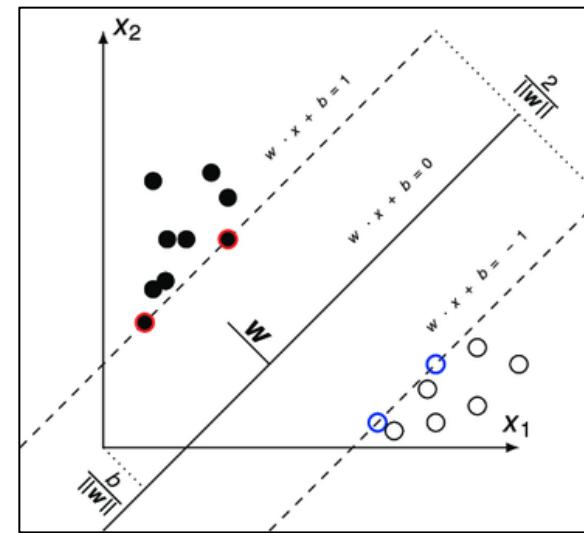
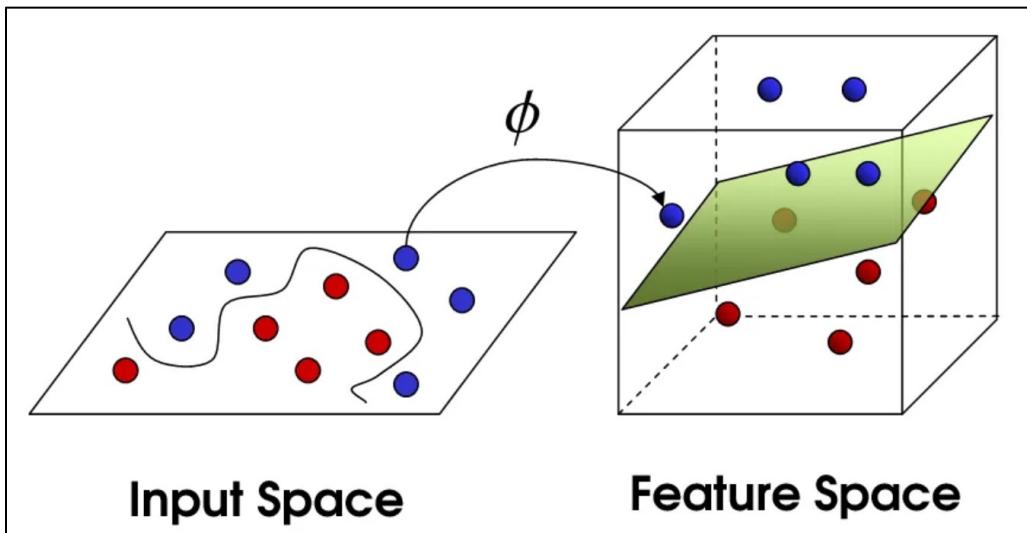


# Support Vector Machine (Another View)

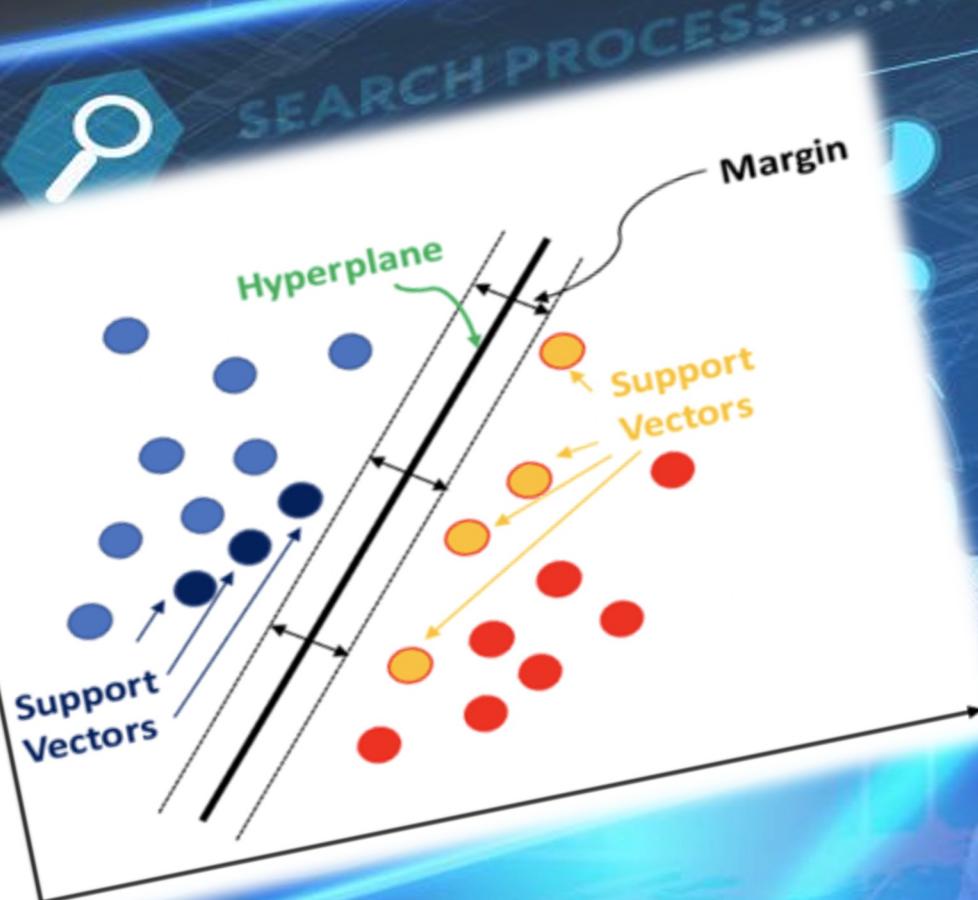


$$\begin{array}{ll} \text{Minimize}_{\mathbf{w}} & \frac{1}{2} \|\mathbf{w}\|^2 \\ \text{subject to} & y_i(\langle \mathbf{w}, \mathbf{x}_i \rangle + b) \geq 1 \end{array}$$

$$Y \cdot (\mathbf{X}\mathbf{w} + b) \geq 1^n$$
$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \cdot \left( \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1d} \\ x_{21} & x_{22} & \dots & x_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nd} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{bmatrix} + b \right) \geq 1^n$$

Vinh Dinh Nguyen  
PhD in Computer Science

# WHAT IS A **SUPPORT VECTOR MACHINE?**



# Outline

- Linear Regression to Logistic Regression
- Logistic Regression to Support Vector Machine
- Example

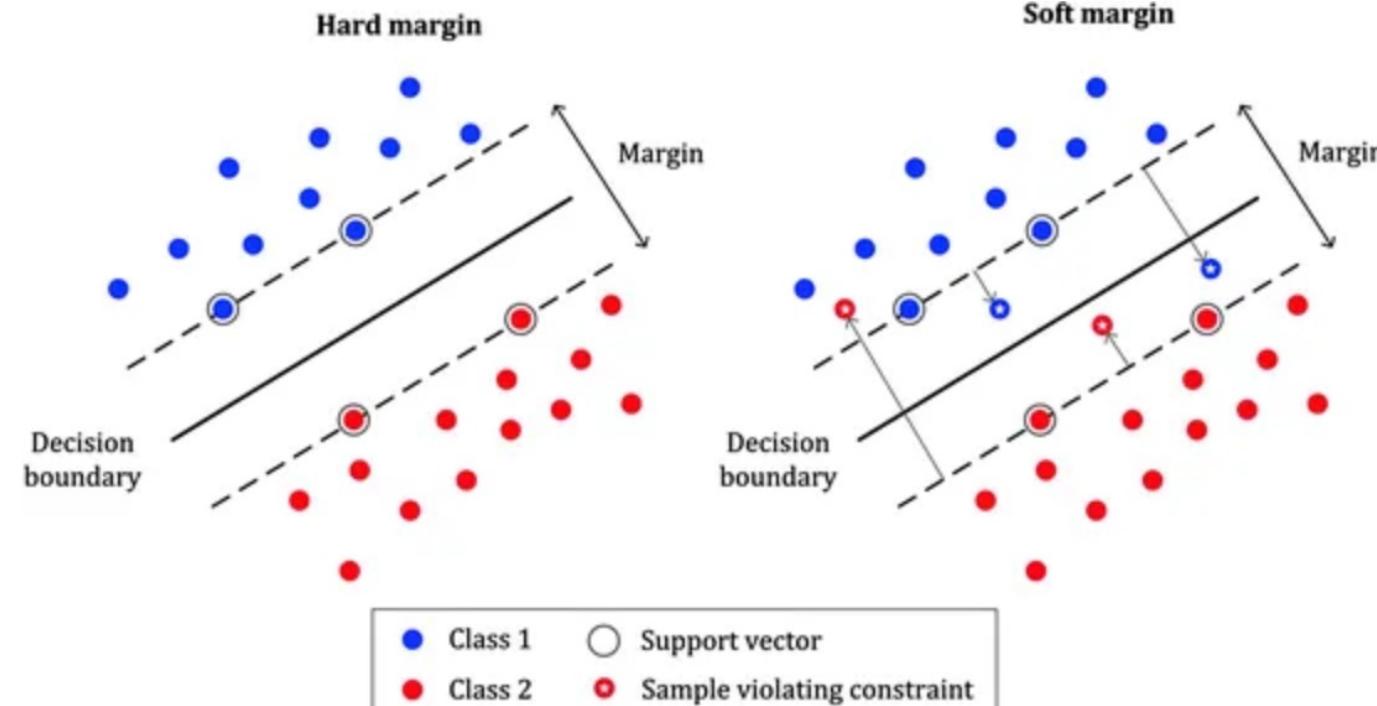
# Outline

- Linear Regression to Logistic Regression
- Logistic Regression to Support Vector Machine
- Example

# Review – Linear SVM

Primal Problem

$$\min \frac{1}{2} \|W\|_2^2 + C \sum_i \varepsilon_i \text{ such that } Y * (w^T X + b) \geq 1 - \varepsilon_i \quad \varepsilon_i \geq 0$$



# Review- Linear SVM

Primal Problem

$$\min \frac{1}{2} \|W\|_2^2 + C \sum_i^N \varepsilon_i \text{ such that } Y * (w^T X + b) \geq 1 - \varepsilon_i$$

$$\varepsilon_i \geq 0$$

Objective Function :  $\min_{\beta, b, \xi_i} \left\{ \frac{\|\beta\|^2}{2} + C \sum_{i=1}^n (\xi_i)^k \right\}$

s.t Linear Constraint :  $y_i(\beta^T x_i + b) \geq 1 - \xi_i$ , where  $\xi_i \geq 0$

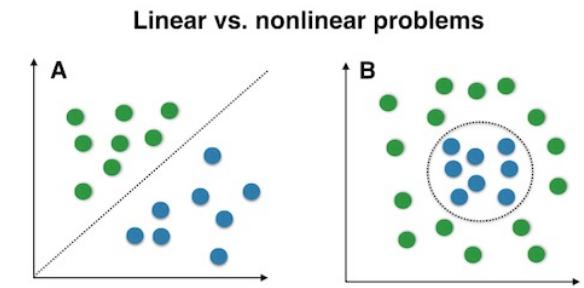
Another form:

**C** and **k** are constants which balances the cost of misclassification. The  $\sum_{i=1}^n (\xi_i)^k$  is the loss term and C is a HyperParameter which controls the trade-off between maximizing the margin and minimizing the loss.

The HyperParameter C is also called as *Regularization Constant*.

If k = 1 , then the loss is named as Hinge Loss and if k =2 then its called Quadratic Loss

How bout Non-Linear Case?



# Review- Linear SVM

Primal Problem

$$\min \frac{1}{2} \|W\|_2^2 + C \sum_i^N \varepsilon_i \text{ such that } Y * (w^T X + b) \geq 1 - \varepsilon_i$$

$$\varepsilon_i \geq 0$$

Objective Function :  $\min_{\beta, b, \xi_i} \left\{ \frac{\|\beta^2\|}{2} + C \sum_{i=1}^n (\xi_i)^k \right\}$

s.t Linear Constraint :  $y_i(\beta^T x_i + b) \geq 1 - \xi_i$ , where  $\xi_i \geq 0$

Another form:

C and k are constants which balances the cost of misclassification. The  $\sum_{i=1}^n (\xi_i)^k$  is the loss term and C is a HyperParameter which controls the trade-off between maximizing the margin and minimizing the loss.

The HyperParameter C is also called as *Regularization Constant*.

If k = 1 , then the loss is named as Hinge Loss and if k =2 then its called Quadratic Loss

However, this way we won't be able use the objective function to solve for **non-linear cases**. Hence, we will find an equivalent problem named **Dual Problem** and solve that using **Lagrange Multipliers**.

# Duality

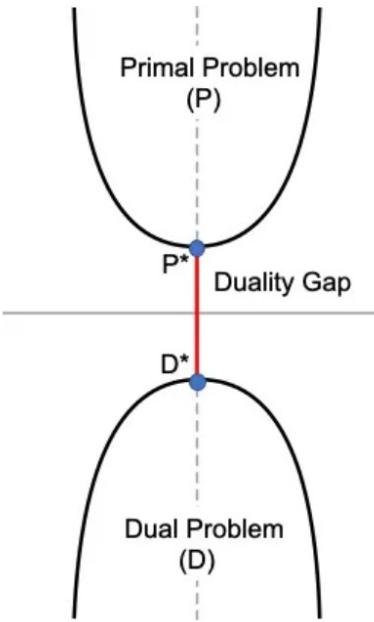
## Duality

“ In mathematical optimization theory, **duality** means that optimization problems may be viewed from either of two perspectives, the primal problem or the dual problem (**the duality principle**). The solution to the dual problem provides a lower bound to the solution of the primal (minimization) problem.

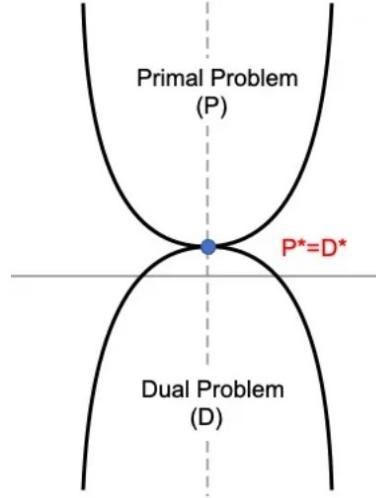
Wikipedia

# Dual Problem (Bài toán đối ngẫu)

- Primal Problem is something that we want to minimize. In the diagram,  $P^*$  minimizes the Primal Objective  $\mathbf{P}$ .
- Dual Problem is something we want to maximize. Here we want to convert the Primal Problem ( $\mathbf{P}$ ) to a Dual Problem ( $\mathbf{D}$ ).  $D^*$  maximizes the Dual Objective  $\mathbf{D}$ .
- Sometimes solving the Dual Problem is same as solving the Primal Problem.
- $(P^* - D^*)$  is called as the **Duality Gap**.
- If  $P^* - D^* > 0$  we can say **weak duality holds**.



- The goal will be to find in which condition we can have  $P^* - D^* = 0$  and we can say that **strong duality holds**.
- In this picture, we can see that maximizing the Dual Problem is same as Minimizing the Primal Problem.
- There are some conditions named **KKT condition**, needs to hold in order to have  $P^* - D^* = 0$
- Next, we will talk about **Lagrange Multiplier**, which will help us to determine when we can find strong duality.



# Warm-up

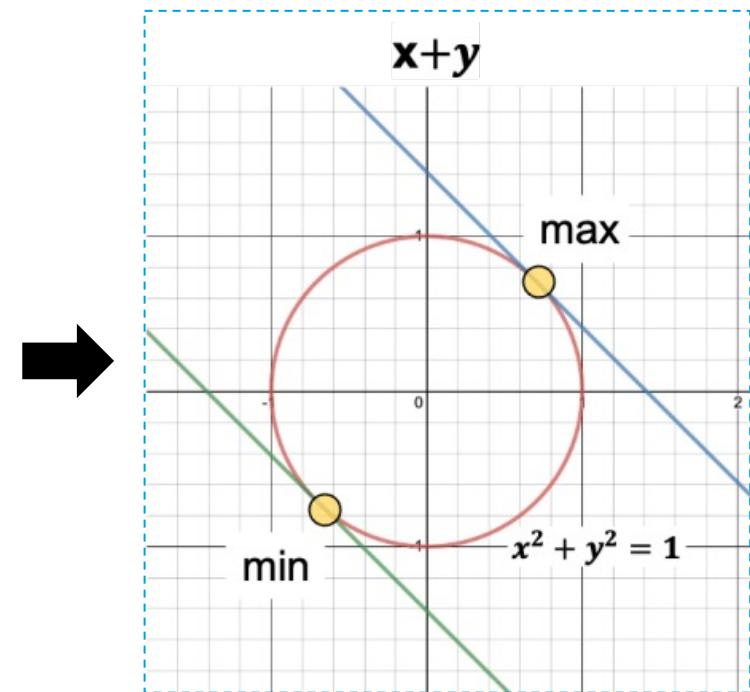
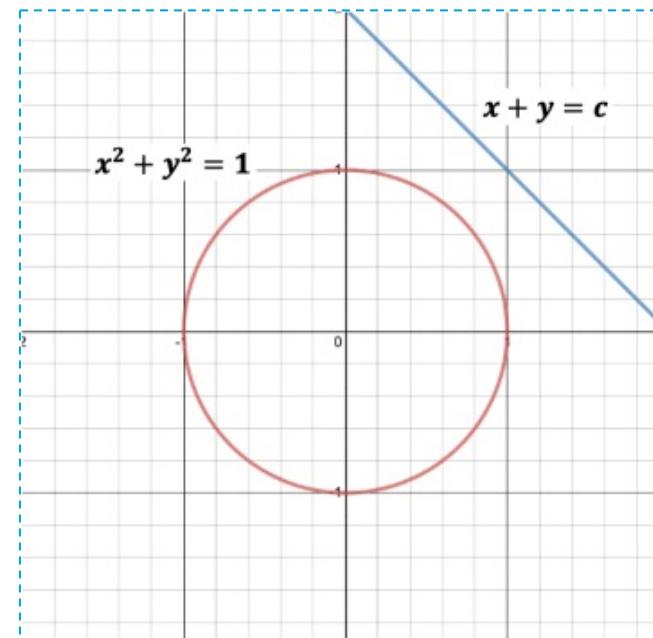


## Primal Problem

$$\begin{aligned} & \max_{x,y} x + y \\ \text{s.t. } & x^2 + y^2 = 1 \end{aligned}$$

- Here is the plot of the constraint  $x^2 + y^2 = 1$  in red, which is a circle.
- $x + y$  is basically the entire plane.
- The blue line is an arbitrary representation of  $x + y = c$

Primal Problem is helpful in solving Linear SVM using SGD. Remember we need to optimize D+1 ( where D is the dimension ) parameters in Primal Problem.



