Assessing Covid-19 Situation in Vietnam Using a Data-Driven Epidemiological Compartmental Model

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Outline

1. Introduction
2. Literature review
3. Methodologies
4. Results
5. Discussion
6. Conclusion

Outline for Introduction

1. Introduction

The Covid-19 disease

- Health impacts
  - 230 million global infections
  - 4.7 million global deaths

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- Health impacts
  - 230 million global infections
  - 4.7 million global deaths
- Economic impacts
  - Delayed shipments
  - Increased transaction cost

Why do we need a Covid-19 model?

- Lack of research on the impacts of Non-Pharmaceutical Intervention (NPI) in Vietnam

Why do we need a Covid-19 model?

- Lack of research on the impacts of Non-Pharmaceutical Intervention (NPI) in Vietnam
- Need of a simple tool for assessing the disease current and future progression

Goals

- A model that works with the data availability in Vietnam

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- An explainable model that can be used by experts

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- An explainable model that can be used by experts
- Assess the effectiveness of mobility data in predicting future cases

Outline for Literature review

2. Literature review
   - Related works
   - Artificial neural networks

Mathematical models: Compartmental

Susceptible-Exposed-Infective-Removed (SEIR) model [1]

$$
\begin{aligned}
S' &= -\beta(N)SI \\
E' &= \beta(N)SI - \kappa E \\
I' &= \kappa E - \alpha I \\
R' &= f\alpha I \\
N' &= -(1-f)\alpha I
\end{aligned}
$$

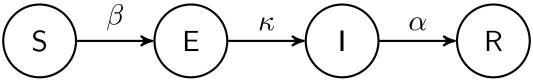
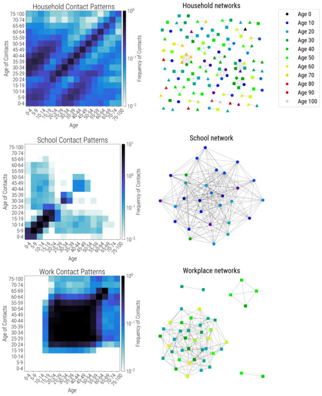
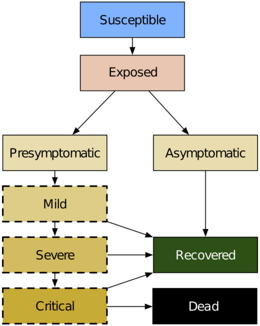


Figure: States graph for the SEIR model

Mathematical models: Agent-based



(a) Transmission networks



(b) States graph

Figure: Covasim model [2]

Mathematical models: Pros & cons

Pros

- Explainable
- Based on many years of research
- Easy to understand and implement

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- Explainable
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- Easy to understand and implement

Cons

- Low representational capabilities
- The represented dynamics are stationary
- Unrealistic assumptions about real-world scenarios

Data-driven models

- Autoregressive Integrated Moving Average (ARIMA) models [3]–[5]

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- Explainable Artificial Neural Network (ANN) encoder [6]

Data-driven models

- Autoregressive Integrated Moving Average (ARIMA) models [3]–[5]
- Explainable Artificial Neural Network (ANN) encoder [6]
- Recurrent Neural Network (RNN) [7], [8]
    - Long Short Term Memory (LSTM)
    - Bidirectional Long Short Term Memory (Bi-LSTM)
    - Gated Recurrent Unit (GRU)

Data-driven models: Pros & cons

Pros

- High prediction accuracy
- Allow for modeling without needing domain knowledge

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Pros

- High prediction accuracy
- Allow for modeling without needing domain knowledge

Cons

- Unexplainable
- Relied on large amount of data
- Inability to capture the true disease dynamics

