DGQT TECHNICAL ASSESSMENT

HUNG LE

1. Buffet

Problem 1.1.
Solution:

Let **P1**, **P2** be 2 players of the game, with **P1** playing first. Let k be the probability that the person that plays first wins. It follows that:

(1) The probability that **P1** wins is k
(2) The probability that the second player wins is $(1-k)$

On the first turn, if **P1** flips H, he then wins. If **P1** flips T, the game essentially resets with **P2** playing first and **P1** becomes the second player, at which point his probability of winning is $(1-k)$.

Therefore:

$$\begin{aligned}
k &= P(\text{P1 flips H}) \times P(\text{P1 wins}|\text{P1 flips H}) \\
&\quad + P(\text{P1 flips T}) \times P(\text{P1 wins}|\text{P1 flips T}) \\
&= \frac{1}{2} \times 1 + \frac{1}{2} \times (1-k) \\
&= 1 - \frac{k}{2} \\
\frac{3k}{2} &= 1 \\
\Rightarrow k &= \frac{2}{3}
\end{aligned}$$

Therefore the probability that **P1** wins is $\frac{2}{3}$, that **P2** wins is $1 - \frac{2}{3} = \frac{1}{3}$ (probability of neither player winning $= \lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$). Since $\frac{2}{3} > \frac{1}{3}$, it does matter who goes first (the first player has a higher probability of winning) and I would thus prefer to go first.

Problem 1.4.
Solution:

Using put-call parity:

$$\begin{aligned}
c + Ke^{-rT} &= p + S_0 \\
12.35 + 140e^{-4\% \times \frac{3}{12}} &= p + 142.16 \\
p &\approx 8.79698
\end{aligned}$$

Thus the price of said put option is \$8.79698.

Problem 1.5.
Solution:

Following the Black-Scholes Differential Equation, the parameter drift rate μ is absent from the equation itself, therefore does not affect the solution. Since all other parameters are equal, current prices $f_1 = f_2$.

2. Challenge Problems

Problem 2.2.
Solution:

Given:

$$\begin{cases} S_0 & = & 50 \\ \sigma & = & 0.25 \end{cases}$$

(a) Annual expected return $= (1 + 14\%)^4 - 1 \approx 0.68896 \approx 68.90\%$

(b) Since the price follows a geometric Brownian motion:

$$dS = \mu S dt + \sigma S dW$$

Integrating both sides:

$$\begin{aligned} S_t &= S_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t} \\ &= 50 e^{(0.68896 - \frac{0.25^2}{2})t + 0.25 W_t} \\ &= 50 e^{0.65771t + 0.25 W_t} \end{aligned} \tag{2.1}$$

(c) From Equation (2.1) it follows that:

$$\ln\left(\frac{S_t}{50}\right) = 0.65771t + 0.25W_t$$

Since $W_t \sim \mathcal{N}(0, t)$:

$$\Rightarrow \ln\left(\frac{S_t}{50}\right) \sim \mathcal{N}(0.65771t, 0.25^2 t)$$

At 9 months, $t = \frac{9}{12} = 0.75$. Therefore:

$$\ln\left(\frac{S_{0.75}}{50}\right) \sim \mathcal{N}(0.65771 \times 0.75, 0.25^2 \times 0.75) = \mathcal{N}(0.4932825, 0.046875)$$

$$\Rightarrow \ln(S_{0.75}) \sim \mathcal{N}(\ln 50 + 0.4932825, 0.046875)$$

Thus after 9 months the stock price will follow a lognormal distribution with the above parameters.

(d) The probability that the stock price will be greater than \$65 in 9 months:

$$\begin{aligned} \mathbb{P}(S_{0.75} > 65) &= \mathbb{P}(\ln S_{0.75} > \ln 65) \\ &= 1 - \mathit{normcdf}(\ln 65, \ln 50 + 0.4932825, 0.046875) \\ &= 0.856916 \end{aligned}$$

(e) At t = 0.75:

$$
\begin{aligned}
\mathbb{E}[X_t] &= X_0 e^{\mu t} \\
&= 50e^{0.68896\times 9\div 12} \\
&\approx 83.8260 \\
\Rightarrow \text{Expected payoff} &\approx \$83.8260 - \$65 \\
&= \$18.8260
\end{aligned}
$$