1. If f is continuous on $[0,1]$ and $\int_0^1 f(x)x^n\,dx=0$ for $n=0,1,\ldots$ prove that $f(x)=0$ on $[0,1]$

2. Let K be the unit circle in the complex plane, ie, the set of all $z\in\mathbb{C}$ st $|z|=1$, and consider the algebra $\mathcal{A}$ of all functions of the form

$$f(e^{i\theta})=\sum_{n=0}^{N} c_n e^{in\theta} \quad (\theta \text{ real}).$$

Show that $\mathcal{A}$ separates points on K and $\mathcal{A}$ vanishes at no point of K, but nevertheless there are continuous functions on K which are not in the uniform closure of $\mathcal{A}$.

3. Let $P_0=0$ and define, for $n=0,1,2,\ldots,$ $P_{n+1}(x)=P_n(x)+\dfrac{x^2-P_n^2(x)}{2}$.

Prove that $\lim_{n\to\infty} P_n(x)=|x|$ uniformly on $[-1,1]$.

4. problem 55 from Chapter 4 of Pugh.

The rest of the problems do not rely on Stone-Weierstrass. - you may have already done some or all of them.
from Pugh - Chapter 4

53, 54, 57, 58, 60, 65