Novel compartmental models: Using mobility data

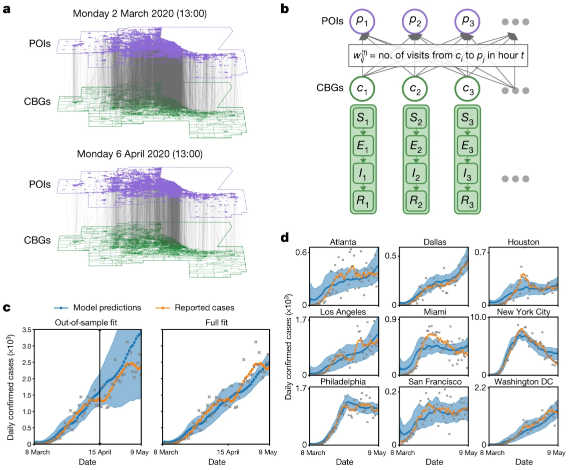
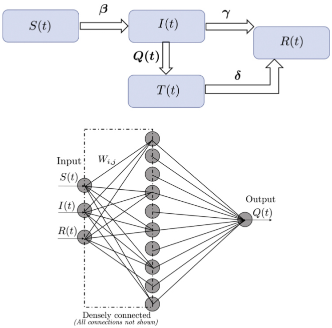
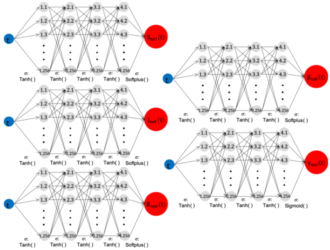


Figure: SEIR model informed with mobility data [9]

Novel compartmental models: Using neural networks



(a) QSIR [10]



(b) Time-dependent SIR [11]

Figure: Predict the parameters of compartmental models with Artificial Neural Networks (ANNs)

Multi-layer perceptron: Graph representation

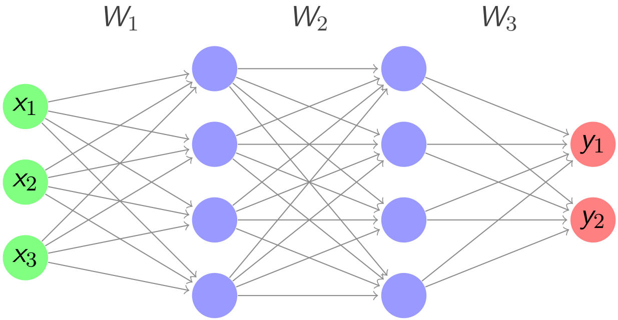


Figure: A Multi-Layer Perceptron (MLP) with four layers

Multi-layer perceptron: Mathematical representation

Definition

$$g(X) = \phi_n(W_n \phi_{n-1}(\cdots(W_2 \phi_1(W_1 X + b_1) + b_2) + \cdots) + b_n)$$

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Theorem

Given appropriate weights, ANNs can approximate any arbitrary function $f : \mathbb{R}^M \to \mathbb{R}^N$ *[12]–[14]*

Training neural networks: Back-propagation

Mean Squared Error (MSE)

$$\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N}(g(X_i) - Y_i)^2$$

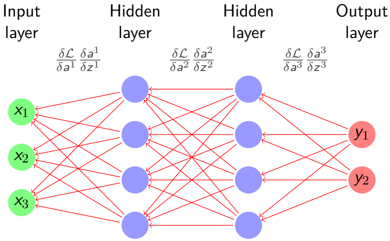

Figure: Graph representation of the back-propagation algorithm

Physics-informed neural networks

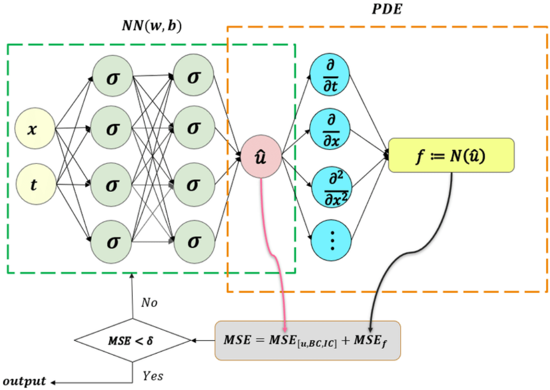

Figure: The schematic of Physics-Informed Neural Networks (PINNs) for solving Partial Differential Equations (PDEs) [15].

