1. Let $E = \{u \in C([0,1];\mathbb{R});\ u(0)=0\}$ with the norm $\|u\| = \max_{t\in[0,1]} |u(t)|$.
Consider the linear functional $f: u \in E \to f(u) = \int_0^1 u(t)\,dt$.
- show that $f \in E^*$ and compute $\|f\|_{E^*}$.
- is there some $u \in E$ st $\|u\| = 1$ and $f(u) = \|f\|_{E^*}$? Why?

2. Consider the space $E = c_0$ with its usual norm ℓ^∞ norm
For every element $u = (u_1, u_2, u_3, \ldots)$ in E define $f(u) = \sum_{n=1}^{\infty} \frac{1}{2^n} u_n$.
- check that $f \in E^*$ and compute $\|f\|_{E^*}$.
- is there $u \in E$ st $\|u\| = 1$ and $f(u) = \|f\|_{E^*}$? Why?

3. Let E be a n.v.s. with norm $\| \,\|$. Let $C \subset E$ be an open convex subset of E such that $0 \in C$. Let p denote the gauge of C.
- assume that C is symmetric, i.e., $-C = C$, and C is bounded. Prove that p is a norm which is equivalent to $\| \,\|$.
- Let $E = C([0,1];\mathbb{R})$ with the norm $\|u\| = \max_{t\in[0,1]} |u(t)|$,
Let $C = \{u \in E;\ \int_0^1 |u(t)|^2 dt < 1\}$.
Check that C is convex and symmetric and that $0 \in C$. Is C bounded in E? Compute the gauge p of C and show that p is a norm on E. Is p equivalent to $\| \,\|$?

4. Let E, F be Banach spaces and $(T_n)_{n\in\mathbb{N}}$ a sequence in $\mathcal{L}(E,F)$. Assume that, for every $x\in E$, $T_n x$ converges as $n\to\infty$ to a limit Tx. Show that, if $x_n \to x$ in E, then $Tx_n \to Tx$ in F.

5. Let $E = C([0,1])$ with norm $\|u(t)\| = \max_{t\in[0,1]} |u(t)|$. Consider the operator $A: D(A) \subset E \to E$ defined by

$$D(A) = C^1([0,1]) \text{ and } Au = u' = \frac{du}{dt}.$$

- Chech that $\overline{D(A)} = E$.
- Is A closed?
- Consider $B: D(B) \subset E \to E$ defined by $D(B) = C^2([0,1])$ & $Bu = u'$. Is B closed? Why?