

Exercices de calcul littéral

Exercice 1

1(a) Pour $a=4$: $3a^2 - 2a + 5 = 3 \cdot 4^2 - 2 \cdot 4 + 5 = 3 \cdot 16 - 8 + 5 = 48 - 3 = 45$.

Pour $a=-2$: $3a^2 - 2a + 5 = 3 \cdot (-2)^2 - 2 \cdot (-2) + 5 = 3 \cdot 4 + 4 + 5 = 12 + 9 = 21$.

Pour $a=-1$: $3a^2 - 2a + 5 = 3 \cdot (-1)^2 - 2 \cdot (-1) + 5 = 3 \cdot 1 + 2 + 5 = 3 + 7 = 10$.

(b) Pour $x=3$, $3x^2 - 5x + 7 = 3 \cdot 3^2 - 5 \cdot 3 + 7 = 3 \cdot 9 - 15 + 7 = 27 - 8 = 19$.

Pour $x=5$, $3x^2 - 5x + 7 = 3 \cdot 5^2 - 5 \cdot 5 + 7 = 3 \cdot 25 - 25 + 7 = 75 - 18 = 57$.

Pour $x=-3$, $3x^2 - 5x + 7 = 3 \cdot (-3)^2 - 5 \cdot (-3) + 7 = 3 \cdot 9 + 15 + 7 = 27 + 22 = 49$.

(c) Pour $x=5$, $4x^3 - 12x^2 - 4x + 7 = 4 \cdot 5^3 - 12 \cdot 5^2 - 4 \cdot 5 + 7$
 $= 4 \cdot 125 - 12 \cdot 25 - 20 + 7$
 $= 500 - 300 - 13$
 $= 187$.

Pour $x=-3$, $4x^3 - 12x^2 - 4x + 7 = 4 \cdot (-3)^3 - 12 \cdot (-3)^2 - 4 \cdot (-2) + 7$
 $= 4 \cdot (-27) - 12 \cdot 9 + 8 + 7$
 $= -108 - 108 + 15$
 $= -201$.

Pour $x=\frac{1}{2}$, $4x^3 - 12x^2 - 4x + 7 = 4 \cdot \left(\frac{1}{2}\right)^3 - 12 \cdot \left(\frac{1}{2}\right)^2 - 4 \cdot \frac{1}{2} + 7$
 $= 4 \cdot \frac{1}{8} - 12 \cdot \frac{1}{4} - \frac{4}{2} + 7$
 $= \frac{4}{8} - \frac{12}{4} - 2 + 7$
 $= \frac{1}{2} - 3 - 2 + 7$
 $= \frac{1}{2} + 2 = 2,5 \text{ ou } \frac{5}{2}$.

①

$$(d): \text{Pour } x=+3: \frac{7x^2}{2} - \frac{8x-6}{3} + \frac{3x+7}{4} = \frac{7 \cdot 3^2}{2} - \frac{8 \cdot 3 - 6}{3} + \frac{3 \cdot 3 + 7}{4}$$

$$= \frac{63}{2} - \frac{24-6}{3} + \frac{9+7}{4}$$

$$= \frac{63}{2} - \frac{18}{3} + \frac{16}{4} = 31,5 - 6 + 4 = \underline{\underline{29,5}} \left(= \frac{59}{2}\right).$$

$$\text{Pour } x=-2: \frac{7x^2}{2} - \frac{8x-6}{3} + \frac{3x+7}{4} = \frac{7(-2)^2}{2} - \frac{8(-2)-6}{3} + \frac{3(-2)+7}{4}$$

$$= \frac{7 \cdot 4}{2} - \frac{-16-6}{3} + \frac{-6+7}{4}$$

$$= 14 - \frac{-22}{3} + \frac{1}{4}$$

$$= \frac{14 \cdot 12}{12} + \frac{22 \cdot 4}{3 \cdot 4} + \frac{1 \cdot 3}{4 \cdot 3}$$

$$= \frac{168+88+3}{12} = \underline{\underline{25,3}}$$

$$\text{Pour } x=+5: \frac{7x^2}{2} - \frac{8x-6}{3} + \frac{3x+7}{4} = \frac{7 \cdot 5^2}{2} - \frac{8 \cdot 5 - 6}{3} + \frac{3 \cdot 5 + 7}{4}$$

$$= \frac{7 \cdot 25}{2} - \frac{40-6}{3} + \frac{15+7}{4}$$

$$= \frac{175}{2} - \frac{34}{3} + \frac{22}{4}$$

$$= \frac{175}{2} - \frac{11}{2} - \frac{34}{3}$$

$$= \frac{186}{2} - \frac{34}{3} = \frac{186 \cdot 3}{2 \cdot 3} - \frac{34 \cdot 2}{3 \cdot 2} = \frac{558}{6} - \frac{68}{6} = \underline{\underline{\frac{490}{6}}}$$

$$= \frac{245 \cdot 2}{3 \cdot 2} = \underline{\underline{\frac{245}{3}}}$$

$$\begin{aligned}
 \text{(e): Pour } x=+4: \frac{7-3x}{12} + \frac{2(x-2)}{3} + \frac{5}{4} &= \frac{7-3 \cdot 4}{12} + \frac{2 \cdot (4-2)}{3} + \frac{5}{4} \\
 &= \frac{7-12}{12} + \frac{2 \cdot 2}{3} + \frac{5}{4} = \frac{-5}{12} + \frac{4}{3} + \frac{5}{4} = \frac{-5}{12} + \frac{4 \cdot 4}{3 \cdot 4} + \frac{5 \cdot 3}{4 \cdot 3} \\
 &= \frac{-5}{12} + \frac{16}{12} + \frac{15}{12} = \frac{26}{12} = \underline{\underline{\frac{13}{6}}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Pour } x=+2: \frac{7-3x}{12} + \frac{2(x-2)}{3} + \frac{5}{4} &= \frac{7-3 \cdot 2}{12} + \frac{2 \cdot (2-2)}{3} + \frac{5}{4} \\
 &= \frac{7-6}{12} + \frac{2 \cdot 0}{3} + \frac{5}{4} = \frac{1}{12} + 0 + \frac{5 \cdot 3}{4 \cdot 3} = \frac{1}{12} + \frac{15}{12} = \frac{16}{12} \\
 &= \underline{\underline{\frac{4}{3}}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Pour } x=-3: \frac{7-3x}{12} + \frac{2(x-2)}{3} + \frac{5}{4} &= \frac{7-3 \cdot (-3)}{12} + \frac{2 \cdot (-3-2)}{3} + \frac{5}{4} \\
 &= \frac{7+9}{12} + \frac{2 \cdot (-5)}{3} + \frac{5}{4} = \frac{16}{12} - \frac{10}{3} + \frac{5}{4} \\
 &= \frac{16}{12} - \frac{10 \cdot 4}{3 \cdot 4} + \frac{5 \cdot 3}{4 \cdot 3} = \frac{16}{12} - \frac{40}{12} + \frac{15}{12} = \frac{-9}{12} = \frac{-3 \cdot 3}{4 \cdot 3} = \underline{\underline{-\frac{3}{4}}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(f): Pour } x=-1: (x^2+1)^2 - x^4 - 2x^2 &= (-1^2+1)^2 - (-1)^4 - 2 \cdot (-1)^2 \\
 &= (1+1)^2 - 1 - 2 \cdot 1 \\
 &= 2^2 - 1 - 2 = 4 - 3 = \underline{\underline{1}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Pour } x=\frac{1}{2}: (x^2+1)^2 - x^4 - 2x^2 &= \left(\left(\frac{1}{2}\right)^2+1\right)^2 - \left(\frac{1}{2}\right)^4 - 2 \cdot \left(\frac{1}{2}\right)^2 \\
 &= \left(\frac{1}{4}+1\right)^2 - \frac{1}{16} - 2 \cdot \frac{1}{4} \\
 &= \left(\frac{5}{4}\right)^2 - \frac{1}{16} - \frac{2}{4} \\
 &= \frac{25}{16} - \frac{1}{16} - \frac{8}{16} = \frac{16}{16} = \underline{\underline{1}}.
 \end{aligned}$$

$$Rq: (x^2+1)^2 = (x^2)^2 + 2 \cdot 1 \cdot x^2 + 1^2 \\ = x^4 + 2x^2 + 1$$

donc $(x^2+1)^2 - x^4 - 2x^2 = 1$.

