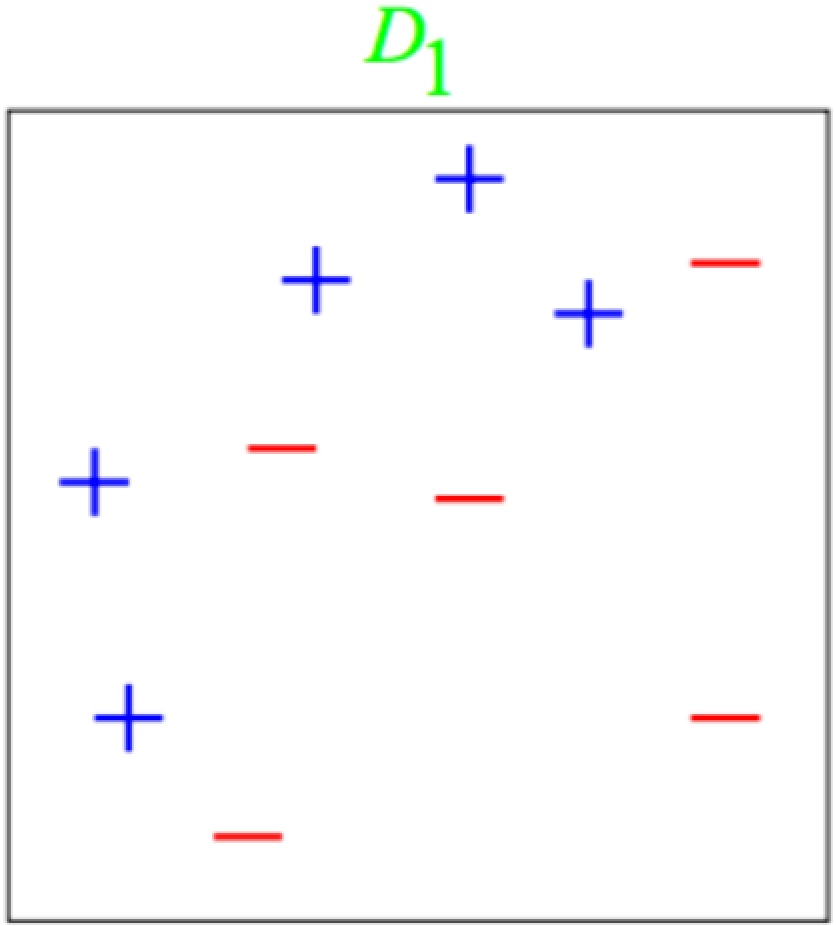
