

Corrigé exercices

Jour 1

$$a) \frac{2}{3} + \frac{3}{4} = \frac{2 \times 4}{3 \times 4} + \frac{3 \times 3}{4 \times 3} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

$$\frac{2}{3} - \frac{3}{4} = \frac{8}{12} - \frac{9}{12} = \frac{-1}{12}$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{\cancel{2} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 2} = \frac{1}{2}$$

$$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$$

$$b) 2 - (x - 3) = 2 - x + 3 = -x + 2 + 3 = -x + 5$$

$$3 - (2x + 2) = 3 - 2x - 2 = -2x + 3 - 2 = -2x + 1$$

$$2 \times (5 + x) + x = 2 \times 5 + 2 \times x + x = 10 + 2x + x = 10 + 3x$$

$$c) V = (8 \text{ cm}) \times (5 \text{ cm}) \times (4 \text{ cm}) = 160 \text{ cm}^3 = 0,160 \text{ dm}^3 = 0,16 \text{ L}$$

Jour 2

$$a) \frac{2}{3} + \frac{3}{4} \times \frac{5}{9} = \frac{2}{3} + \frac{3 \times 5}{4 \times 9} = \frac{2}{3} + \frac{\cancel{3} \times 5}{\cancel{4} \times 3 \times 3} = \frac{2 \times 4}{3 \times 4} + \frac{5}{4 \times 3} = \frac{8}{12} + \frac{5}{12} = \frac{13}{12}$$

$$\frac{2}{3} : \frac{1}{4} - \frac{3}{4} = \frac{2}{3} \times \frac{4}{1} - \frac{3}{4} = \frac{8}{3} - \frac{3}{4} = \frac{8 \times 4}{3 \times 4} - \frac{3 \times 3}{4 \times 3} = \frac{32}{12} - \frac{9}{12} = \frac{23}{12}$$

$$b) [2(x+2)] + [3(x-3)] \\ = [2 \times x + 2 \times 2] + [3 \times x - 3 \times 3] \\ = 2x + 4 + 3x - 9 \\ = (2+3)x + 4 - 9 \\ = 5x - 5$$

$$3 + 2(2x+2) = 3 + [2 \cdot 2x + 2 \cdot 2]$$

$$= 3 + 4x + 4$$

$$= 4x + 7$$

$$2x(5+x) - (x-1) = [2 \cdot 5 + 2 \cdot x] - x + 1$$

$$= 10 + 2x - x + 1$$

$$= 2x - x + 10 + 1$$

$$= x + 11$$

c)  $V = \pi R^2 H = \pi \times (3 \text{ cm})^2 \times (5 \text{ cm})$

$$= 45\pi \text{ cm}^3$$

$$= 45\pi \text{ mL } \approx 141 \text{ mL}$$

Jour 3

a)  $\frac{-1}{4} + \left(-\frac{3}{2}\right) + \frac{5}{2} = \frac{-1}{4} + \frac{-6}{4} + \frac{10}{4} = \frac{(-1) + (-6) + 10}{4} = \frac{3}{4}$

$$-7 - (-9) - 17 + (-41) + 1 = (-7) + 9 + (-17) + (-41) + 1$$

$$= [9 + 1] - (7 + 17 + 41)$$

$$= 10 - 65$$

$$= -55$$

b)  $2(x+2) - (x-3) = [2x + 2 \cdot 2] - x + 3$

$$= 2x + 4 - x + 3$$

$$= 2x - x + 4 + 3$$

$$= x + 7$$

$$3 - 2(2x+2) = 3 - [2 \cdot 2x + 2 \cdot 2]$$

$$= 3 - [4x + 4]$$

$$= 3 - 4x - 4$$

$$= -4x + 3 - 4$$

$$= -4x - 1$$

$$c) V = \frac{1}{3} \times \pi R^2 H = \frac{1}{3} \times \pi \times (3 \text{ cm})^2 \times (5 \text{ cm})$$

$$= 15\pi \frac{\text{cm}^3}{\text{mL}} \approx 47 \text{ cm}^3 = 47 \text{ mL}$$