$$\text{Pour } x = -\frac{1}{2}, (x^2+1)^2 - x^4 - 2x^2 = \left(\left(-\frac{1}{2}\right)^2 + 1\right)^2 - \left(-\frac{1}{2}\right)^4 - 2 \cdot \left(-\frac{1}{2}\right)^2 \\ = \left(\frac{1}{4} + 1\right)^2 + \frac{1}{16} - 2 \cdot \frac{1}{4}$$

= 1. (reprendre le calcul précédent).

$$(3): \text{Pour } a = -3, (a^2+1)(a^2-1) + 4a^2 = (-3^2+1)(-3^2-1) + 4 \cdot (-3)^2 \\ = (9+1)(9-1) + 4 \cdot 9 \\ = 10 \cdot 8 + 36 \\ = 80 + 36 = \underline{\underline{116}}.$$

$$\text{Pour } a = +3, (a^2+1)(a^2-1) + 4a^2 = (3^2+1)(3^2-1) + 4 \cdot 3^2 \\ = (9+1)(9-1) + 36 = \underline{\underline{116}}. \quad (\text{voir ci-dessous}).$$

$$\text{Pour } a = \frac{1}{3}, (a^2+1)(a^2-1) + 4a^2 = \left(\frac{1}{3}^2+1\right)\left(\frac{1}{3}^2-1\right) + 4 \cdot \left(\frac{1}{3}\right)^2 \\ = \left(\frac{1}{9}+1\right)\left(\frac{1}{9}-1\right) + 4 \cdot \frac{1}{9} \\ = \frac{10}{9} \cdot \frac{-8}{9} + \frac{4}{9} = \frac{-80}{81} + \frac{4}{9} \\ = \frac{-80}{81} + \frac{36}{81} = \underline{\underline{\frac{44}{81}}}.$$

$$Rq: (x+y)^2 - x^2 - y^2 \quad (1): \text{Pour } x = -5 \text{ et } y = 2: \frac{(x+y)^2 - (x^2+y^2)}{xy} = \frac{(-5+2)^2 - ((-5)^2+2^2)}{-5 \cdot 2} \\ = x^2 + 2xy + y^2 - x^2 - y^2 = 2xy$$

donc $\frac{(x+y)^2 - (x^2+y^2)}{xy} = \frac{2xy}{xy} = 2$.

$$Rq: \frac{a^2-ab}{a^2-2ab+b^2} \quad (2): \text{Pour } a = +7 \text{ et } b = -2: \frac{a^2-ab}{a^2-2ab+b^2} = \frac{7^2 - 7 \cdot (-2)}{7^2 - 2 \cdot 7 \cdot (-2) + (-2)^2} = \frac{49 + 14}{49 + 28 + 4} = \frac{63}{81} = \frac{9 \cdot 7}{9 \cdot 9} = \underline{\underline{\frac{7}{9}}}.$$

$$\frac{a(a-b)}{(a-b)^2} = \frac{a \cdot (a-b)}{(a-b) \cdot (a-b)}$$

$$= \frac{a}{a-b} \rightarrow \frac{7}{7+2} = \frac{7}{9}.$$

④

$$2. \text{ Pour } a=-5 \text{ et } b=3: \quad a^2 - ab + b^2 = (-5)^2 - (-5) \cdot 3 + 3^2 \\ = 25 + 15 + 9 \\ = \underline{\underline{49}}.$$

$$\frac{a^3 + b^3}{a+b} = \frac{(-5)^3 + 3^3}{-5+3} = \frac{-125 + 27}{-2} = \frac{-98}{-2} = \underline{\underline{49}}.$$

$$(a+b)^2 - 3ab = (-5+3)^2 - 3 \cdot (-5) \cdot 3 = (-2)^2 + 45 = 4 + 45 = \underline{\underline{49}}.$$

Remarque: ce n'est pas une coïncidence: $(a+b)^2 - 3ab = a^2 + 2ab + b^2 - 3ab \\ = a^2 - ab + b^2$

$$\text{Et } (a+b)(a^2 - ab + b^2) = a \cdot a^2 - a \cdot ab + a \cdot b^2 + b \cdot a^2 - b \cdot ab + b^3 \\ = a^3 - a^2b + ab^2 + ba^2 - ab^2 + b^3 \\ = a^3 + b^3$$

$$\text{Donc: } \frac{a^3 + b^3}{a+b} = \frac{(a+b)(a^2 - ab + b^2)}{a+b} = a^2 - ab + b^2.$$

$$3. \text{ Pour } a=+4 \text{ et } b=-1: \quad (a+b)^2(a-b) = (4+(-1))^2(4-(-1)) \\ = 3^2 \cdot 5 = 9 \cdot 5 = \underline{\underline{45}}.$$

$$(a^2 - b^2)(a+b) = (4^2 - (-1)^2)(4+(-1)) = (16-1)(3) = 15 \cdot 3 = \underline{\underline{45}}.$$

$$\frac{a^4 + b^4 - 2a^2b^2}{a-b} = \frac{4^4 + (-1)^4 - 2 \cdot 4^2 \cdot (-1)^2}{4 - (-1)} = \frac{256 + 1 - 32}{5} = \frac{225}{5} = \underline{\underline{45}}.$$

Remarque: ce n'est toujours pas un hasard: $(a+b)^2(a-b) = (a+b)(a+b)(a-b) \\ = (a+b)(a^2 - b^2) \\ = (a^2 - b^2)(a+b)$

$$\text{et } (a^2 - b^2)(a+b) = (a^2 - b^2)(a+b) \times \frac{a-b}{a-b} = \frac{(a^2 - b^2)(a+b)(a-b)}{a-b} = \frac{(a^2 - b^2)(a^2 - b^2)}{a-b} \\ = \frac{(a^2 - b^2)^2}{a-b} = \frac{(a^2)^2 + (b^2)^2 - 2 \cdot a^2 \cdot b^2}{a-b} = \frac{a^4 + b^4 - 2a^2b^2}{a-b}$$

4. Le dénominateur vaut, pour $a=+5$ et $b=-2$: $a(a-1)-bb^2 = 5(5-1)-5 \cdot (-2)^2 = 5 \cdot 4 - 5 \cdot 4 = 0$.

On ne peut donc pas calculer cette fraction qui aurait un dénominateur nul.

5. Pour $a=+1$ et $b=+2$, le dénominateur vaut: $4a^2-b^2 = 4 \cdot 1^2 - 2^2 = 4 - 4 = 0$, donc on ne peut pas évaluer l'expression.

6(a) $\left(\frac{-2}{3}\right)a^3x \times (-3y) \times \left(\frac{+2}{5}\right) = \left(\frac{-2}{3} \cdot 3 \cdot \frac{2}{5}\right)a^3xy = +\frac{12}{15}a^3xy = \frac{4}{5}a^3xy$.

Pour $a=-3$, $x=2$ et $y=-1$, on trouve: $\frac{4}{5}(-3)^2 \times 2 \times (-1) = \frac{-4 \times 9 \times 2}{5} = \frac{-72}{5} (= -14,4)$

(b) $xy \times \left(\frac{-2}{3}\right)x^2 \times \frac{3}{4}a^2 = \left(-\frac{2}{3} \cdot \frac{3}{4}\right)xyx^2a^2 = -\frac{2 \times 3}{3 \times 4}a^2x^3y = -\frac{1}{2}a^2x^3y$.

Pour $a=5$, $x=-2$ et $y=3$, on trouve: $-\frac{1}{2} \times 5^2 \times (-2)^3 \times 3 = -\frac{25 \times (-8) \times 3}{2} = \frac{600}{2} = 300$.

