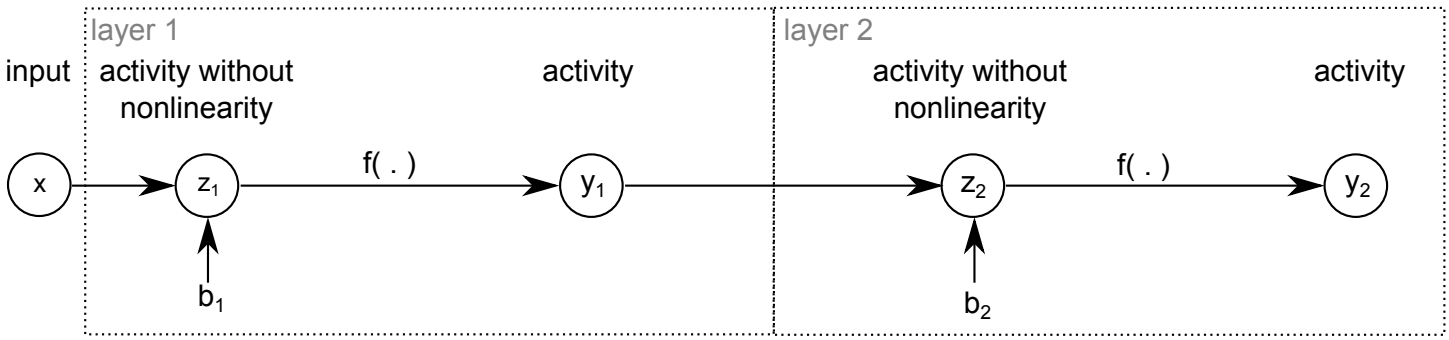

feed forward

$z_1 = V*x+b_1$ $\quad z_2 = W*y_1+b_2$

$y_1 = f(z_1)$ $\quad y_2 = f(z_2)$

back propagation

$cost = 1/2*e^2$

$e = y_2 - target$

$\delta_2 = f'(z_2).*e$ $\quad \delta_1 = f'(z_1).*W'*\delta_2$

$\Delta W = \delta_2 * y_1'$ $\quad \Delta V = \delta_1 * x'$

example

input size, neurons

$$V = \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{bmatrix} \text{ neurons} \qquad W = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \text{ output size}$$

$$z_1 = \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b_1 = \begin{bmatrix} v_{11}*x_1+v_{12}*x_2+v_{13}*x_3+b_1 \\ v_{21}*x_1+v_{22}*x_2+v_{23}*x_3+b_1 \end{bmatrix}$$

$$y_1 = f\left(\begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix}\right) = \begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix}$$

$$z_2 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} * \begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix} + b_2 = \begin{bmatrix} w_{11}*y_{11}+w_{12}*y_{21}+b_2 \\ w_{21}*y_{11}+w_{22}*y_{21}+b_2 \\ w_{31}*y_{11}+w_{32}*y_{21}+b_2 \end{bmatrix}$$

$$y_2 = f\left(\begin{bmatrix} z_{21} \\ z_{22} \\ z_{23} \end{bmatrix}\right) = \begin{bmatrix} y_{21} \\ y_{22} \\ y_{23} \end{bmatrix}$$

example

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \qquad cost = 1/2* \begin{bmatrix} e_1^2 \\ e_2^2 \\ e_3^2 \end{bmatrix}$$

$$\delta_2 = f'\left(\begin{bmatrix} z_{21} \\ z_{22} \\ z_{23} \end{bmatrix}\right). * \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} z_{21}*e_1 \\ z_{22}*e_2 \\ z_{23}*e_3 \end{bmatrix}$$

$$\Delta W = \begin{bmatrix} \delta_{21} \\ \delta_{22} \\ \delta_{23} \end{bmatrix} * \begin{bmatrix} y_{11} & y_{21} \end{bmatrix} = \begin{bmatrix} \delta_{21}*y_{11} & \delta_{21}*y_{21} \\ \delta_{22}*y_{21} & \delta_{22}*y_{21} \\ \delta_{23}*y_{21} & \delta_{23}*y_{21} \end{bmatrix}$$

$$\delta_1 = f'\left(\begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix}\right). * \begin{bmatrix} w_{11} & w_{21} & w_{31} \\ w_{12} & w_{22} & w_{32} \end{bmatrix} * \begin{bmatrix} \delta_{21} \\ \delta_{22} \\ \delta_{23} \end{bmatrix}$$

$$= f'\left(\begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix}\right).* \begin{bmatrix} w_{11}*\delta_{21}+w_{21}*\delta_{22}+w_{31}*\delta_{23} \\ w_{12}*\delta_{21}+w_{22}*\delta_{22}+w_{32}*\delta_{23} \end{bmatrix}$$

$$\Delta V = \begin{bmatrix} \delta_{11} \\ \delta_{12} \end{bmatrix} * \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} \delta_{11}*x_1 & \delta_{11}*x_2 & \delta_{11}*x_3 \\ \delta_{12}*x_1 & \delta_{12}*x_2 & \delta_{12}*x_3 \end{bmatrix}$$