Neural ordinary differential equations: Idea

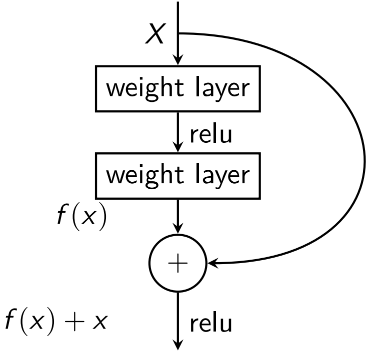

Figure: Skip connection

Neural ordinary differential equations: Idea

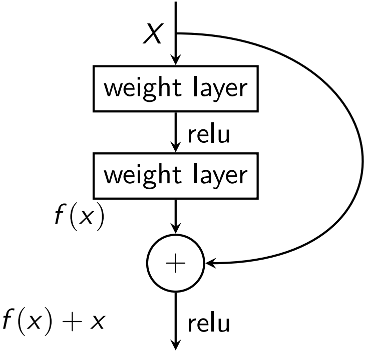

Figure: Skip connection

Observation

$$h_{t+1} = h_t + f(h_t, \theta_t)$$
$$\Leftrightarrow \frac{dh(t)}{dt} = f(h(t), t, \theta)$$

Neural ordinary differential equations: Formulation

Neural Ordinary Differential Equation (NeuralODE) output

$$z(t_1) = z(t_0) + \int_{t_0}^{t_1} f(z(t), t, \theta) dt$$

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- Memory efficiency

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$$z(t_1) = z(t_0) + \int_{t_0}^{t_1} f(z(t), t, \theta)dt$$

- Memory efficiency
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- Scalable and invertible normalizing flows

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- Memory efficiency
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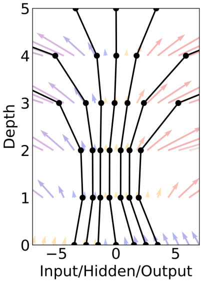
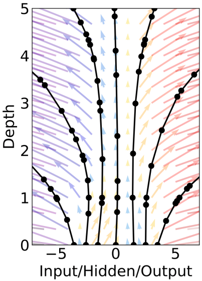
- Scalable and invertible normalizing flows
- Continuous time series models

Neural ordinary differential equations: Outputs



(a) ResNet



(b) NeuralODE

Figure: Comparison between ResNet's discrete state transformations and Neural Ordinary Differential Equation (NeuralODE) continuous state transformations

Neural ordinary differential equations: Gradients

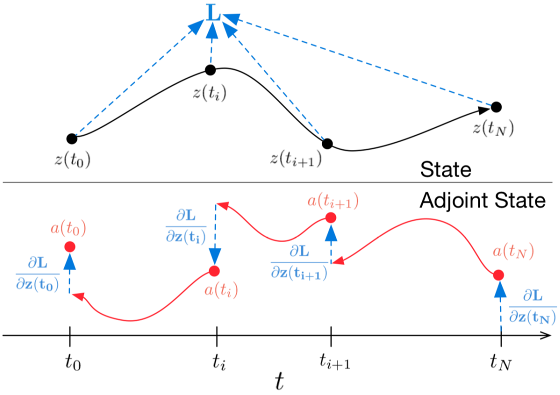


Figure: Reverse-mode differentiation of an Ordinary Differential Equation (ODE) solution. [16]

Outline for Methodologies

3. Methodologies
   - Model definition
   - Datasets
   - Parameters estimation
   - Experiments

Universal differential equation

Universal Differential Equation (UDE) [17] is a neural network architecture based on NeuralODE