(c): $\frac{2}{7}a^2 \times \left(\frac{-3}{4}\right)xy^3 \times \left(\frac{-2}{5}\right)a^3x = \left(\frac{2}{7} \times \frac{-3}{4} \times \frac{-2}{5}\right)a^2xy^3a^3x$
 $= \frac{+2 \times 3}{4 \times 5 \times 7}a^4x^2y^3 = \frac{3}{35}a^4x^2y^3$.

Donc $a=3,5=\frac{7}{2}$, $x=3$, $y=-2$ on trouve: $\frac{3}{35} \times \left(\frac{7}{2}\right)^4 \times 3^2 \times (-2)^3 = \frac{3}{35} \times \frac{7^4}{2^4} \times 9 \times (-2^3)$
 $= \frac{-3 \times 9 \times 2^3 \times 7^4}{5 \times 7 \times 2^4} = \frac{-27 \times 7^3}{5 \times 2}$
 $= \frac{-27 \times 343}{10} = \frac{-9261}{10} = -926,1$.

(d): $\left(\frac{-3}{5}\right)a^2 \times \frac{2}{3}b^3x \times (-x^4) = \left(-\frac{3}{5} \times \frac{2}{3} \times -1\right) \times a^2b^3x^5$
 $= \frac{-3 \times 2 \times -1}{5 \times 3}a^2b^3x^5$
 $= \frac{2}{5}a^2b^3x^5$.

Pour $a=4$, $b=-1$ et $x=-2$, on trouve: $\frac{2}{5}x^2 \times (-1)^2 \times (-2)^5 = \frac{2}{5} \times 1 \times (-32)$
 $= -\frac{32}{5} \times 32 = -\frac{1024}{5}$
 $\underline{\underline{(-204,8)}}$

$$\textcircled{2}) 4x^3 \times (-3y^2) \times \left(-\frac{5}{6}\right)a^2x^3y^5 = \left(4x - 3 + \frac{-5}{6}\right)x^3y^2a^2x^3y^5$$
 $= \frac{+4x-5}{6}a^2x^3y^2y^5$
 $\underline{\underline{= 10a^2x^5y^7.}}$

Pour $a=-\frac{1}{2}$, $x=4$ et $y=\frac{3}{2}$, on obtient: $10 \times \left(\frac{-1}{2}\right)^2 \times 4^5 \times \left(\frac{3}{2}\right)^7$
 $= 10 \times \frac{1}{4} \times \frac{(2^2)^5}{1} \times \frac{3^7}{2^7}$
 $= 10 \times \frac{1 \times 2^{10} \times 3^7}{2^2 \times 1 \times 2^7}$
 $= 10 \times 3^7 \times \frac{2^{10}}{2^9} = 10 \times 3^7 \times 2^{10-9}$
 $\underline{\underline{= 10 \times 3^7 \times 2.}}$

$$\begin{aligned} \text{Enfin } 3^7 &= \underbrace{3 \times 3}_{= 27} \times \underbrace{3 \times 3}_{= 27} \times \underbrace{3 \times 3}_{= 27} \times 3 \\ &= \underbrace{27 \times 27}_{= (25+2)^2} \times 3 \\ &= (25^2 + 2 \times 2 \times 25 + 2^2) \times 3 \\ &= (625 + 100 + 4) \times 3 \\ &= 729 \times 3 = 2187 \end{aligned}$$

D'après on obtient finalement $10 \times 2187 \times 2$
 $= 21870 \times 2$
 $\underline{\underline{= 43740.}}$

7. a) $\frac{2}{3}ax - \frac{1}{2}ax + \frac{3}{4}ax - \frac{5}{6}ax = \left[\frac{2}{3} - \frac{1}{2} + \frac{3}{4} - \frac{5}{6}\right]ax$
 $= \left[\frac{8}{12} - \frac{6}{12} + \frac{9}{12} - \frac{10}{12}\right]ax = \frac{1}{12}ax.$

(7)

$$(b): -\frac{3}{5}a^2bx + \frac{1}{4}a^3bx - \frac{7}{2}a^3bx + \frac{1}{10}a^3bx = \left(-\frac{3}{5} + \frac{1}{4} - \frac{7}{2} + \frac{1}{10}\right)a^3bx$$

$$= \left(-\frac{12}{20} + \frac{5}{20} - \frac{70}{20} + \frac{2}{20}\right)a^3bx$$

$$= \frac{-75}{20}a^3bx = \underline{\underline{-\frac{15}{4}a^3bx}}$$

$$(c): -\frac{4}{7}a^2b^3x + \frac{5}{2}a^2b^3x - \frac{5}{4}a^2b^3x = \left(-\frac{4}{7} + \frac{5}{2} - \frac{5}{4}\right)a^2b^3x$$

$$= \left[-\frac{16}{28} + \frac{70}{28} - \frac{35}{28}\right]a^2b^3x$$

$$= \underline{\underline{\frac{19}{28}a^2b^3x}}$$

$$(d): \frac{3}{4}a^2b^3x^4y - \frac{2}{3}a^2b^3x^4y + \frac{1}{4}a^2b^3x^4y = \left[\frac{3}{4} - \frac{2}{3} + \frac{1}{4}\right]a^2b^3x^4y$$

$$= \left[1 - \frac{2}{3}\right]a^2b^3x^4y = \underline{\underline{\frac{1}{3}a^2b^3x^4y}}$$

Exercise 2

$$\begin{aligned}
 1. (a) & -\frac{3}{2}x + \frac{5}{4}x - 3x^2 + \frac{x}{6} - \frac{5}{2}x^2 + 5 + 4x^2 \\
 & = -3x^2 + 4x^2 - \frac{5}{2}x^2 - \frac{3}{2}x + \frac{5}{4}x + \frac{x}{6} + 5 \\
 & = \left(-3 + 4 - \frac{5}{2}\right)x^2 + \left(-\frac{3}{2} + \frac{5}{4} + \frac{1}{6}\right)x + 5 \\
 & = \left(1 - \frac{5}{2}\right)x^2 + \left(-\frac{18}{12} + \frac{15}{12} + \frac{2}{12}\right)x + 5 \\
 & = \underline{\underline{-\frac{3}{2}x^2 - \frac{1}{12}x^2 + 5}}
 \end{aligned}$$

$$\begin{aligned}
 (b): & \frac{3}{2}x^2 + xy + y^2 - 2xy + \frac{x^2}{3} - \frac{3}{2}x^2 \\
 & = \frac{3}{2}x^2 + \frac{x^2}{3} - \frac{3}{2}x^2 + xy - 2xy + y^2 \\
 & = \underline{\underline{\frac{1}{3}x^2 - xy + y^2}}
 \end{aligned}$$

$$\begin{aligned}
 (c) & 4a^2 - \frac{2}{3}a - \frac{3}{5}a^2 + \frac{1}{3}a - 5a - \frac{2}{15}a^2 = 4a^2 - \frac{3}{5}a^2 - \frac{2}{15}a^2 - \frac{2}{3}a + \frac{1}{3}a - 5a \\
 & = \left(4 - \frac{3}{5} - \frac{2}{15}\right)a^2 + \left(-\frac{2}{3} + \frac{1}{3} - 5\right)a \\
 & = \left(\frac{60}{15} - \frac{9}{15} - \frac{2}{15}\right)a^2 + \left(-\frac{1}{3} - \frac{15}{3}\right)a \\
 & = \underline{\underline{\frac{43}{15}a^2 - \frac{16}{3}a}}
 \end{aligned}$$

$$\begin{aligned}
 (d) & 3x^2 + \frac{4}{5} - \frac{5}{3}x - 2x^2 - \frac{3}{5}x^3 + 4 - 2x^2 + 7x = -\frac{3}{5}x^3 + 3x^2 - 2x^2 - 2x^2 - \frac{5}{3}x + 7x + \frac{4}{5} + 4 \\
 & = -\frac{3}{5}x^3 + (3 - 2 - 2)x^2 + \left(-\frac{5}{3} + 7\right)x + \left(\frac{4}{5} + \frac{20}{5}\right) \\
 & = -\frac{3}{5}x^3 - 1x^2 + \left(-\frac{5}{3} + \frac{21}{3}\right)x + \frac{24}{5} = \underline{\underline{-\frac{3}{5}x^3 - x^2 + \frac{16}{3}x + \frac{24}{5}}}
 \end{aligned}$$