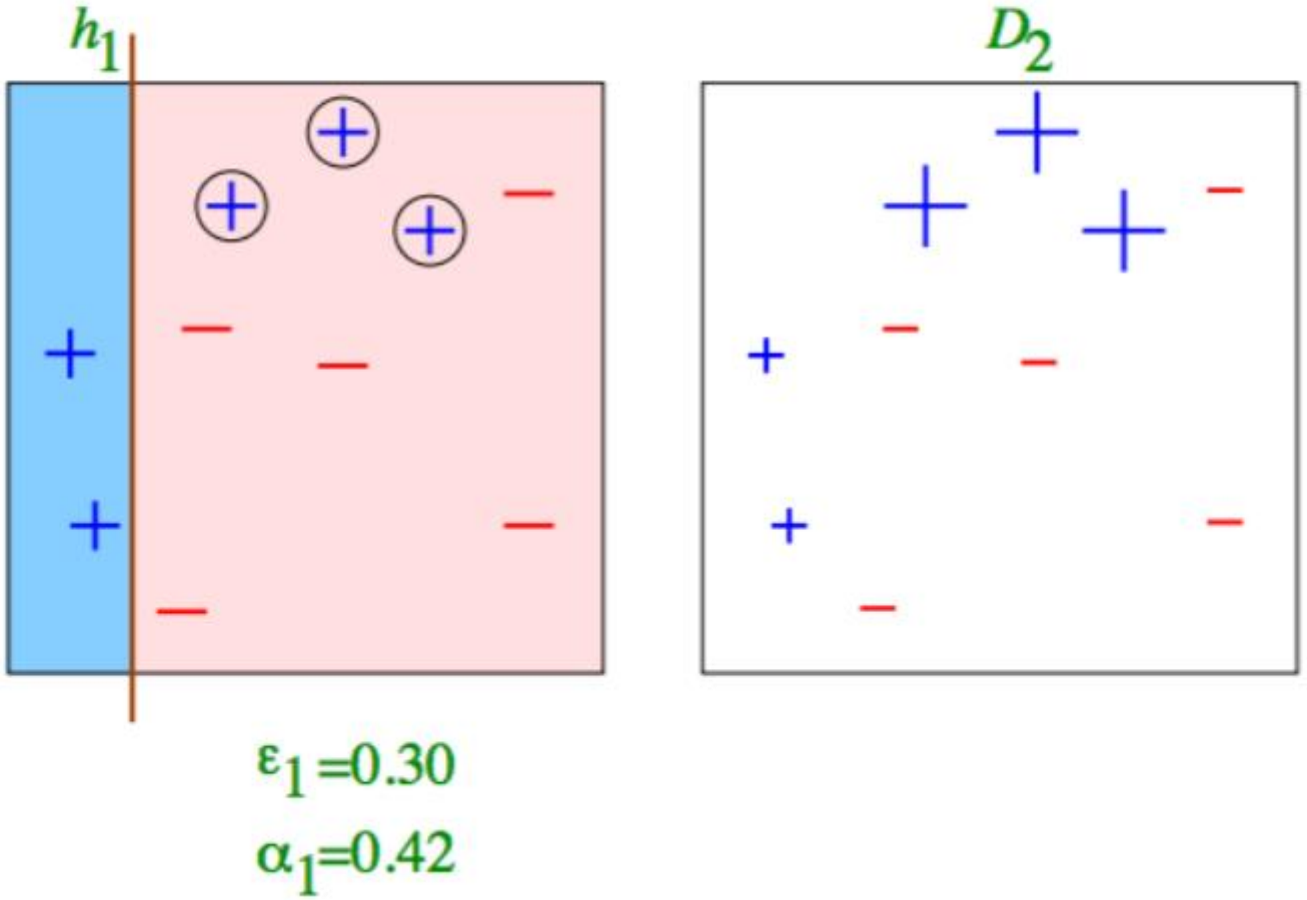