Definition

$$u' = f(u, t, U_\theta(u, t))$$

System of ODEs

Definition

$$
\begin{aligned}
S' &= -\frac{\beta(t)SI}{N} \\
E' &= \frac{\beta(t)SI}{N} - \gamma E \\
I' &= \gamma E - \lambda I \\
R' &= (1-\alpha(t)) * \lambda * I \\
D' &= \alpha(t) * \lambda * I \\
N' &= -\alpha * \lambda * I \\
C' &= -C + \gamma * E \\
T' &= \gamma * E
\end{aligned}
$$

System parameters

Box constraints for time-independent parameters

$$\gamma = \gamma_L + (\gamma_U - \gamma_L) * \sigma(\gamma')$$
$$\lambda = \lambda_L + (\lambda_U - \lambda_L) * \sigma(\lambda')$$

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Neural networks for time-dependent parameters

$$\beta(t) = \mathcal{NN}_{\theta_1}(\mathcal{F})$$
$$\alpha(t) = \mathcal{NN}_{\theta_2}\left(\frac{t}{t_{\text{max}}}, \frac{I(t-1)}{N(t-1)}, \frac{R(t-1)}{N(t-1)}, \frac{D(t-1)}{N(t-1)}\right)$$

- Hidden layers activation: mish [18]
- Output layer activation $\nu_i = \nu_{i,L} + (\nu_{i,U} - \nu_{i,L}) * \sigma(z_i)$

Neural network input features

1st. set of input features

$$\mathcal{F}_1(t) = \left\{ \frac{t}{t_{\max}}, \frac{S(t-1)}{N(t-1)}, \frac{E(t-1)}{N(t-1)}, \frac{I(t-1)}{N(t-1)} \right\}$$

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2nd. set of input features

$$\mathcal{F}_2(t) = \mathcal{F}_1(t) \cup \{\text{MovementRange}(t)\}$$

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2nd. set of input features

$$\mathcal{F}_2(t) = \mathcal{F}_1(t) \cup \{\text{MovementRange}(t)\}$$

3rd. set of input features

$$\mathcal{F}_3(t) = \mathcal{F}_2(t) \cup \{\text{SocialProximityToCases}(t)\}$$

Covid-19 cases time series

date	infective	confirmed	recoveries	deaths
1/22/20	0	0	0	0
1/23/20	2	2	0	0
1/24/20	2	2	0	0
...	...	...	...	...

Table: Structure of the processed Covid-19 time series datasets

.

- John Hopkins University (JHU) Covid-19 public datasets
- *VNExpress* Covid-19 public dashboard
- *VnCDC* Covid-19 public dashboard

Facebook's movement range maps dataset

ds	country	polygon	rel. change	stay put ratio
...	...	...	...	...
2021-01-01	VNM	VNM.1.10_1	0.125	0.270
2021-01-02	VNM	VNM.1.10_1	0.052	0.259
2021-01-03	VNM	VNM.1.10_1	0.185	0.269
...	...	...	...	...

Table: Structure of Facebook's Movement Range Maps (MRMs) dataset.

Facebook's social connectedness index

Social Connectedness Index (SCI)

$$\text{SCI}_{i,j} = \frac{\text{FB connections}_{i,j}}{\text{FB users}_i * \text{FB users}_j},$$

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Social Proximity to Cases (SPC) index [19]

$$\text{SPC}_{i,t} = \sum_{j}^{C} \text{Cases per 10k}_{j,t} \frac{\text{SCI}_{i,j}}{\sum_h^C \text{SCI}_{i,h}},$$

Population data

ID_1	NAME_1	AVGPOPULATION
3	Ha Noi	8.2466e6
62	Vinh Phuc	1.1712e6
...	...	...
1001.0	Autauga, Alabama, US	55869
1003.0	Baldwin, Alabama, US	223234
...	...	...

Table: Structure of the processed average population datasets

.

