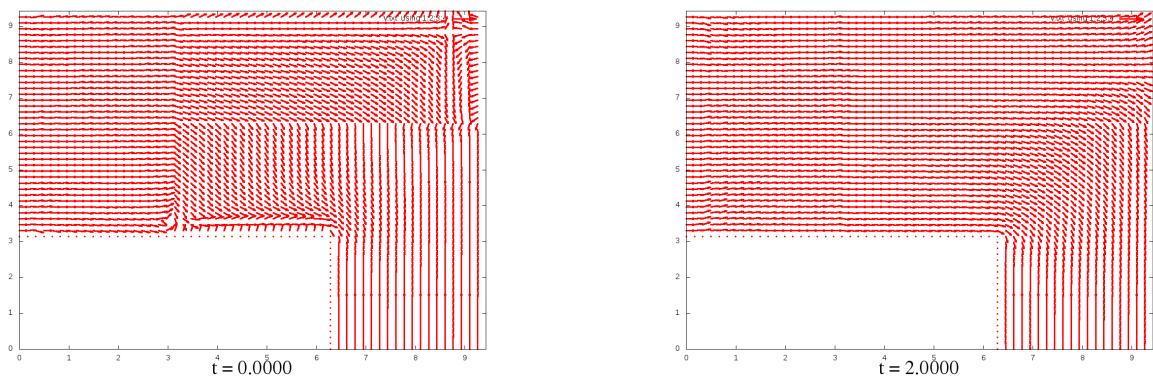
4.1 Графики для $\mu = 0.1$

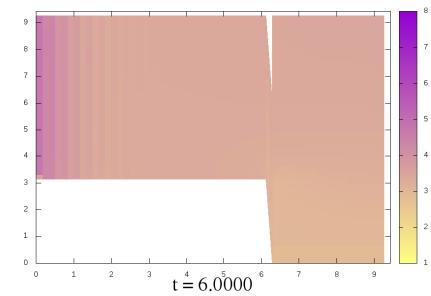
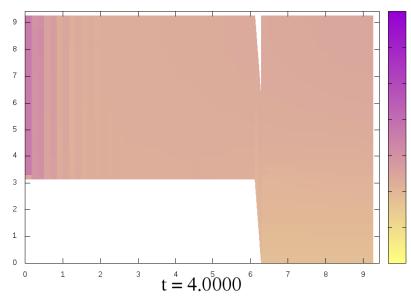
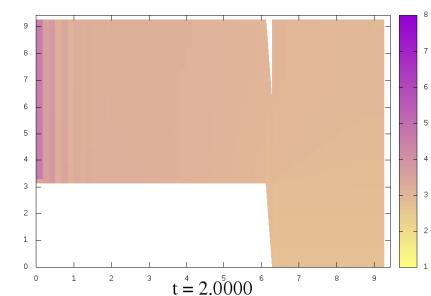
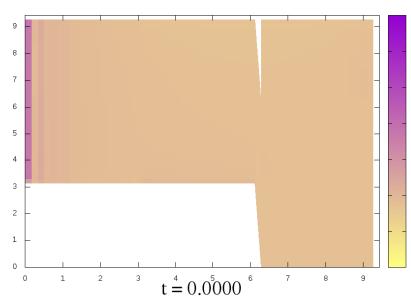
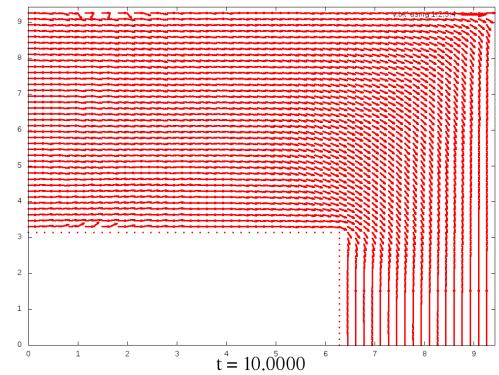
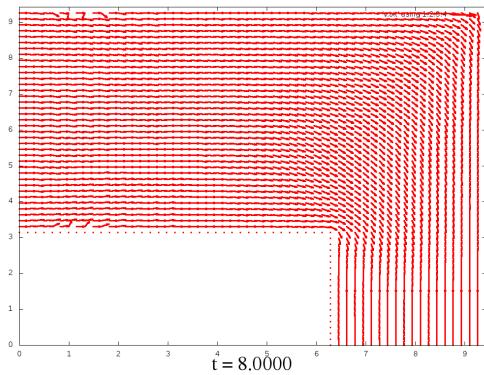
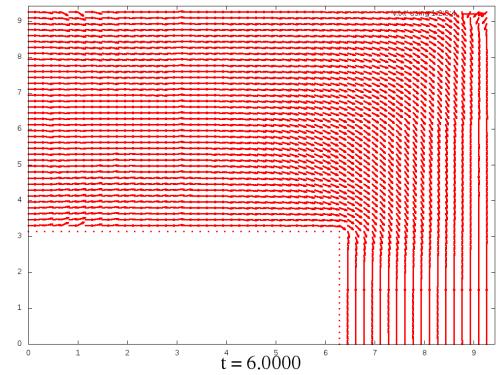
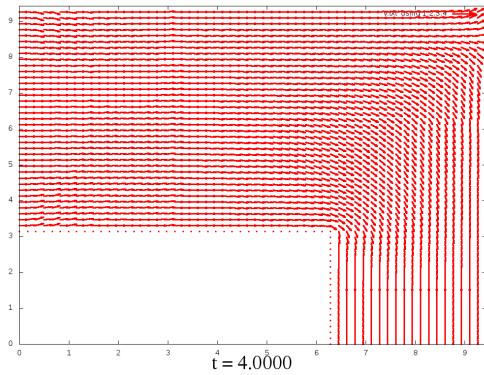
4.1.1 $M_x = 10; M_y = 10; T = 10$

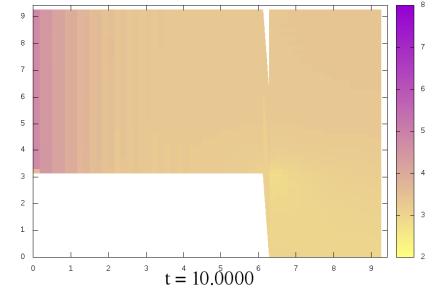
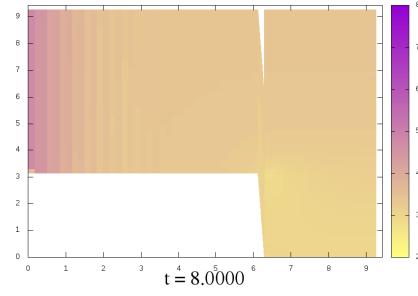




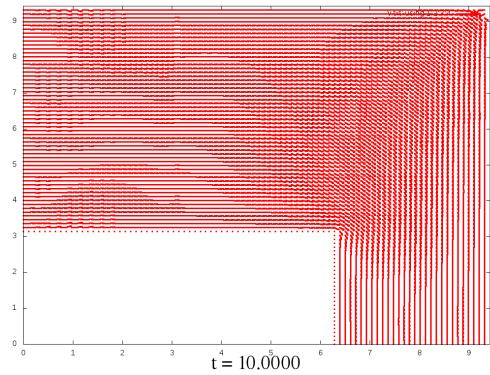
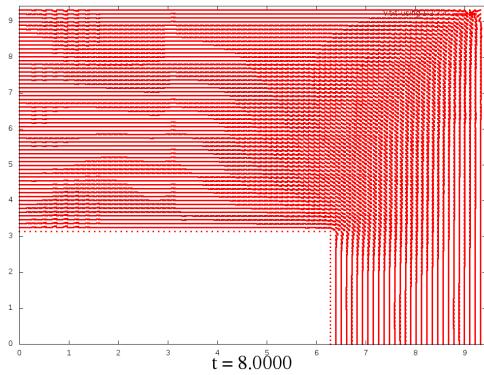
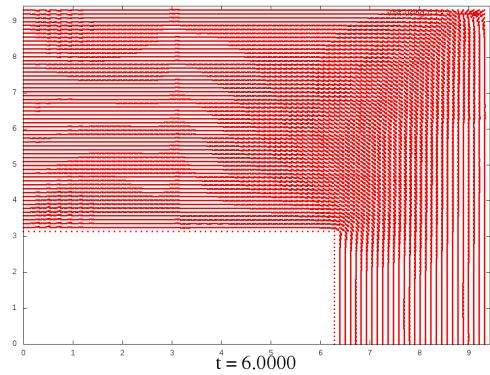
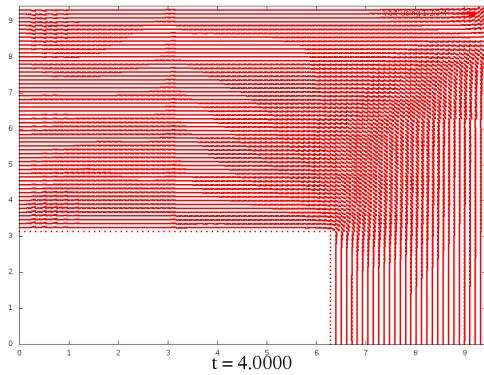
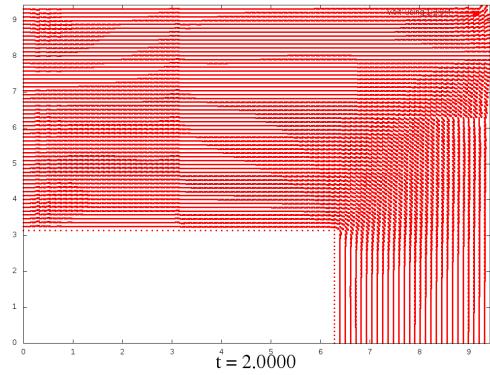
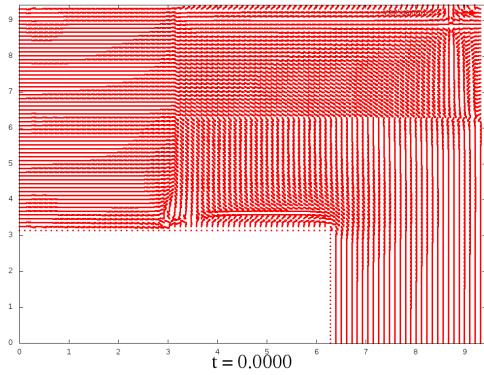
4.1.2 $M_x = 20; M_y = 20; T = 10$

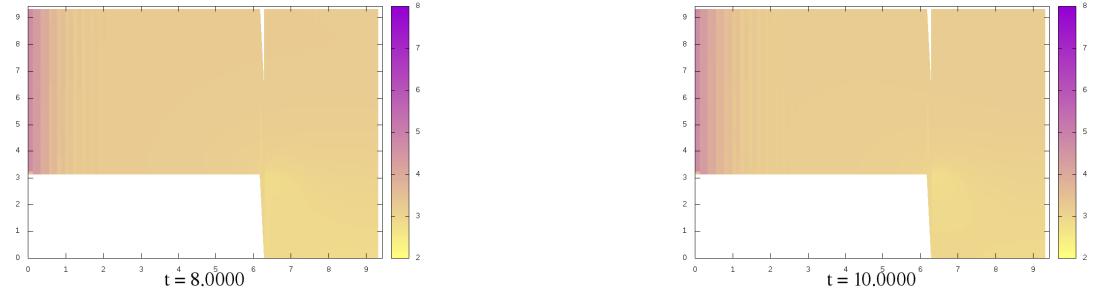
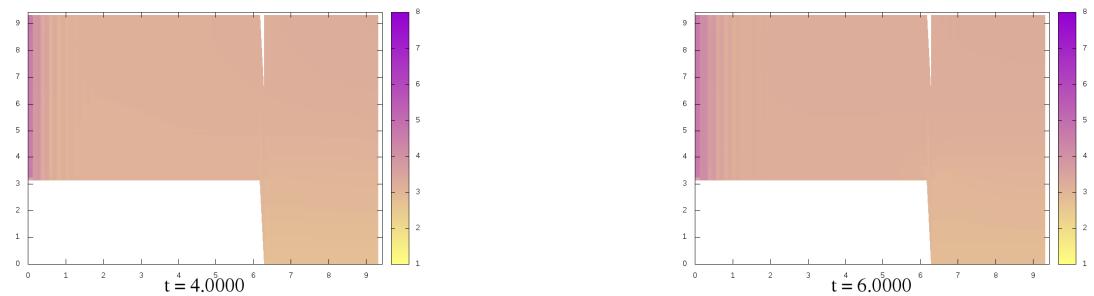
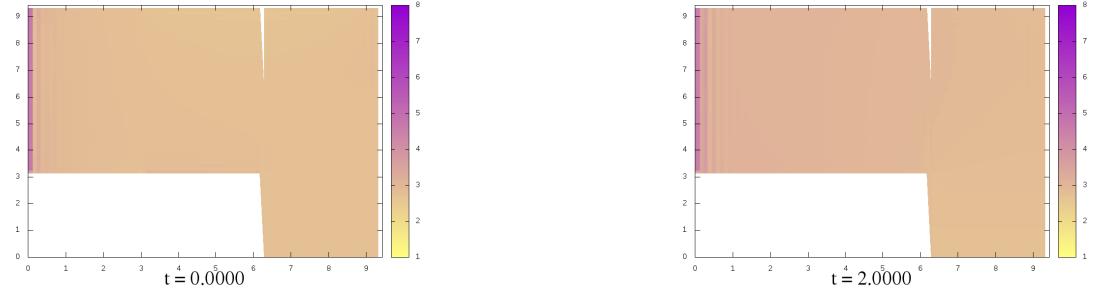




