1 Week 8 Reading

1. (Page 183) I get the proof of Plancherel's Theorem, but would you have any remarks on why it *should* be true?

2. (Page 185, middle of the page, under 2.) "By approximation the same formula is true if $D^\alpha u \in L^2(\mathbb{R}^n)$". I assume this approximation refers to the case when u is not smooth and has compact support? But I'm not sure.

3. (Page 185, under 3.) Application of Fubini's Theorem? In this case $u, v \in L^1(\mathbb{R}^n) \cap L^2(\mathbb{R}^n) \Rightarrow (u * v) \in L^1(\mathbb{R}^n)$ so it's all good?

4. (Page 187, line 3) I looked up the errata file and the first sentence was corrected to be "Even though $\hat{B}$ is not in L^1 or L^2 for large n, we may proceed to compute B". I guess we're still allowed to use the relationship $B = (\hat{B})\check{}$ and treat B being L^1 (the condition for the previous equation to hold) like an "ansatz"? And eventually either prove that B as computed above is actually L^1 so that the equation in the first place was valid or prove that the re-computed $\hat{B}$ using "guessed" B actually aligned with the initial $\hat{B}$?

5. (Page 187, above equation (12)) "Deforming Γ into the real axis". I know it's a little bit too much of complex analysis at this point but could you somehow provide an intuition for why the two integrals should be equal? On a relevant note, perhaps remarks on what analytic functions are and why they provide nice properties too.

1.1 Some more questions

(Sorry for jumping everywhere with my questions)

1. (Page 184, line 6) Why does $\hat{w} \geq 0$ matter as we send $\varepsilon \to 0^+$?

2. (Another convergence question, Page 186, line 5) Lebesgue point

3. (Page 188, 189, Example 2, 3) Equation (17) can only be used if $g \in L^1 \bigcap L^2$? (cf. Theorem 2 (iii)) So we need "nice enough" initial conditions for this to work? The fundamental solution approach didn't have such requirements on g?

4. On the same note, I guess I never really thought about this. Do convolutions run into "convergence problems" (if that's a valid question)? Let's say, equation (18) page 188