Book: Pugh's Real Mathematical Analysis

1 Real Number

2 A Taste of Topology

Definition 2.1 (Convergence on metric space)

Let (M, d) be a metric space. Sequence $(p_n) \in M$ converges to $p \in M$ if $\forall\, \varepsilon > 0\; \exists\, N \in \mathbb{N}$ such that $\forall\, n \geq N, d(p_n, p) < \varepsilon$

Definition 2.2 (Continuous map between metric spaces)

Let M, N be metric spaces. $f : M \to N$ is continuous if it preserves sequential convergence. i.e., if sequence $p_n \to p$ in M, then sequence $f(p_n) \to f(p)$ in N.
This condition is equivalent to standard ε, δ definition of continuity, that is:
$f : M \to N$ is continuous iff $\forall\, \varepsilon > 0,\ \forall\, p \in M,\ \exists\, \delta = \delta(\varepsilon, p)$ such that $d(p, x) < \delta \Rightarrow d(fp, fx) < \varepsilon$.

Definition 2.3 (Continuous map between topological spaces)

$f : M \to N$. f is continuous if the preimage of each closed set in N is closed in M, or equivalently, the preimage of each open set in N is closed in M.

Definition 2.4 (Homeomorphism - the Isomorphism of Metric Spaces)

Let M, N be metric spaces. $f : M \to N$ is a homeomorphism if it is bijective, continuous and its inverse f^{-1} is also continuous. Write $M \cong N$.