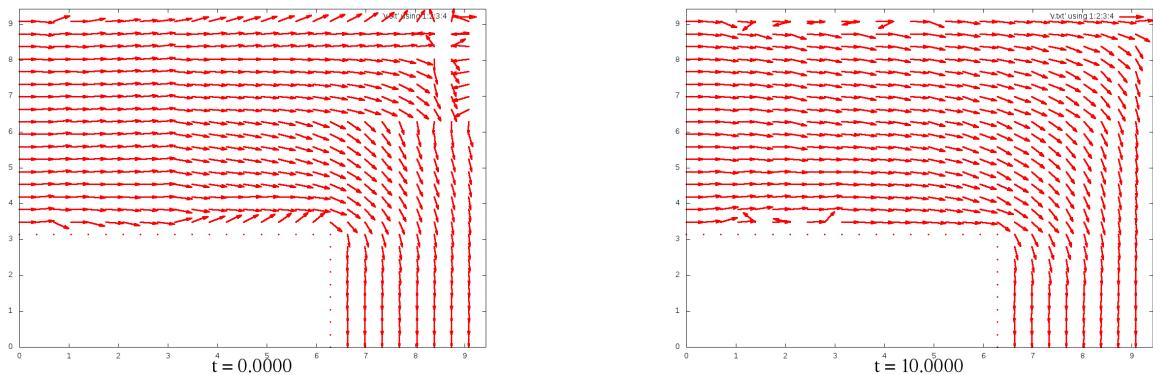


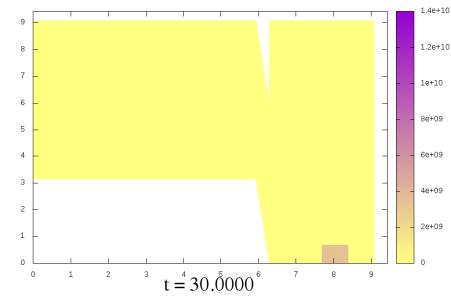
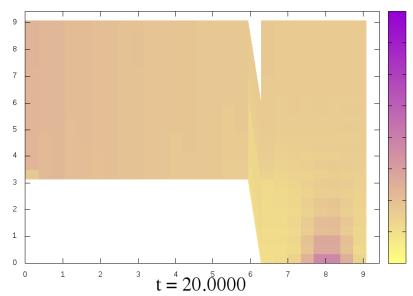
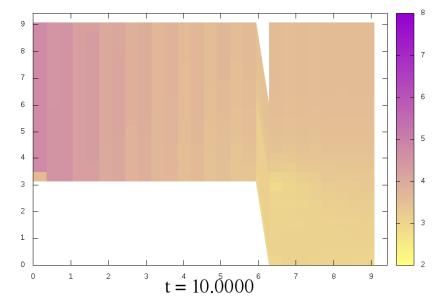
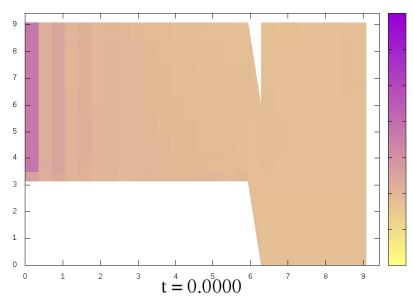
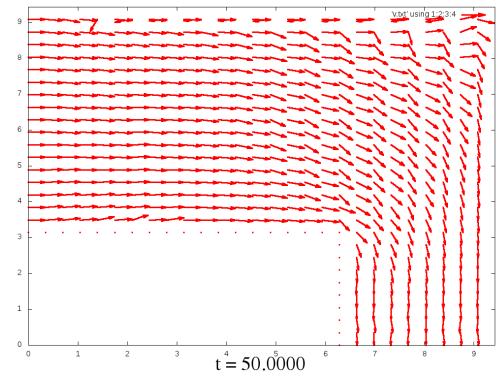
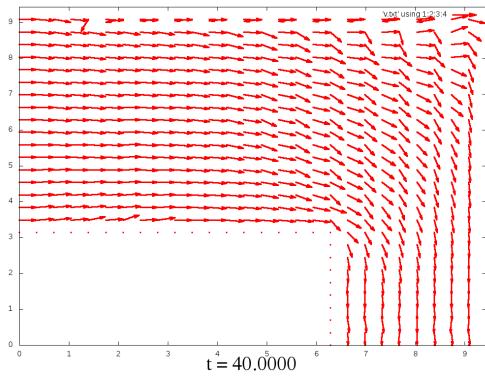
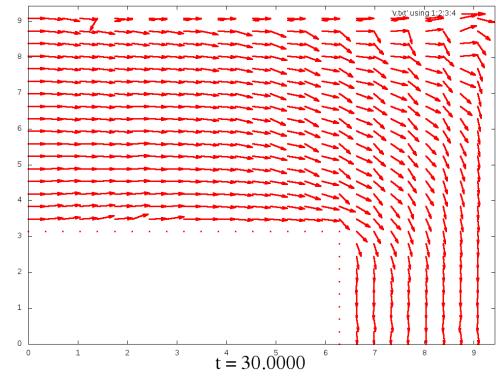
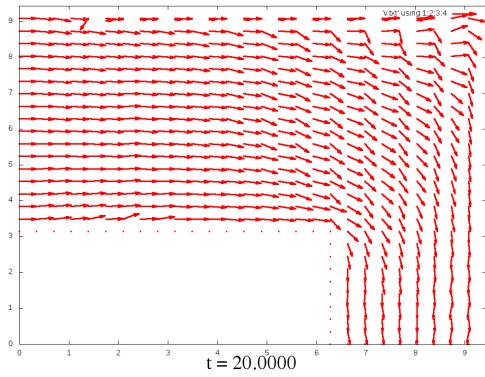
4.1.3 $M_x = 30; M_y = 30; T = 10$

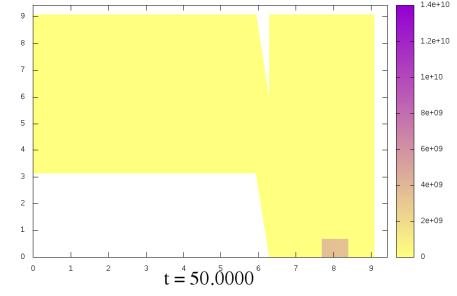
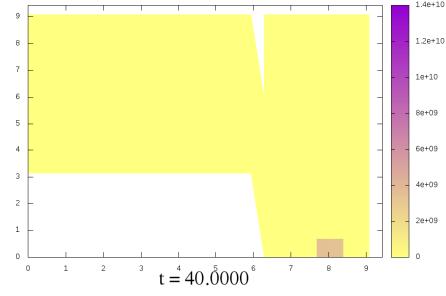




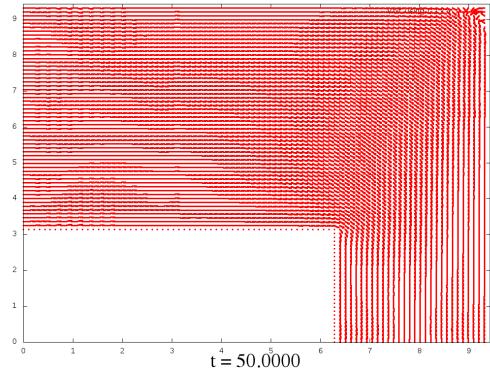
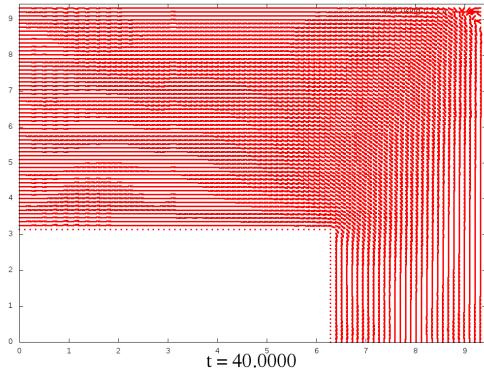
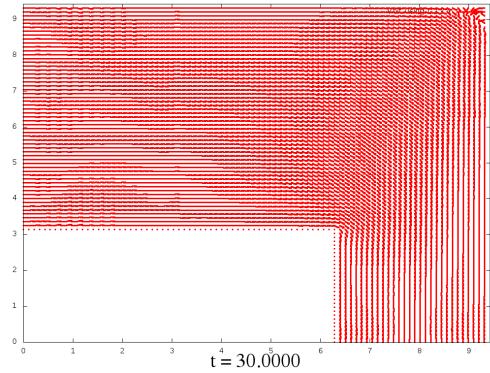
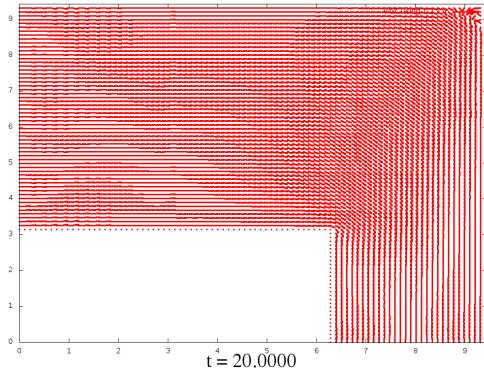
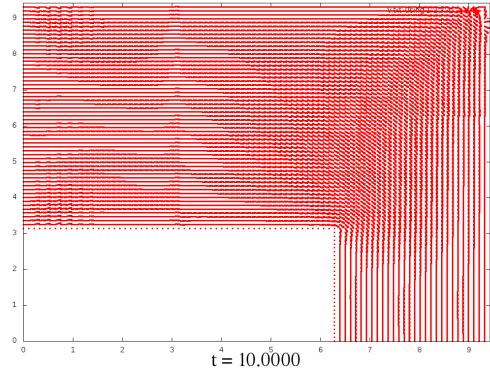
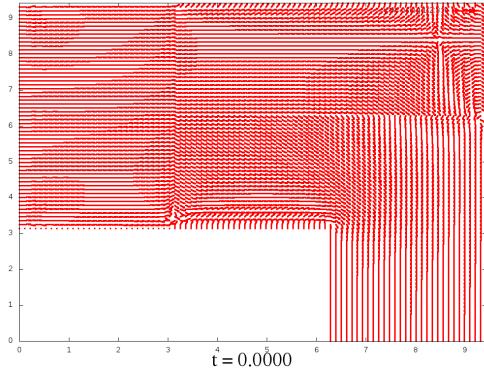
4.1.4 $M_x = 10$; $M_y = 10$; $T = 50$