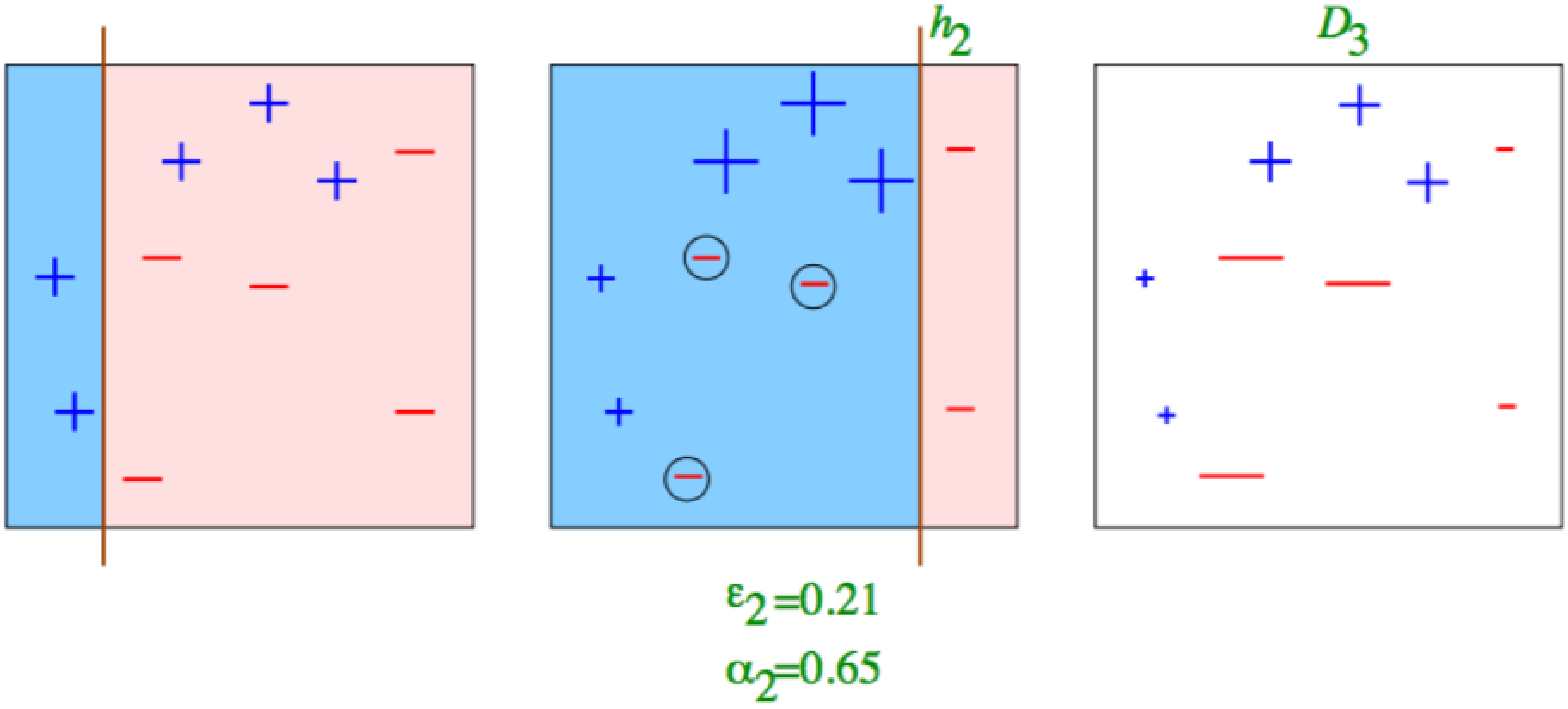