# Lagrange Multiplier



Primal Problem

$$\begin{aligned} & \max_{x,y} x + y \\ \text{s.t. } & x^2 + y^2 = 1 \end{aligned}$$

$$\max_{x,y} f(x, y)$$

such that  $g(x, y) = c$

$$\begin{aligned} L(x, y, \alpha) &= f(x, y) - \alpha(g(x, y) - c) \\ &= x + y - \alpha(x^2 + y^2 - 1) \end{aligned}$$

Since we are trying to maximize  $x + y$ , we will consider only the positive values. **Final Result:**

$$\delta_x L(x, y, \alpha) = 1 - 2\alpha x = 0$$

$$x = \frac{1}{2\alpha}$$

$$\delta_y L(x, y, \alpha) = 1 - 2\alpha y = 0$$

$$y = \frac{1}{2\alpha}$$

$$\delta_\alpha L(x, y, \alpha) = x^2 + y^2 - 1 = 0$$

$$\frac{1}{2\alpha}^2 + \frac{1}{2\alpha}^2 - 1 = 0$$

$$\frac{1}{2\alpha^2} = 1$$

$$\alpha = \pm \frac{1}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}}$$

$$y = \frac{1}{\sqrt{2}}$$

$$\begin{aligned} f(x, y) &= x + y \\ &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} & \max_x f(x) \\ & \text{s.t. } g(x) = 0 \end{aligned}$$



Assume  $x$  has  $D$  dimensions, then there will be total  $D+1$  unknowns for the following function,

However, in order to find the solution for the variables, it wont be enough only to take the gradients and set those to zero due to the Inequality Constraints.

Setting  $\Delta_{x,\lambda} L = 0$  still gives two of the conditions, but for the Inequality Constraint, we need to have 3 additional conditions. Hence instead of total 3, we will now have total 5 conditions.

$$\begin{aligned} & \alpha_i g_i(x) = 0, \forall i = 1..n \\ & g_i(x) \leq 0, \forall i = 1..n \\ & \alpha_i \geq 0, \forall i = 1..n \\ & \delta_{x_d} L = 0, \forall d = 1..D \\ & \delta_{\lambda_j} L = 0, \forall j = 1..m \end{aligned}$$

These above five conditions are called as KKT (Karush–Kuhn–Tucker) conditions and they must be met for strong duality, i.e for  $P^* - D^* = 0$  to be true.

If you have  $n$  constraints then there will be total  $D+n$  unknowns.

$$\begin{aligned} & \max_x f(x) \\ & \text{s.t. } g_1(x) = 0, g_2(x) = 0, \dots, g_n(x) = 0 \end{aligned}$$

We can then define the **Lagrangian** as following,

$$L(x_1, \dots, x_D, \alpha_1, \dots, \alpha_n) = f(x) - \sum_{i=1}^n \alpha_i g_i(x)$$



Inequality Constraint

$$\begin{aligned} & \max_x f(x) \\ & \text{s.t. } g_i(x) \leq 0, \forall i = 1..n \\ & h_i(x) = 0, \forall j = 1..m \end{aligned}$$



We can define the **Lagrangian** as (One constant for each constraint),

$$L(x, \alpha, \lambda) = f(x) + \sum_{i=1}^n \alpha_i g_i(x) + \sum_{j=1}^m \lambda_j h_j(x)$$

# Duality: Hard Margin Classifier

Change *Hard Margin Classifier's Objective Function* from *Primal Problem* to *Dual Problem* using Lagrange Multiplier and KKT Conditions.

$$\text{Objective Function : } \min_{\beta, b} \left\{ \frac{\|\beta^2\|}{2} \right\}$$

$$\text{s.t Linear Constraint : } y_i(\beta^T x_i + b) \geq 1, \forall x_i \in D$$

The constraint can be redefined as following (This is required for the 2<sup>nd</sup> KKT

$$\text{Condition } g_i(x) \leq 0,$$

$$\text{s.t Linear Constraint : } 1 - y_i(\beta^T x_i + b) \leq 0, \forall x_i \in D$$

KKT (Karush–Kuhn–Tucker)  
conditions

$$\begin{aligned} \alpha_i g_i(x) &= 0, \forall i = 1..n \\ g_i(x) &\leq 0, \forall i = 1..n \\ \alpha_i &\geq 0, \forall i = 1..n \\ \delta_{x_d} L &= 0, \forall d = 1..D \\ \delta_{\lambda_j} L &= 0, \forall j = 1..m \end{aligned}$$

The **Lagrangian** can be defined as following,

$$\begin{aligned} \min L &= \frac{\|\beta^2\|}{2} + \sum_{i=1}^n \alpha_i (1 - y_i(\beta^T x_i + b)) \\ &= \frac{\|\beta^2\|}{2} - \sum_{i=1}^n \alpha_i (y_i(\beta^T x_i + b) - 1) \end{aligned}$$

$$\begin{aligned} \delta_\beta L &= \beta - \sum_{i=1}^n \alpha_i y_i x_i = 0 \\ \beta &= \sum_{i=1}^n \alpha_i y_i x_i \\ \text{and } \delta_b L &= \sum_{i=1}^n \alpha_i y_i = 0 \end{aligned}$$

$$\begin{aligned} \alpha_i (1 - y_i(\beta^T x_i + b)) &= 0 \\ \text{and } \alpha_i &\geq 0 \\ 1 - y_i(\beta^T x_i + b) &\leq 0, \forall x_i \in D \end{aligned}$$

# Duality: Hard Margin Classifier

Change Hard Margin Classifier's Objective Function from Primal Problem to Dual Problem using Lagrange Multiplier and KKT Conditions.

The **Lagrangian** can be defined as following,

$$\begin{aligned}\min L &= \frac{\|\beta^2\|}{2} + \sum_{i=1}^n \alpha_i(1 - y_i(\beta^T x_i + b)) \\ &= \frac{\|\beta^2\|}{2} - \sum_{i=1}^n \alpha_i(y_i(\beta^T x_i + b) - 1)\end{aligned}$$

$$\delta_\beta L = \beta - \sum_{i=1}^n \alpha_i y_i x_i = 0$$

$$\beta = \sum_{i=1}^n \alpha_i y_i x_i$$

$$\text{and } \delta_b L = \sum_{i=1}^n \alpha_i y_i = 0$$

$$\begin{aligned}\alpha_i(1 - y_i(\beta^T x_i + b)) &= 0 \\ \text{and } \alpha_i &\geq 0 \\ 1 - y_i(\beta^T x_i + b) &\leq 0, \forall x_i \in D\end{aligned}$$



Dual Lagrangian Objective Function

$$\begin{aligned}L_{dual} &= \frac{\|\beta^2\|}{2} - \sum_{i=1}^n \alpha_i(y_i(\beta^T x_i + b) - 1) \\ &= \frac{1}{2} \beta^T \beta - \sum_{i=1}^n \alpha_i y_i \beta^T x_i - \sum_{i=1}^n \alpha_i y_i b + \sum_{i=1}^n \alpha_i \\ &= \frac{1}{2} \beta^T \beta - \beta^T \left( \sum_{i=1}^n \alpha_i y_i x_i \right) - b \left( \sum_{i=1}^n \alpha_i y_i \right) + \sum_{i=1}^n \alpha_i \\ &= \frac{1}{2} \beta^T \beta - \beta^T (\beta) - b(0) + \sum_{i=1}^n \alpha_i \\ &= -\frac{1}{2} \beta^T \beta + \sum_{i=1}^n \alpha_i \\ &= \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j x_i^T x_j\end{aligned}$$

# Duality: Hard Margin Classifier

$$\begin{aligned}
 L_{dual} &= \frac{\|\beta^2\|}{2} - \sum_{i=1}^n \alpha_i(y_i(\beta^T x_i + b) - 1) \\
 &= \frac{1}{2} \beta^T \beta - \sum_{i=1}^n \alpha_i y_i \beta^T x_i - \sum_{i=1}^n \alpha_i y_i b + \sum_{i=1}^n \alpha_i \\
 &= \frac{1}{2} \beta^T \beta - \beta^T \left( \sum_{i=1}^n \alpha_i y_i x_i \right) - b \left( \sum_{i=1}^n \alpha_i y_i \right) + \sum_{i=1}^n \alpha_i \\
 &= \frac{1}{2} \beta^T \beta - \beta^T (\beta) - b(0) + \sum_{i=1}^n \alpha_i \\
 &= -\frac{1}{2} \beta^T \beta + \sum_{i=1}^n \alpha_i \\
 &= \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j x_i^T x_j
 \end{aligned}$$

We can now find the optimal Hyperplane given  $\beta$  and  $b$ . For any new point  $\mathbf{z}$ , we can predict the class using following,

$$\hat{y} = \text{sign}(\beta^T z + b)$$

$L$  should be minimized w.r.t  $\beta$  and  $b$ , and should be maximized w.r.t  $\alpha_i$ . Hence, instead of **minimizing** the **Primal Problem**, we can now **maximize** the **Dual Problem** ( $P^*=D^*$  as the KKT conditions are now satisfied). So we can write the following,

**Objective Function:**  $\max_{\alpha} L_{dual} = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j x_i^T x_j$

**Linear Constraints:**  $\alpha_i \geq 0, \forall i \in D$ , and  $\sum_{i=1}^n \alpha_i y_i = 0$

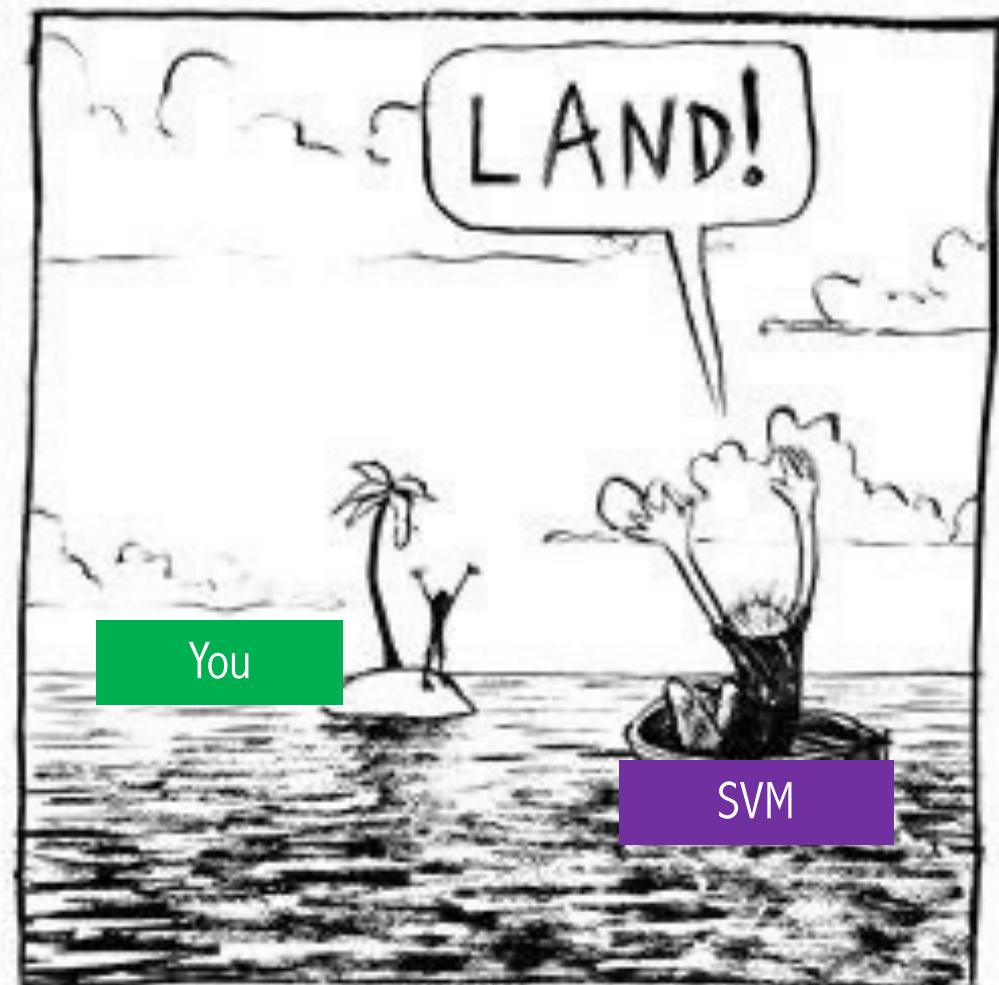
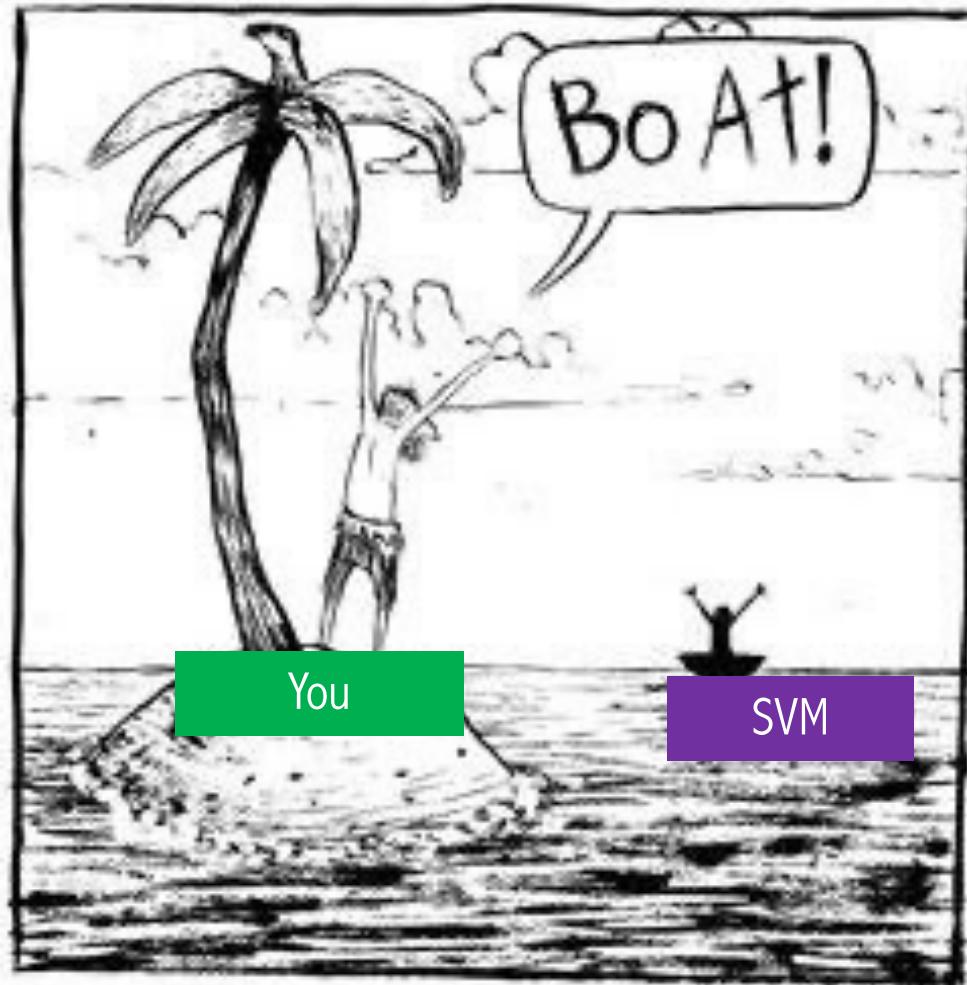


Once we have got the  $\alpha_i$  values, we can calculate the weight vector  $\beta$  and bias  $b$ . As per the KKT conditions we had,

$$\alpha_i(1 - y_i(\beta^T x_i + b)) = 0$$

$$\beta = \sum_{i=1}^n \alpha_i y_i x_i$$

# Another View of the SVM



# Linear Regression

Develop a program to predict the price of the house based on the area of the house. Can we use Linear Regression to solve this problem?

Hypothesis:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

$$h_{\theta}(x) = [-\infty, +\infty]$$

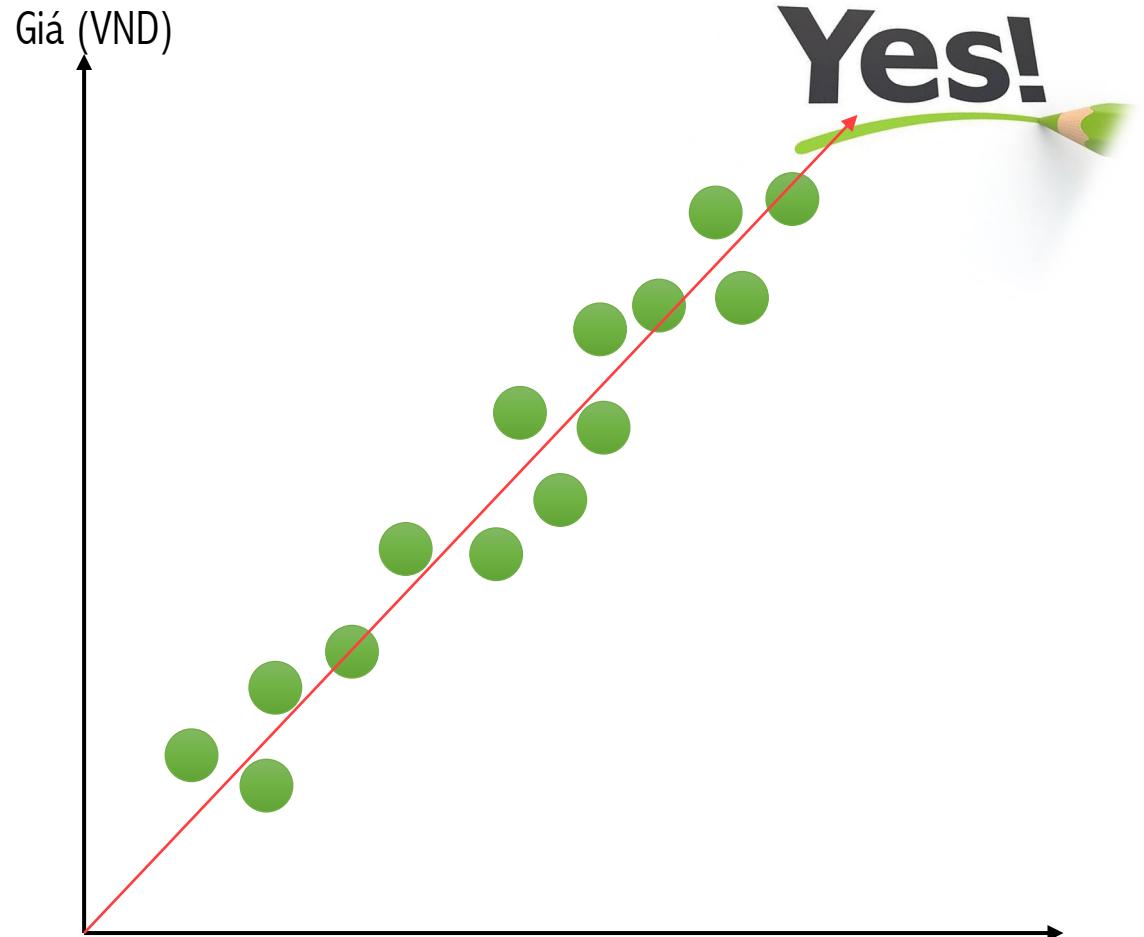
Parameters:

$$\theta_0, \theta_1$$

Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

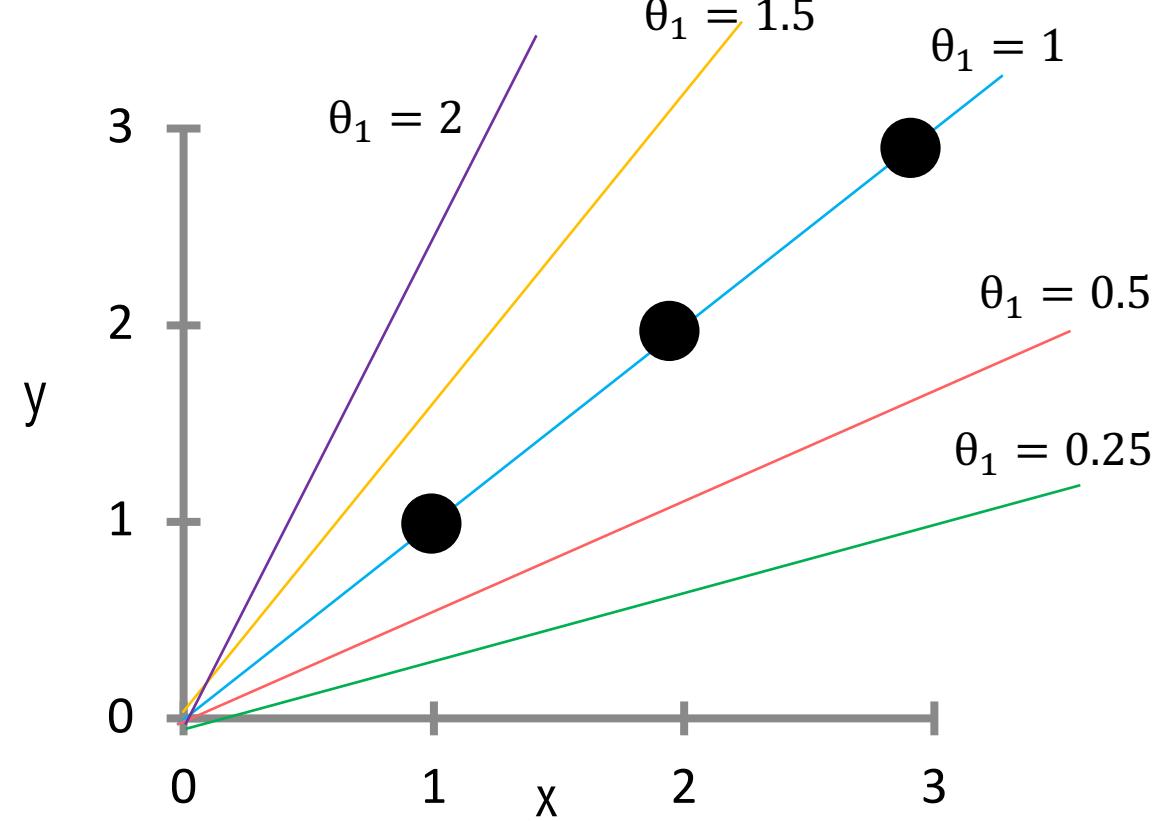
Goal: minimize  $J(\theta_0, \theta_1)$



# Linear Regression: Cost Function

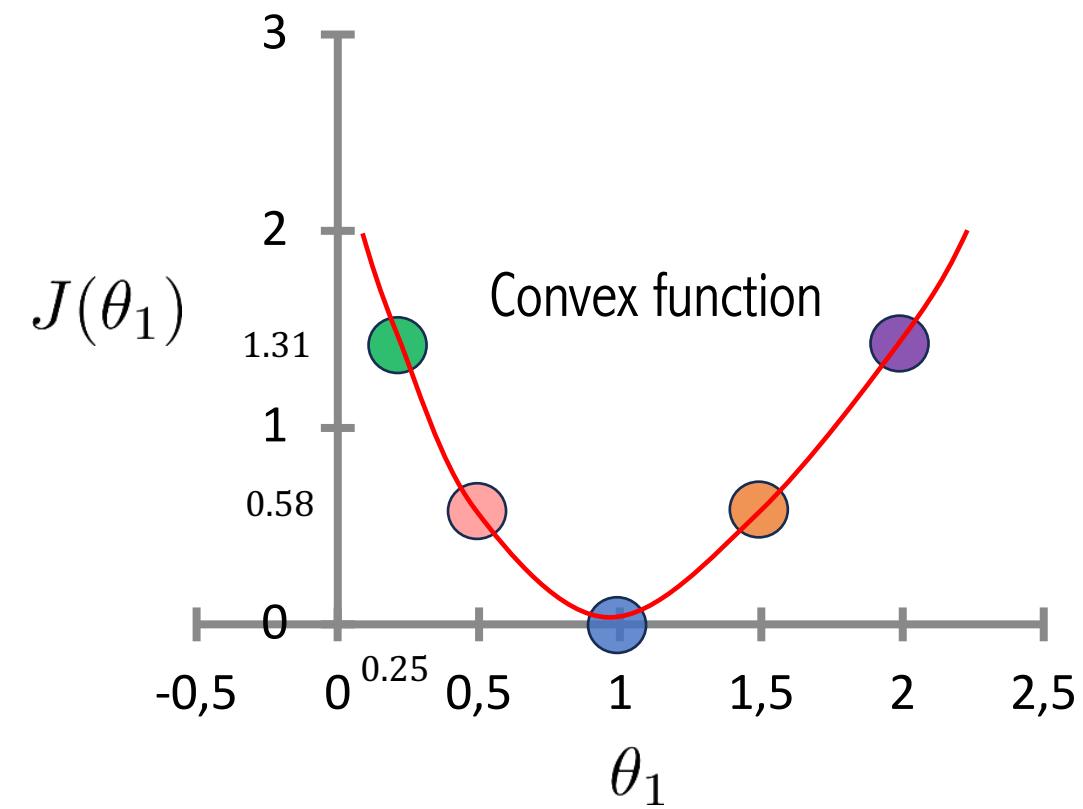
$$h_{\theta}(x) = \theta_1 x$$

(for fixed  $\theta_1$ , this is a function of  $x$ )



$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

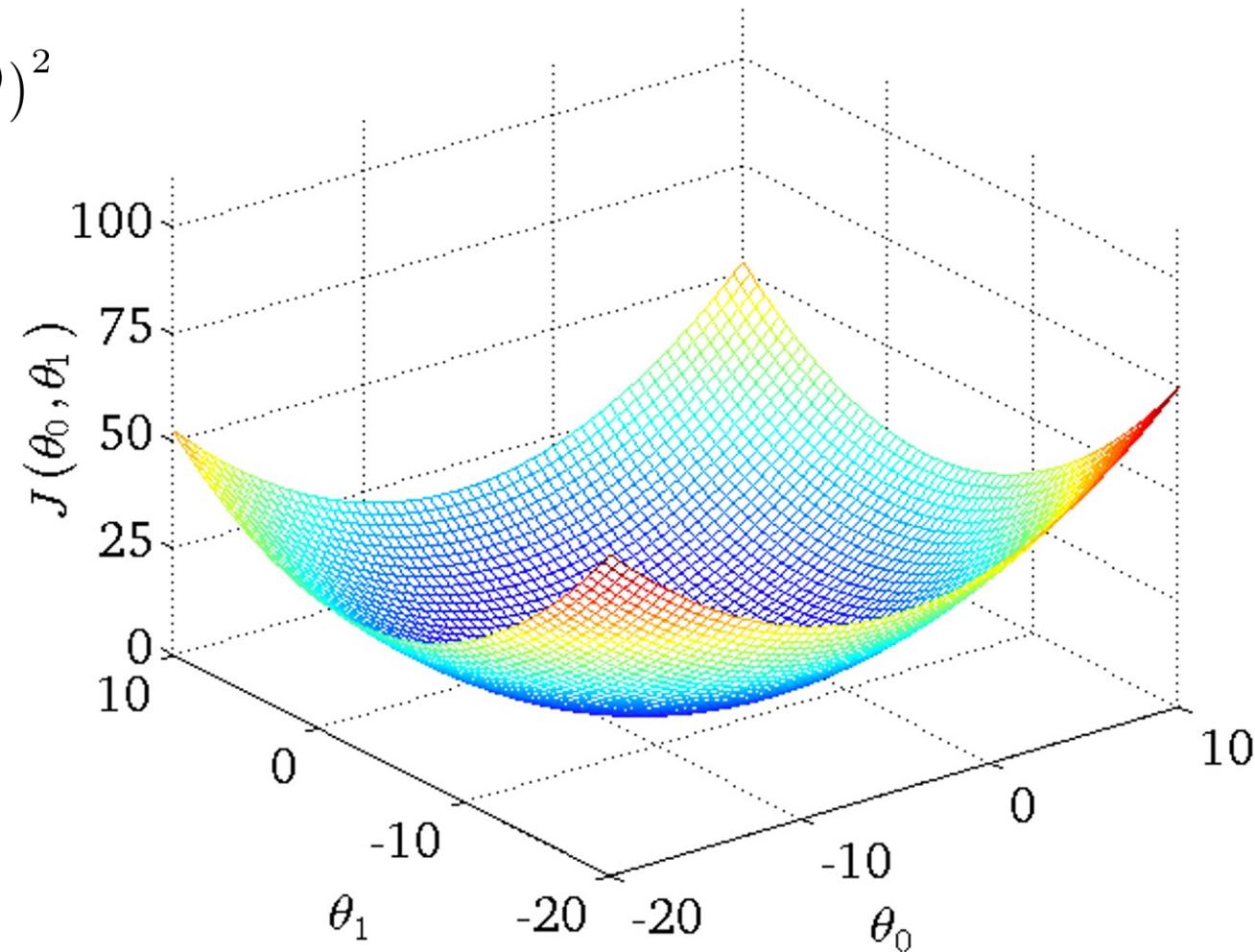
(function of the parameter  $\theta_1$ )



# Linear Regression: Cost Function

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$



# Linear Regression: Cost Function

Hypothesis

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

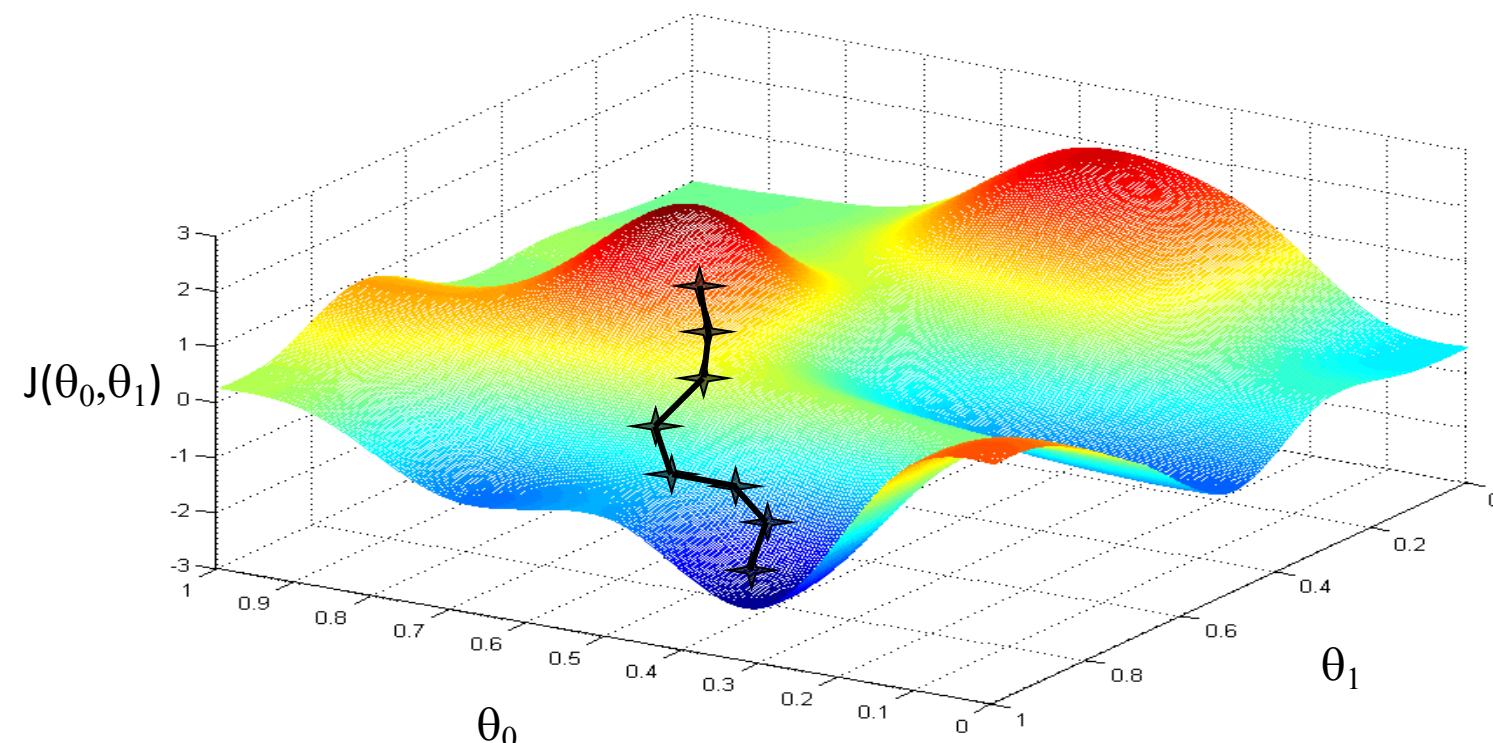
Gradient Descent

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x^{(i)}$$

}



# Linear Regression: Cost Function

Hypothesis

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

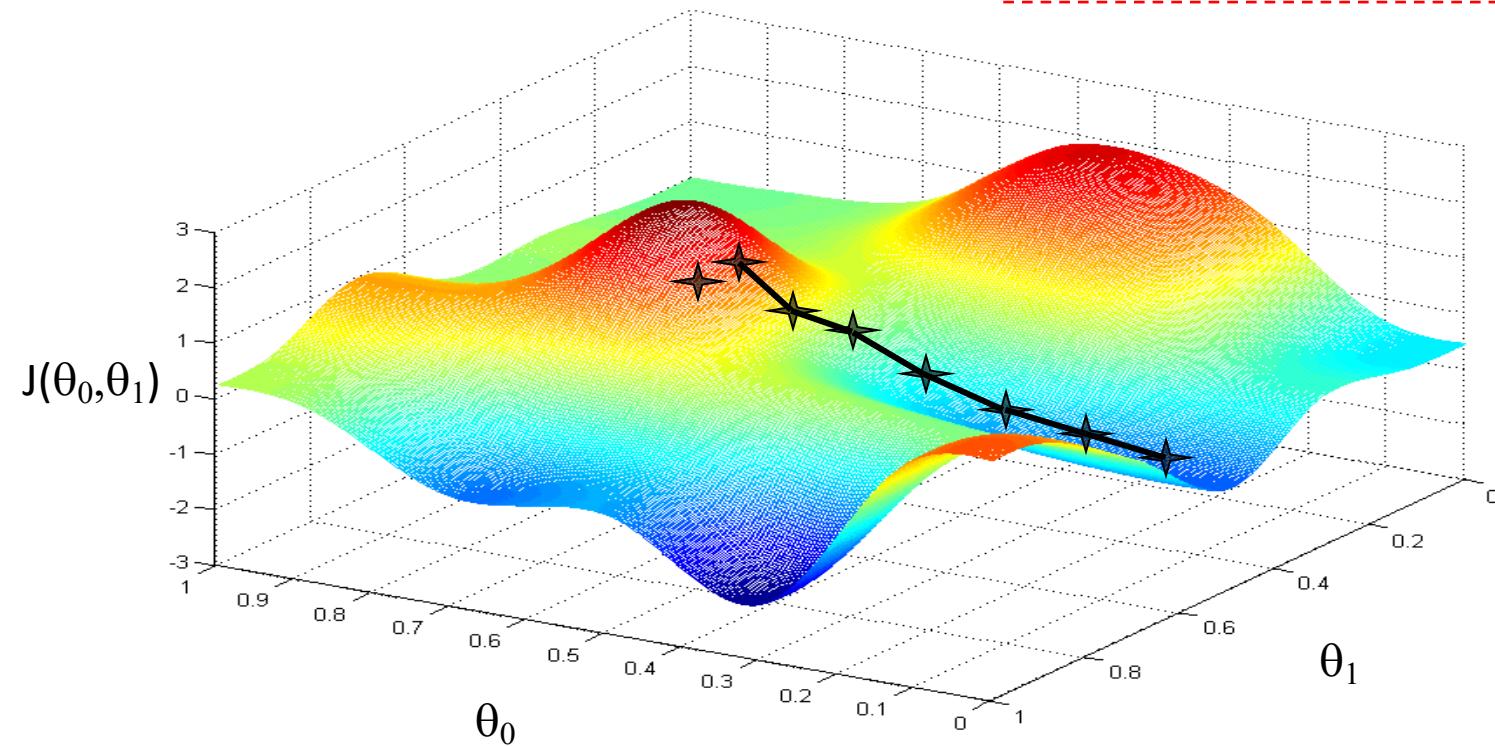
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repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x^{(i)}$$

}



# Linear Regression for Classification

Develop a program to classify whether a tumor is malignant or not based on its size. Can we use Linear Regression to solve this problem?

Hypothesis:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

$$h_{\theta}(x) = [-\infty, +\infty]$$

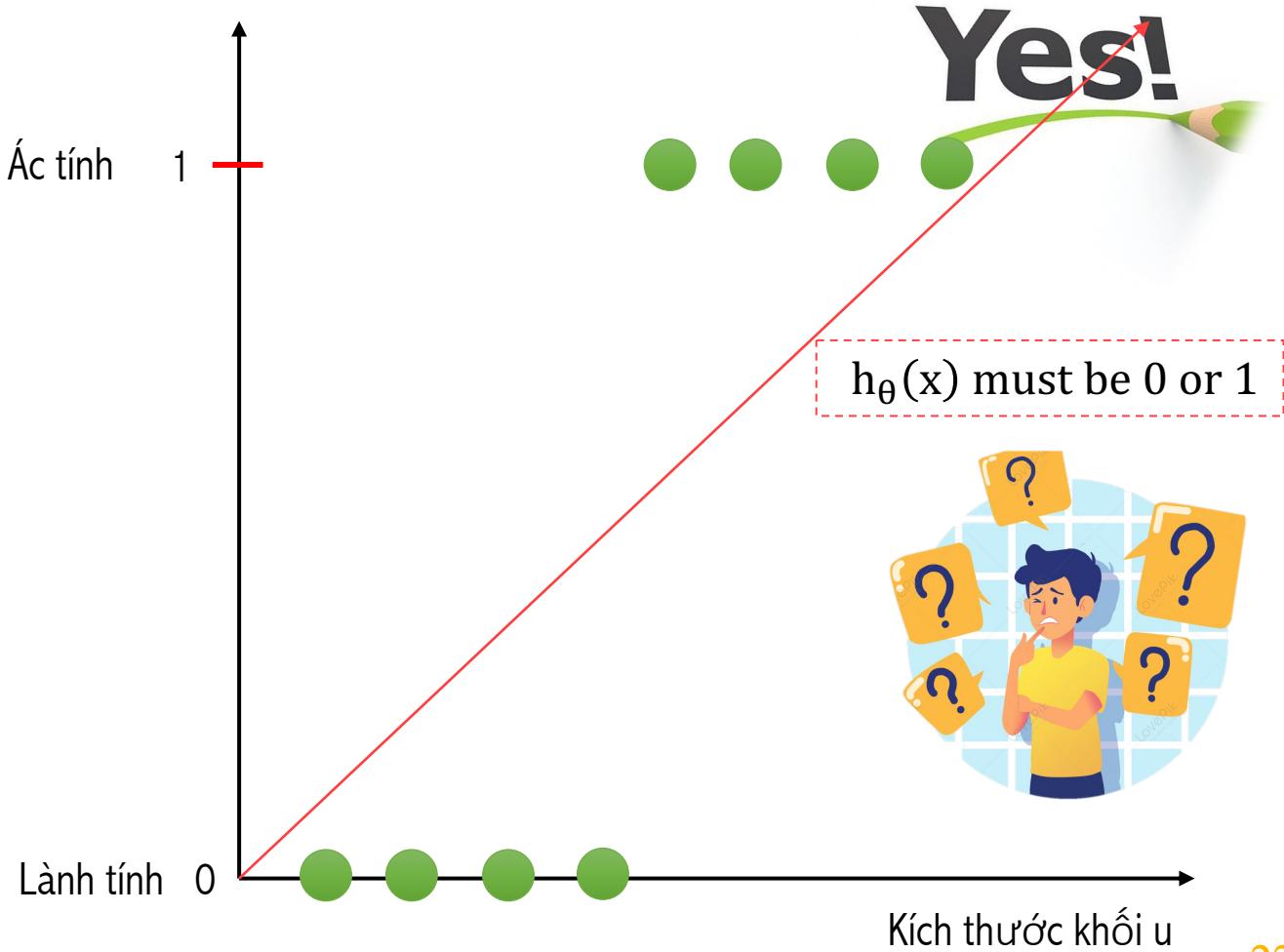
Parameters:

$$\theta_0, \theta_1$$

Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Goal: minimize  $J(\theta_0, \theta_1)$



# Linear Regression for Classification

Develop a program to classify whether a tumor is malignant or not based on its size. Can we use Linear Regression to solve this problem?