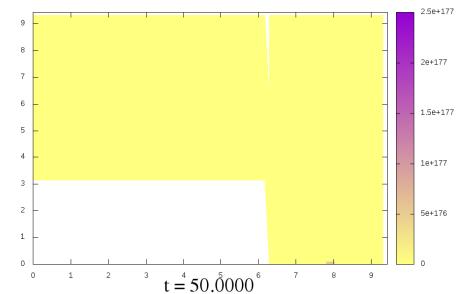
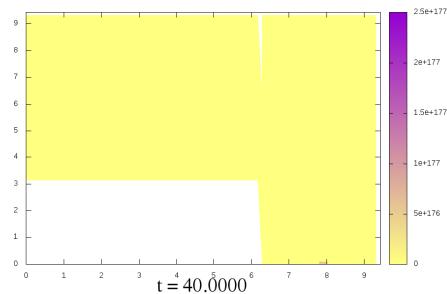
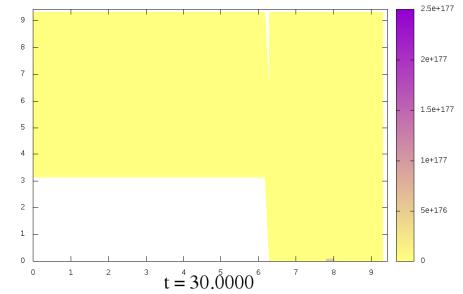
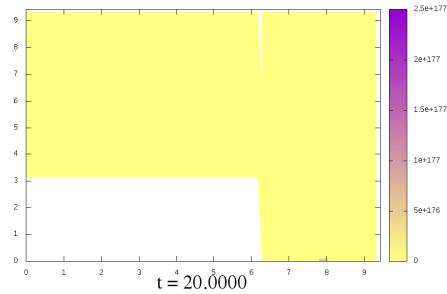
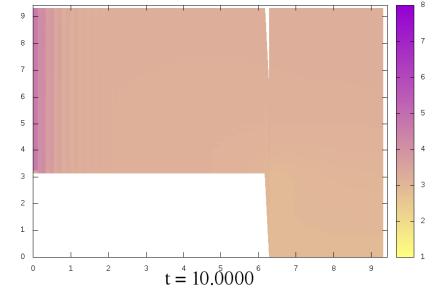
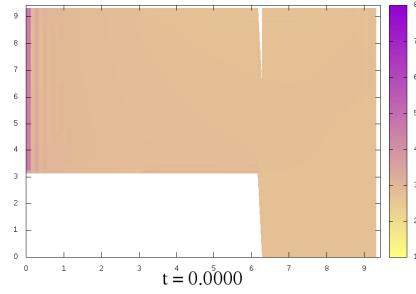






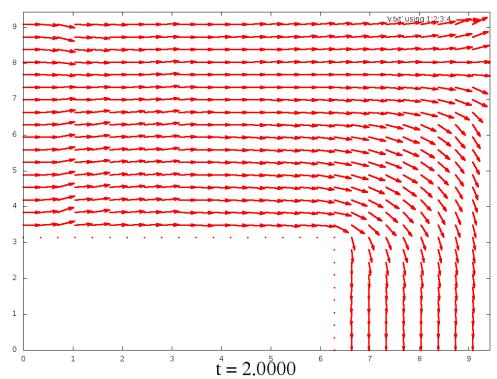
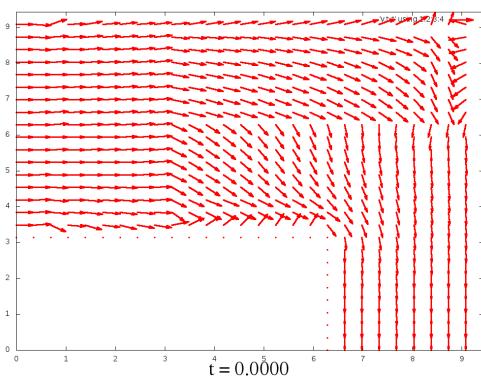
4.1.5 $M_x = 30$; $M_y = 30$; $T = 50$

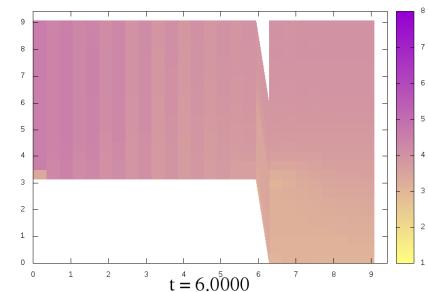
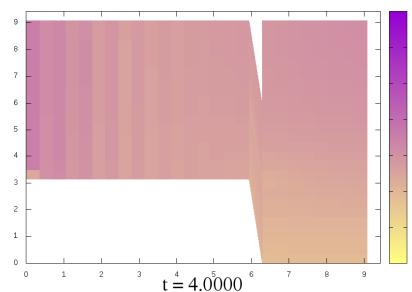
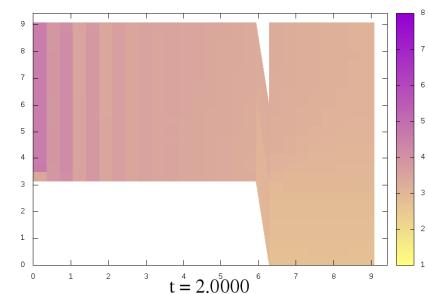
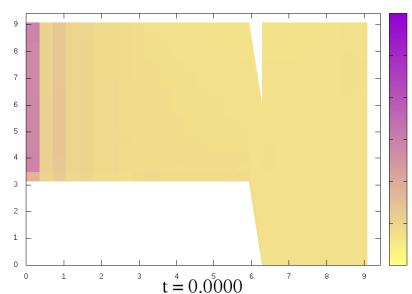
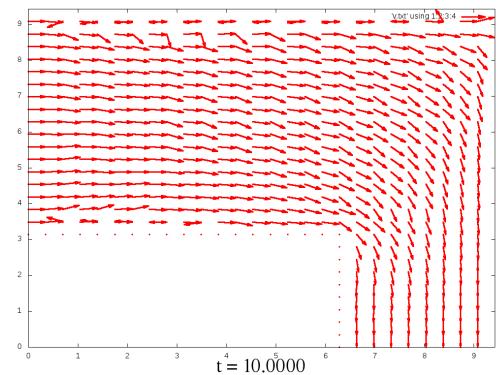
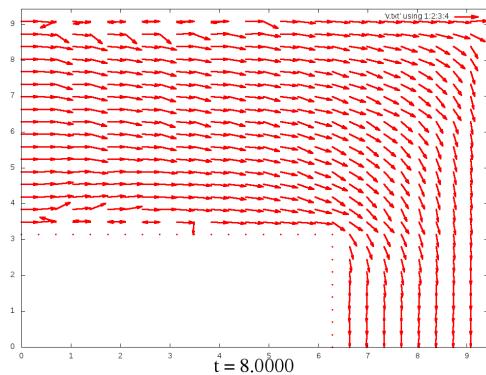
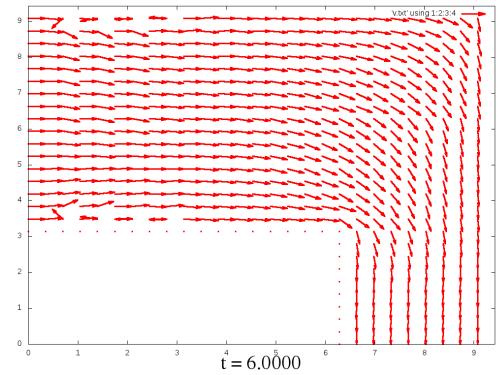
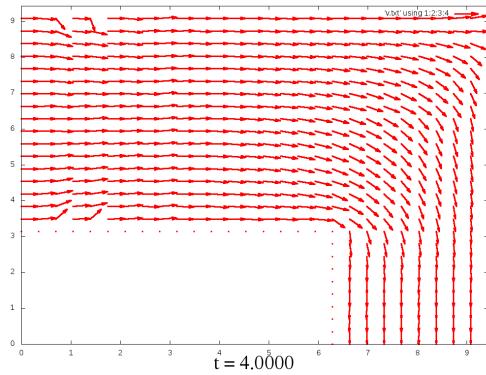


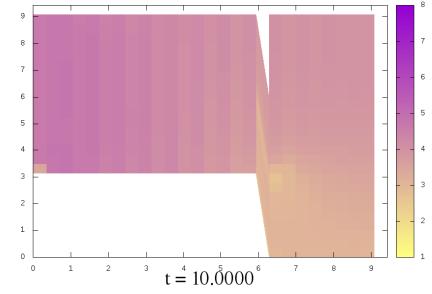
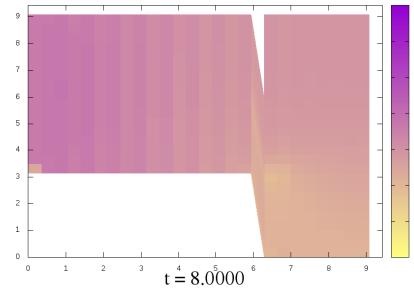


4.2 Графики для $\mu = 0.01$

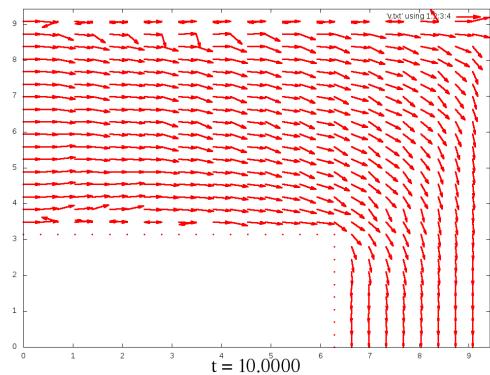
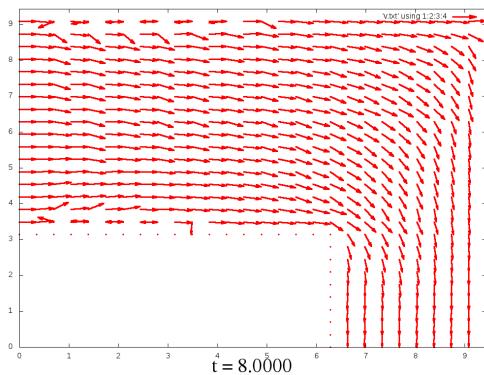
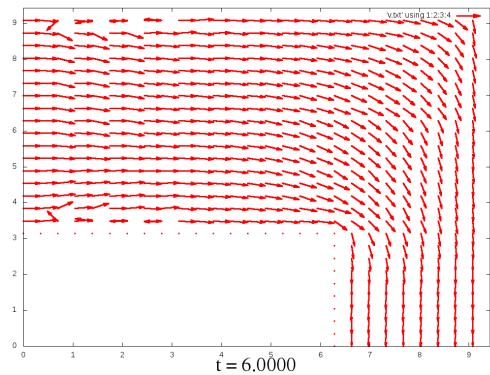
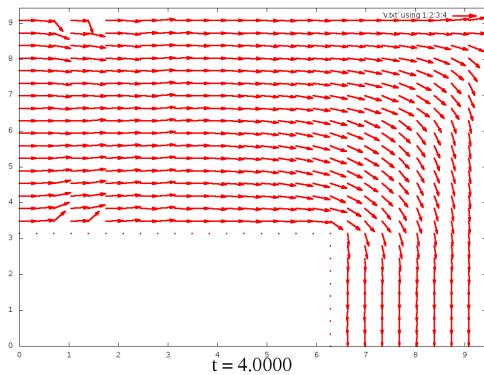
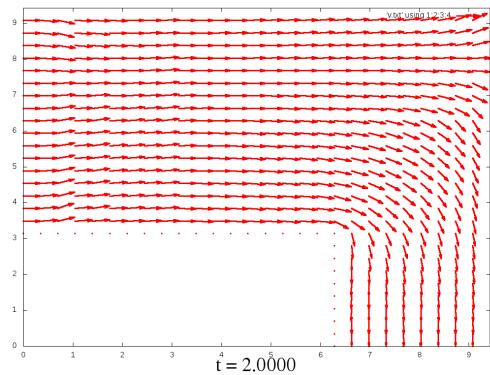
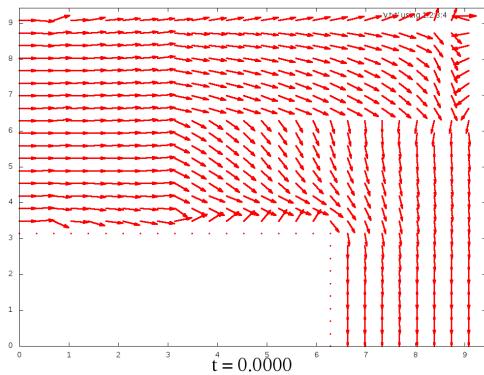
4.2.1 $M_x = 10; M_y = 10; T = 10$