$$\begin{aligned}
 & (e) 4x^2 - \frac{7}{2} + \frac{3}{5}x - \frac{5}{2}x^2 + \frac{4}{3}x^3 - 5 + \frac{3}{2}x^3 + 7 - 2x \\
 & = \frac{4}{3}x^3 + \frac{3}{2}x^3 + 4x^2 - \frac{5}{2}x^2 + \frac{3}{5}x - 2x - \frac{7}{2} - 5 + 7 \\
 & = \left(\frac{4}{3} + \frac{3}{2}\right)x^3 + \left(4 - \frac{5}{2}\right)x^2 + \left(\frac{3}{5} - 2\right)x - \frac{7}{2} + \frac{4}{2} \\
 & = \frac{17}{6}x^3 + \frac{3}{2}x^2 - \frac{7}{5}x - \frac{3}{2}.
 \end{aligned}$$

$$\begin{aligned}
 & (f) \frac{2}{5}a^3b + 3a^2b^3 - 4ab^3 + \frac{5}{2}a^2b + \frac{7}{2}b^3 - b^3 + 2ab^2 \\
 & = 3a^3 + \frac{2}{5}a^2b + \frac{5}{2}a^2b - 4ab^2 + 2ab^2 + \frac{7}{2}b^3 - b^3 \\
 & = 3a^3 + \left(\frac{2}{5} + \frac{5}{2}\right)a^2b + (-4 + 2)ab^2 + \left(\frac{7}{2} - 1\right)b^3 \\
 & = 3a^3 + \left(\frac{4}{10} + \frac{25}{10}\right)a^2b + (-2)ab^2 + \left(\frac{7}{2} - \frac{2}{2}\right)b^3 \\
 & = 3a^3 + \frac{29}{10}a^2b - 2ab^2 + \frac{5}{2}b^3.
 \end{aligned}$$

$$\begin{aligned}
 3. (a) A+B &= (-4x^3 - 3x + 2) + (4x - 6x^3 + 5x^2 - 2) \\
 &= -4x^3 + 5x^2 - 6x^3 - 2x + 4x + 2 - 2 \\
 &= (-4 + 5)x^3 - 6x^3 + (-2 + 4)x \\
 &= x^3 - 6x^3 + 2x.
 \end{aligned}$$

$$\begin{aligned}
 3. (b) \text{ Si } x=2, \text{ on obtient: } A &= -4(2)^3 - 2 \cdot 2 + 2 = -4 \cdot 8 - 4 + 2 \\
 &= -32 - 2 = \underline{-34}.
 \end{aligned}$$

$$\begin{aligned}
 B &= 4 \cdot 2 - 6 \cdot 2^2 + 5 \cdot 2^3 - 2 \\
 &= 8 - 6 \cdot 4 + 5 \cdot 8 - 2 \\
 &= 8 - 24 + 40 - 2 \\
 &= 48 - 26 = \underline{22}.
 \end{aligned}$$

$$\text{Donc } A+B = -34 + 22 = \underline{-12}.$$

D'autre part, $(B)^3 - 6 \cdot 2^2 + 3 \cdot 2 = 8 - 6 \cdot 4 + 4 = 12 - 24 = \underline{-12}$, ce qui est donc cohérent.

$$\begin{aligned}
 \text{(c)} \quad A - B &= (-4x^3 - 2x + 2) - (4x - 6x^2 + 5x^3 - 2) \\
 &= -4x^3 - 2x + 2 - 4x + 6x^2 - 5x^3 + 2 \\
 &= -4x^3 - 5x^3 + 6x^2 - 2x - 4x + 2 + 2 \\
 &= (-4 + -5)x^3 + 6x^2 + (-2 - 4)x + 4 \\
 &= -9x^3 + 6x^2 - 6x + 4.
 \end{aligned}$$

(d) Pour $x = 3$, on obtient $A = -4 \times 3^3 - 2 \times 3 + 2 = -4 \times 27 - 6 + 2 = -108 - 4 = \underline{-112}$.
 $B = 4 \times 3 - 6 \times 3^2 + 5 \times 3^3 - 2 = 12 - 6 \times 9 + 5 \times 27 - 2 = 10 - 54 + 135 = \underline{91}$.

donc $A - B = -112 - 91 = \underline{-203}$.

D'autre part, $-9 \times 3^3 + 6 \times 3^2 - 6 \times 3 + 4 = -9 \times 27 + 6 \times 9 - 18 + 4$
 $= -81 - 14 = \underline{-95}$, ce qui est cohérent.

$$\begin{aligned}
 \text{3. (a)} \quad &\left(-5x^4 + 3 - \frac{4}{5}x^3\right) + \left(-\frac{2}{3}x^3 - 2x\right) - \left(7x^2 - \frac{4}{5}x + 5x^4\right) \\
 &= -5x^4 + 3 - \frac{4}{5}x^3 - \frac{2}{3}x^3 - 2x - 7x^2 + \frac{4}{5}x - 5x^4 \\
 &= -5x^4 - 5x^4 - \frac{4}{5}x^3 - \frac{2}{3}x^3 - 7x^2 - 2x + \frac{4}{5}x + 3 \\
 &= (-5 - 5)x^4 + \left(-\frac{4}{5} - \frac{2}{3}\right)x^3 - 7x^2 + \left(-2 + \frac{4}{5}\right)x + 3 \\
 &= -10x^4 + \left(-\frac{12}{15} - \frac{10}{15}\right)x^3 - 7x^2 + \left(-\frac{10}{5} + \frac{4}{5}\right)x + 3 \\
 &= -10x^4 - \frac{22}{15}x^3 - 7x^2 - \frac{6}{5}x + 3.
 \end{aligned}$$

$$\begin{aligned}
 \text{(b).} \quad &(12x^3 + 2x^2 - 5x + 13) + (3x + 5 - 4x^3) - (5x^3 - 8 + 2x^2) \\
 &= 12x^3 + 2x^2 - 5x + 13 + 3x + 5 - 4x^3 - 5x^3 + 8 - 2x^2 \\
 &= 12x^3 - 4x^3 - 5x^3 + 2x^2 - 2x^2 - 5x + 3x + 13 + 5 + 8 \\
 &= (12 - 4 - 5)x^3 + (2 - 2)x^2 + (-5 + 3)x + 26 \\
 &= 3x^3 - 2x + 26
 \end{aligned}$$

$$\begin{aligned}
 & (c). (a^3 - 3a^2b + 3ab^2 - b^3) + (a^3 + 3a^2b + 3ab^2 + b^3) - (6ab^2 - 3a^3) \\
 &= a^3 - 3a^2b + 3ab^2 - b^3 + a^3 + 3a^2b + 3ab^2 + b^3 - 6ab^2 + 3a^3 \\
 &= a^3 + a^3 + 3a^3 - 3a^2b + 3a^2b + 3ab^2 + 3ab^2 - 6ab^2 - b^3 + b^3 \\
 &= (1+1+3)a^3 + (-3+3)a^2b + (3+3-6)ab^2 + (-1+1)b^3 \\
 &= \underline{\underline{5a^3}}.
 \end{aligned}$$

$$\begin{aligned}
 & 4.(a) (3x-5) + [2x-5 - (3x-2y+4) - (4x-3y-9)] \\
 &= 3x-5 + [2x-5 - 3x+2y-4 - 4x+3y+9] \\
 &= 3x-5 + 2x-5 - 3x+2y-4 - 4x+3y+9 \\
 &= 3x+2x-3x-4x+2y+3y-5-5-4+9 \\
 &= (3+2-3-4)x + (2+3)y - 5 \\
 &= \underline{\underline{-2x+5y-5}}.
 \end{aligned}$$

