# 1 Week 5 Reading

I've finished the Martingale notes and cleared the question that I was going to ask last week. I'm now reading Evans' PDE book.

1. (Page 24) I think this might be a dumb question, but how do we assert the bounds on

   $$||D^2 f||_{L^\infty} \text{ (equation (12)) and } ||Df||_{L^\infty} \text{ (equation (14))}$$

   which per page 618 (Appendix) I think is their $\operatorname{ess\,sup}$. Does this have anything to do with the assumption on page 23 right above Theorem 1, that $f \in C_c^2(\mathbb{R}^n)$? And the importance of $f$ having compact support as part of the assumption for simplicity.

2. (Page 27, Remark at the bottom of page)

   " The strong maximum principle asserts in particular that if $U$ is connected and $u \in C^2(U) \cap C(\bar{U})$ satisfies

   $$\begin{cases} \Delta u = 0 & \text{in } U \\ u = g & \text{on } \partial U \end{cases}$$

   where $g \geq 0$, then $u$ is positive *everywhere* if $g$ is positive *somewhere*"

   I can kinda hand wave the reason why this is an implication of the maximum principle but would like to hear more.

3. In general I'm running into heavy computations involving "analysis" type estimates, which I'm not so sure if there's a point in scrutinizing through to see what's going on. I've gone through transport equation and mostly through Laplace's equation subsection in Section 2 of Evans' book, and plan to probably read through Heat equation too to re-see what Prof. Lawler covered in a different light - though, again, some computations and mentioning of higher level analysis concepts (most notably $L^p$ norms) have been hindering progress quite a bit. But also with the rigor, it's nice to see some things written out more explicitly than in Lawler's notes, e.g. mollifiers and convolution to show $u$ from $C^2$ to $C^\infty$, so that was good to see.

   I'm wondering if you have any thoughts or recommendations on what to do.