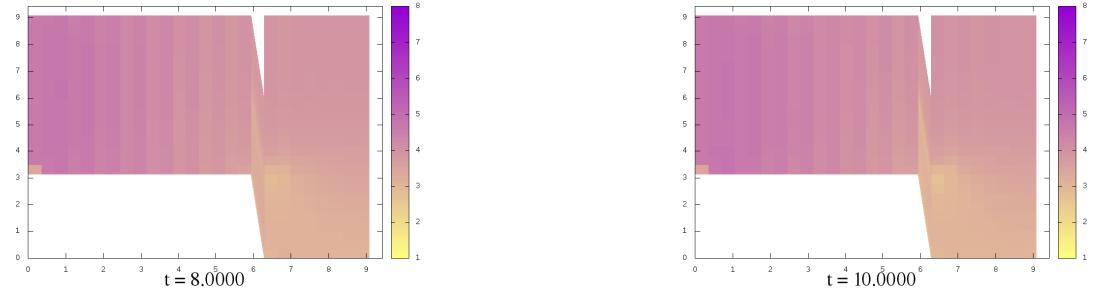
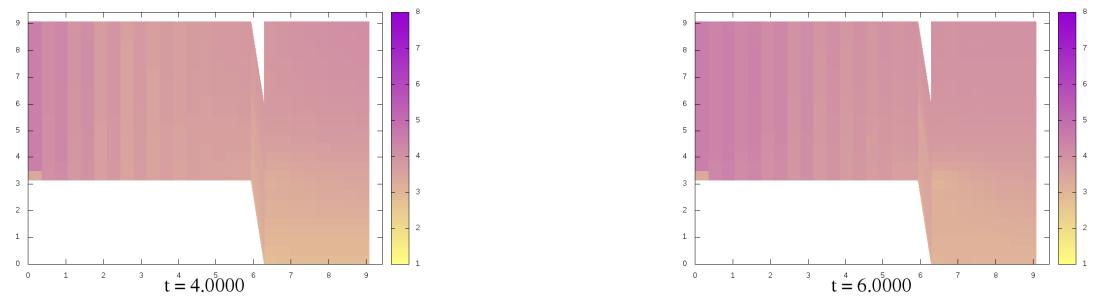
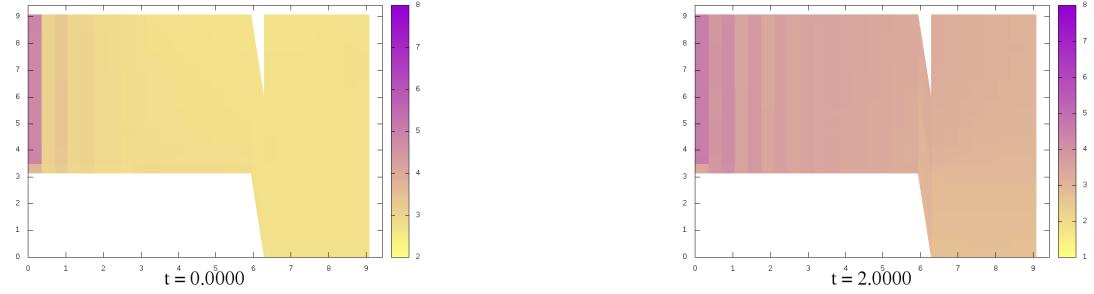




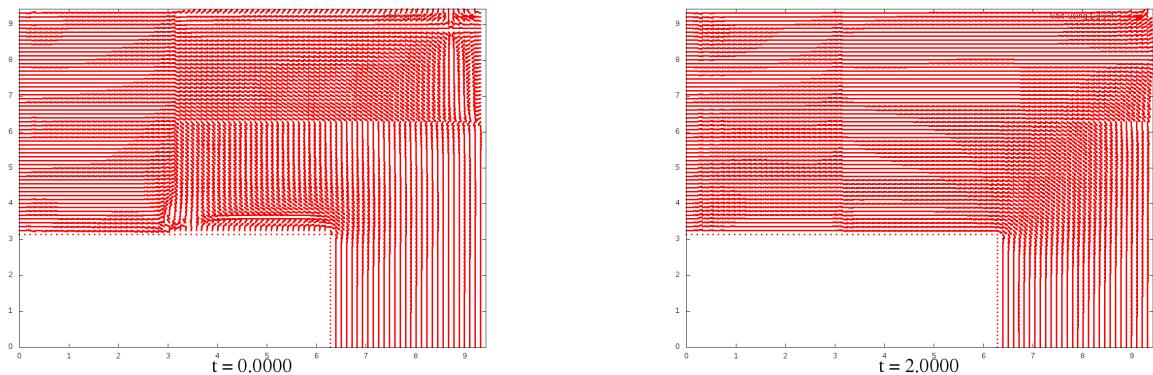


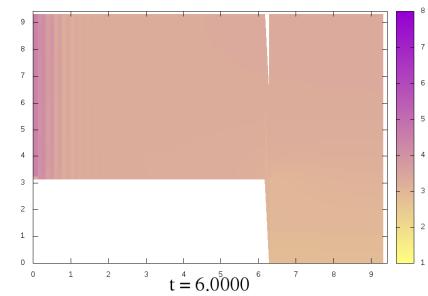
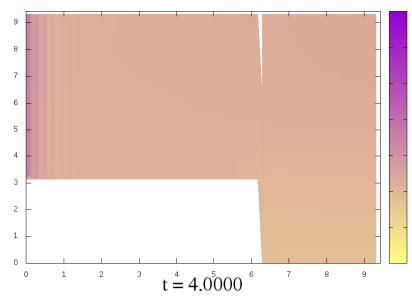
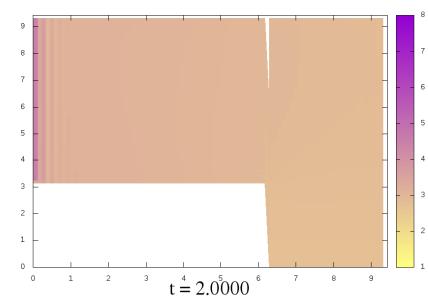
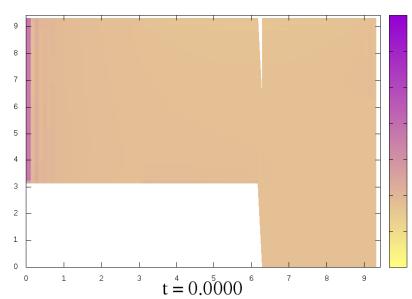
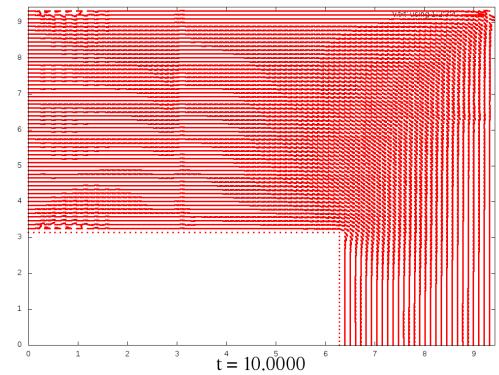
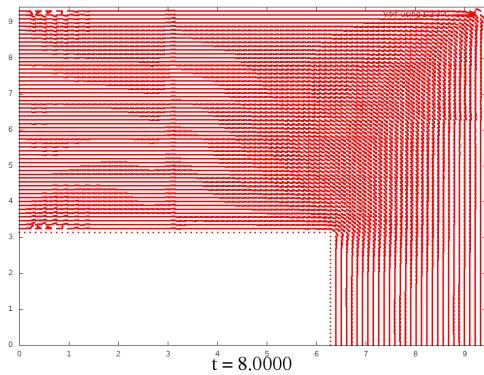
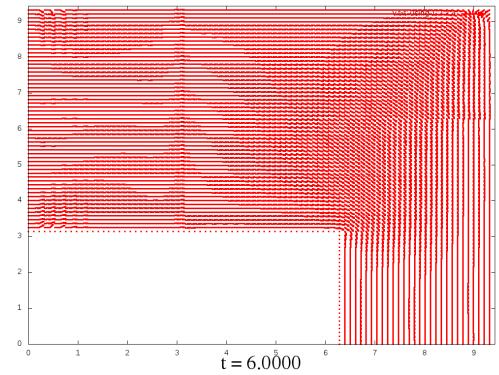
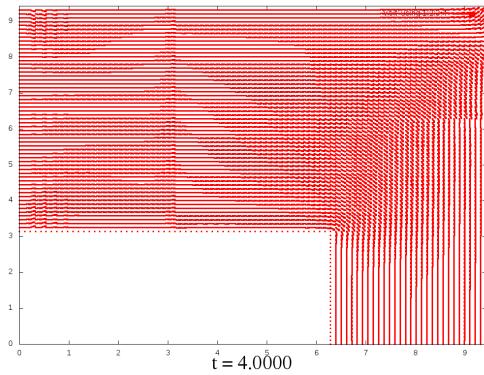
4.2.2 $M_x = 20; M_y = 20; T = 10$

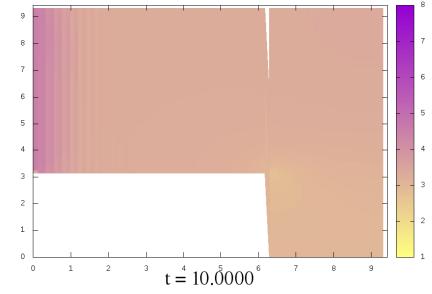
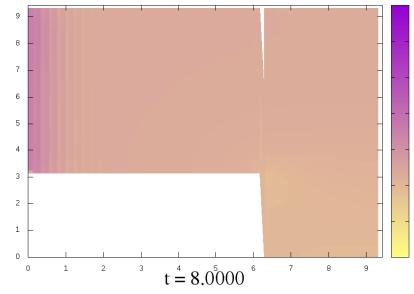




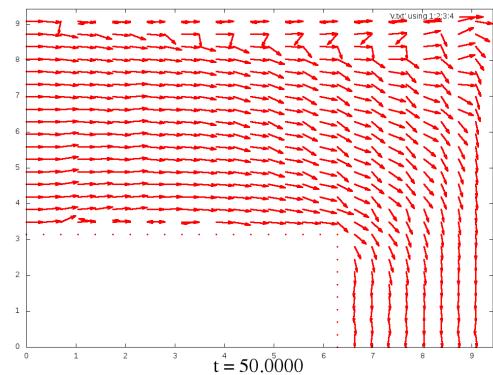
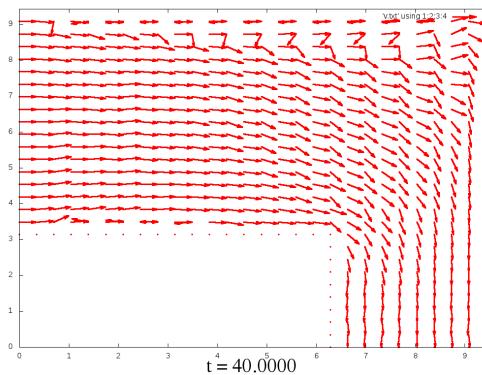
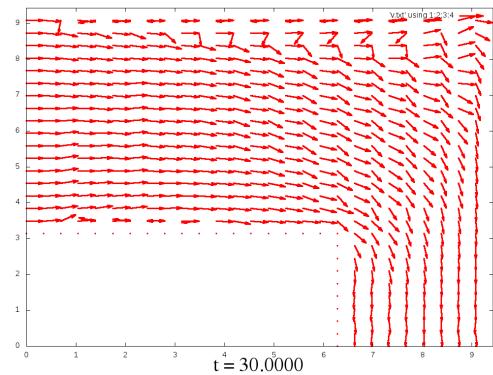
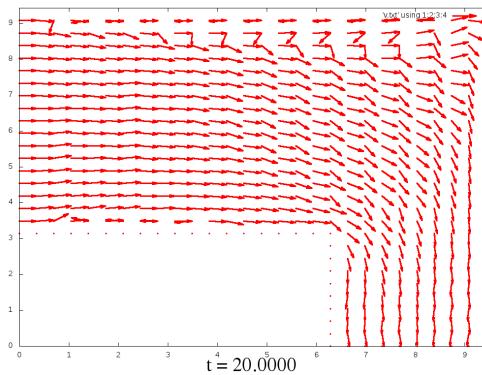
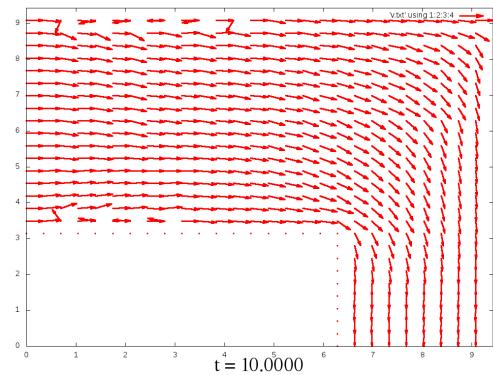
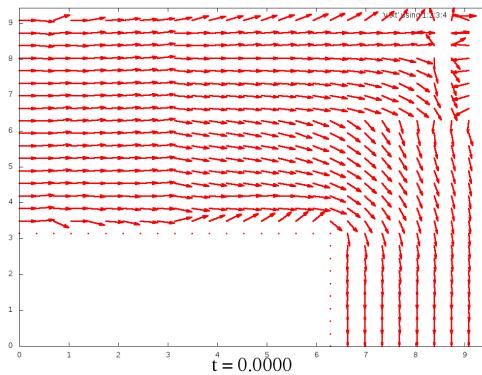
4.2.3 $M_x = 30; M_y = 30; T = 10$