$$\begin{aligned}
 & (b). (2x-5y+7) - [(3x+2y-3) - (4x+4y-2)] - [2x-3y+4] \\
 &= 2x-5y+7 - [3x+2y-3 - 4x-4y+2] - [2x-3y+4] \\
 &= 2x-5y+7 - 3x-2y+3+4x+4y-2 - 2x+3y+4 \\
 &= 2x-3x+4x-2x-5y-2y+4y+3y+7+3-2+4 \\
 &= (2-3+4-2)x + (-5-2+4+3)y + 12 \\
 &= 1.x + 0.y + 12 \\
 &= \underline{\underline{x+12}}.
 \end{aligned}$$

$$\begin{aligned}
 & (c). [(x-2y+5) - (3x+2y+7)] - [(2x+3) - (4y-2)] \\
 &= [x-2y+5-3x-2y-7] - [2x+3-4y+2] \\
 &= x-2y+5-3x-2y-7-2x-3+4y-2 \\
 &= x-3x-2x-2y-2y+4y+5-7-3-2 \\
 &= (1-3-2)x + (-2-2+4)y-7 \\
 &= -4x+0y-7 \\
 &= \underline{\underline{-4x-7}}.
 \end{aligned}$$

$$\begin{aligned}
 5. A+B+C &= (3x^2 - 4x + 5) + (2x^2 + 5x - 4) + (4x^2 - x + 3) \\
 &= 3x^2 - 4x + 5 + 2x^2 + 5x - 4 + 4x^2 - x + 3 \\
 &= 3x^2 + 2x^2 + 4x^2 - 4x + 5x - x + 5 - 4 + 3 \\
 &= (3+2+4)x^2 + (-4+5-1)x + 4 \\
 &= \underline{\underline{8x^2 + 4}}.
 \end{aligned}$$

$$\begin{aligned}
 A+B-C &= (3x^2 - 4x + 5) + (2x^2 + 5x - 4) - (4x^2 - x + 3) \\
 &= 3x^2 - 4x + 5 + 2x^2 + 5x - 4 - 4x^2 + x - 3 \\
 &= 3x^2 + 2x^2 - 4x^2 - 4x + 5x + x + 5 - 4 - 3 \\
 &= (3+2+1)x^2 + (-4+5+1)x - 2 \\
 &= \underline{\underline{x^2 + 2x - 2}}.
 \end{aligned}$$

$$\begin{aligned}
 A-B+C &= (3x^2 - 4x + 5) - (2x^2 + 5x - 4) + (4x^2 - x + 3) \\
 &= 3x^2 - 4x + 5 - 2x^2 - 5x + 4 + 4x^2 - x + 3 \\
 &= 3x^2 - 2x^2 + 4x^2 - 4x - 5x - x + 5 + 4 + 3 \\
 &= (3-2+4)x^2 + (-4-5-1)x + 12 \\
 &= \underline{\underline{5x^2 - 10x + 12}}.
 \end{aligned}$$

$$\begin{aligned}
 -A+B+C &= -(3x^2 - 4x + 5) + (2x^2 + 5x - 4) + (4x^2 - x + 3) \\
 &= -3x^2 + 4x - 5 + 2x^2 + 5x - 4 + 4x^2 - x + 3 \\
 &= -3x^2 + 2x^2 + 4x^2 + 4x + 5x - x - 5 - 4 + 3 \\
 &= (-3+2+4)x^2 + (4+5-1)x + (-6) \\
 &= \underline{\underline{3x^2 + 8x - 6}}.
 \end{aligned}$$

$$\begin{aligned}
 6. A-B-C &= (5a^2 - 3ab + 7b^2) - (6a^2 - 8ab + 3b^2) - (4a^2 - 3ab - 7b^2) \\
 &= 5a^2 - 3ab + 7b^2 - 6a^2 + 8ab - 3b^2 - 4a^2 + 3ab + 7b^2 \\
 &= 5a^2 - 6a^2 - 4a^2 - 3ab + 8ab + 3ab + 7b^2 - 9b^2 + 7b^2 \\
 &= (5-6-4)a^2 + (-3+8+3)ab + (7-9+7)b^2 \\
 &= \underline{\underline{-5a^2 + 8ab + 5b^2}}.
 \end{aligned}$$

$$\begin{aligned}
 -A - B + C &= -(5a^2 - 3ab + 7b^2) - (6a^2 - 8ab + 8b^2) + (4a^2 - 3ab - 7b^2) \\
 &= -5a^2 + 3ab - 7b^2 - 6a^2 + 8ab - 8b^2 + 4a^2 - 3ab - 7b^2 \\
 &= -5a^2 - 6a^2 + 4a^2 + 3ab + 8ab - 3ab - 7b^2 - 8b^2 - 7b^2 \\
 &= \underline{-7a^2 + 8ab - 23b^2}.
 \end{aligned}$$

$$\begin{aligned}
 -A + B - C &= -(5a^2 - 3ab + 7b^2) + (6a^2 - 8ab + 8b^2) - (4a^2 - 3ab - 7b^2) \\
 &= -5a^2 + 3ab - 7b^2 + 6a^2 - 8ab + 8b^2 - 4a^2 + 3ab + 7b^2 \\
 &= -5a^2 + 6a^2 - 4a^2 + 3ab - 8ab + 3ab - 7b^2 + 8b^2 + 7b^2 \\
 &= \underline{-3a^2 - 2ab + 8b^2}.
 \end{aligned}$$

7. $(P+Q)-(R+S) = ?$

$$\begin{aligned}
 P+Q &= (2x^5 - 3x^3 + 4x) + (4x^3 - 5x^2 + 2x - 1) \\
 &= 2x^5 + 4x^3 - 3x^2 + 6x - 1
 \end{aligned}$$

$$\begin{aligned}
 R+S &= (4x^5 - 2x^3 + 3x - 1) + (3x^2 + 2x - 5) \\
 &= 4x^5 - 2x^3 + 3x^2 + 5x - 6
 \end{aligned}$$

$$\begin{aligned}
 20x(P+Q)-(R+S) &= (2x^5 + 4x^3 - 8x^2 + 6x - 1) - (4x^5 - 2x^3 + 3x^2 + 5x - 6) \\
 &= 2x^5 + 4x^3 - 8x^2 + 6x - 1 - 4x^5 + 2x^3 - 3x^2 - 5x + 6 \\
 &= 2x^5 - 4x^5 + 4x^3 + 2x^3 - 8x^2 - 3x^2 + 6x - 5x - 1 + 6 \\
 &= \underline{-2x^5 + 6x^3 - 11x^2 + x + 5}.
 \end{aligned}$$

Exercice 3

Exercices de calcul: correction

$$1(a) \quad 443: \left(3a^3b^3\right)\left(\frac{2}{3}ab^5\right) = \left(3 \times \frac{2}{3}\right) a^2 a b^3 b^5 = 2 \cdot a^3 b^8.$$

$$(b) \quad 444: \left(\frac{4}{5}a^3b^2c\right)\left(-\frac{3}{4}abc^4\right) = \left(\frac{4}{5} \times -\frac{3}{4}\right) a^3 a b^2 b c c^4 = \frac{-4 \times 3}{5 \times 4} a^4 b^3 c^5 = -\frac{3}{5} a^4 b^3 c^5.$$

$$(c) \quad 445: \left(\frac{4}{7}a^2xy^3\right)\left(-\frac{5}{2}a^3y^4\right) = \left(\frac{4}{7} \times -\frac{5}{2}\right) a^2 a^3 x y^3 y^4 = \frac{-4 \times 5}{7 \times 2} a^5 x y^7 = -\frac{10}{7} a^5 x y^7.$$

$$(d) \quad 446: \left(-\frac{3}{4}x^2y\right)\left(\frac{3}{5}a^3y^5\right) = \left(-\frac{3}{4} \times \frac{3}{5}\right) a^3 x^2 y y^5 = -\frac{9}{20} a^3 x^2 y^6.$$

$$(e) \quad 447: \left(\frac{9}{4}a^4x^3y^3\right)\left(\frac{1}{3}ax^2\right) = \left(\frac{9}{4} \times \frac{1}{3}\right) a^5 x^2 y^3 = -\frac{9}{3} a^5 x^4 y^3 = -3 a^5 x^4 y^3.$$

