MATH 16300
Honors Calculus III

Cong Hung Le Tran

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Section: 43
Professor: Minjae Park
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Course materials: Calculus by Spivak (4th Edition), Calculus On Manifolds by Spivak

Disclaimer: This document will inevitably contain some mistakes, both simple typos and serious logical and mathematical errors. Take what you read with a grain of salt as it is made by an undergraduate student going through the learning process himself. If you do find any error, I would really appreciate it if you can let me know by email at conghungletran@gmail.com.

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Lecture 3

Uniform Convergence

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Motivation. We want to elevate the concept of "convergence" to beyond sequences (which are essentially maps of $\mathbb{N} \to \mathbb{R}$) to a higher level of abstraction:

$$f : \mathbb{N} \to \mathcal{F} = \{g : A \to \mathbb{R}\}$$

$f(1) = f_1, f(2) = f_2, \ldots$ then become functions $A \to \mathbb{R}$.

Convergence can be explained via **"measuring closeness"**. For reals, this is intuitive and trivial:

$$d(a, b) = |a - b|$$

However, for functions, this is not clear.

> **Example.** For $\mathbb{R}^2$, one way to measure distance between $x = (a, b), y = (c, d)$ is
>
> $$d(x, y) = \sqrt{(a-c)^2 + (b-d)^2}$$
>
> But this is not the only way! One might also measure distance via the Manhattan Distance
>
> $$d(x, y) = |a - c| + |b - d|$$
>
> Therefore we must be very careful about "distance" and "closeness".

Recall. $a_n \to a$ if $\forall\, \varepsilon > 0,\ \exists\, N \in \mathbb{N}$ such that $n > N \Rightarrow |a_n - a| < \varepsilon$.

More generally and abstractly, the condition can be written as $d(a_n, a) = |a_n - a| < \varepsilon$.