Hypothesis:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

$$h_{\theta}(x) = [-\infty, +\infty]$$

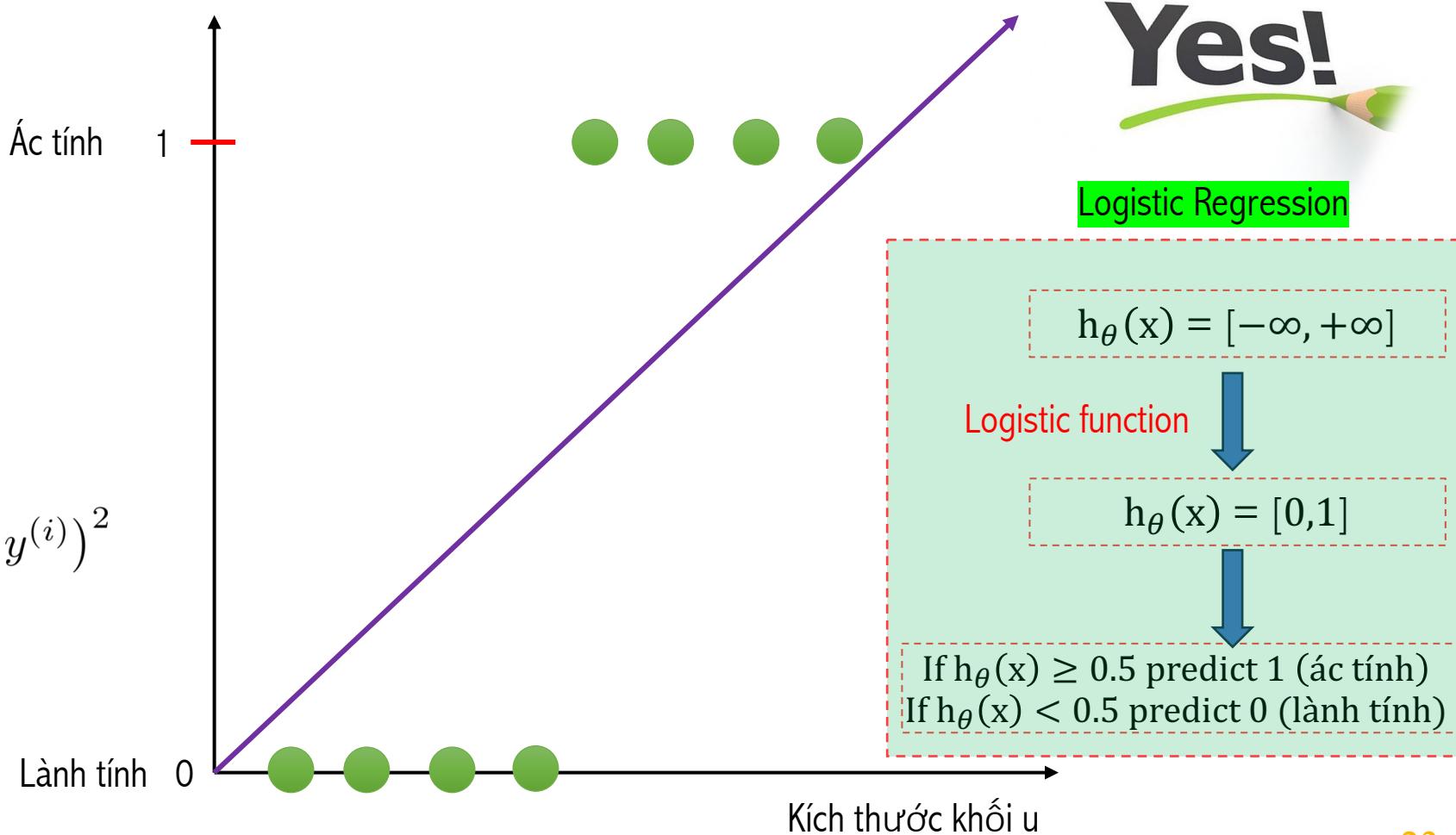
Parameters:

$$\theta_0, \theta_1$$

Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Goal: minimize  $J(\theta_0, \theta_1)$



# Logistic Regression for Classification

Hypothesis:

$$h_{\theta}(x) = g(\theta^T x)$$

$$g(z) = \frac{1}{1+e^{-z}}$$

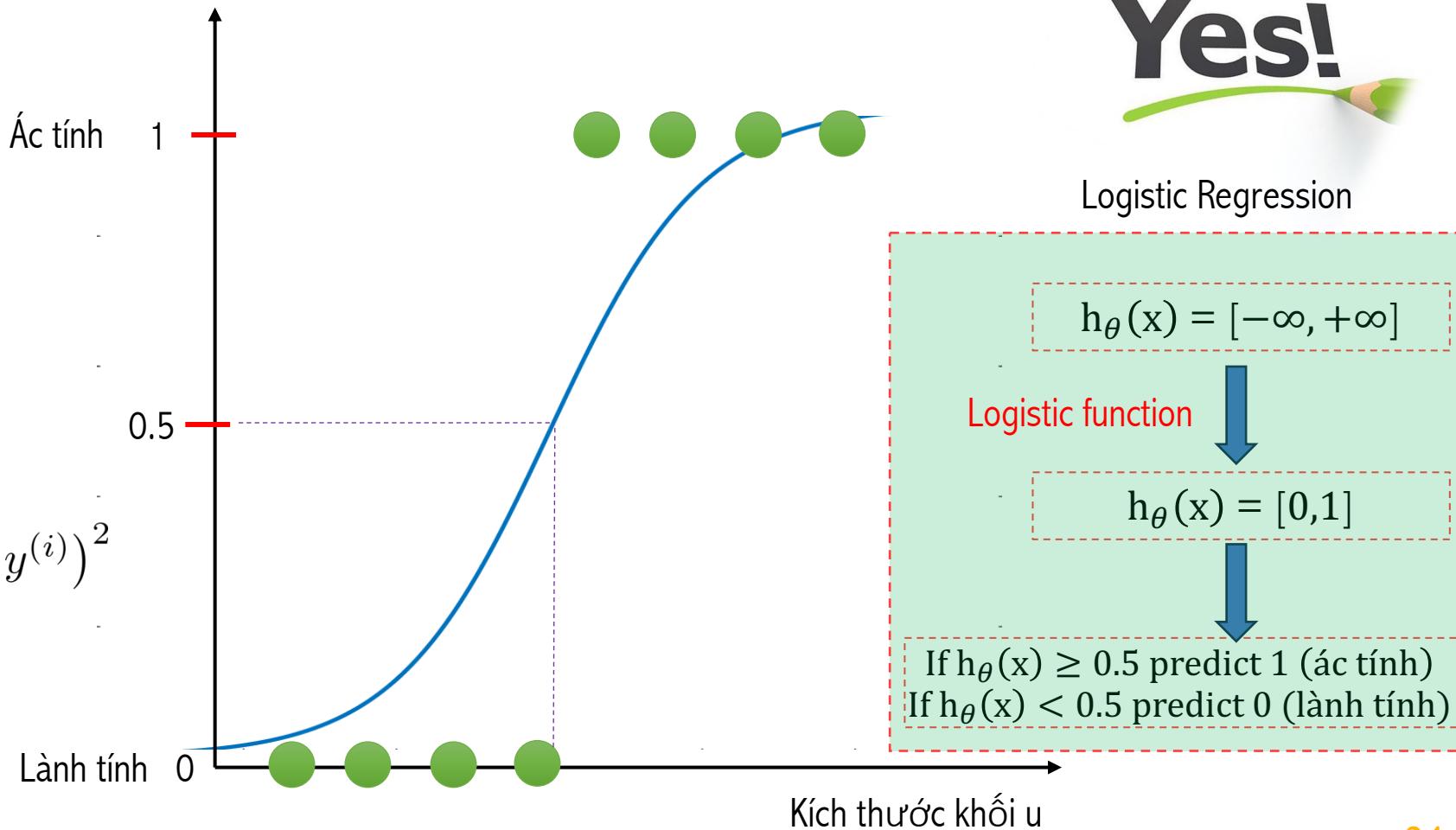
Parameters:

$$\theta_0, \theta_1$$

Cost Function:

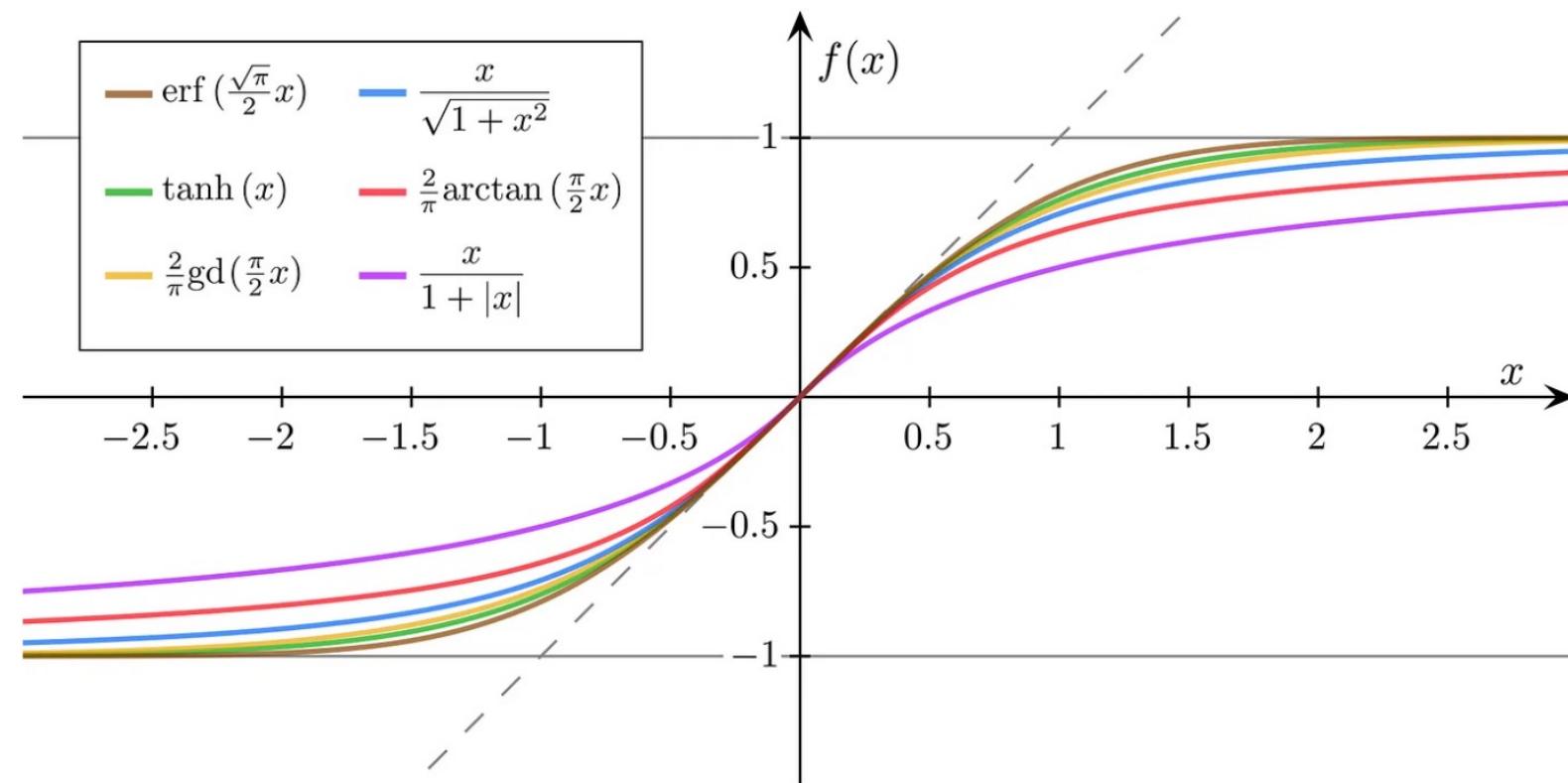
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Goal: minimize  $J(\theta_0, \theta_1)$



# Sigmoid Function

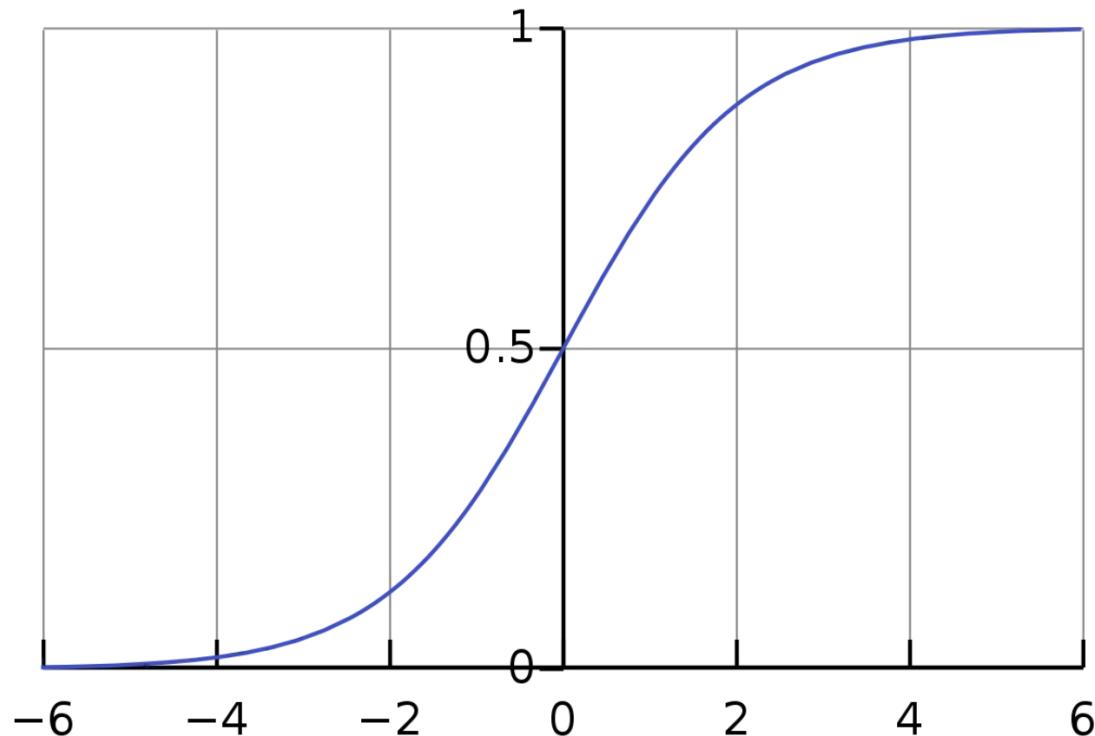
Sigmoid functions are general mathematical functions that share similar properties: have S-shaped curves, just as the figure below shows.



Members of Sigmoid Functions Family, from [Wikipedia](#)

# Logistic Function

The most common sigmoid function used in machine learning is Logistic Function, as the formula below.



$$\sigma(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{e^x + 1} = 1 - \sigma(-x).$$

The Curve of a Logistic Function, from Wikipedia

# Logistic Regression

Hypothesis:

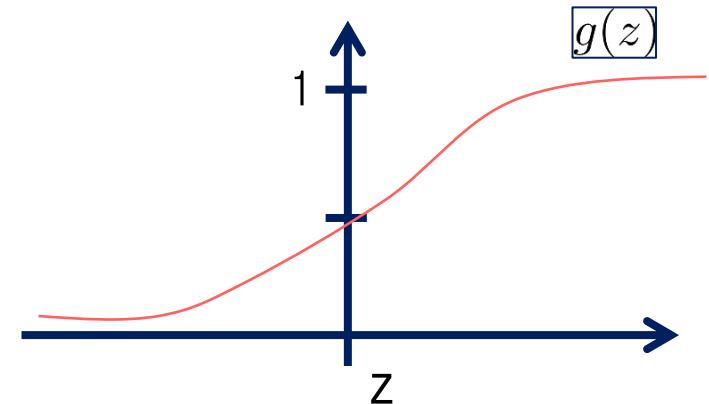
$$h_{\theta}(x) = g(\theta^T x)$$

$$g(z) = \frac{1}{1+e^{-z}}$$

$$Z = \theta^T x \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \quad x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}$$

predict “ $y = 1$ “ if  $h_{\theta}(x) \geq 0.5 \Rightarrow g(z) \geq 0.5$  when  $Z = \theta^T x \geq 0$

predict “ $y = 0$ “ if  $h_{\theta}(x) < 0.5 \Rightarrow g(z) < 0.5$  when  $Z = \theta^T x < 0$