$$(f) \quad 448: \left(\frac{14}{3}a^2b^3x\right)\left(-\frac{6}{7}a^2b^5\right) = -\frac{14 \times 6}{3 \times 7} a^2 a^2 b^3 b^5 x = -4 a^4 b^8 x.$$

$$(g) \quad 449: \left(-\frac{7}{2}ax^2y\right)\left(-\frac{8}{15}b^3xy^2\right)\left(\frac{5}{21}abx^3\right) = \left(-\frac{7}{2} \times -\frac{8}{15} \times \frac{5}{21}\right) a a b^3 b^2 x^2 x^3 y y^2 \\ = + \frac{7 \times 8 \times 5}{2 \times 15 \times 21} a^2 b^4 x^6 y^3 \\ = \frac{\textcircled{1} \textcircled{2} \textcircled{3} \textcircled{4}}{\textcircled{2} \textcircled{3} \textcircled{5} \textcircled{6} \textcircled{7}} a^2 b^4 x^6 y^3 = \frac{4}{9} a^2 b^4 x^6 y^3.$$

$$(h) \quad 450: \left(\frac{2}{3}xy^2\right)^2 (-4x^3y) = \left(\frac{2}{3}xy^2\right) \left(\frac{2}{3}xy^2\right) (-4x^3y) = \frac{(2)(2)(-4)}{3 \times 3} x x x x^2 y^2 y y y \\ = -\frac{8}{9} x^4 y^5.$$

$$(i) \quad 451: \left(\frac{5}{12}a^2b^2x\right)\left(-\frac{2}{7}ax^2y\right)\left(-\frac{14}{5}a^2x^2y^4\right) = \frac{5 \times (-2) \times (-14)}{12 \times 7 \times 5} a^4 a b^2 b^2 x^2 x^3 y^3 y^4 \\ = \frac{\textcircled{1} \textcircled{2} \textcircled{3} \textcircled{4} \textcircled{5}}{\textcircled{2} \textcircled{3} \textcircled{4} \textcircled{5} \textcircled{6}} a^5 b^2 x^4 y^7 = \frac{1}{3} a^5 b^2 x^4 y^7.$$

$$(j) \quad 452: \left(\frac{3}{5}x^3y\right)^3 \left(-\frac{5}{4}xy\right) = \frac{3 \times 3 \times 3}{5 \times 5 \times 5} \times -\frac{5}{4} x^2 x^2 x^2 y y y x y = -\frac{27}{100} x^7 y^4.$$

$$(a) 453. \left(-\frac{2}{5}ab^3\right)^2 = \left(-\frac{2}{5}ab^3\right)\left(-\frac{2}{5}ab^3\right) = \frac{4}{25}a^2b^6.$$

$$(b) 454. \left(\frac{5}{3}a^2b^3x^4\right)^2 = \left(\frac{5}{3}a^2b^3x^4\right)\left(\frac{5}{3}a^2b^3x^4\right) = \frac{25}{9}a^4b^6x^8.$$

$$(c) 455. \left(\frac{-3}{2}a^4b^3y^3\right)^3 = \left(\frac{-3}{2}a^4b^3y^3\right)\left(a^3\right)^3\left(b^3\right)^3\left(y^3\right)^3 = \frac{-27}{8}a^{12}b^9y^9.$$

$$(d) 456. \left(\frac{7}{2}a^3b^2x^3\right)^3 = \frac{49}{4}a^6b^{10}x^6.$$

$$(e) 457. \left(-\frac{9}{4}a^4b^2x^5\right)^2 = \frac{36}{16}a^8b^4x^{10}.$$

$$(f) 458. \left(-\frac{6}{5}ax^3y^5\right)^3 = \frac{(-6)^3}{5^3}a^3(x^3)^3(y^5)^3 = \frac{-216}{125}a^3x^{12}y^{15}.$$

$$\begin{aligned} (g) 459. & \left(\frac{3}{2}a^2b - \frac{5}{4}ab + 3a\right)\left(-\frac{4}{3}a^2b^3\right) = \left(\frac{3}{2}a^2b\right)\left(-\frac{4}{3}a^2b^3\right) + \left(\frac{-5}{4}ab\right)\left(-\frac{4}{3}a^2b^3\right) + 3a\left(-\frac{4}{3}a^2b^3\right) \\ &= \left(\frac{3}{2} \times -\frac{4}{3}\right)a^2b^2a^2b^3 + \left(\frac{-5}{4} \times -\frac{4}{3}\right)a^2ba^2b^3 + \left(3 \times -\frac{4}{3}\right)a^2b^3 \\ &= -2a^4b^4 + \frac{5}{3}a^3b^4 - 4a^3b^3. \end{aligned}$$

$$\begin{aligned} (h) 460. & \left(\frac{5}{4}ax^2 + \frac{3}{2}bx^2 - 4c\right)\left(-\frac{4}{5}abx^5\right) = \left(\frac{5}{4}ax^2\right)\left(-\frac{4}{5}abx^5\right) + \left(\frac{3}{2}bx^2\right)\left(-\frac{4}{5}abx^5\right) + (-4c)\left(-\frac{4}{5}abx^5\right) \\ &= \left(\frac{5}{4} \times -\frac{4}{5}\right)ax^2abx^5 + \left(\frac{3}{2} \times -\frac{4}{5}\right)bx^2abx^5 + \left(4 \times \frac{4}{5}\right)cabx^5 \\ &= -a^2bx^7 - \frac{6}{5}ab^2x^6 + \frac{16}{5}abcx^5. \end{aligned}$$

$$\begin{aligned} (i) 461. & \left(\frac{2}{5}a^2x - 3ay - 4by\right)(4a^3x^2y) = \left(\frac{2}{5} \times 4\right)(a^2x)(a^3x^2y) - (3 \times 4)aya^3x^2y - 4 \times 4bya^3x^2y \\ &= \frac{8}{5}a^5x^3y - 12a^4x^3y^2 - 16a^3bx^2y^2. \end{aligned}$$

$$\begin{aligned} (j) 462. & \left(-\frac{3}{2}x^5 + \frac{15}{4}x^3 - \frac{3}{5}x\right)\left(-\frac{20}{3}x^4\right) = \left(-\frac{3}{2}x^5\right)\left(-\frac{20}{3}x^4\right) + \left(\frac{15}{4}x^3\right)\left(-\frac{20}{3}x^4\right) + \left(-\frac{3}{5}x\right)\left(-\frac{20}{3}x^4\right) \\ &= +10x^9 - 25x^7 + \frac{8}{3}x^5. \end{aligned}$$

$$(e) 463. (2x-3y)(4x-2) = \cancel{2x}x + \cancel{x} + \cancel{2x}(-2) + (-3y)(4x) + (-3y)(-2)$$

$$= 8x^2 - 4x - 12xy + 6y.$$

$$(f) 464. (2a+3b)(-4a+6b) = \cancel{2a}a + \cancel{2a}6b + \cancel{3b}(-4a) + \cancel{3b}6b$$

$$= -8a^2 + 12ab - 12ab + 18b^2$$

$$= -8a^2 + 18b^2.$$

$$(g) 465. (-4x+3y+1)(y-3) = (-4x)y + (3y)y + \cancel{1}y + (-4x)(-3) + \cancel{3y}(-3) + \cancel{1}(-3)$$

$$= -4xy + 3y^2 + y + 12x - 9y - 3$$

$$= 12x - 4xy + y - 9y + 3y^2 - 3$$

$$= 12x - 4xy - 8y + 3y^2 - 3.$$

$$(h) 466. (-2a+3b-5)(a-b) = \cancel{-2aa} + \cancel{-2a}(-b) + \cancel{3ba} + \cancel{3b}(-b) + (-5)a + (-5)(-b)$$