Tour +

$$\begin{aligned} a) & (5 - 3 + 7 - 1) + (-9 + 4 - 1) - (-3 - 7 + 2) \\ &= (5 + 7 - 3 - 1) + (-9 - 1) - (2 - 3 - 7) \\ &= (12 - 4) + (-10) - (-2 - 10) \\ &= 8 + (-6) - (-8) \\ &= 2 + 8 \\ &= 10 \end{aligned}$$

$$\begin{aligned} & \left( -\frac{2}{3} - \frac{3}{5} + 1 \right) - \left( \frac{1}{3} + \frac{4}{5} - 2 \right) + \left( -1 + \frac{7}{3} \right) \\ &= -\frac{2}{3} - \frac{3}{5} + 1 - \frac{1}{3} - \frac{4}{5} + 2 + (-1) + \frac{7}{3} \\ &= 1 + 2 + (-1) + \left( -\frac{2}{3} \right) - \frac{1}{3} + \frac{7}{3} - \frac{3}{5} - \frac{4}{5} \\ &= 2 + \frac{7 - 2 - 1}{3} + \frac{-3 - 4}{5} \\ &= 2 + \frac{4}{3} - \frac{7}{5} \quad \cancel{=} \\ &= \frac{2 \times 15}{1 \times 15} + \frac{4 \times 5}{3 \times 5} - \frac{7 \times 3}{5 \times 3} \\ &= \frac{30}{15} + \frac{20}{15} - \frac{21}{15} \\ &= \frac{17}{15} \quad \cancel{-29} \end{aligned}$$

$$\begin{aligned}
 b) & [12 - (14 - 5 + 1)] + [-14 + (3 - 2)] \\
 & = 12 - (14 - 5 + 1) + (-14) + (3 - 2) \\
 & = 12 - 14 + 5 - 1 - 14 + 3 - 2
 \end{aligned}$$

$$\begin{aligned}
 & [(5 - 9) + (3 - 5)] - [(7 + 3 - 5) - (7 - 10)] \\
 & = [5 - 9 + 3 - 5] - [7 + 3 - 5 - 7 + 10] \\
 & = 5 - 9 + 3 - 5 - 7 - 3 + 5 + 7 - 10
 \end{aligned}$$

$$\begin{aligned}
 c) & 2h15\text{ min} + 11h47\text{ min} = 13h62\text{ min} \\
 & 13h62\text{ min} - 3h17\text{ min} = 10h45\text{ min}
 \end{aligned}$$

### Jour 5

$$\begin{aligned}
 a) & \left(\frac{4}{5}\right)^3 - \left(\frac{2}{3}\right)^{-2} = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} - \frac{1}{\frac{2}{3}} \times \frac{1}{\frac{2}{3}} = \frac{4^3}{5^3} - \frac{3}{2} \times \frac{3}{2} \\
 & = \frac{64}{125} - \frac{9}{4} \\
 & = \frac{64 \times 4}{125 \times 4} - \frac{9 \times 125}{4 \times 125} \\
 & = \frac{256}{500} - \frac{1125}{500} = \frac{-869}{500}
 \end{aligned}$$

$$\begin{aligned}
 b) & (a - b + c) - (d - e - f) + (b - a) = a - b + c - d + e + f + b - a \\
 & (= c - d + e + f)
 \end{aligned}$$

$$\begin{aligned}
 & [(a - b) - (a - 5)] + [b - 7 - (a - 3)] = [a - b - a + 5] + [b - 7 - a + 3] \\
 & = a - b - a + 5 + b - 7 - a + 3 \\
 & (= -a + 1)
 \end{aligned}$$

$$\begin{aligned}
 & [12 - (a - b) + 6] - [15 + (b - a - 13)] = [12 - a + b + 6] - [15 + b - a - 13] \\
 & = 12 - a + b + 6 - 15 - b + a + 13 \\
 & (= 16)
 \end{aligned}$$

c) Ces étapes donnent:

$$x$$

$$x+4$$

$$(x+4) \times 2$$

Le programme renvoie donc  $2(x+4)$  (ou  $2x+8$ )

$$\begin{aligned} d) \quad 2+x+4(1-x) &= 2+x+4 \cdot 1 - 4 \cdot x \\ &= 2+x+4-4x \\ &= x-4x+2+4 \\ &= (1-4)x+6 \\ &= -3x+6 \end{aligned}$$

$$\begin{aligned} 4-x+x(1-2x) &= 4-x+x \cdot 1 - x \cdot 2x \\ &= 4-x+x-2x^2 \\ &= 4-2x^2 \end{aligned}$$

$$\begin{aligned} 5(1+x^3)-x^2(3-x) &= 5 \cdot 1 + 5 \cdot x^3 - [x^2 \cdot 3 - x^2 \cdot x] \\ &= 5+5x^3 - [3x^2-x^3] \\ &= 5+5x^3-3x^2+x^3 \\ &= 6x^3-3x^2+5 \end{aligned}$$