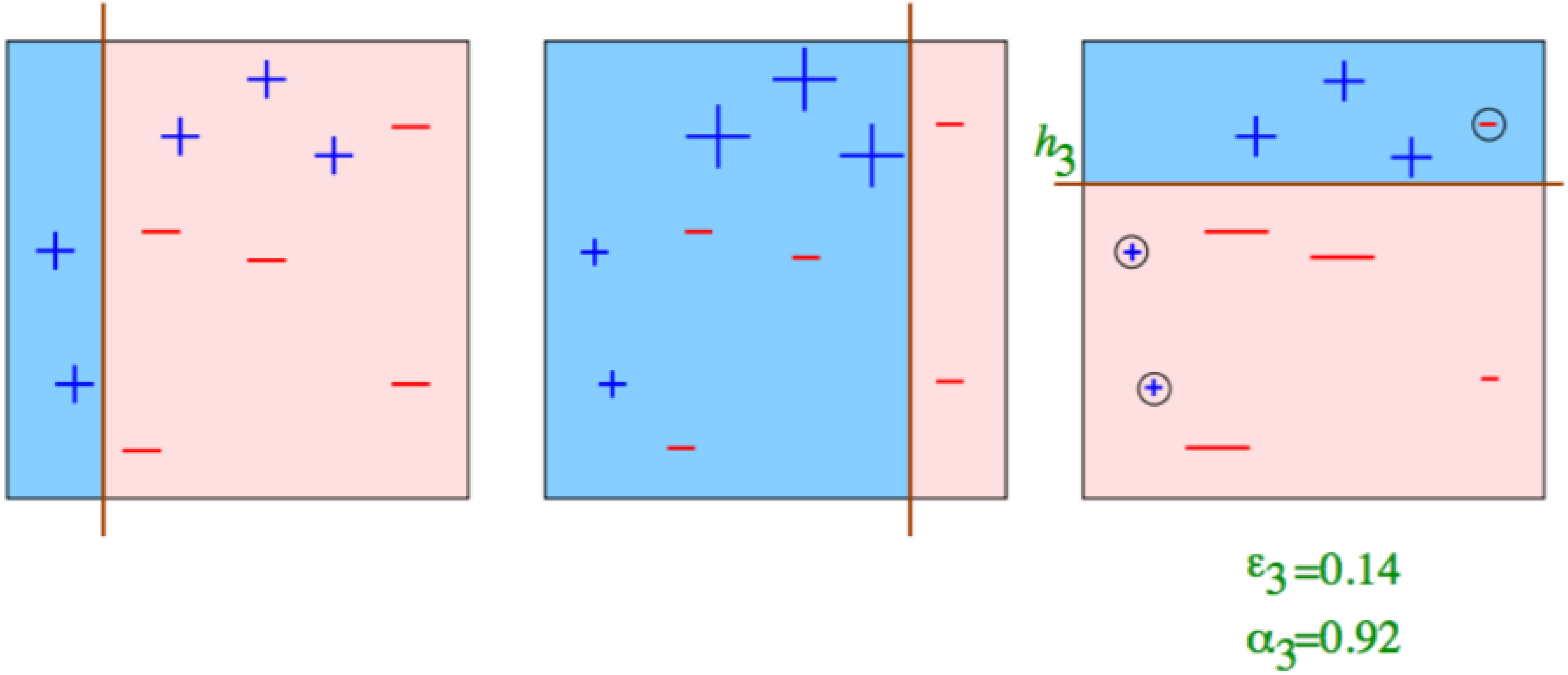