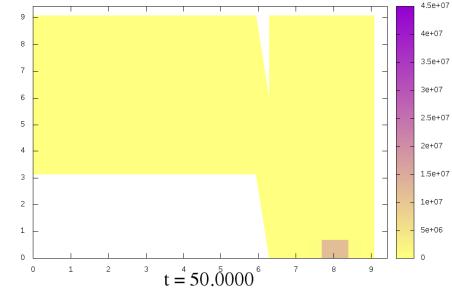
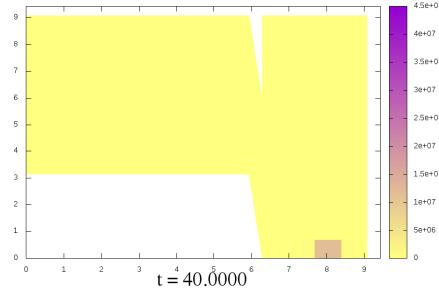
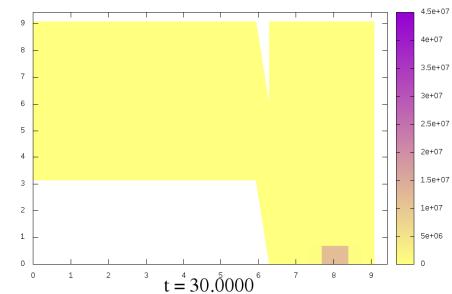
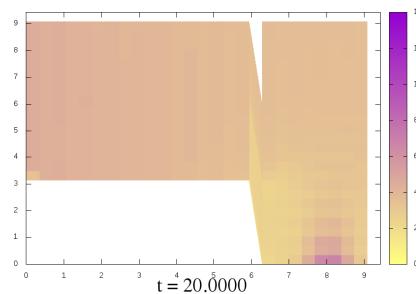
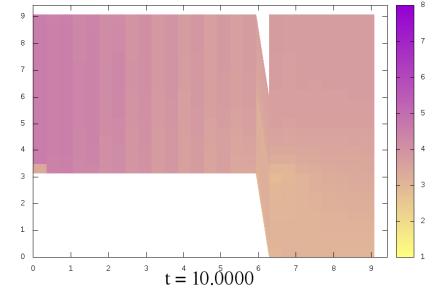
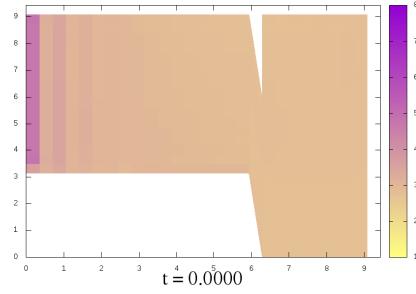




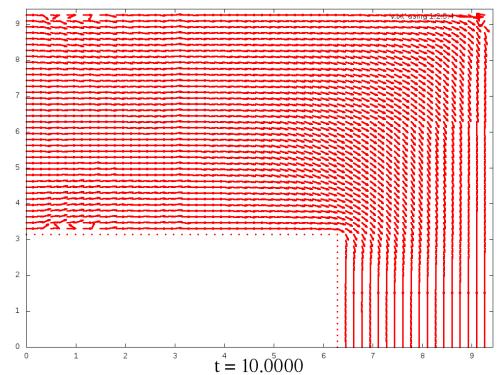
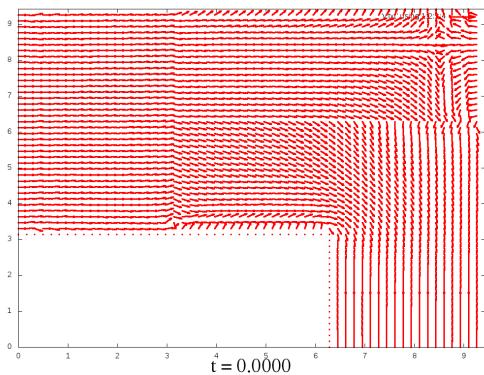


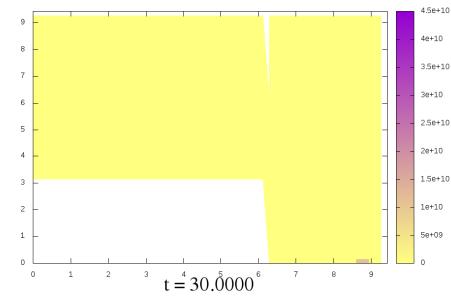
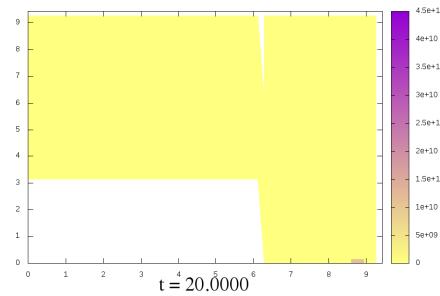
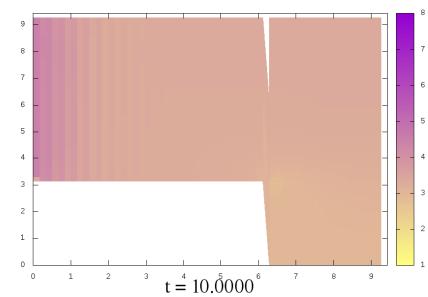
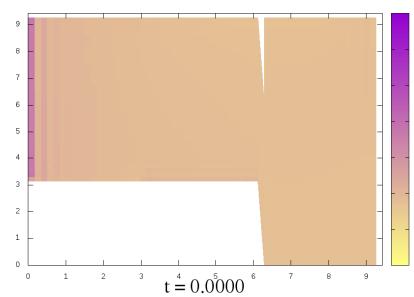
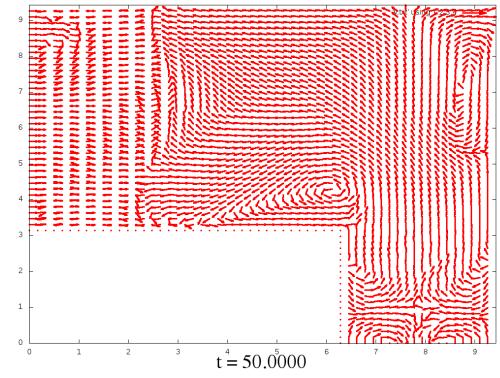
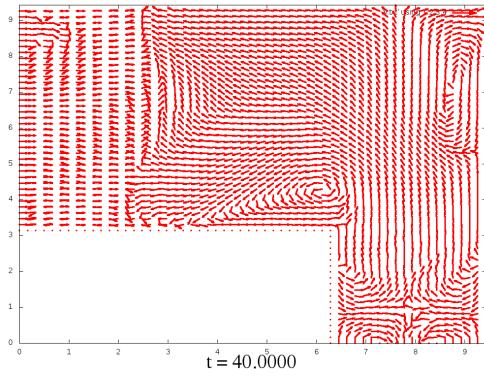
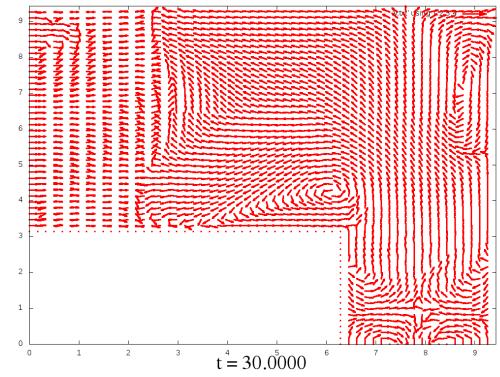
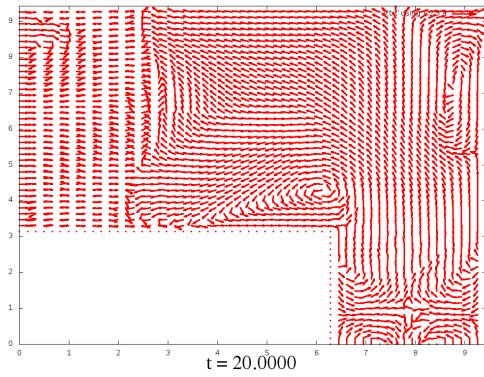
4.2.4 $M_x = 10; M_y = 10; T = 50$

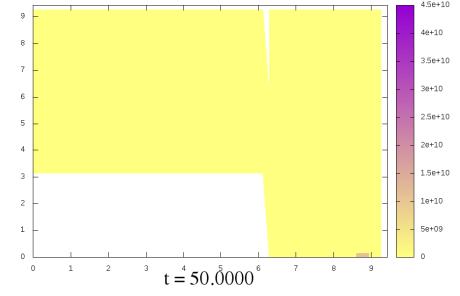
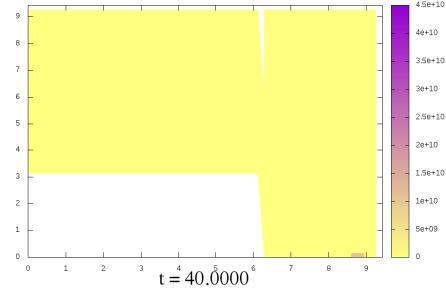




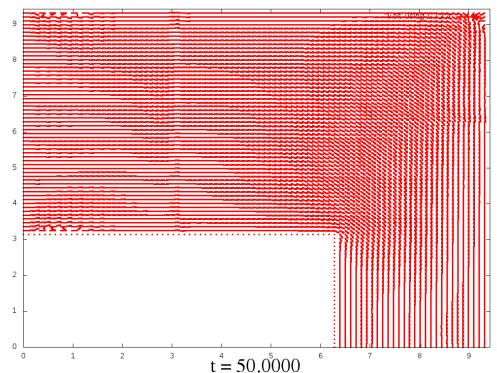
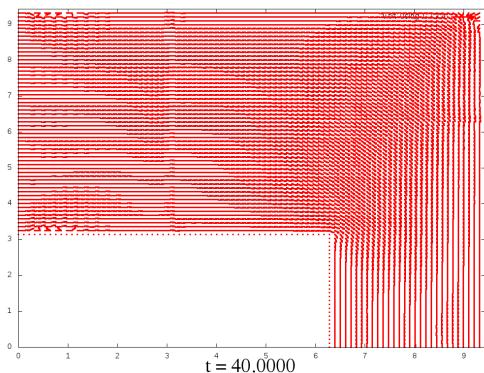
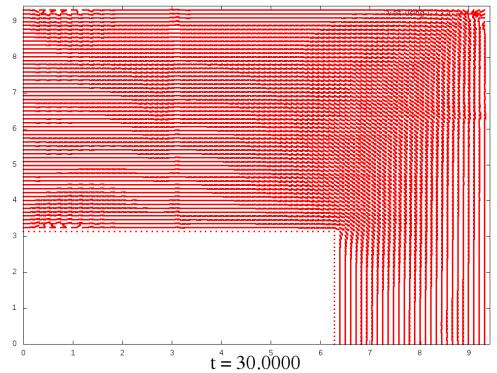
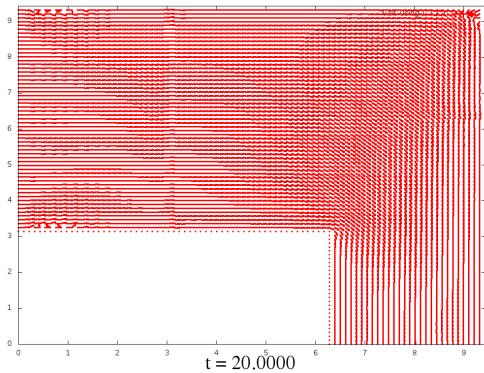
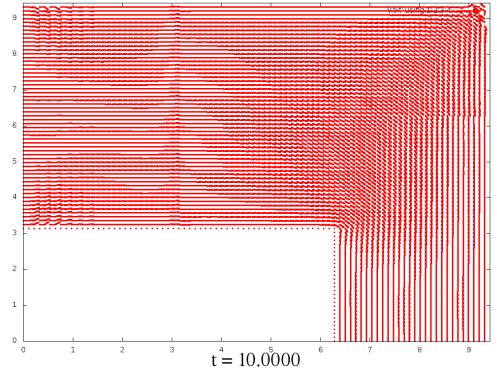
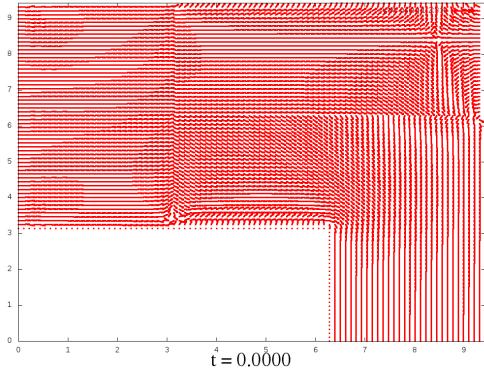
4.2.5 $M_x = 20; M_y = 20; T = 50$