# Cost Function

## Linear Regression

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

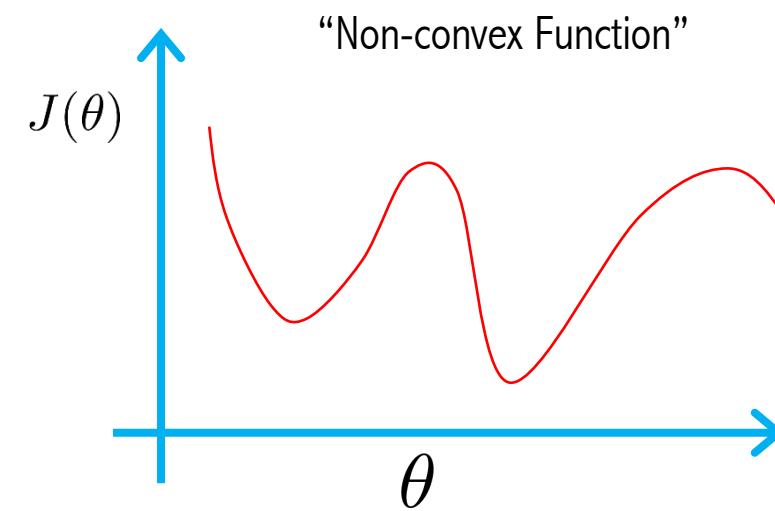
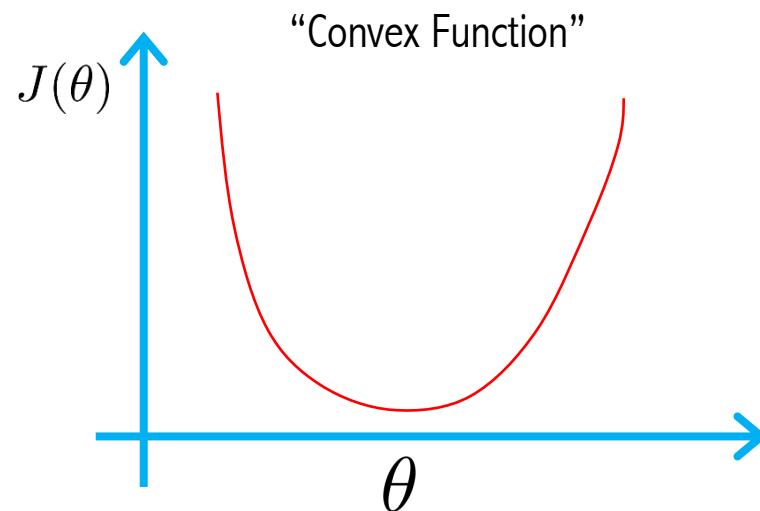
$$\text{Cost}(h_\theta(x^{(i)}), y^{(i)}) = \frac{1}{2} (h_\theta(x^{(i)}) - y^{(i)})^2$$

## Logistic Regression

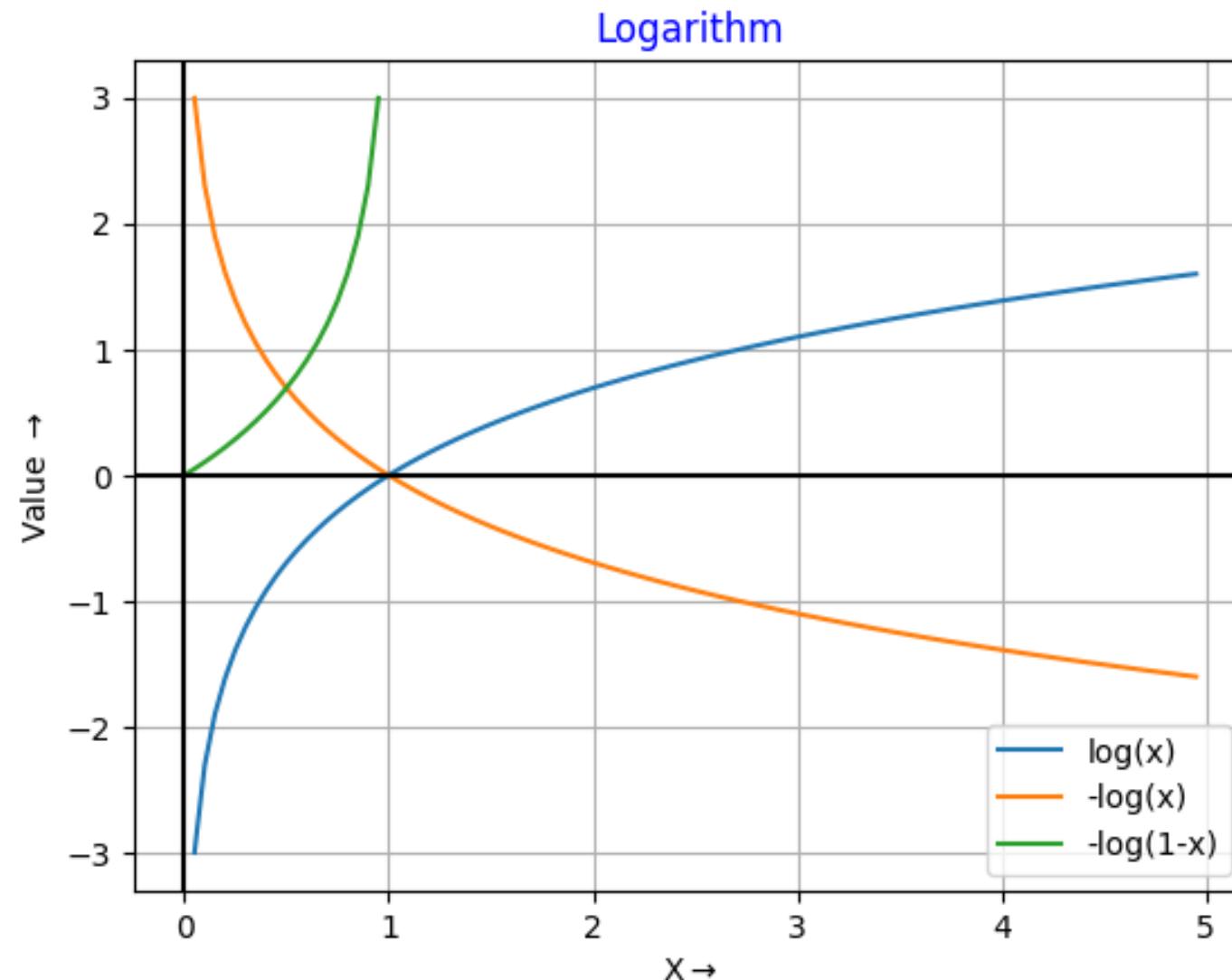
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

$$h_\theta(x) = g(\theta^T x) \quad g(z) = \frac{1}{1+e^{-z}}$$

$$\text{Cost}(h_\theta(x^{(i)}), y^{(i)}) = \frac{1}{2} (h_\theta(x^{(i)}) - y^{(i)})^2$$



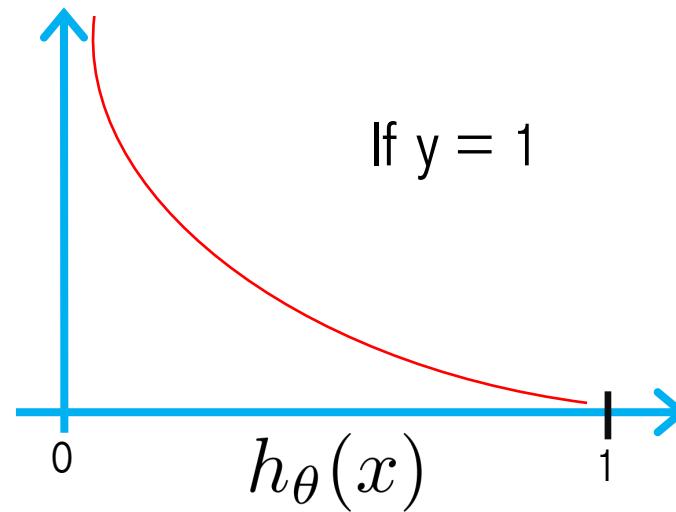
# Logarithm Function



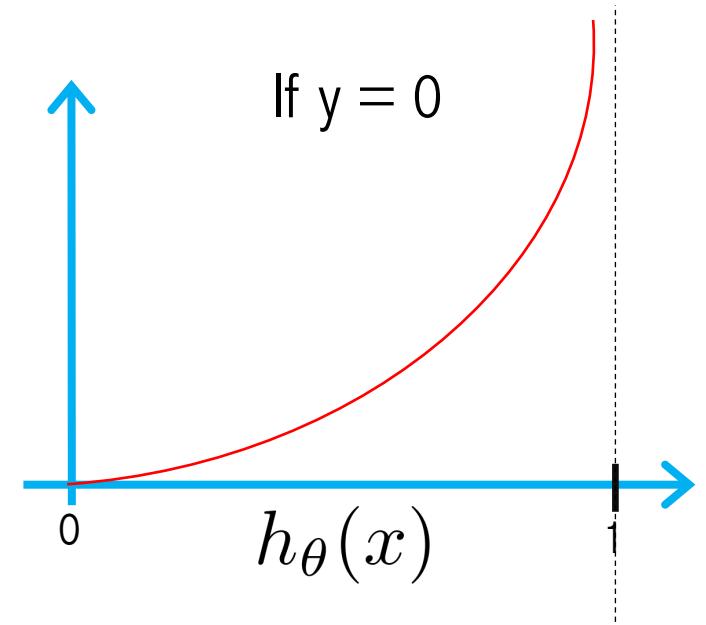
# Logistic Regression: Cost Function

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Cost = 0 if  $y = 1, h_\theta(x) = 1$



Cost = 0 if  $y = 0, h_\theta(x) = 0$



# Logistic Regression: Cost Function

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



$$\text{Cost}(h_\theta(x), y) = -y\log(h_\theta(x)) - (1 - y)\log(1 - h_\theta(x))$$

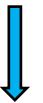
$$\text{If } y = 1: \text{Cost}(h_\theta(x), y) = -\log(h_\theta(x))$$

$$\text{If } y = 0: \text{Cost}(h_\theta(x), y) = -\log(1 - h_\theta(x))$$

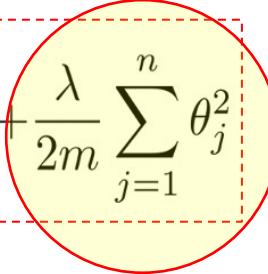


$$\begin{aligned} J(\theta) &= \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_\theta(x^{(i)}), y^{(i)}) \\ &= -\frac{1}{m} \left[ \sum_{i=1}^m y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)})) \right] \end{aligned}$$

Prevent Overfitting



$$\min_{\theta} \frac{1}{m} \left[ \sum_{i=1}^m y^{(i)} \left( -\log h_\theta(x^{(i)}) \right) + (1 - y^{(i)}) \left( -\log(1 - h_\theta(x^{(i)})) \right) \right] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$



# Behind The Scene

$$J(\theta) = -\frac{1}{m} \left[ \sum_{i=1}^m y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)})) \right]$$

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

$$\sigma = \frac{1}{(1+e^{-x})}$$

$$\sigma' = \sigma(x) \times (1 - \sigma(x))$$

$$z = \theta^T x$$

For  $y = \ln(u)$

$$\frac{dy}{dx} = \frac{1}{u} \frac{du}{dx}$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = -\frac{1}{m} \sum_{i=1}^m \left[ y^{(i)} \times \frac{1}{h_\theta(x^{(i)})} \times \frac{\partial J(h_\theta(x^{(i)}))}{\partial \theta_j} \right] + \frac{1}{m} \sum_{i=1}^m \left[ (1 - y^{(i)}) \times \frac{1}{(1 - h_\theta(x^{(i)}))} \times \frac{\partial J(1 - h_\theta(x^{(i)}))}{\partial \theta_j} \right]$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = -\frac{1}{m} \sum_{i=1}^m \left[ y^{(i)} \times \frac{1}{h_\theta(x^{(i)})} \times (\sigma(z)(1 - \sigma(z)) \times \frac{\partial J(z)}{\partial \theta_j}) \right] + \frac{1}{m} \sum_{i=1}^m \left[ (1 - y^{(i)}) \times \frac{1}{(1 - h_\theta(x^{(i)}))} \times (-\sigma(z)(1 - \sigma(z)) \times \frac{\partial J(z)}{\partial \theta_j}) \right]$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = -\frac{1}{m} \sum_{i=1}^m \left[ y^{(i)} \times \frac{1}{h_\theta(x^{(i)})} \times (h_\theta(x^{(i)})(1 - h_\theta(x^{(i)})) \times x_j^i) \right] + \frac{1}{m} \sum_{i=1}^m \left[ (1 - y^{(i)}) \times \frac{1}{(1 - h_\theta(x^{(i)}))} \times (-h_\theta(x^{(i)})(1 - h_\theta(x^{(i)})) \times x_j^i) \right]$$

# Behind The Scene

$$J(\theta) = -\frac{1}{m} \left[ \sum_{i=1}^m y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)})) \right]$$

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

$$\sigma = \frac{1}{(1+e^{-x})}$$

$$\sigma' = \sigma(x) \times (1 - \sigma(x))$$

$$z = \theta^T x$$

For  $y = \ln(u)$

$$\frac{dy}{dx} = \frac{1}{u} \frac{du}{dx}$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = -\frac{1}{m} \sum_{i=1}^m [y^{(i)} \times (1 - h_\theta(x^{(i)})) \times x_j^i - (1 - y^{(i)}) \times h_\theta(x^{(i)}) \times x_j^i]$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = -\frac{1}{m} \sum_{i=1}^m [y^{(i)} - y^{(i)} \times h_\theta(x^{(i)}) - h_\theta(x^{(i)}) + y^{(i)} \times h_\theta(x^{(i)})] \times x_j^i$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = -\frac{1}{m} \sum_{i=1}^m [y^{(i)} - h_\theta(x^{(i)})] \times x_j^i$$

The gradient descent is the same as Linear Regression

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

$$\sigma = \frac{1}{(1+e^{-x})}$$

# Outline

- Linear Regression to Logistic Regression
- Logistic Regression to Support Vector Machine
- Example

# Another View of Logistic Regression

Hypothesis:

$$h_{\theta}(x) = g(\theta^T x)$$

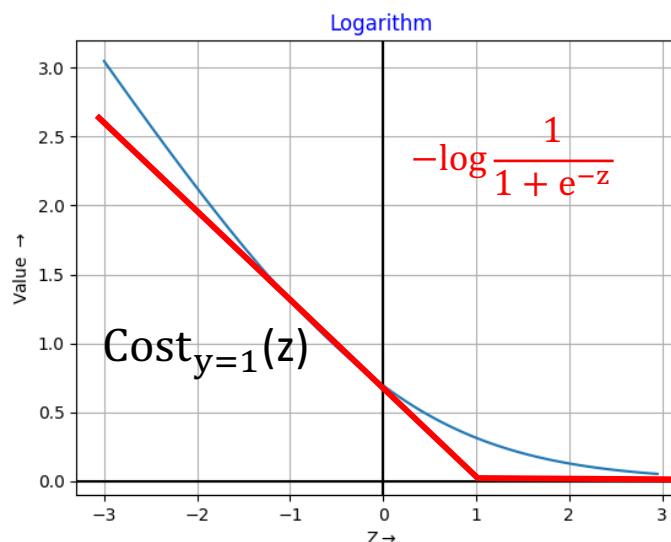
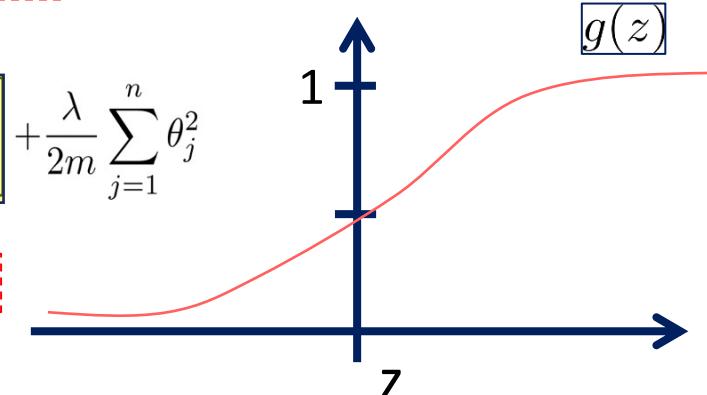
$$g(z) = \frac{1}{1+e^{-z}}$$

$$Z = \theta^T x$$

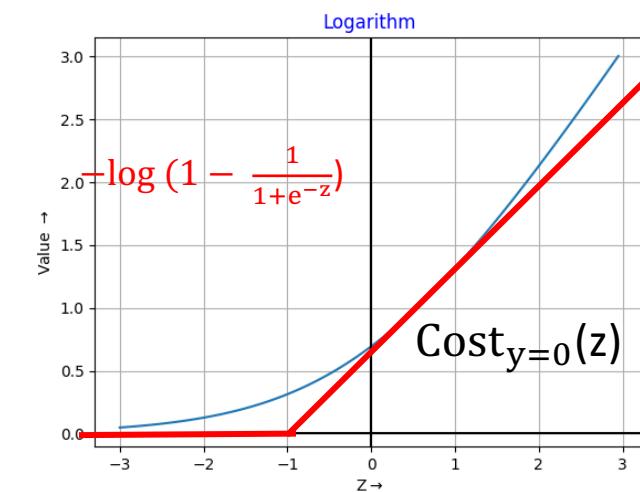
$$\text{Cost}_{y=1}(\theta^T x^i)$$

$$\text{Cost}_{y=0}(\theta^T x^i)$$

$$\text{Cost}(h_{\theta}(x), y) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x))$$



If  $y = 1$ , we want  $h_{\theta}(x) \approx 1$ ,  $Z = \theta^T x \gg 0$



If  $y = 0$ , we want  $h_{\theta}(x) \approx 0$ ,  $Z = \theta^T x \ll 0$

# Logistic Regression to Support Vector Machine

## Logistic Regression

$$\min_{\theta} \frac{1}{m} \left[ \sum_{i=1}^m y^{(i)} \left( -\log h_{\theta}(x^{(i)}) \right) + (1 - y^{(i)}) \left( -\log(1 - h_{\theta}(x^{(i)})) \right) \right] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

Cost<sub>y=1</sub>( $\theta^T x^i$ )

Sắp xỉ

Cost<sub>y=0</sub>( $\theta^T x^i$ )

Sắp xỉ

A

B

$$\min (A + \lambda B)$$

## Support Vector Machine

$$\min_{\theta} C \sum_{i=1}^m -y_i \text{Cost}_{y=1}(\theta^T x^i) + (1 - y) \text{Cost}_{y=0}(\theta^T x^i) + \frac{1}{2} \sum_{j=1}^n \theta_j^2$$

$$\min (CA + B)$$

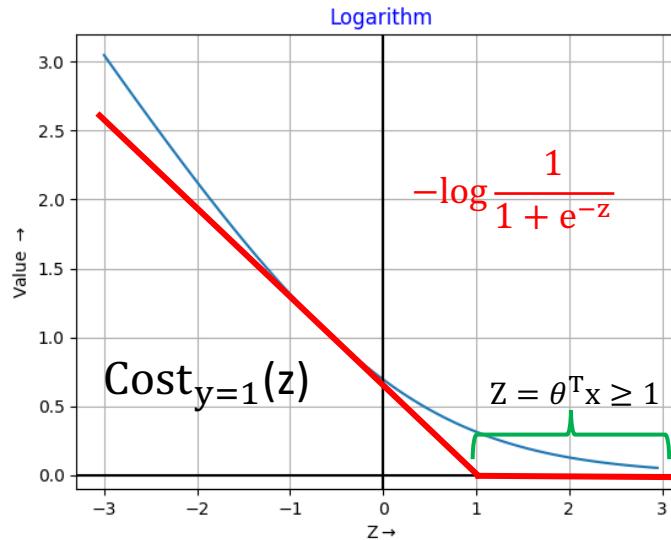
$$C = 1 / \lambda$$

A

B

# Support Vector Machine

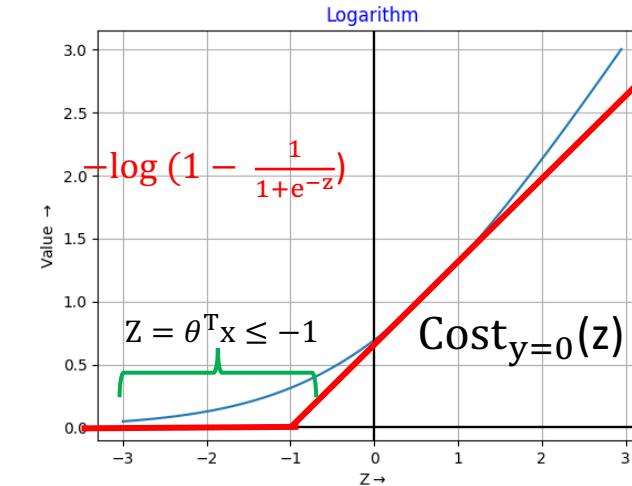
$$\min_{\theta} C \sum_{i=1}^m -y_i \text{Cost}_{y=1}(\theta^T x^i) + (1-y) \text{Cost}_{y=0}(\theta^T x^i) + \frac{1}{2} \sum_{j=1}^m \theta_j^2$$



Logistic Regression  
If  $y = 1$ , we want  $h_\theta(x) \approx 1, Z = \theta^T x > 0$

Support Vector Machine  
If  $y = 1$ , we want  $h_\theta(x) \approx 1, Z = \theta^T x \geq 1$

Supposing that  $C$  is very very large (100,000)  
Does it affect to the optimization?



