

Prescott's RBC Model

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- Finn Kydland and Edward Prescott were awarded the 2004 Nobel Memorial Prize in Economics, in part, for their contribution to our understanding of the driving forces behind business cycles.
- They pioneered a new approach to studying business cycles that became known as *real business cycle* or *RBC* theory.

- Edward Prescott's 1986 article "Theory Ahead of Business Cycle Measurement" provides an overview of models he developed with Kydland.
- The model described on pages 11-17 – which I'll call *Prescott's model* – explains the endogenous co-movement of output, employment, consumption, and investment over the business cycle.

Introduction

- Prescott's model is *decentralized*: the household makes some decisions and profit-maximizing firms make others and allocations are settled in markets with prices.
- Below, I describe a *centralized* version of Prescott's model: the representative household owns all resources and makes all allocation decisions
- The only consequence of the difference is that Prescott is able to model the equilibrium prices of labor and capital.

Centralized RBC Model with Labor

- A *representative household* lives for an infinite number of periods.
- Each period: consumes C_t and allocates share of available time L_t to labor activities.
- Flow of utility in period t :

$$\log(C_t) + \varphi \log(1 - L_t), \quad (1)$$

where $1 - L_t$ is the share of the household's time spent enjoying leisure.

Centralized RBC Model with Labor

- The *expected present value of lifetime utility* to the household from consuming C_0, C_1, C_2, \dots is denoted by U_0 :

$$\begin{aligned} U_0 = & \log(C_0) + \varphi \log(1 - L_0) \\ & + \beta [E_0 \log(C_1) + \varphi \log(1 - L_1)] \\ & + \beta^2 [E_0 \log(C_2) + \varphi \log(1 - L_2)] \\ & + \dots \end{aligned} \tag{2}$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t [\log(C_t) + \varphi \log(1 - L_t)], \tag{3}$$

Centralized RBC Model with Labor

- The household enters period 0 with capital $K_0 > 0$.
- Production in period t :

$$F(A_t, K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha} \quad (4)$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \quad (5)$$

- The household's resource constraint is:

$$C_t + K_{t+1} = A_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t, \quad (6)$$

for $t = 0, 1, 2, 3, \dots$

Centralized RBC Model with Labor

- **Optimization problem:** Each period the household chooses:
 - ① Consumption for the current period
 - ② Labor the current period
 - ③ Capital for the subsequent periodto maximize its expected present value of lifetime utility.
- We'll solve the problem for period 0 and then generalize the solution to apply to all periods $0, 1, 2, \dots$

Centralized RBC Model with Labor

- In period 0, the household solves:

$$\max_{C_0, L_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t [\log(C_t) + \varphi \log(1 - L_t)] \quad (7)$$

$$\text{s.t.} \quad C_t + K_{t+1} = A_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t$$

- The problem can be written as a choice of L_0 and K_1 only:

$$\begin{aligned} \max_{L_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t & [\log(A_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t - K_{t+1}) \\ & + \varphi \log(1 - L_t)] \end{aligned} \quad (8)$$

Centralized RBC Model with Labor

- The first-order condition with respect to L_0 is:

$$\frac{(1 - \alpha)A_0K_0^\alpha L_0^{-\alpha}}{C_0} = \frac{\varphi}{1 - L_0} \quad (9)$$

- The first-order condition with respect to K_1 is:

$$\frac{1}{C_0} = \beta E_0 \left[\frac{\alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha} + 1 - \delta}{C_1} \right] \quad (10)$$

Centralized RBC Model with Labor

- The household solves the same problem in periods $1, 2, 3, \dots$
- So given $K_0 > 0$ and A_0 , the equilibrium paths for labor, consumption, capital, and TFP are described by:

$$\frac{\varphi}{1 - L_t} = \frac{(1 - \alpha)A_t K_t^\alpha L_t^{1-\alpha}}{C_t} \quad (11)$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta}{C_{t+1}} \right] \quad (12)$$

$$C_t + K_{t+1} = A_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t \quad (13)$$

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \quad (14)$$

Centralized RBC Model without Labor

- Computing numeric values for consumption and capital is not trivial. Recall the Euler equation:

$$\frac{1}{C_t} = \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta}{C_{t+1}} \right] \quad (15)$$

- Consumption at date t depends on the *expectation* of consumption at date $t + 1$ which in turn depends on the expectation of consumption at date $t + 2$ and so on.
- Solving the problem requires numerical methods like those employed in the `linearsolve` Python package.

Figure 1: Kydland and Prescott RBC model with labor. Impulse responses to a one percent shock to TFP in period 5.