- Vietnam General Statistics Office (GSO)
- JHU Covid-19 datasets

Loss function

Regularized MSE with scaled outputs

$$\mathcal{L}(\hat{y}, y) = \frac{1}{T} \sum_{i=1}^{N} \sum_{t=0}^{T-1} \left[e^{\zeta t} \left(\frac{\hat{y}_{i,t} - y_{i,t}}{max(y_i) - min(y_i)} \right)^2 \right] + \frac{\lambda}{2T} (\|\theta_1\|_2^2 + \|\theta_2\|_2^2)$$

Training process

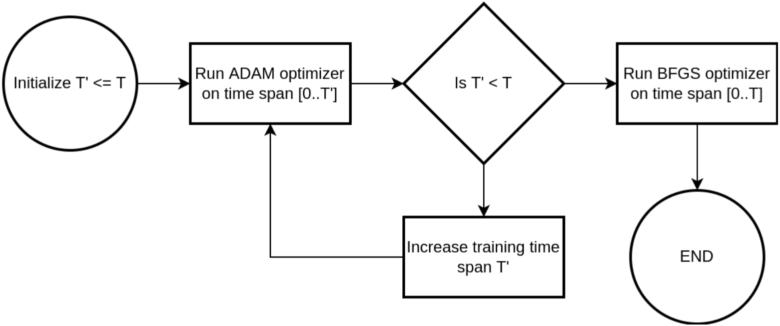

Figure: Model training procedure

Data preprocessing

- Applied 7-day moving average to all datasets
- Applied min-max scaling on the Movement Range Map (MRM) dataset and Social Proximity to Cases (SPC) index

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  - Vietnam: First date when the total confirmed cases passed 500
  - United States: 1st July 2021

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- Training period: 48 days
  - Vietnam: First date when the total confirmed cases passed 500
  - United States: 1st July 2021
- Testing period: 28 days

Differential equation solver configurations

- Tsit5 solver (Tsitouras Runge-Kutta 5/4 method [20])
- InterpolatingAdjoint technique for approximating gradients in NeuralODE [21]

Optimizer configurations

- Initial time span of 4 days
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- ADAM optimizer
  - Learning rate 0.05
    - decay rate 0.5
    - decay step 1000
    - decay limit 0.00001
  - 1000 iterations on each time span

Optimizer configurations

- Initial time span of 4 days
- Time span increment of 4 days
- Time weighting parameter $\zeta = -0.001$
- ADAM optimizer
  - Learning rate 0.05
    - decay rate 0.5
    - decay step 1000
    - decay limit 0.00001
  - 1000 iterations on each time span
- BFGS optimizer
  - *Initial stepnorm* 0.01
  - 1000 iterations on the full time span

Initial conditions

Location	$S(0)$	$E(0)$	$I(0)$	$R(0)$
Vietnam	9.75e7	25	5	2817
Ho Chi Minh city	9.22e6	173.5	34.7	360.1
Binh Duong	2.58e6	206.4	41.2	314.7
Dong Nai	3.17e6	295	59	194.8
Long An	1.71e6	217.8	43.5	300.5
United States	2.99e8	71890	14378	3.31e7
Los Angeles, California	8.78e6	2500	500	1.22e6
Cook, Illinois	4.59e6	615	123	546508
Harris, Texas	4.30e6	930	186	396430
Maricopa, Arizona	3.92e6	1325	265	549899

Table: Initial conditions for the system of ODEs

Initial parameters

Parameter	Value	Lower bound	Upper bound
β	*N/A*	0.05	1.67
γ	1/4	1/4	1/4
λ	1/14	1/14	1/14
α	*N/A*	0.005	0.05
θ_1	*glorot_normal*	*N/A*	*N/A*
θ_2	*glorot_normal*	*N/A*	*N/A*

Table: Initial parameters for the system of ODEs

Outline for Results

4. Results
   - Model's outputs for Vietnam and the United States
   - Model's outputs for counties in the United States
   - Model's outputs for provinces in Vietnam