Logistic Regression  
If  $y = 1$ , we want  $h_\theta(x) \approx 1, Z = \theta^T x < 0$

Support Vector Machine  
If  $y = 1$ , we want  $h_\theta(x) \approx 1, Z = \theta^T x \leq -1$

# SVM Decision Boundary

Supposing that C is very very large (100,000)

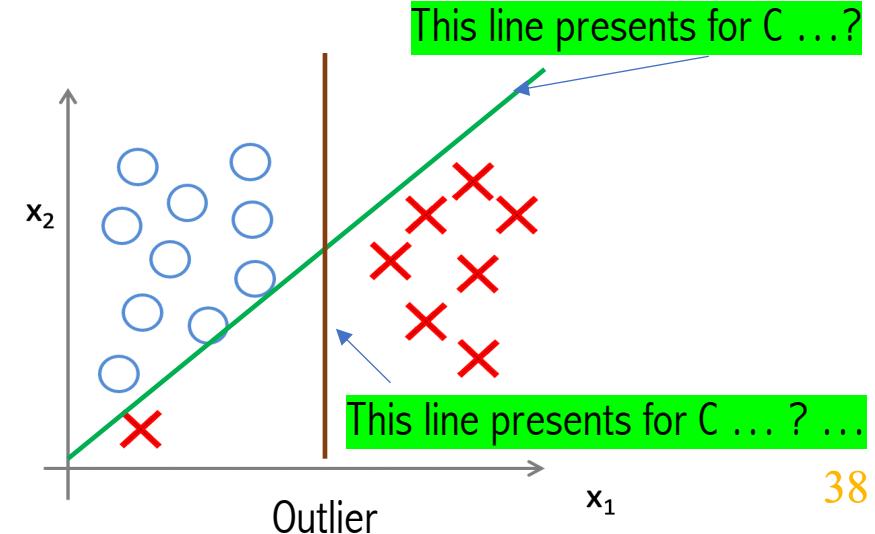
This term should be ....

$$\min_C \sum_{i=1}^m -y_i \text{Cost}_{y=1}(\theta^T x^i) + (1 - y) \text{Cost}_{y=0}(\theta^T x^i) + \frac{1}{2} \sum_{j=1}^m \theta_j^2$$

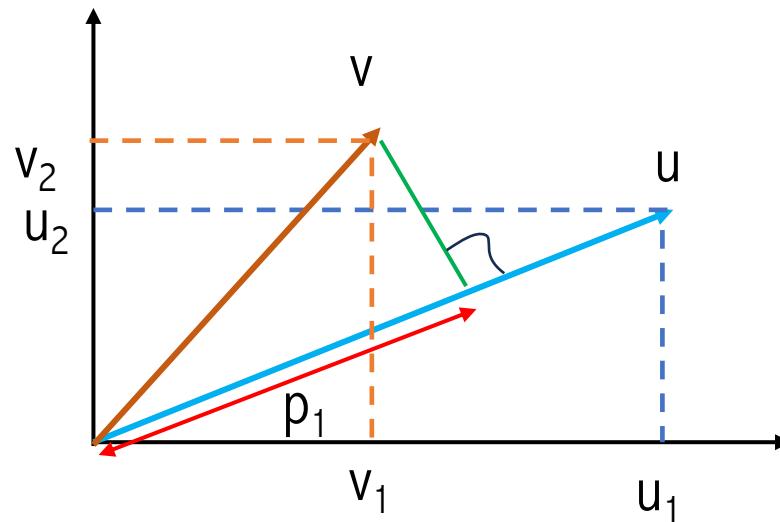
3

$$\min_C \frac{1}{2} \sum_{j=1}^m \theta_j^2$$

Linear separable case



# Vector Inner Product



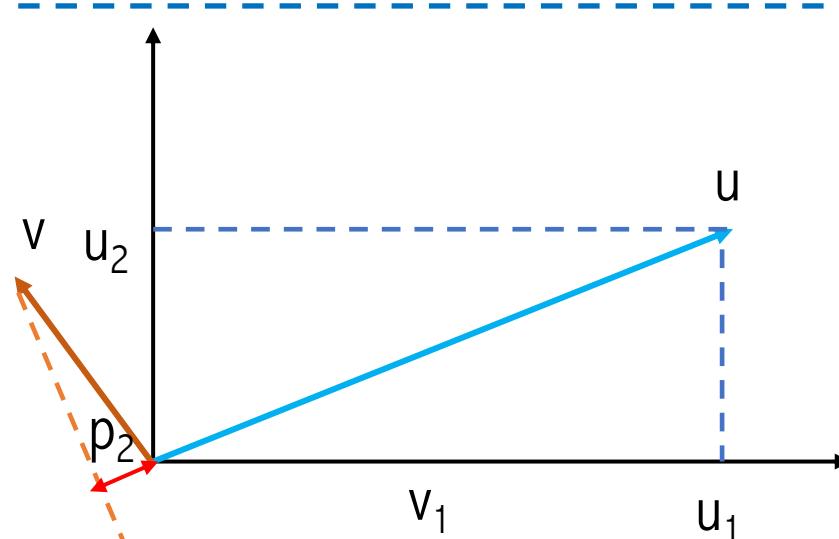
$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Length of vector  $u$

$$\|u\| = \sqrt{u_1^2 + u_2^2}$$

$$u^T v = p_1 \cdot \|u\| = u_1 v_1 + u_2 v_2$$

$p_1$ : Length of projection of vector  $v$  on vector  $u$



$$u^T v = p_2 \cdot \|u\| = u_1 v_1 + u_2 v_2$$

What are the value of  $p$  in two cases?

- a.  $p_1 > 0 \text{ & } p_2 < 0$
- b.  $p_1 < 0 \text{ & } p_2 > 0$
- c.  $p_1 > 0 \text{ & } p_2 > 0$
- d.  $p_1 < 0 \text{ & } p_2 < 0$

# SVM Decision Boundary

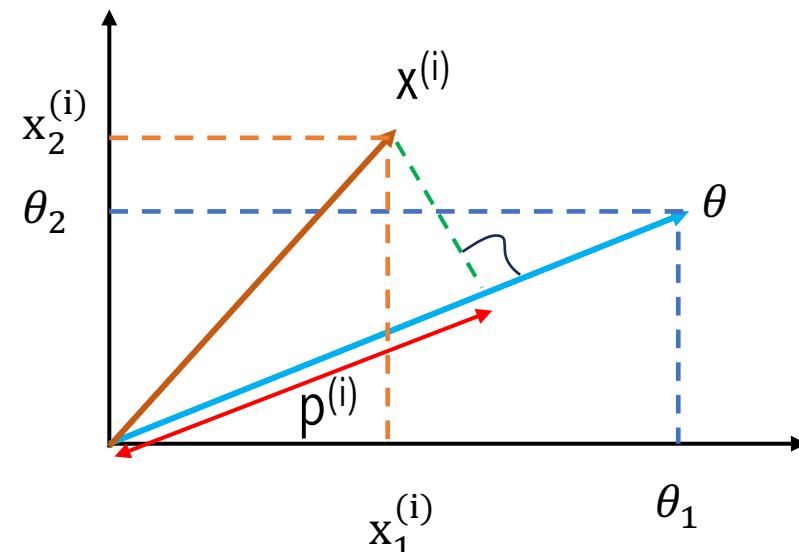
$$\min C \frac{1}{2} \sum_{j=1}^m \theta_j^2 = \frac{1}{2} (\theta_0^2 + \theta_1^2 + \theta_2^2) = \frac{1}{2} \left( \sqrt{\theta_0^2 + \theta_1^2 + \theta_2^2} \right)^2 = \frac{1}{2} \|\theta\|^2$$

Subjective to

$$\begin{aligned}\theta^T x^{(i)} &\geq 1 && \text{If } y^{(i)} = 1 \\ \theta^T x^{(i)} &\leq -1 && \text{If } y^{(i)} = 0\end{aligned}$$

Simplification:

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} \quad \theta_0 = 0 \quad n = 2 \quad x^{(i)} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \end{bmatrix} \quad x_0^{(i)} = 1$$



$$\theta^T x^{(i)} = p^{(i)} \|\theta\| = \theta_1 x_1^{(i)} + \theta_2 x_2^{(i)}$$

# SVM Decision Boundary

$$\min C \frac{1}{2} \sum_{j=1}^m \theta_j^2 = \frac{1}{2} (\theta_0^2 + \theta_1^2 + \theta_2^2) = \frac{1}{2} \left( \sqrt{\theta_0^2 + \theta_1^2 + \theta_2^2} \right)^2 = \frac{1}{2} \|\theta\|^2 \quad (1)$$

## Subjective to

$$p^{(i)} \|\theta\| \geq 1 \quad \text{if } y^{(i)} = 1$$

$$p^{(i)} \|\theta\| \leq -1 \quad \text{if } y^{(i)} = 0$$

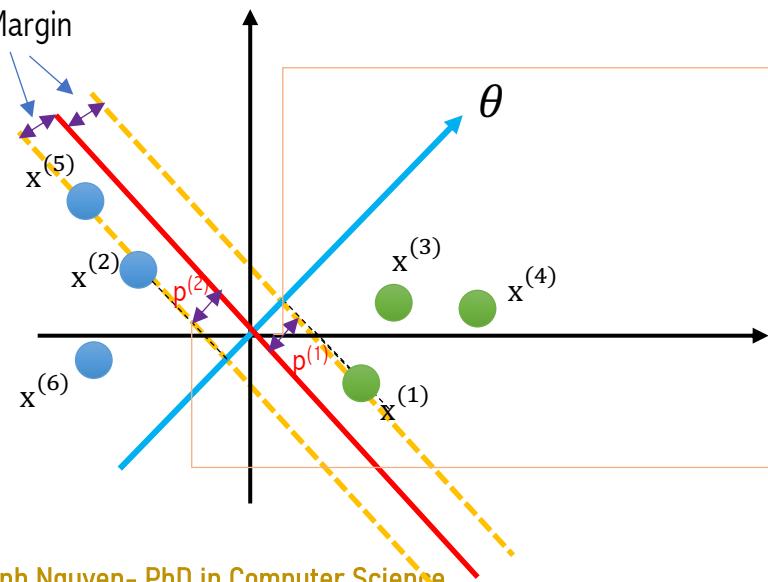
$p^{(i)}$  is the projection of  $x^{(i)}$  onto the vector  $\theta$

## Simplification:

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\kappa^{(i)} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \end{bmatrix} x_0^{(i)} = 1$$

## Margin



$$p^{(i)} \|\theta\| \geq 1$$

$\|\theta\|$  is large

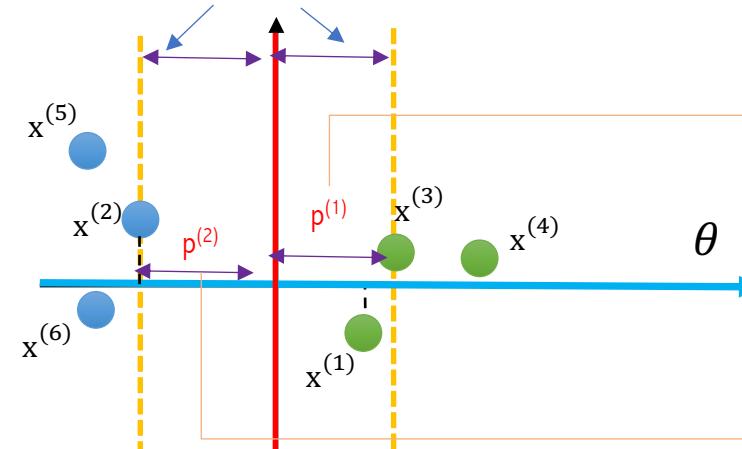
## Violate (1)

$$p^{(i)} \|\theta\| \leq -1$$

$\|\theta\|$  is large

## Violate (1)

## Maximize Margin



$$p^{(i)} \|\theta\| \geq 1$$

$\|\theta\|$  is small

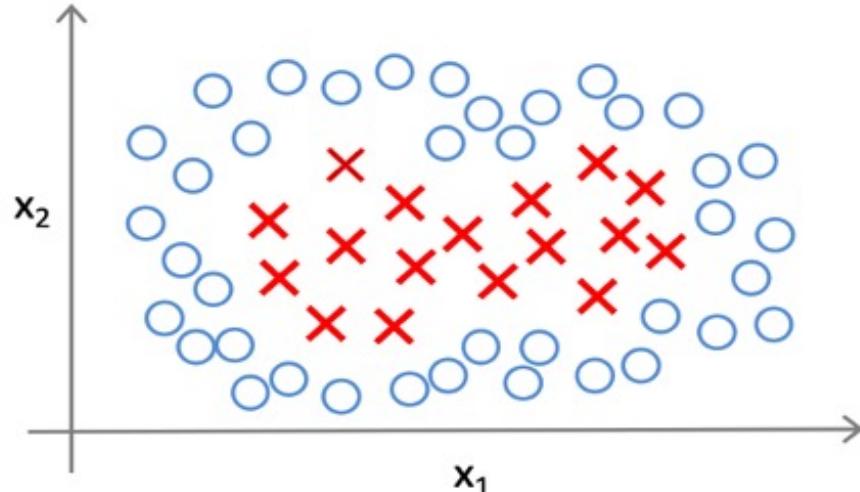
Satisfy (1)

$$p^{(i)} \|\theta\| \leq -1$$

$\|\theta\|$  is small

Satisfy (1)<sup>4</sup>

# Non-Linear Decision Boundary



Có phương pháp nào để xây dựng features  $f_1, f_2, f_3$ ,  
tố hơn không không ?

Linear case

$$z = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4 + \theta_5 x_5 + \dots$$

$$h_{\theta}(x) = \begin{cases} 1 & z \geq 1 \\ 0 & z \leq -1 \end{cases}$$

Polynomial case

$$z = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1 x_2 + \theta_4 x_1^2 + \theta_5 x_2^2 + \dots$$

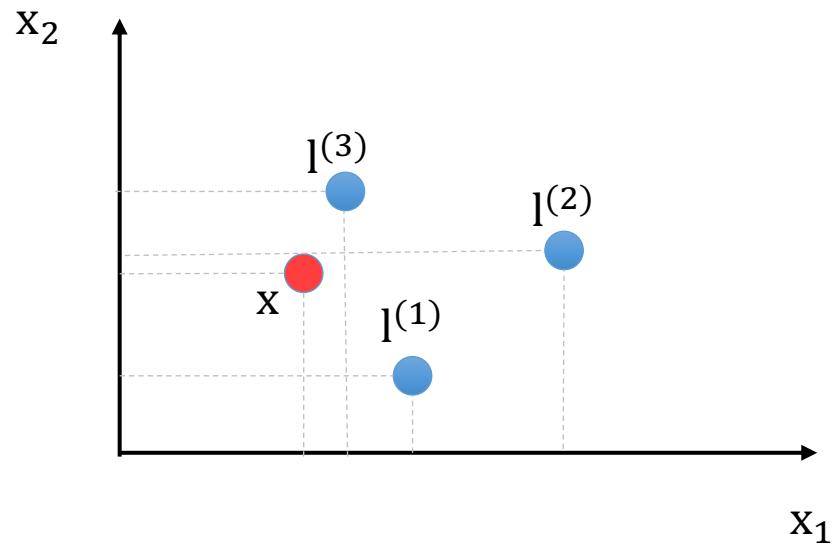
$$h_{\theta}(x) = \begin{cases} 1 & z \geq 1 \\ 0 & z \leq -1 \end{cases}$$

Polynomial case

$$z = \theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 + \theta_4 f_4 \dots$$

$$f_1 = x_1, f_2 = x_2, f_3 = x_1 x_2, f_4 = x_1^2, f_5 = x_2^2, \dots$$

# Kernel and Similarity



X and  $l^{(1)}$  Relationship

$$\text{If } x \text{ is near to } \approx l^{(1)}: f_1 = e^{\left(-\frac{\|x-l^{(1)}\|^2}{2\sigma^2}\right)} \approx 1$$

$$\text{If } x \text{ is far from } l^{(1)}: f_1 = e^{\left(-\frac{\|x-l^{(1)}\|^2}{2\sigma^2}\right)} \approx 0$$

Given sample  $x [x_1, x_2]$ , compute new feature depending on proximity to landmarks  $l^{(1)}, l^{(2)}, l^{(3)}$

Generate new feature

$$f_1 = \text{similarity}(x, l^{(1)}) = e^{\left(-\frac{\|x-l^{(1)}\|^2}{2\sigma^2}\right)}$$

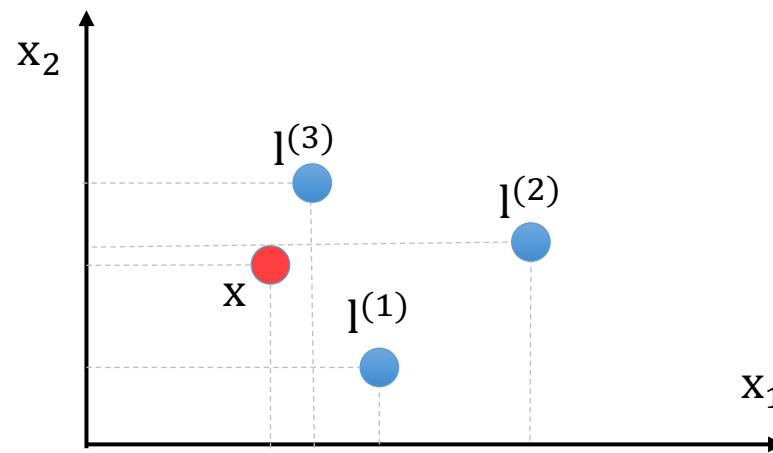
$$f_2 = \text{similarity}(x, l^{(2)}) = e^{\left(-\frac{\|x-l^{(2)}\|^2}{2\sigma^2}\right)}$$

$$f_3 = \text{similarity}(x, l^{(3)}) = e^{\left(-\frac{\|x-l^{(3)}\|^2}{2\sigma^2}\right)}$$