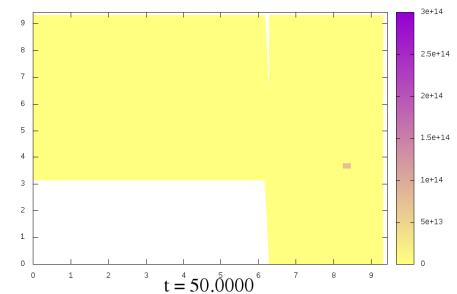
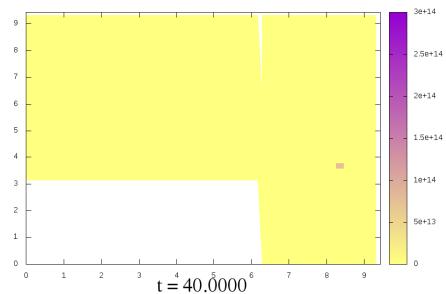
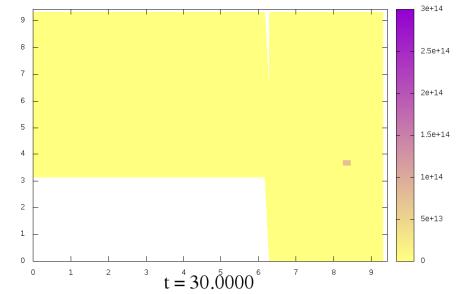
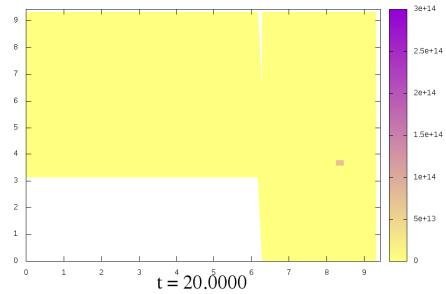
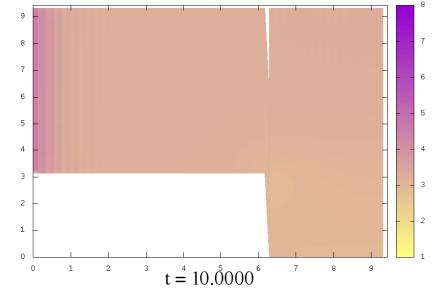
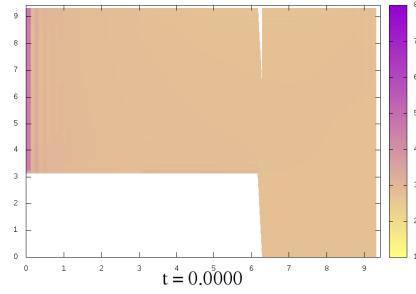






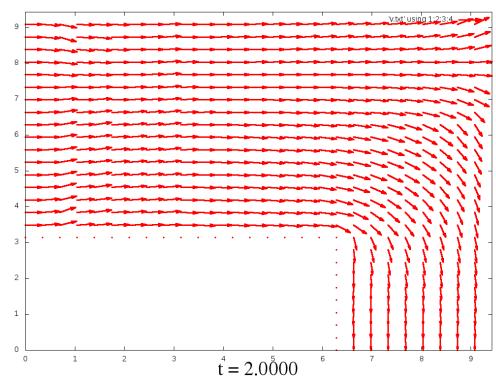
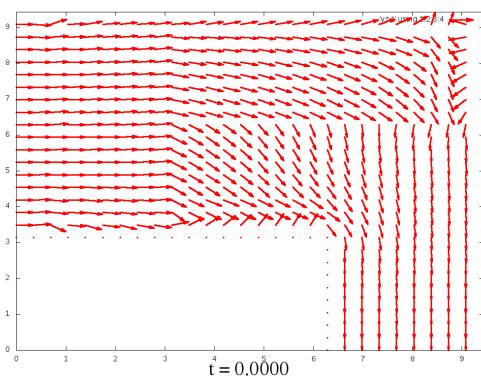
4.2.6 $M_x = 30$; $M_y = 30$; $T = 50$

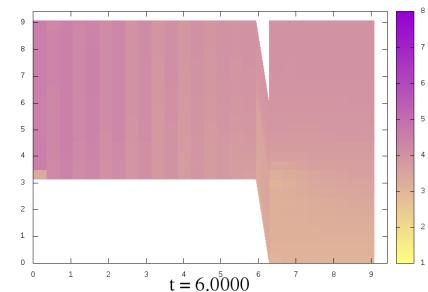
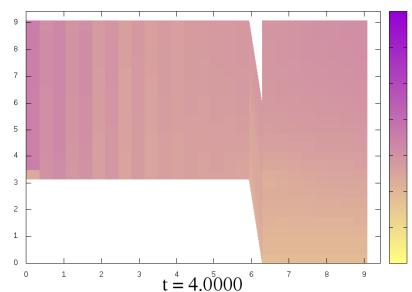
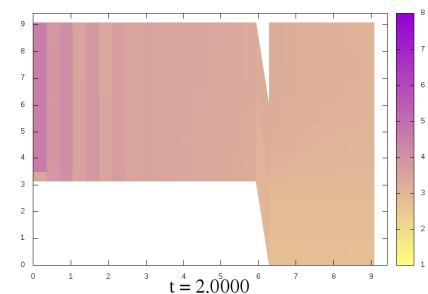
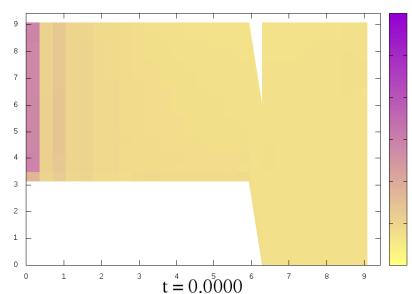
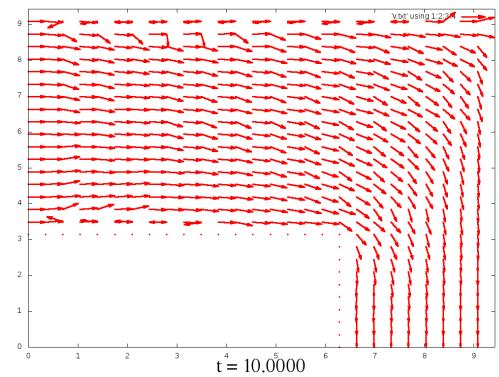
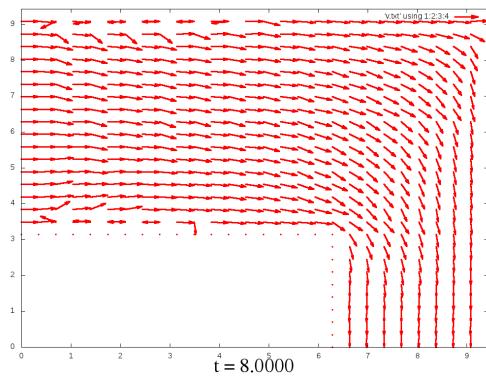
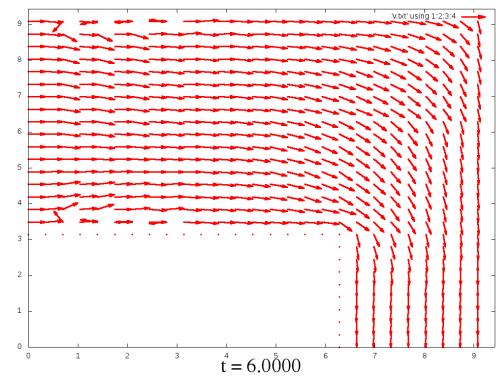
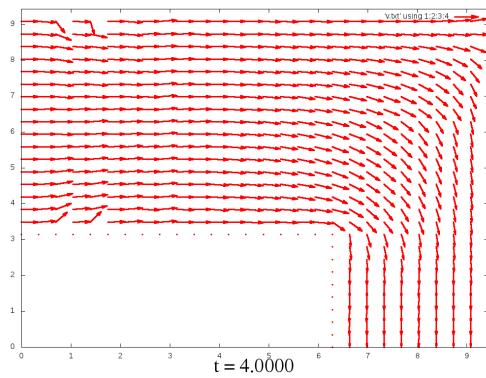


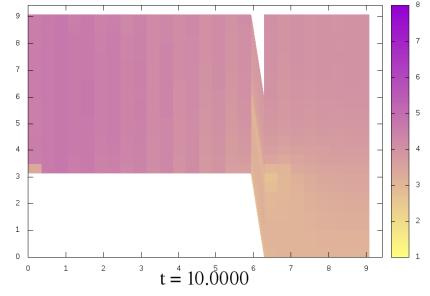
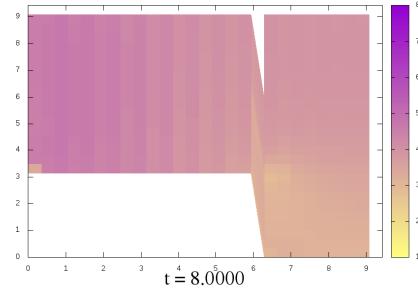


4.3 Графики для $\mu = 0.001$

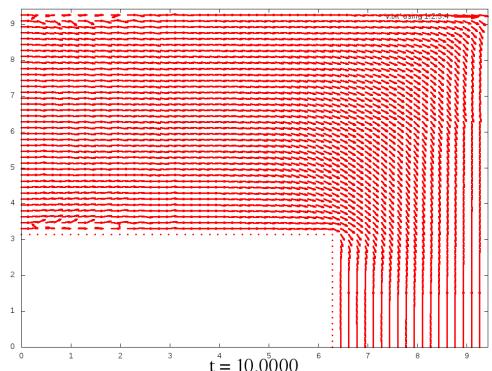
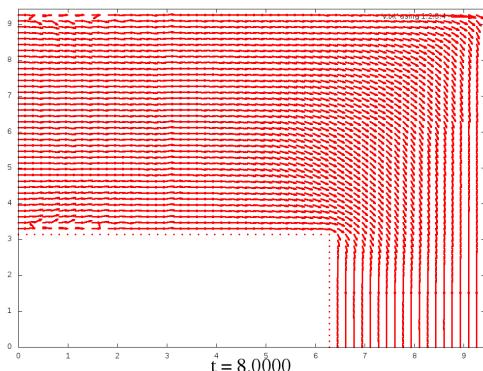
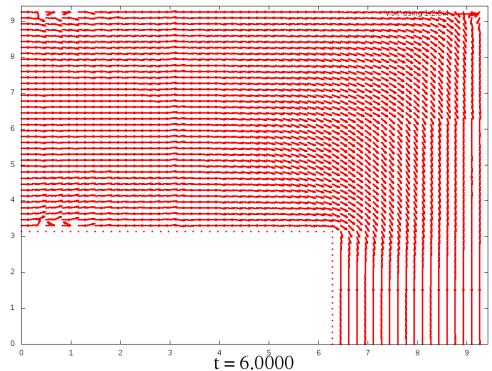
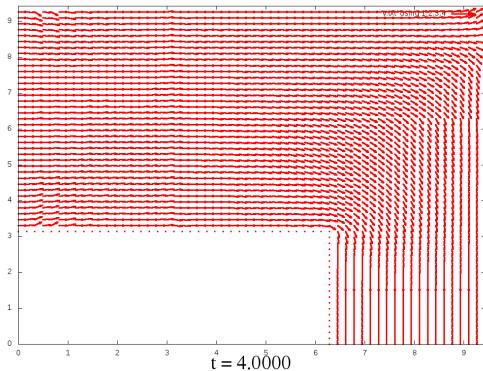
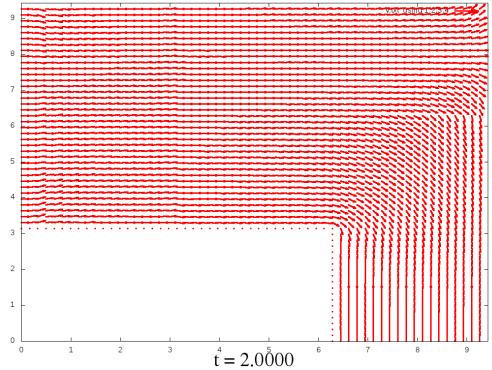
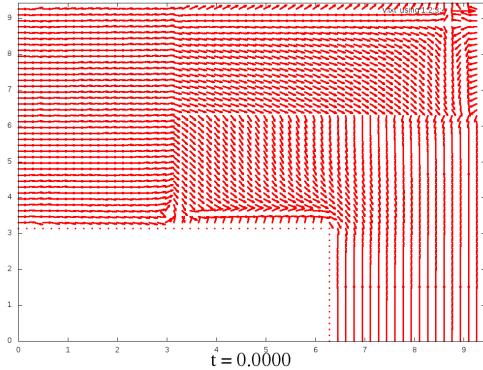
4.3.1 $M_x = 10; M_y = 10; T = 10$