## Tour 6

$$\begin{aligned} a) \quad -\frac{3}{2}x + \frac{5}{4}x - 3x^2 + \frac{x}{6} - \frac{5}{2}x^2 + 5 + 4x^2 \\ &= -3x^2 - \frac{5}{2}x^2 + 4x^2 - \frac{3}{2}x + \frac{5}{4}x + \frac{1}{6}x + 5 \\ &= \left(-3-\frac{5}{2}+4\right)x^2 + \left(\frac{-3}{2}+\frac{5}{4}+\frac{1}{6}\right)x + 5 \\ &= \left(1-\frac{5}{2}\right)x^2 + \left(\frac{-18}{12}+\frac{15}{12}+\frac{2}{12}\right)x + 5 \\ &= \frac{3}{2}x^2 - \frac{1}{12}x + 5 \end{aligned}$$

$$\begin{aligned}
 & \frac{3}{2}x^2 + xy + y^2 - 2yx + \frac{x^2}{3} - \frac{3}{2}x^2 \\
 &= \frac{3}{2}x^2 + \frac{x^2}{3} - \frac{3}{2}x^2 + y^2 + xy - 2xy \quad (\text{car } xy = yx) \\
 &= \left(\frac{3}{2} + \frac{1}{3} - \frac{3}{2}\right)x^2 + y^2 + (1-2)xy \\
 &= \frac{1}{3}x^2 + y^2 - xy \quad \text{car } (-1)xy = -xy
 \end{aligned}$$

$$\begin{aligned}
 & \frac{2}{5}a^2b + 3a^3 - 4ab^2 + \frac{5}{2}a^2b + \frac{7}{2}b^3 - b + 2ba \\
 &= 3a^3 + \frac{2}{5}a^2b + \frac{5}{2}a^2b - 4ab^2 + 2ab^2 + \frac{7}{2}b^3 - b^3 \\
 &= 3a^3 + \left(\frac{2}{5} + \frac{5}{2}\right)a^2b + (2-4)ab^2 + \left(\frac{7}{2} - 1\right)b^3 \\
 &= 3a^3 + \left(\frac{4}{10} + \frac{25}{10}\right)a^2b + (-2)ab^2 + \left(\frac{7}{2} - \frac{2}{2}\right)b^3 \\
 &= 3a^3 - \frac{29}{10}a^2b - 2ab^2 + \frac{5}{2}b^3
 \end{aligned}$$

b)  $(x+y)(x-y)$  (produit de la somme  $x+y$  par la différence  $x-y$ )

Somme: c)  $(-4x^3 - 2x + 2) + (4x - 6x^2 + 5x^3 - 2)$

$$\begin{aligned}
 &= -4x^3 + 5x^3 - 6x^2 - 2x + 4x + 2 - 2 \\
 &= (5 - 4)x^3 - 6x^2 + (4 - 2)x + 0 \\
 &= x^3 - 6x^2 + 2x
 \end{aligned}$$

Différence:  $(-4x^3 - 2x + 2) - (4x - 6x^2 + 5x^3 - 2)$

$$\begin{aligned}
 &= -4x^3 - 2x + 2 - 4x + 6x^2 - 5x^3 + 2 \\
 &= -4x^3 - 5x^3 + 6x^2 - 2x - 4x + 2 + 2 \\
 &= (-4 - 5)x^3 + 6x^2 + (-2 - 4)x + 4 \\
 &= -9x^3 + 6x^2 - 6x + 4
 \end{aligned}$$

On calcule la somme des deux expressions trouvées:

$$\begin{aligned}(x^3 - 6x^2 + 2x) + (-3x^3 + 6x^2 - 6x + 4) \\= x^3 - 3x^3 - 6x^2 + 6x^2 + 2x - 6x + 4 \\= (1-3)x^3 + (-6+6)x^2 + (2-6)x + 4 \\= -2x^3 + 0x^2 + (-4)x + 4 \\= -2x^3 - 4x + 4\end{aligned}$$

En divisant par 2:  $\frac{-8x^3 - 4x + 4}{2} = \frac{-8}{2}x^3 - \frac{4}{2}x + \frac{4}{2} = -4x^3 - 2x + 2$ .