D_1

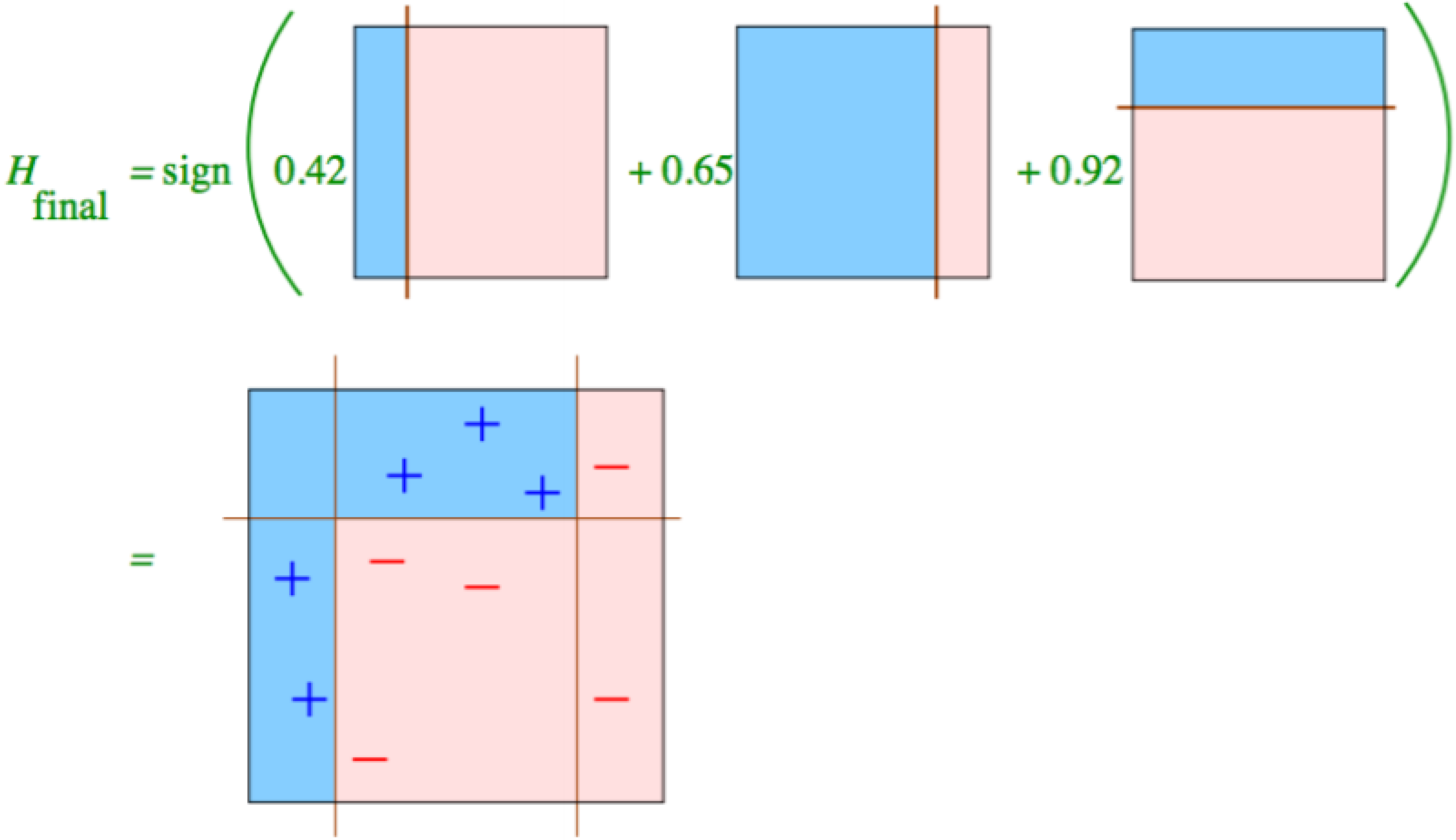

h_1
D_2
ε_1=0.30
α_1=0.42



h_2
D_3
ε_2=0.21
α_2=0.65



h_3
ε_3=0.14
α_3=0.92



H final = sign
0.42
+ 0.65
+ 0.92
=
+
−

Training set: $\{(x_1, y_1), \ldots, (x_m, y_m)\} \subseteq \mathcal{X} \times \{-1,1\}$

Let $D_1(i) = \frac{1}{m}$.

At each iteration t:

1. Find weak learner h_t minimizing

$$\varepsilon_t = \sum_{i=1}^{m} D_t(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i)\,.$$

2. Set $D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \exp\big(-\alpha_t y_i h_t(x_i)\big)$, where $\alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$

and $Z_t = \sum_{j=1}^{m} D_t(j) \exp\left(-\alpha_t y_j h_t(x_j)\right)$

Why?

2. Set $D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \exp\left(-\alpha_t y_i h_t(x_i)\right)$, where $\alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$

and $Z_t = \sum_{j=1}^{m} D_t(j) \exp\left(-\alpha_t y_j h_t(x_j)\right)$

Why?

Adaboost uses this weighting mechanism to “force” the weak learner to **focus on the problematic examples** in the next iteration.

Formally,

$$\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i) = \frac{1}{2}$$

Question.

$D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \exp\left(-\alpha_t y_i h_t(x_i)\right)$, where $\alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$.

What is $D_{t+1}(i)$ when $h_t(x_i) = y_i$?

Your answer should only be in terms of ε_t, $D_t(i)$, and Z_t.

Question.

$D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \exp\big(-\alpha_t y_i h_t(x_i)\big)$, where $\alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$.

What is $D_{t+1}(i)$ when $h_t(x_i) = y_i$?

Your answer should only be in terms of ε_t, $D_t(i)$, and Z_t.

