SOMETHING HARMONIC FUNCTION

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Abstract. This is the abstract

Contents

1. Harmonic Function

Definition 1.1 (Harmonic Function)**.** Recall that a function f on $\mathbb{Z}^d$ is harmonic at x if $f(x)$ equals the average of f on its nearest neighbors. If U is an open subset of $\mathbb{R}^d$, we will say that f is **harmonic** in U if and only if it is continuous and satisfies the following **mean value property**: for every $x \in U$, and every $0 < \epsilon < dist(x, \partial U)$,

$$f(x) = MV(f; x, \epsilon) = \int_{|y-x|=\epsilon} f(y) ds(y) \tag{1.2}$$

Remark 1.3. Value at x equals to average of ball radius ϵ around x for all ϵ

Definition 1.4 (Laplacian)**.**

$$\Delta f(x) = \lim_{e \to 0} \frac{1}{\epsilon^2} \sum_{y \in \mathbb{Z}^d, |y|=1} [f(x + \epsilon y) - f(x)]$$

Remark 1.5. Just taking in each direction, not the whole ball!

Proposition 1.6 (Representing Laplacian in partial derivatives)**.** *Suppose f is C^2 in a neighborhood of x in $\mathbb{R}^d$. Then $\Delta f(x)$exists at x and*

$$\Delta f(x) = \sum_{j=1}^{d} \partial_{jj} f(x)$$

Proof. This comes naturally from the above definition of $\Delta f(x)$, as well as the approximation one can make from the C^2 smoothness of $f(x)$. □

Proposition 1.7. *If f is C^2 in a neighborhood of x, then*

$$\frac{1}{2d} \Delta f(x) = \lim_{\epsilon \to 0} \frac{MV(f; x, \epsilon) - f(x)}{\epsilon^2} \tag{1.8}$$

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Theorem 1.9. ***BIG THEOREM! Stating the equivalence of a harmonic function with its Laplacian operator***

A function in a domain U is harmonic if and only if f is C^2 with $\Delta f(x) = 0 \; \forall \; x \in U$

Acknowledgments

You should thank anyone who deserves thanks, and for sure you should thank your mentor. “It is a pleasure to thank my mentor, his/her name, for .... ”. Or add anyone else, for example “I thank [another participant] for helping me understand [something or other]”

2. Bibliography

References

[1] http://www.ams.org/publications/authors/tex/amslatex
[2] Michael Downes. Short Math Guide for LaTeX. http://tex.loria.fr/general/downes-short-math-guide.pdf
[3] J. P. May. A Concise Course in Algebraic Topology. University of Chicago Press. 1999.
[4] Tobias Oekiter, Hubert Partl, Irene Hyna and Elisabeth Schlegl. The Not So Short Introduction to LaTeX2e. https://tobi.oetiker.ch/lshort/lshort.pdf