C'est la première des expressions de départ.

On forme maintenant la différence:  $(x^3 - 6x^2 + 2x) - (-3x^3 + 6x^2 - 6x + 4)$

$$\begin{aligned}= x^3 - 6x^2 + 2x + 3x^3 - 6x^2 + 6x - 4 \\= x^3 + 3x^3 - 6x^2 - 6x^2 + 2x + 6x - 4 \\= (1+3)x^3 + (6-6)x^2 + (2+6)x - 4 \\= 10x^3 - 12x^2 + 8x - 4\end{aligned}$$

Divisant par 2 il vient:  $\frac{10x^3 - 12x^2 + 8x - 4}{2} = \frac{10}{2}x^3 - \frac{12}{2}x^2 + \frac{8}{2}x - \frac{4}{2}$   
 $= 5x^3 - 6x^2 + 4x - 2$ .

C'est la seconde des expressions de départ.

## Jour 7

Programme A:

2) somme:  $0+10 = 10$  ]  $\rightarrow$  demi-somme:  $\frac{10+(-10)}{2} = \frac{0}{2} = 0$ .  
différence  $0-10 = -10$  ]

Programme B:

somme:  $0+10 = 10$  ]  $\rightarrow$  demi-différence:  $\frac{10-(-10)}{2} = \frac{10+10}{2} = 10$ .  
différence:  $0-10 = -10$  ]

On retrouve les nombres de départ.

$$\text{b) A: } \left[ (-3)+4 = 1 \right] \rightarrow \frac{1+(-7)}{2} = \frac{-6}{2} = -3$$

$$\left[ -3-4 = -7 \right]$$

$$\text{B: } \left[ (-3)+4 = 1 \right] \rightarrow \frac{1-(-7)}{2} = \frac{1+7}{2} = \frac{8}{2} = 4$$

$$\left[ -3-4 = -7 \right]$$

On récupère encore les nombres initiaux

$$\text{c) A: } \left[ \left( -\frac{2}{3} \right) + \left( \frac{3}{7} \right) = \frac{-14}{21} + \frac{9}{21} = \frac{-5}{21} \right] \rightarrow \frac{\frac{-5}{21} + \frac{-23}{21}}{2} = \frac{\frac{-28}{21}}{2} = \frac{-28}{21} \times \frac{1}{2} = \frac{-14}{21}$$

$$\left[ \left( -\frac{2}{3} \right) - \left( \frac{3}{7} \right) = \frac{-14}{21} - \frac{9}{21} = \frac{-23}{21} \right] \rightarrow \frac{\frac{-23}{21}}{2} = \frac{-23}{21} \times \frac{1}{2} = \frac{-2}{3}$$

$$\text{B donne: } \left[ \frac{-5}{21} - \left( \frac{-23}{21} \right) \right] \div 2 = \left[ \frac{-5}{21} + \frac{23}{21} \right] \times \frac{1}{2}$$

$$= \frac{18}{21} \times \frac{1}{2} = \frac{9}{21} = \frac{3}{7}$$

On récupère encore les deux nombres initiaux.

$$\text{d) } \boxed{\text{A}}: \left[ x+y \right] \rightarrow \frac{(x+y)+(x-y)}{2} = \frac{x+x+y-y}{2} = \frac{2x}{2} = x$$

$$\left[ x-y \right]$$

Le programme A renvoie toujours le <sup>1<sup>er</sup></sup> nombre de départ.

$$\text{e) } \boxed{\text{B}}: \left[ x+y \right] \rightarrow \frac{(x+y)-(x-y)}{2} = \frac{x+y-x+y}{2} = \frac{2y}{2} = y$$

$$\left[ x-y \right]$$

Le programme B renvoie toujours le second nombre de départ.