Answer.

$$\frac{1}{Z_t} D_t(i) \exp\big(-\alpha_t y_i h_t(x_i)\big) = \frac{1}{Z_t} D_t(i) \exp(-\alpha_t) = \frac{1}{Z_t} D_t(i) \sqrt{\frac{\varepsilon_t}{1-\varepsilon_t}}.$$

Question.

$D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \sqrt{\frac{\varepsilon_t}{1-\varepsilon_t}}$ when $h_t(x_i) = y_i$.

What is $\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) = y_i)$?

Your answer should only be in terms of Z_t and ε_t.

$$\varepsilon_t = \sum_{i=1}^{m} D_t(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i)$$

Question.

$D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \sqrt{\frac{\varepsilon_t}{1-\varepsilon_t}}$ when $h_t(x_i) = y_i$.

What is $\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) = y_i)$?

Your answer should only be in terms of Z_t and ε_t.

$$\varepsilon_t = \sum_{i=1}^{m} D_t(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i)$$

Answer.

$$\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) = y_i) = \frac{\sqrt{\varepsilon_t(1 - \varepsilon_t)}}{Z_t}$$

Question.

$D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \exp\left(-\alpha_t y_i h_t(x_i)\right)$, where $\alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$.

What is $D_{t+1}(i)$ when $h_t(x_i) \neq y_i$?

Your answer should only be in terms of ε_t, $D_t(i)$, and Z_t.

Question.

$D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \exp\big(-\alpha_t y_i h_t(x_i)\big)$, where $\alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$.

What is $D_{t+1}(i)$ when $h_t(x_i) \neq y_i$?

Your answer should only be in terms of ε_t, $D_t(i)$, and Z_t.

Answer.

$$\frac{1}{Z_t} D_t(i) \exp\big(-\alpha_t y_i h_t(x_i)\big) = \frac{1}{Z_t} D_t(i) \exp(\alpha_t) = \frac{1}{Z_t} D_t(i) \sqrt{\frac{1-\varepsilon_t}{\varepsilon_t}}.$$

Question.

$D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \sqrt{\frac{1-\varepsilon_t}{\varepsilon_t}}$ when $h_t(x_i) \neq y_i$.

What is $\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i)$?

Your answer should only be in terms of Z_t and ε_t.

$$\varepsilon_t = \sum_{i=1}^{m} D_t(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i)$$

Question.

$D_{t+1}(i) = \frac{1}{Z_t} D_t(i) \sqrt{\frac{1-\varepsilon_t}{\varepsilon_t}}$ when $h_t(x_i) \neq y_i$.

What is $\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i)$?

Your answer should only be in terms of Z_t and ε_t.

$$\varepsilon_t = \sum_{i=1}^{m} D_t(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i)$$

Answer.

$$\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i) = \frac{\sqrt{\varepsilon_t(1-\varepsilon_t)}}{Z_t}$$

Question.

We saw that

$$\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i) = \sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) = y_i)$$

$$= \frac{\sqrt{\varepsilon_t(1-\varepsilon_t)}}{Z_t}$$

Why does this mean that $\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i) = \frac{1}{2}$?

Answer.

$$z_1 = \sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i) = \mathbb{P}_{(x,y)\sim D_{t+1}}[\mathbf{1}(h_t(x) \neq y)]$$

$$z_2 = \sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) = y_i) = \mathbb{P}_{(x,y)\sim D_{t+1}}[\mathbf{1}(h_t(x) = y)]$$

$z_1 + z_2 = 1$ and $z_1 = z_2$.
Therefore,

$$z_1 = \sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i) = \frac{1}{2}.$$

Adaboost uses this weighting mechanism to "force" the weak learner to **focus on the problematic examples** in the next iteration.

Therefore,

$$\sum_{i=1}^{m} D_{t+1}(i) \cdot \mathbf{1}(h_t(x_i) \neq y_i) = \frac{1}{2}.$$