Forecasts: Country-level data

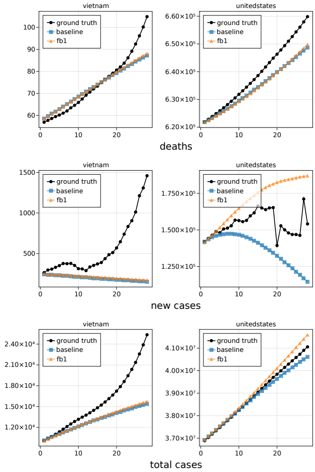
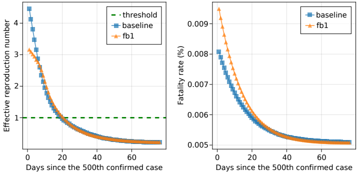
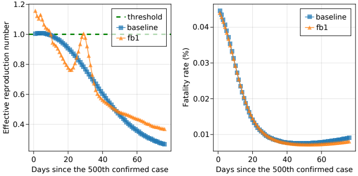


Figure: Forecasts made by different versions of the model

R_0 and fatality rate: Country-level data



(a) Viet nam



(b) United States

Figure: Disease metrics learned by different versions of the model

MAPE: Country-level data

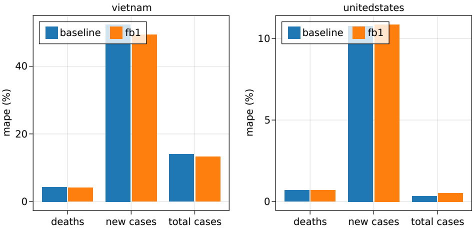


Figure: MAPE for 28-day forecast horizon

Forecasts: Counties in the United States

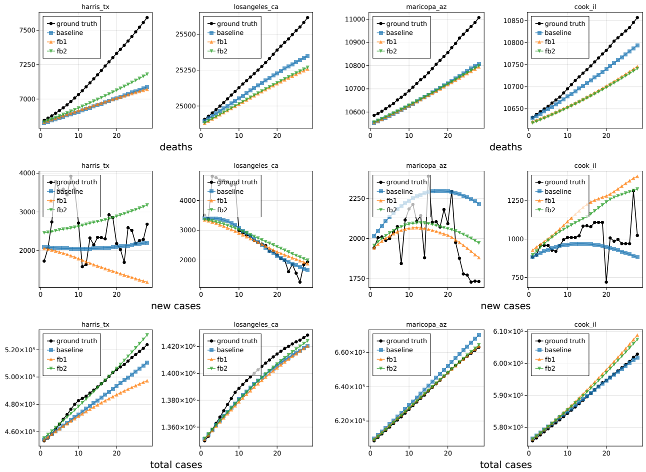
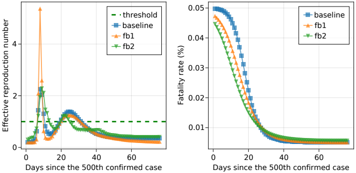
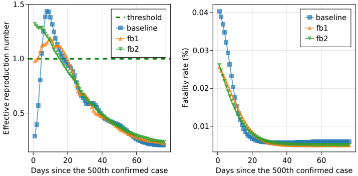


Figure: Forecasts made by different versions of the model

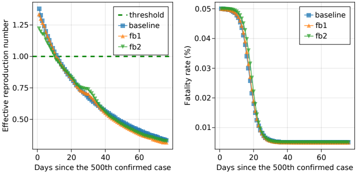
R_0 and fatality rate: Counties in the United States



