1 Week 3 Reading

1. Is the technique of "separation of variables" always possible? e.g. on page 86, example 2.1.3. And if possible could you give me an overview of the technique?

2. (Page 89 - 90, Proposition 2.14) I don't really understand the proof, especially the step:

   Let $r < \rho(x)$ and let
   $$g(y) = f(x + ry)$$
   I'm confused as to what's fixed and what's variable here.

3. (Page 105-107) I understand the example and want to hear your remarks on the failure of convergence (?) / failure of swapping lim and $\mathbb{E}$, swapping lim and $\int$ in this context i.e. like what prevents the swapping in general and what prevents the swapping in this specific case

4. (Page 108, 109) I'm unsure what the remark at the bottom of 108 to start of 109, saying that "there is a lingering effect from the fact that we assumed that the a priori distribution was the uniform distribution". I think Prof. Lawler is referring to the distribution $f_0(x) = 1, 0 < x < 1$ (page 108) here, but I don't really see how the previous distribution $f_n(x \mid k)$ affects the later distribution $f_{n+1}(x \mid k)$.

5. (Page 112, 115) I assume that the requirement $\mathbb{E}(|Y|) < \infty$ (p.112, around midpage under **3.2**), $\mathbb{E}(|M_n|) < \infty$ (p.115, bottom page, under **Definition 3.2.**) is present to enable some expression involving convergence to be well-defined, and want to know more about the theory underlying it.

1.1 Additional Questions

(a) (Page 117-120) Proposition 3.3
$$\mathbb{E}[M_{n\wedge T}] = \mathbb{E}(M_0)$$
and Theorem 3.4 that under conditions
$$\begin{cases} \mathbb{E}(T < \infty) = 1 \\ \mathbb{E}[|M_T|] < \infty \\ \lim_{n\to\infty} \mathbb{E}[M_n 1_{T>n}] = 0 \end{cases}$$
$$\Rightarrow \mathbb{E}[M_T] = \mathbb{E}(M_0)$$

(b) (Page 121) Polya's urn and Lemma 3.5