Kernel

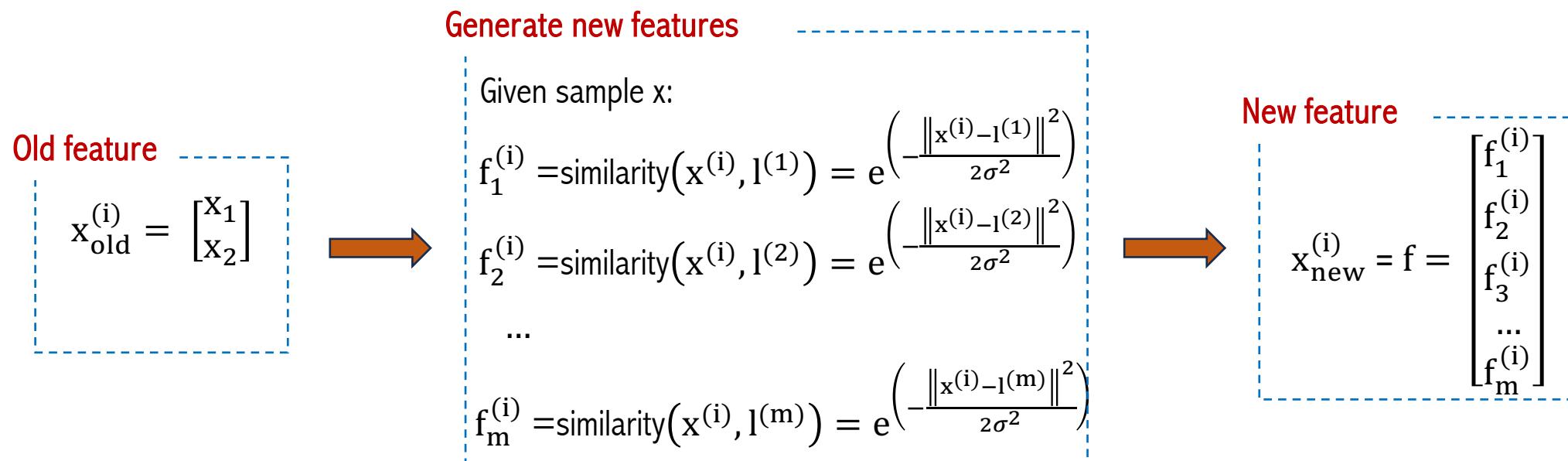
Gaussian kernel

# Landmark Selection

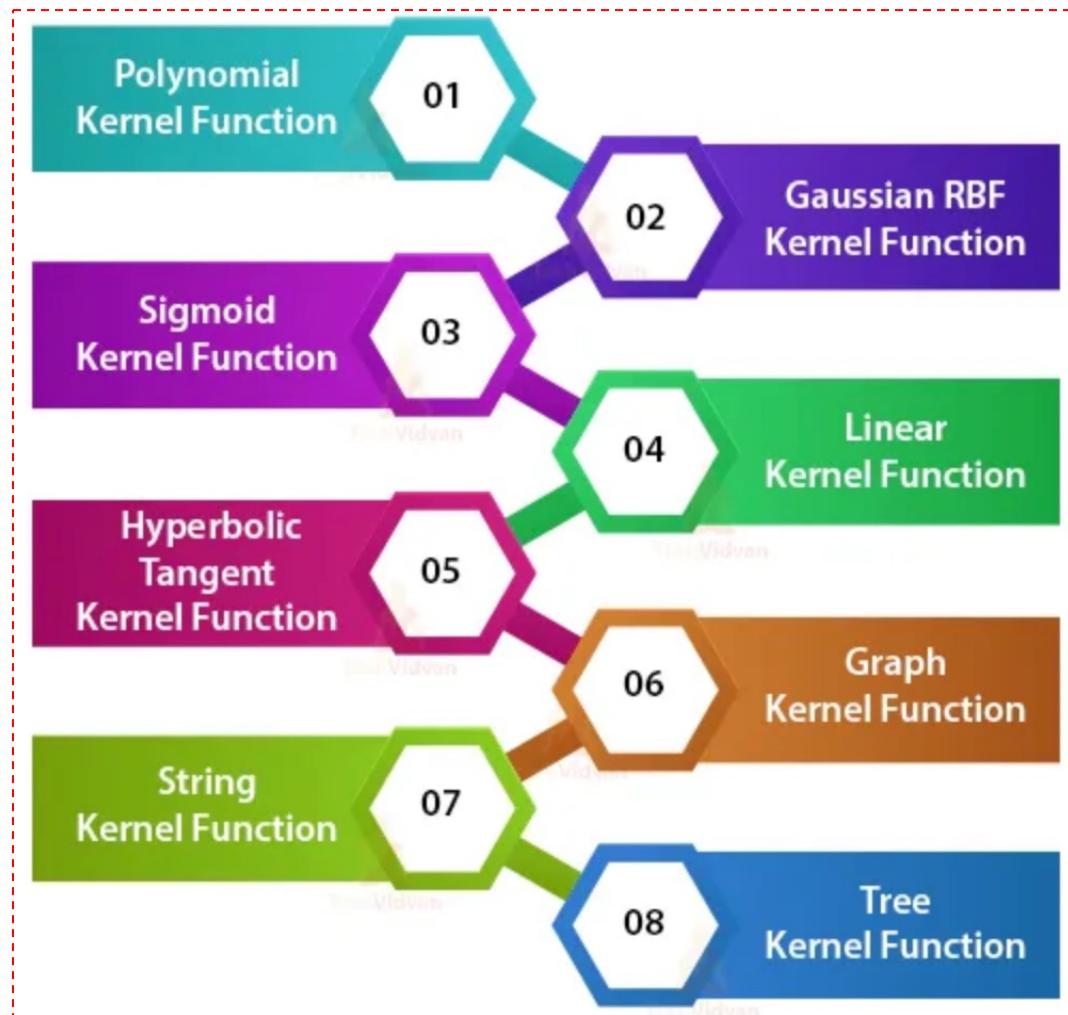


Given samples:  $((x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})$

Choose  $l^{(1)} = x^{(1)}, l^{(2)} = x^{(2)}, \dots, l^{(m)} = x^{(m)}$

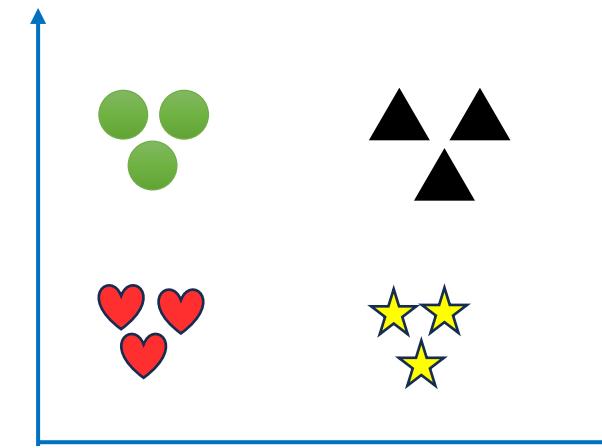


# Other Kernel



Tên	Công thức	kernel	Thiết lập hệ số
linear	$\mathbf{x}^T \mathbf{z}$	'linear'	không có hệ số
polynomial	$(r + \gamma \mathbf{x}^T \mathbf{z})^d$	'poly'	$d: \text{degree}, \gamma: \text{gamma}, r: \text{coef0}$
sigmoid	$\tanh(\gamma \mathbf{x}^T \mathbf{z} + r)$	'sigmoid'	$\gamma: \text{gamma}, r: \text{coef0}$
rbf	$\exp(-\gamma \ \mathbf{x} - \mathbf{z}\ _2^2)$	'rbf'	$\gamma > 0: \text{gamma}$

# Multiple Class Classification



# Outline

- Linear Regression to Logistic Regression
- Logistic Regression to Support Vector Machine
- Example

# Email Spam Classification - 1

Normal Email

```
0028.1999-12-17.farmer.ham.txt ×  
1 Subject: pennzenergy property details  
2 ----- forward  
3 pm  
4 0010.1999-12-14.farmer.ham.txt ×  
5 Subject: duns number changes  
6 fyi  
7 ----- forwarded by e  
8 0007.1999-12-14.farmer.ham.txt ×  
9 Subject: mcmullen gas for 11 / 99  
10 jackie ,  
11 since the inlet to 3 river plant is shut in on 10 / 19  
12 flow ) :  
13 at what meter is the mcmullen gas being diverted to ?  
14 at what meter is hpl buying the residue gas ? ( this is  
15 vastar , vintage , tejones , and swift )  
16 i still see active deals at meter 3405 in path manager  
17 vintage , tejones , and swift  
18 i also see gas scheduled in pops at meter 3404 and 3405  
19 please advice . we need to resolve this as soon as poss  
20 can send out payments .  
21 thanks
```

Spam Email

```
0026.2003-12-18.GP.spam.txt ×  
1 Subject: coca cola , mbna america , nascar partner with otcbb  
2 stock  
3 profile  
4 about  
5 company  
6 invest  
7 highlight  
8 press  
9 12 / 01 / 2003  
10 indianapolis , in - race car simulators ? inks the sale of ei  
11 09 / 17 / 2003  
12 indianapolis , in - nascar silicon motor speedway ? simulator  
13 09 / 05 / 2003  
14 indianapolis , in - nascar silicon motor speedway ? expands t  
15 09 / 02 / 2003  
16 indianapolis , in - nascar silicon motor speedway ? announces  
17 08 / 14 / 2003  
18 indianapolis , in - race car simulators ? and baldacci sign a  
19 08 / 12 / 2003  
...  
0018.2003-12-18.GP.spam.txt ×  
1 Subject: await your response  
2 dear partner ,  
3 we are a team of government officials that belongs to an oil  
4 at the  
5 bear in  
6 1 . th  
7 2 . th  
8 indianapolis , in - race car simulators ? inks the sale of ei  
9 12 / 01 / 2003  
10 indianapolis , in - nascar silicon motor speedway ? simulator  
11 09 / 17 / 2003  
12 indianapolis , in - nascar silicon motor speedway ? expands t  
13 09 / 05 / 2003  
14 indianapolis , in - nascar silicon motor speedway ? announces  
15 09 / 02 / 2003  
16 indianapolis , in - nascar silicon motor speedway ? announces  
17 08 / 14 / 2003  
18 indianapolis , in - race car simulators ? and baldacci sign a  
19 08 / 12 / 2003  
...  
0026.2003-12-18.GP.spam.txt ×  
1 Subject: coca cola , mbna america , nascar partner with otcbb  
2 stock  
3 profile  
4 about  
5 company  
6 investment  
7 highlights  
8 press release  
9 12 / 01 / 2003  
10 indianapolis , in - race car simulators ? inks the sale of ei  
11 09 / 17 / 2003  
12 indianapolis , in - nascar silicon motor speedway ? simulator  
13 09 / 05 / 2003  
14 indianapolis , in - nascar silicon motor speedway ? expands t  
15 09 / 02 / 2003  
16 indianapolis , in - nascar silicon motor speedway ? announces  
17 08 / 14 / 2003  
18 indianapolis , in - race car simulators ? and baldacci sign a  
19 08 / 12 / 2003  
...
```

# Email Spam Classification - 1

Normal Email

```
0028.1999-12-17.farmer.ham.txt ×  
1 Subject: pennzenergy property details  
2 ----- forward  
3 pm -----  
4 dscottl @ . com on 12 / 14 / 99 10 : 56 : 01 am  
5 to : ami chokshi / corp / enron @ enron  
6 cc :  
7 subject : pennzenergy property details  
8 ami , attached is some more details on the devon so  
9 me  
10 know if you have any questions .  
11 david  
12 - devon stx . xls
```

Spam Email

```
0026.2003-12-18.GP.spam.txt ×  
1 Subject: coca cola , mbna america , nascar partner with otcbb  
2 stock  
3 profile  
4 about  
5 company  
6 investment  
7 highlights  
8 press release  
9 12 / 01 / 2003  
10 indianapolis , in - race car simulators ? inks the sale of eid  
11 09 / 17 / 2003  
12 indianapolis , in - nascar silicon motor speedway ? simulators  
13 09 / 05 / 2003  
14 indianapolis , in - nascar silicon motor speedway ? expands to  
15 09 / 02 / 2003  
16 indianapolis , in - nascar silicon motor speedway ? announces  
17 08 / 14 / 2003  
18 indianapolis , in - race car simulators ? and baldacci sign ag  
19 08 / 12 / 2003  
20 indianapolis , in - imts forms new subsidiary for manufacturin  
21 08 / 07 / 2003
```

Text cleanup

Sample Code

```
text_before = "Subject: vastar resources , inc ."  
text_after = text_cleanup(text_before)  
print("before cleanup: ", text_before)  
print("after cleanup: ", text_after)
```

▶ before cleanup: Subject: vastar resources , inc .  
▶ after cleanup: ['subject', 'vastar', 'resources', 'inc']

# Email Spam Classification - 1

Normal Email

```
0028.1999-12-17.farmer.ham.txt ×
1 Subject: pennzenergy property details
2 ----- forward
3 pm -----
4 dscottl @ . com on 12 / 14 / 99 10 : 56 : 01 am
5 to : ami chokshi / corp / enron @ enron
6 cc :
7 subject : pennzenergy property details
8 ami , attached is some more details on the devon so
9 me
10 know if you have any questions .
11 david
12 - devon stx . xls
```

Spam Email

```
0026.2003-12-18.GP.spam.txt ×
1 Subject: coca cola , mbna america , nascar partner with otcbb
2 stock
3 profile
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17 08 / 14 / 2003
18 indianapolis , in - race car simulators ? and baldacci sign ag
19 08 / 12 / 2003
20 indianapolis , in - imts forms new subsidiary for manufacturin
21 08 / 07 / 2003
```

Text cleanup

```
def prepare_dictionary():
    start_time = time.time()

    lmtzr = WordNetLemmatizer()
    k=0
    count = {}

    directory_in_str = "/content/email"
    directory = os.fsencode(directory_in_str)

    for file in os.listdir(directory):
        file = file.decode("utf-8")
        file_name = str(os.getcwd()) + '/email/'
        file_name = file_name + file
```

Build dictionary

word	count
ect	489
subject	294
hou	256
enron	237
deal	119
meter	114
gas	111
com	107
please	105

# Email Spam Classification - 1

Normal Email

```
0028.1999-12-17.farmer.ham.txt X
1 Subject: pennzenergy property details
2 ----- forward
3 pm -----
4 dscottl @ . com on 12 / 14 / 99 10 : 56 : 01 am
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19 08 / 12 / 2003
20 indianapolis , in - imts forms new subsidiary for manufacturin
21 08 / 07 / 2003
```

Text cleanup

Build dictionary

```
▶ def encode_email(fileName, words):
    file_reading = open(fileName,"r",encoding='utf-8', errors='ignore')
    words_list_array = np.zeros(words.size)
    for word in file_reading.read().split():
        word = lmtzr.lemmatize(word.lower())
        if(word in stopwords.words('english') or word in string.punctuation or
           continue
        for i in range(words.size):
            if(words[i]==word):
                words_list_array[i] = words_list_array[i]+1
                break
    return words_list_array
```

Encoding data

```
▶ normal_sample = encode_email('/content/email/0028.1999-12-17.farmer.ham.txt', words)
print(normal_sample)

spam_sample = encode_email('/content/email/0334.2004-01-30.GP.spam.txt', words)
print(spam_sample)

⇒ [0. 1. 0. 3. 0. 0. 0. 1. 0.]
[0. 0. 0. 0. 0. 0. 0. 1. 1.]
```