$$= -2a^2 + 2ab + 3ab - 3b^2 - 5a + 5b$$

$$= -2a^2 + 5ab - 3b^2 - 5a + 5b.$$

$$(i) 467. (2x^3-3y^2+5)(x^2-y) = \cancel{2x^3}x^3 + \cancel{2x^3}(-y) + (-3y^2)x^3 + (-3y^2)(-y) + \cancel{5x^2} + \cancel{5}(-y^2)$$

$$= 2x^5 - 2x^3y - 3x^2y^2 + 3y^4 + 5x^2 - 5y^2$$

$$(j) 468. (4a^3-5b^4+ab)(a^2-b) = \cancel{4a^3}a^2 + \cancel{4a^3}(-b) + (-5b^4)a^2 + \cancel{ab}a^2 + \cancel{ab}(-b)$$

$$= 4a^5 - 4a^3b - 5a^2b^4 + 5b^5 + a^3b - ab^2$$

$$= 4a^5 - 4a^3b + a^3b - 5a^2b^4 - ab^2 + 5b^5$$

$$= 4a^5 - 3a^3b - 5a^2b^4 - ab^2 + 5b^5.$$

$$(k) 469. (5xy+3x-2y)(2x-y) = \cancel{5xy}2x + \cancel{5xy}(-y) + \cancel{3x}2x + \cancel{3x}(-y) + \cancel{2y}(2x) + \cancel{2y}(-y)$$

$$= 10x^2y - 5xy^2 + 6x^2 - 3xy - 4xy - 2y^2$$

$$= 10x^2y + 6x^2 - 5xy^2 - 7xy - 2y^2$$

$$(l) 470. (-3xy+4x-2y)(x+5) = (-3xy)x + (-3xy)5 + \cancel{4x}x + \cancel{4x}5 + (-2y)x + (-2y)5$$

$$= -3x^2y - 15xy + 4x^2 + 20x - 2xy - 10y$$

$$= -3x^2y + 4x^2 + 17xy + 20x - 10y.$$

$$(m) 471. (14a^2b + 5a^2 - b)(a^2 - 2b) = (14a^2b)a^2 + (14a^2b)(-2b) + 5a^2a^2 + 5a^2(-2b) + (-b)a^2 + (-b)(-2b)$$

$$= 14a^4b - 28a^3b^2 + 5a^4 - 10a^2b - a^2b + 2b^2$$

$$= \underline{\underline{14a^4b + 5a^4 - 28a^3b^2 - 11a^2b + 2b^2}}$$

$$(n) 472. (7a^3b - 4b^3 + 2a^3)(2a^3 + 4b^2) = (7a^3b)(2a^3) + (7a^3b)(4b^2) + (-4b^3)2a^3 + (-4b^3)4b^2 + 2a^32a^3 + 2a^34b^2$$

$$= 14a^6b + 28a^3b^3 - 8a^3b^2 - 16b^4 + 4a^6 + 8a^3b^2$$

$$= \underline{\underline{14a^6b + 28a^3b^3 - 16b^4 + 4a^6}}$$

4 (a) 473. 1) $A = -2x^2 + 3x + 5$ et $B = x^2 - x + 3$

$$\text{donc } AB = (-2x^2 + 3x + 5)(x^2 - x + 3)$$

$$= (-2x^2)x^2 + (-2x^2)(-x) + (-2x^2)3 + 3x \cdot x^2 + 3x \cdot (-x) + 3 \cdot 3$$

$$+ 5 \cdot x^2 + 5 \cdot (-x) + 5 \cdot 3$$

$$= -2x^4 + 2x^3 - 6x^2 + 3x^3 - 3x^2 + 9x + 5x^2 - 5x + 15$$

$$= -2x^4 + 2x^3 + 3x^3 - 6x^2 - 3x^2 + 5x^2 + 9x - 5x + 15$$

$$= \underline{\underline{-2x^4 + 5x^3 - 4x^2 + 4x + 15}}$$

(b) 2) Pour $x = -3$: $A = -2x(-3)^2 + 3x(-3) + 5$ $B = (-3)^2 - (-3) + 3$

$$= -2x9 - 9 + 5$$

$$= -18 - 4$$

$$= 15$$

$$= -22$$

$$\text{donc } AB = (-22) \cdot 15 = \underline{\underline{-330}}$$

D'autre part: $-2x^4 + 5x^3 - 4x^2 + 4x + 15$

$$= -2x(-3)^4 + 5x(-3)^3 - 4x(-3)^2 + 4x(-3) + 15$$

$$= -2x81 + 5x(-27) - 4x9 - 12 + 15$$

$$= -162 - 135 - 36 - 12 + 15$$

$$= \underline{\underline{-330}}$$

5(a) 474. 1) $A^2 = (x^2 - 3x + 2)(x^2 + 3x + 2)$

$$= x^2 \cdot x^2 + x^2 \cdot (-3x) + x^2 \cdot 2 + (-3x) \cdot x^2 + (-3x) \cdot (-3x) + (-3x) \cdot 2 + 2 \cdot x^2 + 2 \cdot x \cdot (-3x) + 2 \cdot 2$$

$$= x^4 - 3x^3 + 2x^2 - 3x^3 + 9x^2 - 6x + 2x^2 - 6x + 4$$

$$= x^4 - 3x^3 - 3x^3 + 2x^2 + 9x^2 + 2x^2 - 6x - 6x + 4$$

$$= \underline{\underline{x^4 - 6x^3 + 13x^2 - 12x + 4}}.$$
(18)

$$\begin{aligned}
 A^3 &= A \cdot A^2 = (x^2 - 3x + 2)(x^4 - 6x^3 + 13x^2 - 12x + 4) \\
 &= x^2 \cdot x^4 + x^2 \cdot (-6x^3) + x^2 \cdot 13x^2 + x^2 \cdot (-12x) + x^2 \cdot 4 \\
 &\quad + (-3x) \cdot x^4 + (-3x) \cdot (-6x^3) + (-3x) \cdot (13x^2) + (-3x) \cdot (-12x) + (-3x) \cdot 4 \\
 &\quad + 2x^4 + 2 \cdot (-6x^3) + 2 \cdot 13x^2 + 2 \cdot (-12x) + 2 \cdot 4 \\
 &= x^6 - 6x^5 + 13x^4 - 12x^3 + 4x^2 \\
 &\quad - 3x^5 + 18x^4 - 39x^3 + 36x^2 - 12x \\
 &\quad + 2x^4 - 12x^3 + 26x^2 - 24x + 8 \\
 &= x^6 + (-6 - 3)x^5 + (13 + 18 + 2)x^4 + (-12 - 39 - 12)x^3 + (4 + 36 + 26)x^2 + (-12 - 24)x + 8 \\
 &= \underline{\underline{x^6 - 9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8}}.
 \end{aligned}$$

(b) 29) Pour $x = -4$, $A = (-4)^2 - 3 \cdot (-4) + 2 = 16 + 12 + 2 = 30$
 donc $A^2 = \underline{\underline{30^2 = 900}}$ et $A^3 = \underline{\underline{30^3 = 27000}}$.