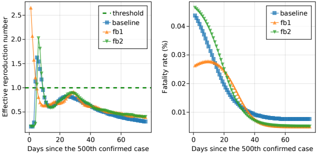
(a) Harris, TX



(b) Los Angeles, CA



(c) Maricopa, AZ



(d) Cook, IL

Figure: Disease metrics learned by different versions of the model

MAPE: Counties in the United States

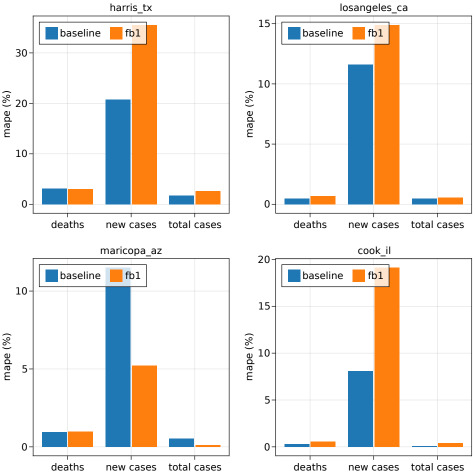


Figure: MAPE for 28-day forecast horizon

Forecasts: Provinces in Vietnam

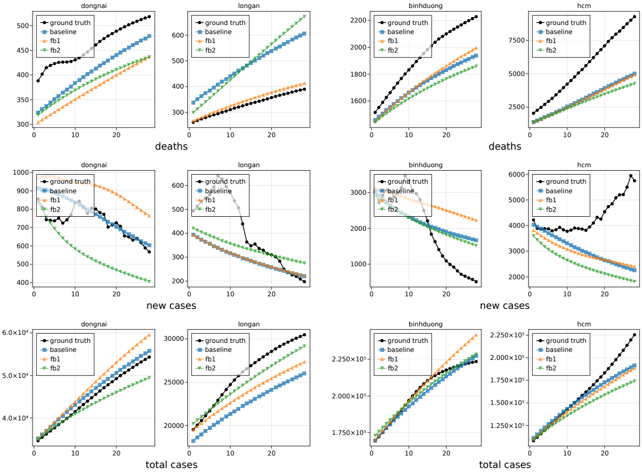
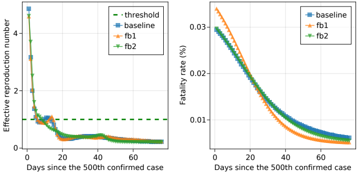
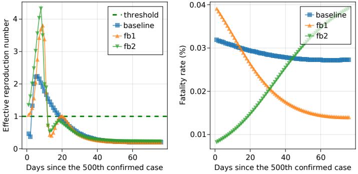


Figure: Forecasts made by different versions of the model

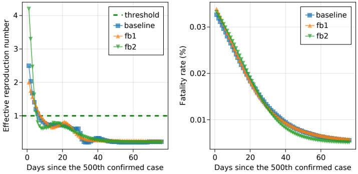
R_0 and fatality rate: Provinces in Vietnam



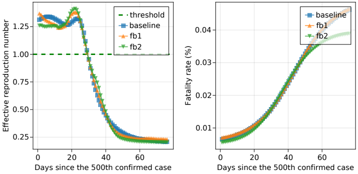
(a) Dong Nai



(b) Long An



(c) Binh Duong



(d) Ho Chi Minh city

Figure: Disease metrics learned by different versions of the model

MAPE: Provinces in Vietnam

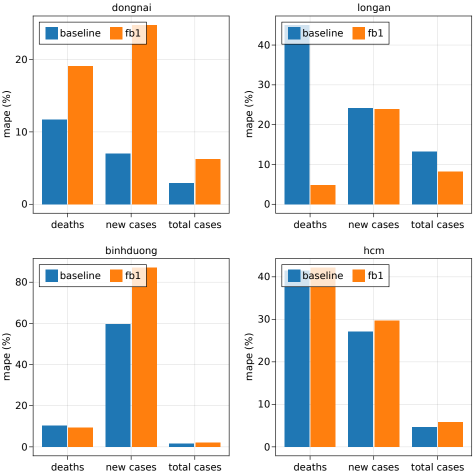


Figure: MAPE for 28-day forecast horizon

Outline for Discussion

5. Discussion
   - Model's convergence and generalizability
   - Model's limitations

About the results

- Could capture the disease trends with high accuracy

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- Accuracy get lower as we extrapolated further into the future

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Outline for Conclusion

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- Add mobility data to the model and show that ineffectiveness of the approach

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- Use methods such as SINDy to find a governing equation and improve the model interpretability

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