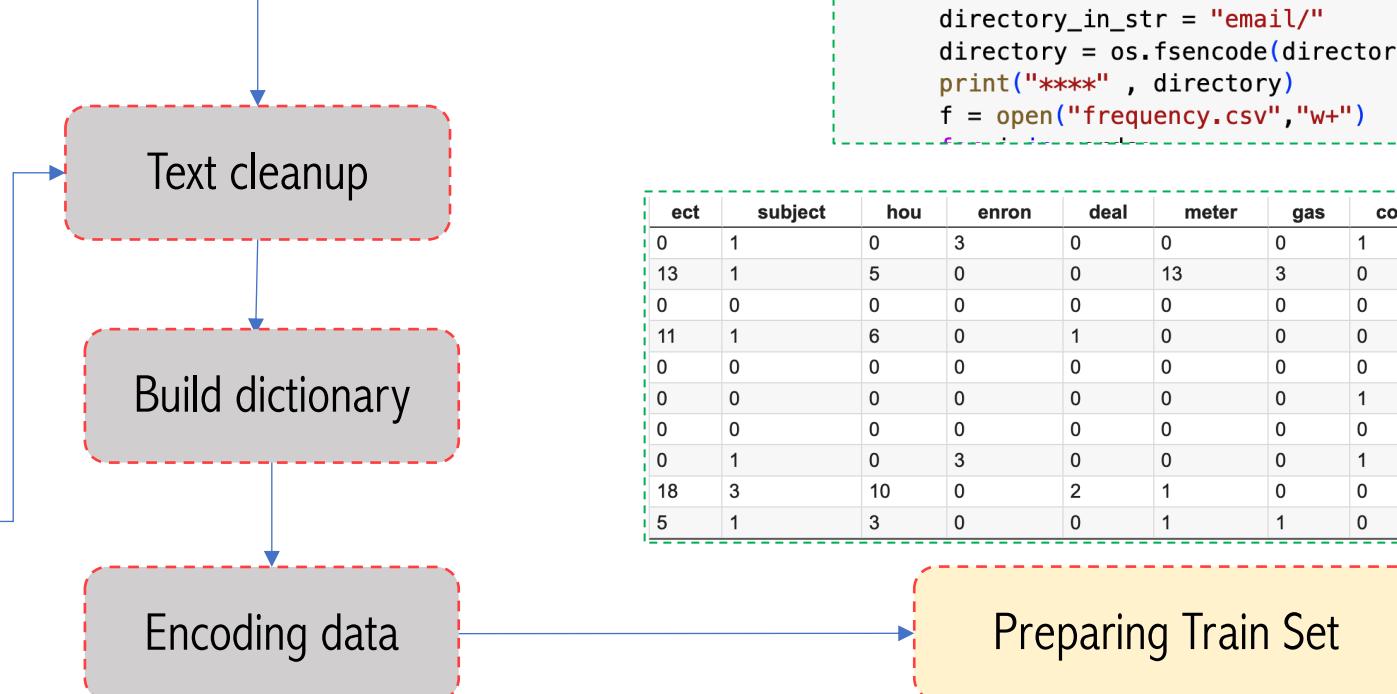
# Email Spam Classification - 1

Normal Email

```
0028.1999-12-17.farmer.ham.txt ×
1 Subject: pennzenergy property details
2 ----- forward
3 pm -----
4 dscottl @ . com on 12 / 14 / 99 10 : 56 : 01 am
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21 08 / 07 / 2003
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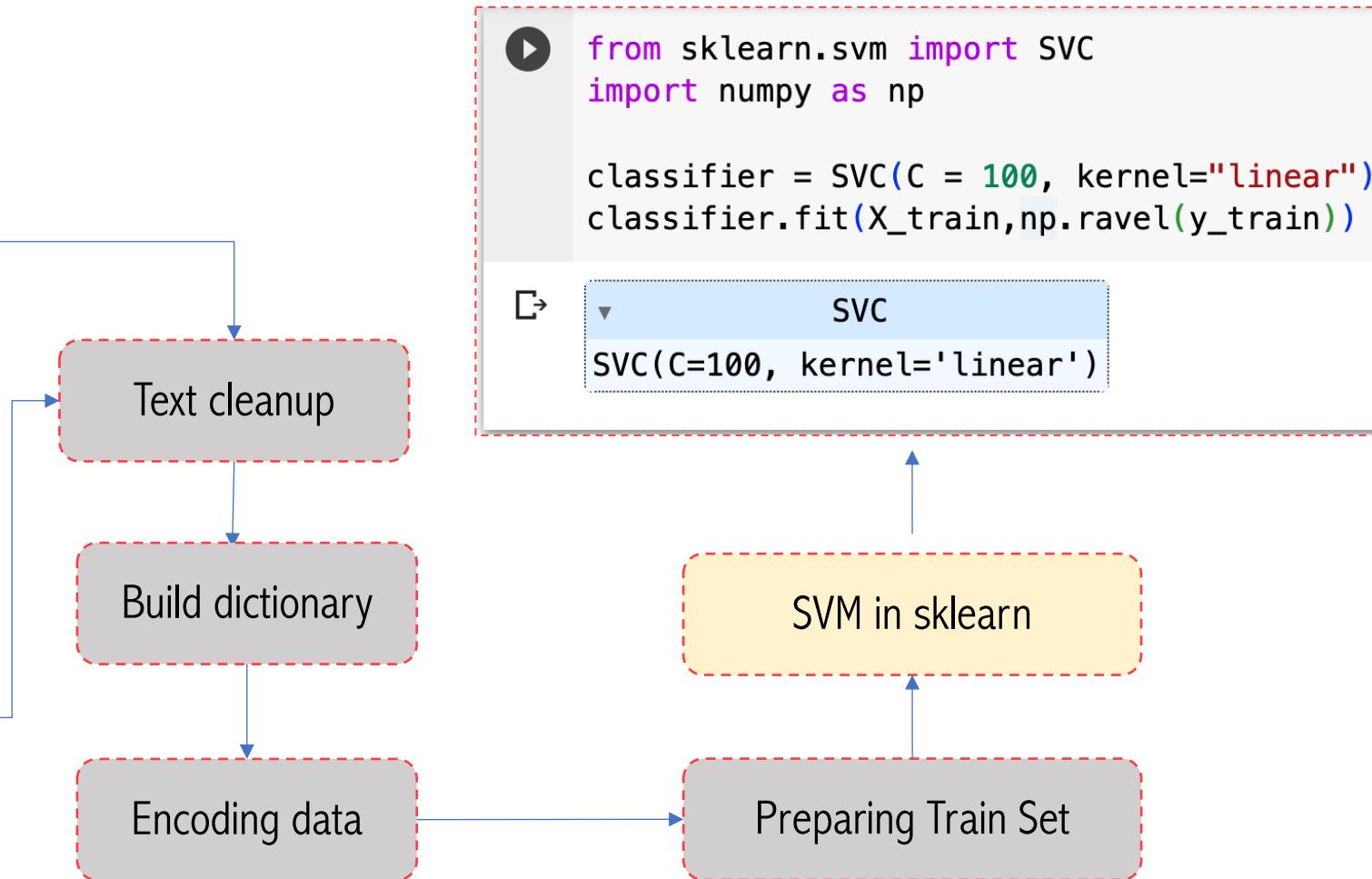
# Email Spam Classification - 1

Normal Email

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4 dscottl @ . com on 12 / 14 / 99 10 : 56 : 01 am
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19 08 / 12 / 2003
20 indianapolis , in - imts forms new subsidiary for manufacturin
21 08 / 07 / 2003
```



## Dataset

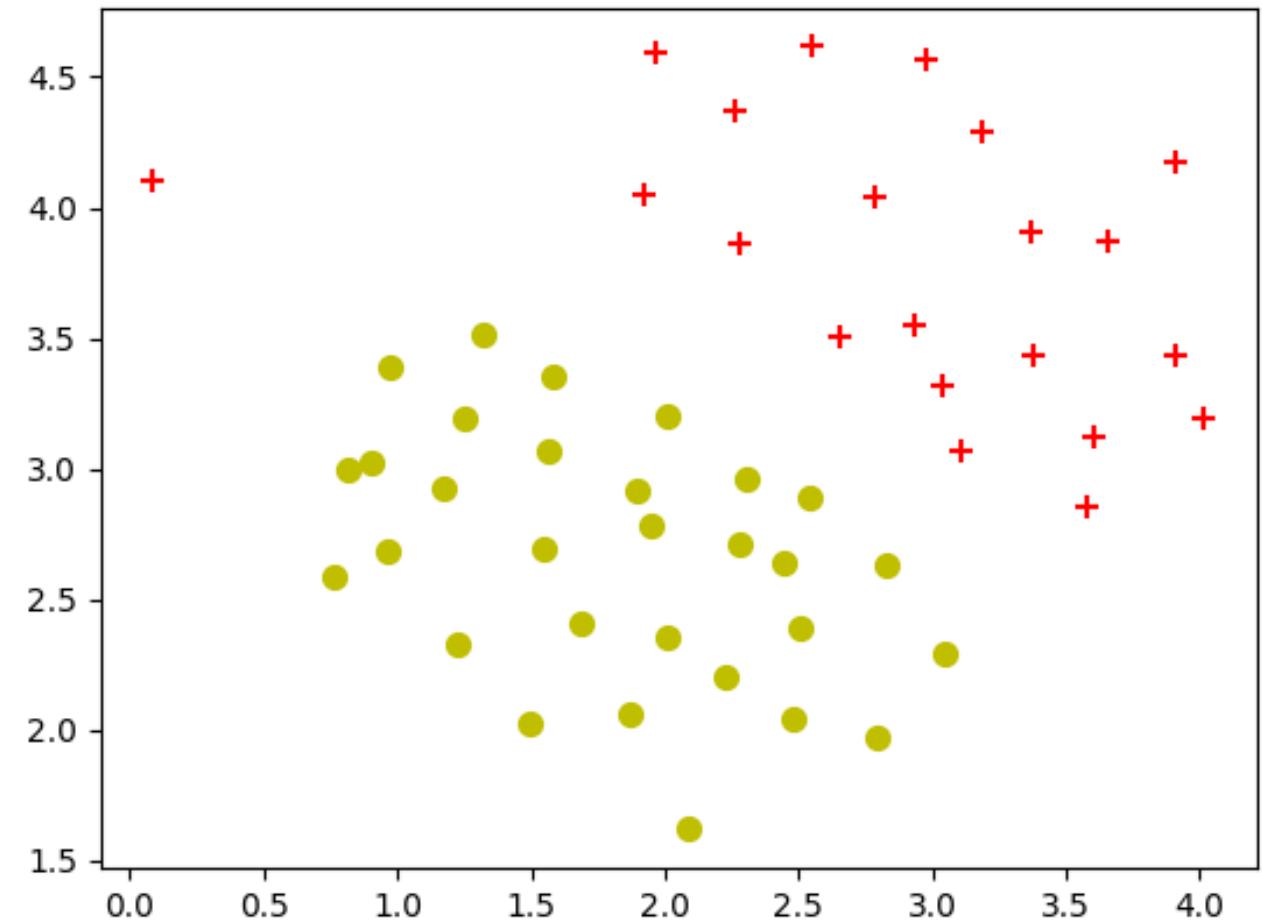
Feature 1	Feature 2	Category
1.9643	4.5957	1
2.2753	3.8589	1
2.9781	4.5651	1
...	...	
...	...	
3.3814	3.4291	0



```
mat = loadmat("/content/product.mat")
X = mat["X"]
y = mat["y"]
```

```
m,n = X.shape[0],X.shape[1]
pos,neg= (y==1).reshape(m,1), (y==0).reshape(m,1)
plt.scatter(X[pos[:,0],0],X[pos[:,0],1],c="r",marker="+",s=50)
plt.scatter(X[neg[:,0],0],X[neg[:,0],1],c="y",marker="o",s=50)
```

## Visualization



## Dataset

Feature 1	Feature 2	Category
1.9643	4.5957	1
2.2753	3.8589	1
2.9781	4.5651	1
...	...	
...	...	
3.3814	3.4291	0



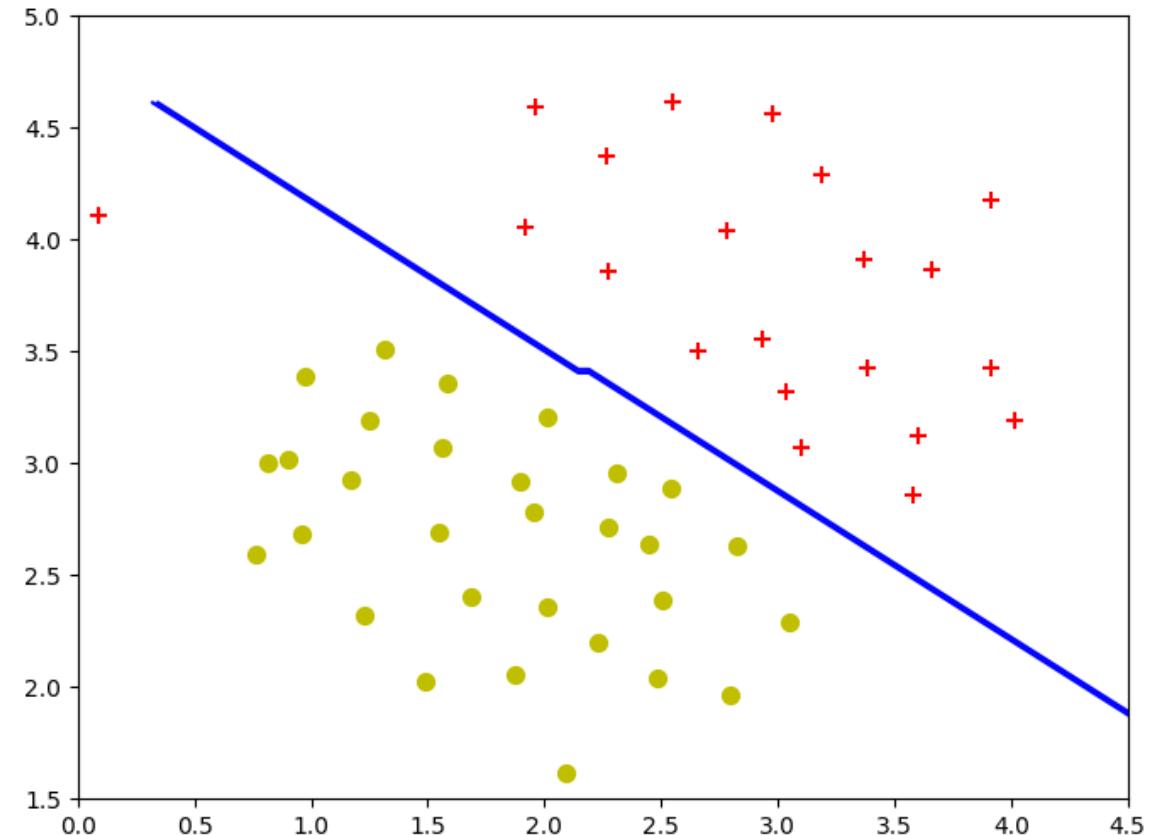
```
from sklearn.svm import SVC
classifier = SVC(kernel="linear")
classifier.fit(X,np.ravel(y))
```



SVC

```
SVC(kernel='linear')
```

## Visualization



## Dataset

Feature 1	Feature 2	Category
1.9643	4.5957	1
2.2753	3.8589	1
2.9781	4.5651	1
...	...	
...	....	
3.3814	3.4291	0

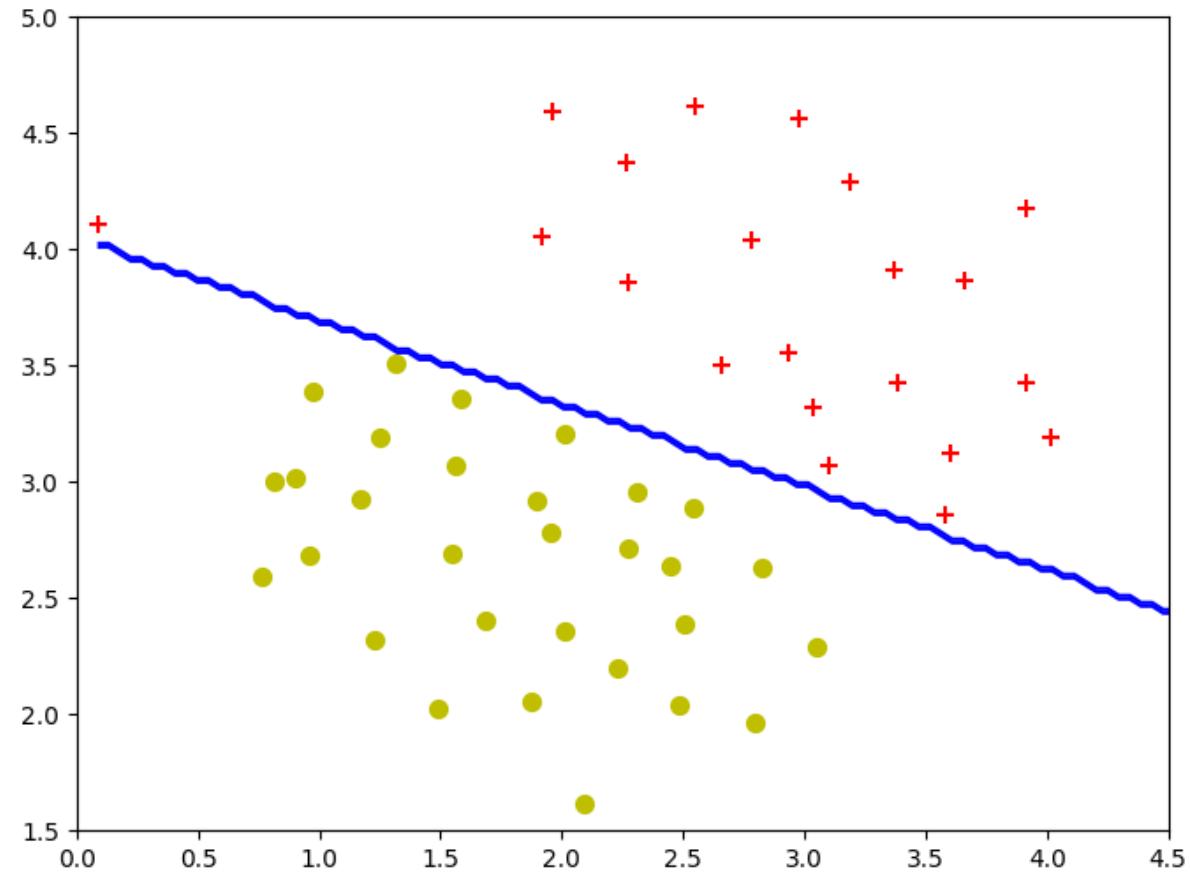


```
# Test C = 100
classifier_c_100 = SVC(C=100,kernel="linear")
classifier_c_100.fit(X,np.ravel(y))
```



```
SVC
SVC(C=100, kernel='linear')
```

## Visualization (C = 100)



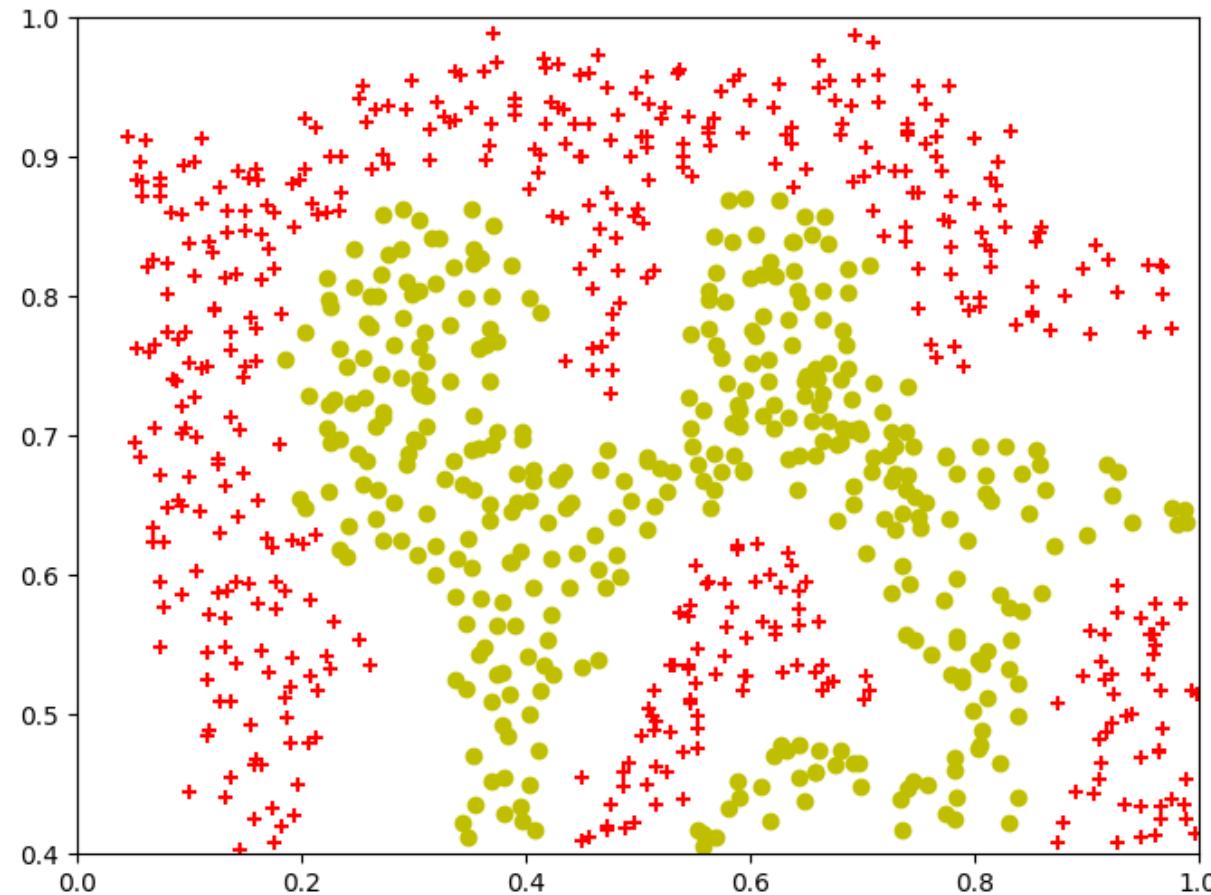
## Dataset

Feature 1	Feature 2	Category
-1.58986e-01	4.23977e-01	1
-3.47926e-01	4.70760e-01	1
-5.04608e-01	3.53801e-01	1
...	...	
...	....	
-5.96774e-01	-5.96774e-01	0

```
▶ mat2 = loadmat("/content/product_c_.mat")
X2 = mat2["X"]
y2 = mat2["y"]
```

```
▶ m2,n2 = X2.shape[0],X2.shape[1]
pos2,neg2= (y2==1).reshape(m2,1), (y2==0).reshape(m2,1)
plt.figure(figsize=(8,6))
plt.scatter(X2[pos2[:,0],0],X2[pos2[:,0],1],c="r",marker="+")
plt.scatter(X2[neg2[:,0],0],X2[neg2[:,0],1],c="y",marker="o")
plt.xlim(0,1)
plt.ylim(0.4,1)
```

## Visualization



## Dataset

Feature 1	Feature 2	Category
-1.58986e-01	4.23977e-01	1
-3.47926e-01	4.70760e-01	1
-5.04608e-01	3.53801e-01	1
...	...	
...	...	
-5.96774e-01	-5.96774e-01	0



```
from sklearn.svm import SVC
classifier = SVC(kernel="rbf", gamma=300)
classifier.fit(X2,y2.ravel())
```

## Visualization