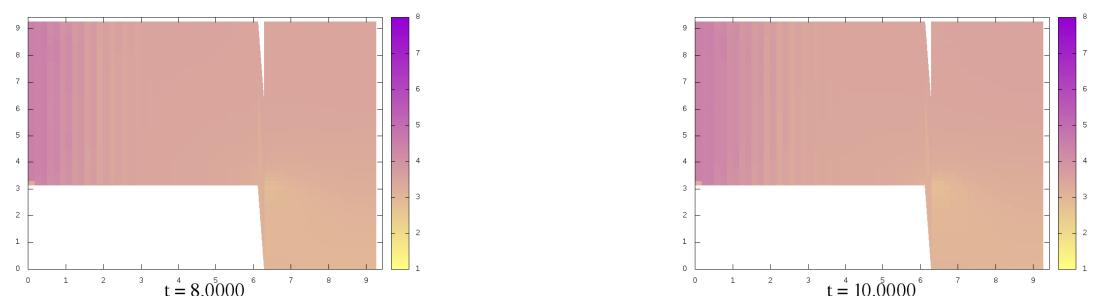
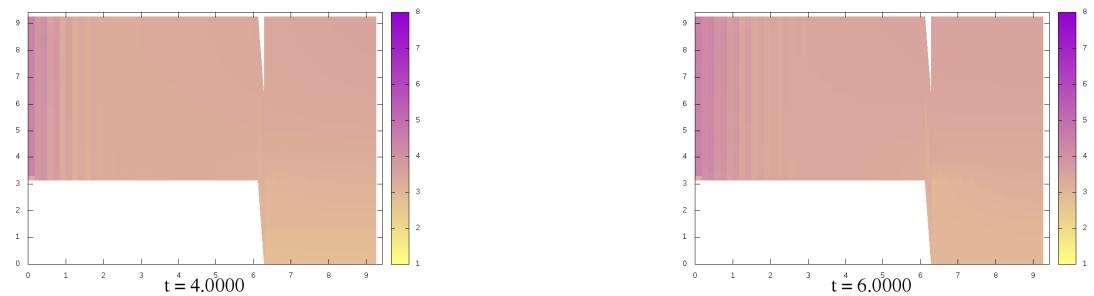
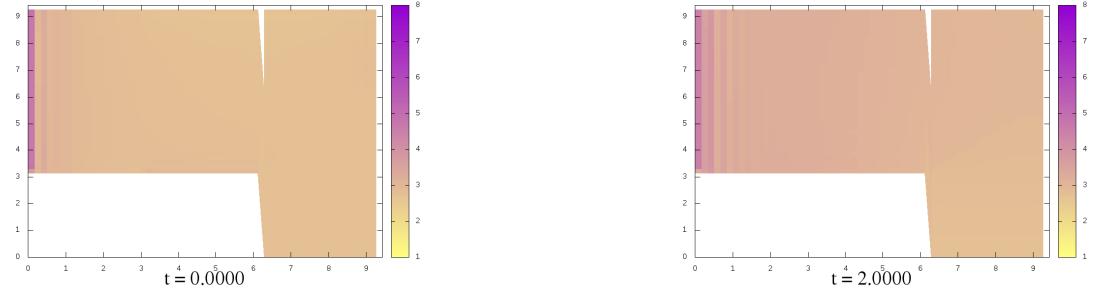




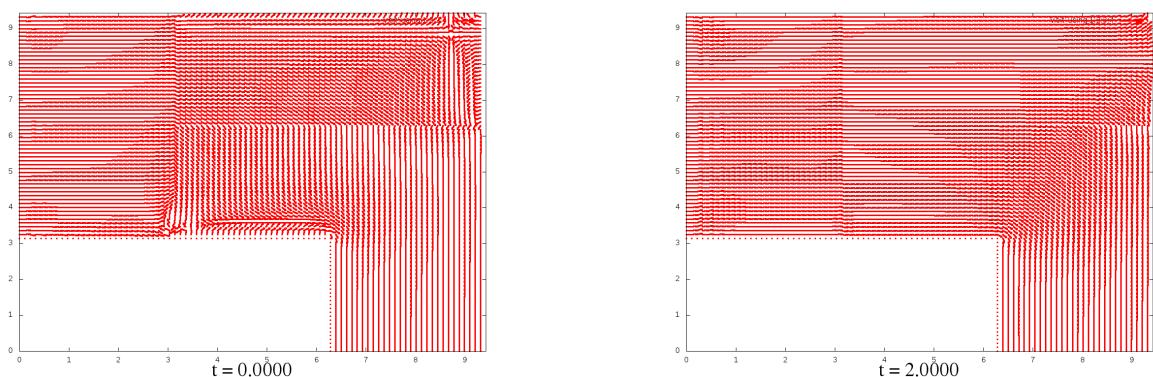


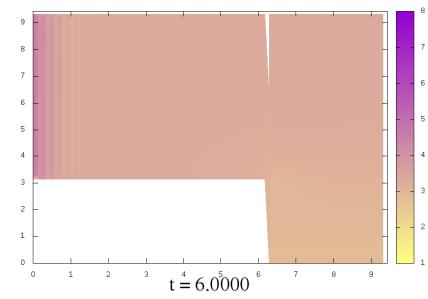
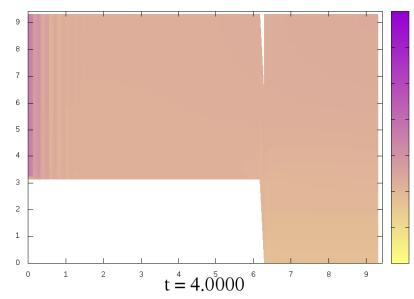
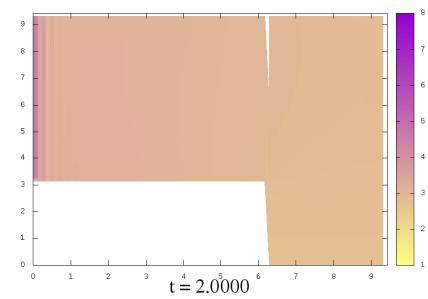
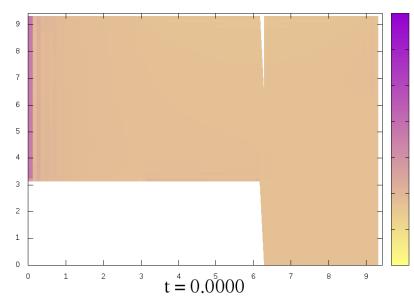
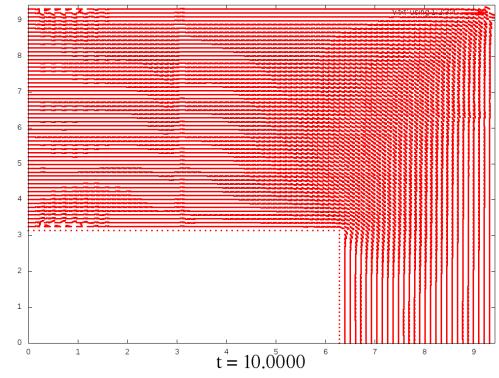
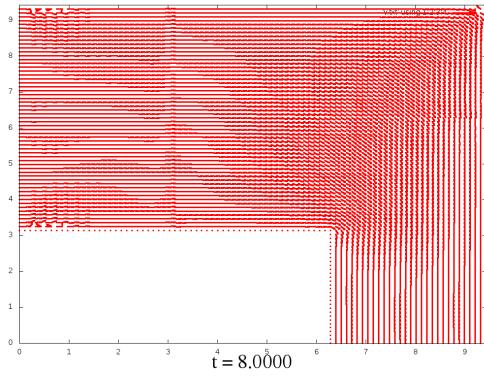
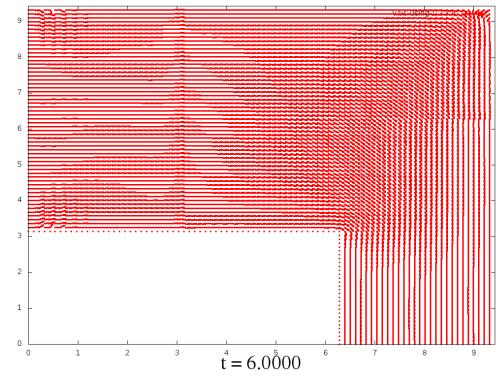
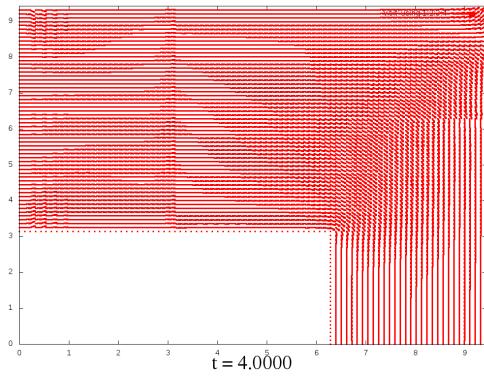
4.3.2 $M_x = 20; M_y = 20; T = 10$

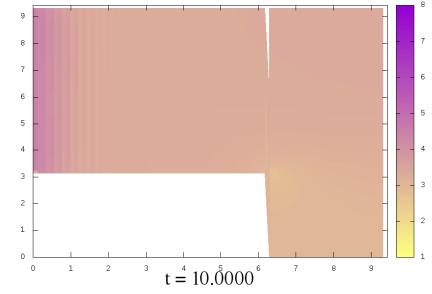
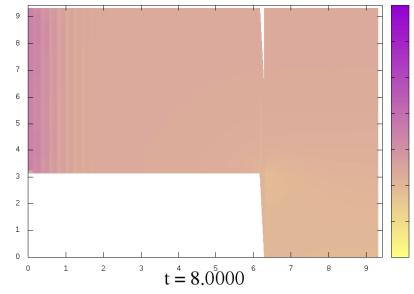