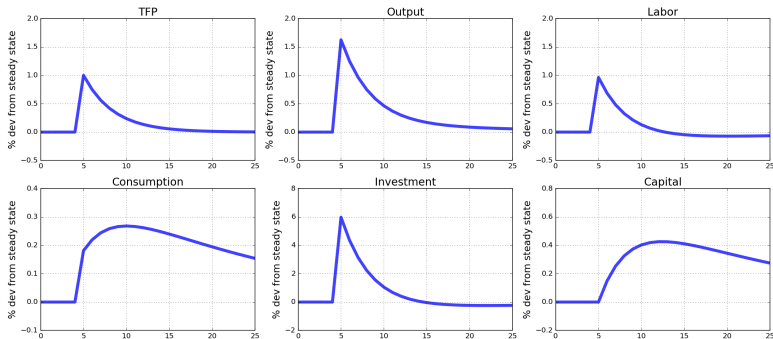


Figure 2: GDP. Actual and simulated data from Prescott's RBC model for the US from April 1948 to April 2022. Source: FRED.

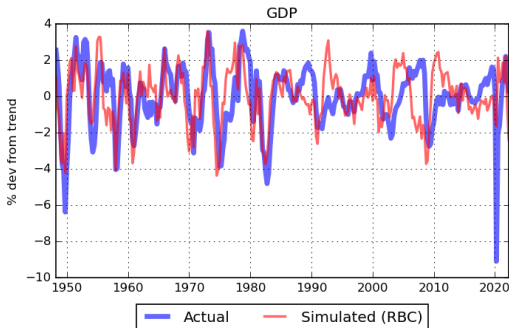


Figure 3: Consumption. Actual and simulated data from Prescott's RBC model for the US from April 1948 to April 2022. Source: FRED.

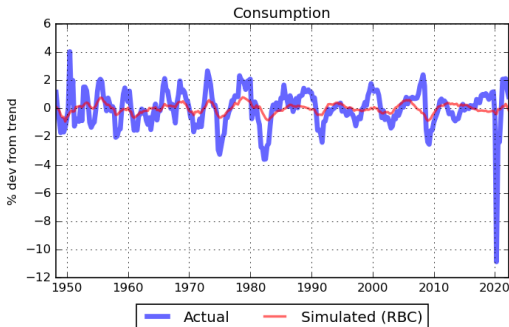


Figure 4: Investment. Actual and simulated data from Prescott's RBC model for the US from April 1948 to April 2022. Source: FRED.

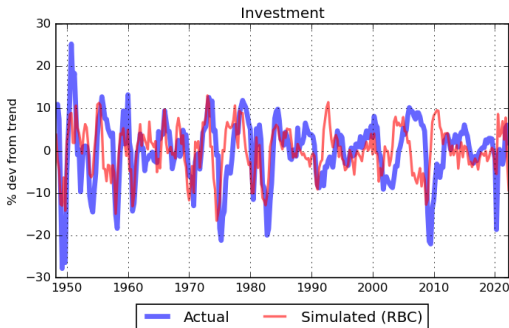


Figure 5: Labor. Actual and simulated data from Prescott's RBC model for the US from April 1948 to April 2022. Source: FRED.

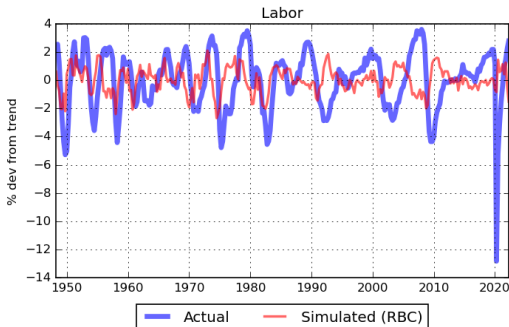


Table 1: Standard deviations. Actual data and data simulated from RBC models with and without labor. Units are percent deviations from trend. Source: FRED.

	Actual	RBC (w/o labor)	RBC (w/ labor)
Output	1.69	0.94	1.53
Consumption	1.34	0.22	0.33
Investment	7.41	3.39	5.60
Labor	2.06	—	0.91

Table 2: Correlations with GDP. Actual data and simulated data. Source: FRED.

	Actual	RBC (w/o labor)	RBC (w/ labor)
Consumption	0.81	0.61	0.59
Investment	0.84	0.99	0.99
Labor	0.88	—	0.98

Centralized RBC Model without Labor

- Summary analysis of the Prescott RBC Model with labor:
 - Substantial improvement in explaining output, consumption, investment, and labor volatility
 - Still under-predicts consumption-GDP correlation and over-predicts investment-GDP correlation
- Criticisms
 - ① Attributes *all* business cycle fluctuations to TFP shocks
 - ② Attributes *all* labor fluctuations to *voluntary* changes in labor supply.

Prescott, Edward C., “Theory Ahead of Business Cycle Measurement,” *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986, 10 (4), 9–22.