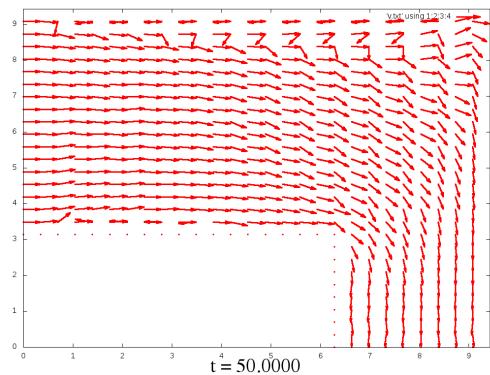
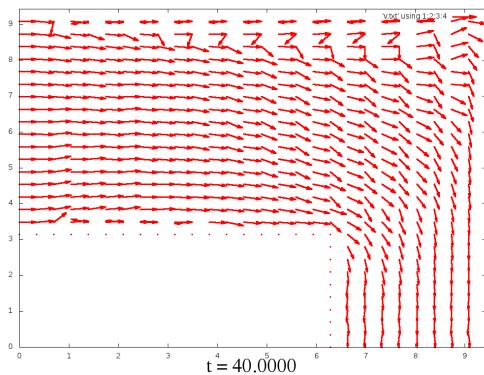
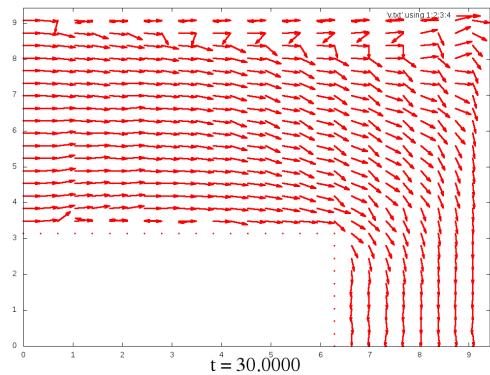
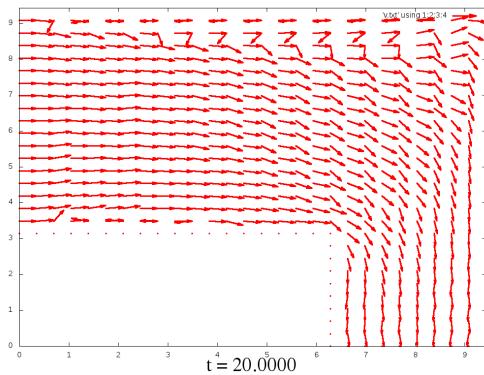
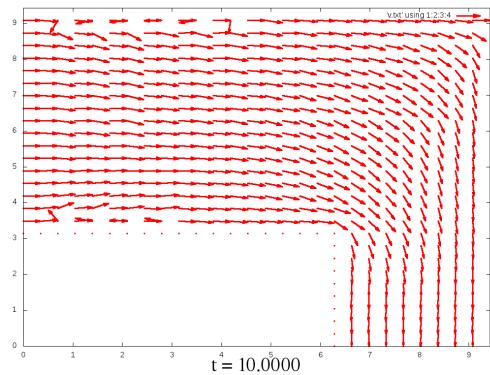
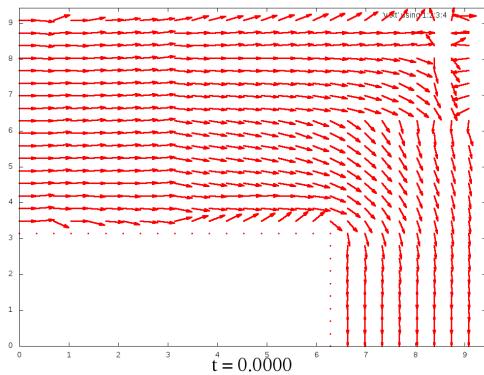
4.3.3 $M_x = 30; M_y = 30; T = 10$

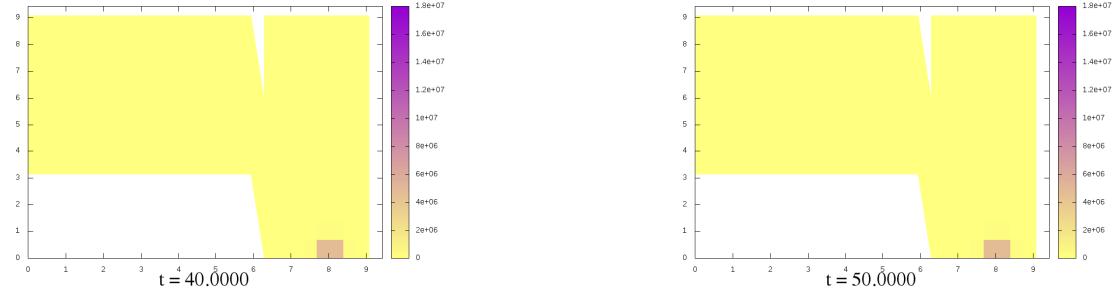
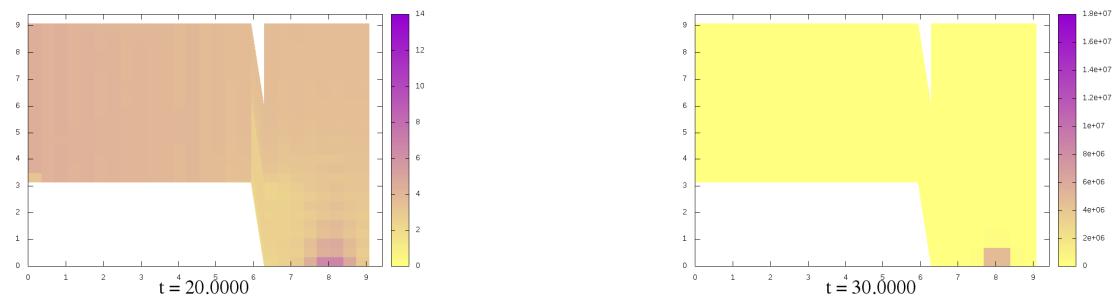
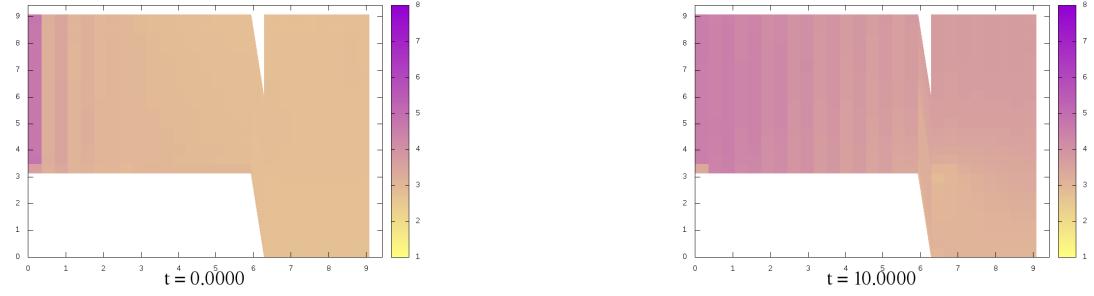




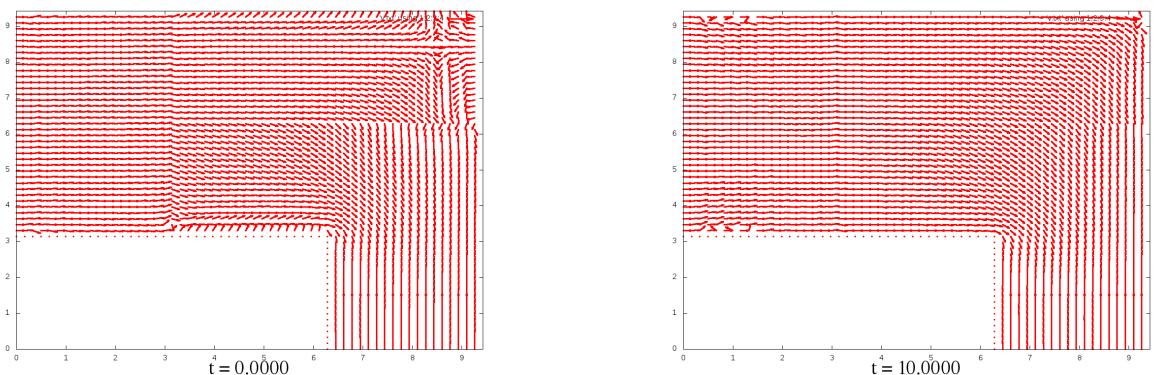


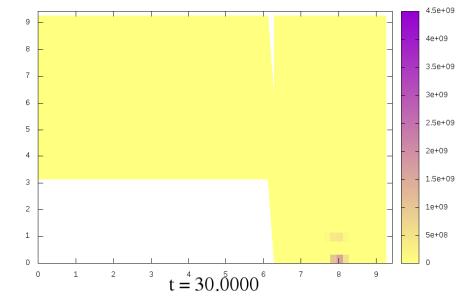
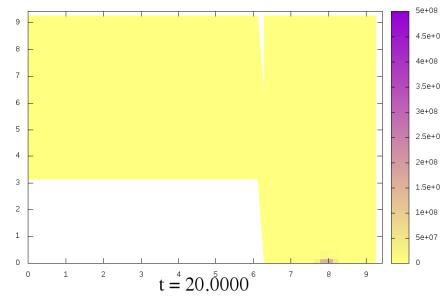
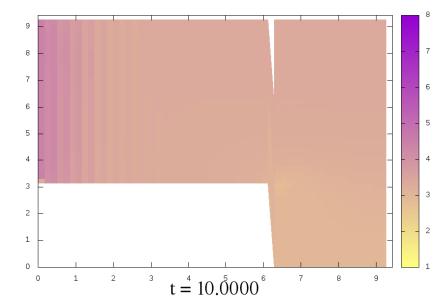
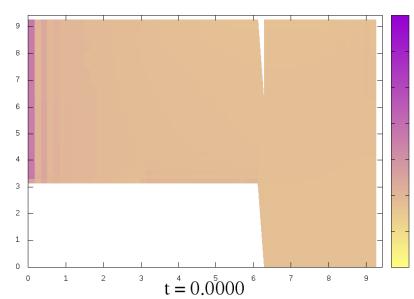
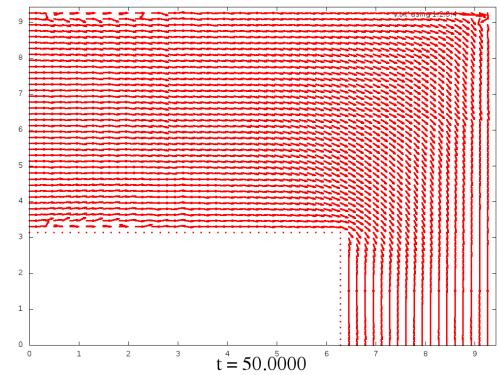
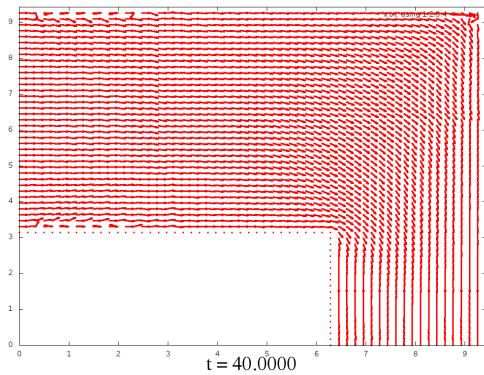
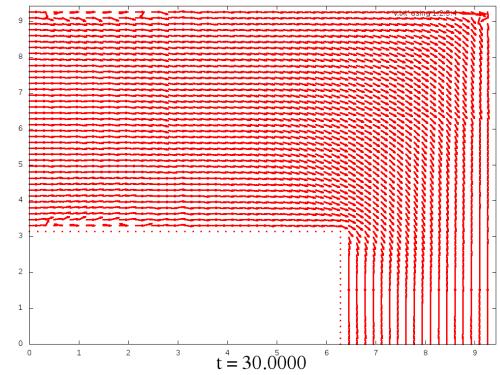
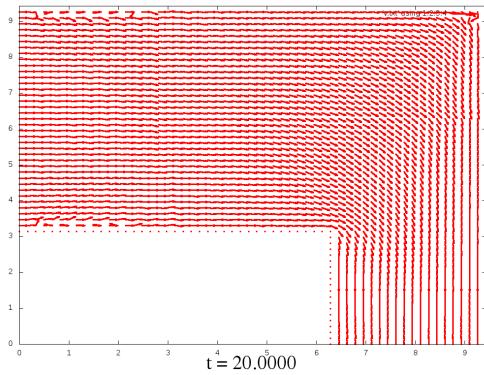
4.3.4 $M_x = 10; M_y = 10; T = 50$

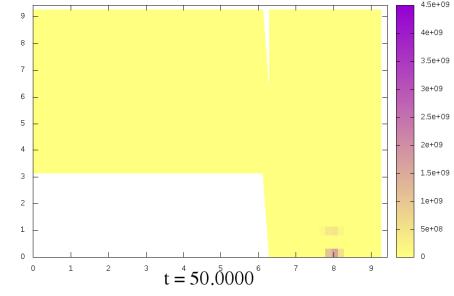
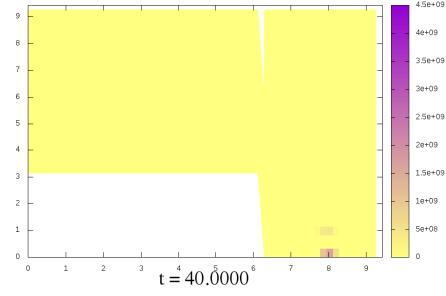




4.3.5 $M_x = 20$; $M_y = 20$; $T = 50$







4.3.6 $M_x = 30$; $M_y = 30$; $T = 50$