D'autre part, $x^4 - 6x^3 + 13x^2 - 12x + 4 = (-4)^4 - 6(-4)^3 + 13 \cdot (-4)^2 - 12 \cdot (-4) + 4$
 $= 256 - 6(-64) + 13 \cdot 16 + 48 + 4$
 $= 256 + 384 + 208 + 48 + 4$
 $= \underline{\underline{900}}.$

$$\begin{aligned}
 \text{et } x^6 - 9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8 \\
 &= (-4)^6 - 9 \cdot (-4)^5 + 33 \cdot (-4)^4 - 63 \cdot (-4)^3 + 66 \cdot (-4)^2 - 36 \cdot (-4) + 8 \\
 &= 4096 - 9(-4096) + 33 \cdot 256 - 63(-64) + 66 \cdot 16 + 144 + 8 \\
 &= 4096 + 9216 + 8448 + 4032 + 1056 + 144 + 8 \\
 &= \underline{\underline{27000}}.
 \end{aligned}$$

6(2) 475. $\underline{(2x-7)(-3x+2)} = (2x) \cdot (-3x) + 2x \cdot 2 + (-7) \cdot (-3x) + (-7) \cdot 2$
 $= -6x^2 + 4x + 21x - 14$
 $= \underline{\underline{-6x^2 + 25x - 14}}.$

(b) 476. $(4x^5 + 7 - 2x^3)(x^3 - 2x) = 4x^5 x^3 + 4x^5 \cdot (-2x) + 7 \cdot x^3 + 7 \cdot (-2x) + (2x^3) x^3 + (-2x^3) \cdot (-2x)$
 $= 4x^8 - 8x^6 + 7x^3 - 14x - 2x^6 + 4x^4$
 $= 4x^8 - 8x^6 - 2x^6 + 4x^4 + 7x^3 - 14x$
 $= \underline{\underline{4x^8 - 10x^6 + 4x^4 + 7x^3 - 14x}}.$

$$(c) 477. (5x^3 - 2x)(3x - 4x^2) = \cancel{5x^3} \cdot 3x + \cancel{5x^3} \cdot (-4x^2) + (-2x) \cdot 3x + (-2x) \cdot (-4x^2)$$

$$= 15x^4 - 20x^5 - 6x^2 + 8x^3$$

$$= \underline{-20x^5 + 15x^4 + 8x^3 - 6x^2}$$

$$(d) 478. (2x - 7x^2 + 5x^3)(3x - 5x^2 + 8) = 2x \cdot 3x + 2x \cdot (-5x^2) + 2x \cdot 8$$

$$+ (-7x^2) \cdot 3x + (-7x^2) \cdot (-5x^2) + (-7x^2) \cdot 8$$

$$+ 5x^3 \cdot 3x + 5x^3 \cdot (-5x^2) + 5x^3 \cdot 8$$

$$= 6x^2 - 10x^3 + 16x - 21x^3 + 35x^4 - 56x^2$$

$$+ 15x^4 - 25x^5 + 40x^3$$

$$= -25x^5 + (15+35)x^4 + (40-10-21)x^3 + (6-56)x^2 + 16x$$

$$= \underline{-25x^5 + 50x^4 + 9x^3 - 50x^2 + 16x}$$

$$(e) 479. \left(-2x + \frac{3}{2}\right)(4x+3) = (-2x) \cdot (+x) + (-2x) \cdot 3 + \frac{3}{2} \cdot 4x + \frac{3}{2} \cdot 3$$

$$= -8x^2 - 6x + 6x + \frac{9}{2}$$

$$= \underline{-8x^2 + \frac{9}{2}}$$

$$(f) 480. \left(\frac{8}{3}x - \frac{3}{2}x^2 + 5\right)(4x^3 - 5x^2 + 7) = \frac{8}{3}x \cdot 4x^3 + \frac{8}{3}x \cdot (-5x^2) + \frac{8}{3}x \cdot 7$$

$$+ \left(\frac{3}{2}x^2\right) \cdot 4x^3 + \left(\frac{3}{2}x^2\right) \cdot (-5x^2) + \left(\frac{3}{2}x^2\right) \cdot 7$$

$$+ 5 \cdot 4x^3 + 5 \cdot (-5x^2) + 5 \cdot 7$$

$$= \underline{\frac{32}{3}x^4 - \frac{40}{3}x^3 + \frac{56}{3}x - 6x^5}$$

$$(g) 481. (7x^4 - 2x^3 + 4x^2)(3x^2 - 5) = 7x^4 \cdot 3x^2 + 7x^4 \cdot (-5) + (-2x^3) \cdot 3x^2 + (-2x^3) \cdot (-5) + (+x^2) \cdot 3x^2$$

$$+ (+x^2) \cdot (-5)$$

$$= 21x^6 - 35x^4 - 6x^5 + 10x^3 + 12x^4 - 20x^2$$

$$= 21x^6 - 6x^5 + 12x^4 - 35x^4 + 10x^3 - 20x^2$$

$$= \underline{21x^6 - 6x^5 - 23x^4 + 10x^3 - 20x^2}$$

$$(h) 482. (2x^3 - 4x^2)(x^2 - 2x) = 2x^2 \cdot x^2 + 2x^2 \cdot (-2x) + (-4x^3) \cdot x^2 + (-4x^3) \cdot (-2x)$$

$$= 2x^4 - 4x^3 - 4x^5 + 8x^4$$

$$= -4x^5 + 2x^4 + 8x^4 - 4x^3$$

$$= \underline{-4x^5 + 10x^4 - 4x^3}$$

$$\begin{aligned}
 (i) 483. (2x^2 - 4 + 2x)(x^2 + 5 - 2x) &= 2x^2 \cdot x^2 + 2x^2 \cdot 5 + 2x^2 \cdot (-2x) \\
 &\quad + (-4)x^2 + (-4)5 + (-4)(-2x) \\
 &\quad + 2x \cdot x^2 + 2x \cdot 5 + 2x \cdot (-2x) \\
 &= 2x^4 + 10x^2 - 4x^3 - 4x^2 - 20 + 8x + 2x^3 + 10x - 4x^2 \\
 &= 2x^4 - 4x^3 + 2x^3 + 10x^2 - 4x^2 - 4x^3 + 8x + 10x - 20 \\
 &= \underline{\underline{2x^4 - 2x^3 + 2x^2 + 18x - 20}}
 \end{aligned}$$

$$\begin{aligned}
 (j) 484. \left(\frac{5}{4}x^3 - 2x + 1\right) \left(\frac{7}{2}x^3 - \frac{2}{3}x + x^2\right) &= \frac{5}{4}x^3 \cdot \frac{7}{2}x^3 + \frac{5}{4}x^3 \cdot \left(-\frac{2}{3}x\right) + \frac{5}{4}x^3 \cdot x^2 \\
 &\quad + (-2x) \cdot \frac{7}{2}x^3 + (-2x) \cdot \left(-\frac{2}{3}x\right) + (-2x) \cdot x^2 \\
 &\quad + \frac{1}{2} \cdot \frac{7}{2}x^6 + \frac{1}{2} \cdot \left(-\frac{2}{3}x\right) + \frac{1}{2}x^2 \\
 &= \frac{35}{8}x^6 - \frac{5}{6}x^4 + \frac{5}{4}x^5 - 7x^4 + \frac{4}{3}x^3 - 2x^3 + \frac{7}{4}x^3 - \frac{1}{3}x + \frac{1}{2}x^2 \\
 &= \frac{35}{8}x^6 + \frac{5}{4}x^5 + \left(\frac{5}{6} + 7\right)x^4 + \left(-2 + \frac{7}{4}\right)x^3 + \left(\frac{4}{3} + \frac{1}{2}\right)x^2 - \frac{1}{3}x \\
 &= \frac{35}{8}x^6 + \frac{5}{4}x^5 + \frac{37}{6}x^4 - \frac{1}{4}x^3 + \frac{11}{6}x^2 - \frac{1}{3}x. \\
 &\underline{\underline{\frac{35}{8}x^6 + \frac{5}{4}x^5 + \frac{37}{6}x^4 - \frac{1}{4}x^3 + \frac{11}{6}x^2 - \frac{1}{3}x.}}
 \end{aligned}$$

$$\begin{aligned}
 7.(a) 485. (2x+3)(3x+2)(x-4) &= (2x \cdot 3x + 2x \cdot 2 + 3 \cdot 3x + 3 \cdot 2)(x-4) \\
 &= (6x^2 + 4x + 9x + 6)(x-4) \\
 &= (6x^2 + 13x + 6)(x-4) \\
 &= 6x^2 \cdot x + 6x^2 \cdot (-4) + 13x \cdot x + 13x \cdot (-4) + 6x + 6 \cdot (-4) \\
 &= 6x^3 - 24x^2 + 13x^2 - 52x + 6x - 24 \\
 &= 6x^3 - 11x^2 - 46x - 24. \\
 &\underline{\underline{6x^3 - 11x^2 - 46x - 24.}}
 \end{aligned}$$

$$\begin{aligned}
 (b) 486. (5x-1)(2x+3)(7+4x) &= (5x \cdot 2x + 5x \cdot 3 + (-1) \cdot 2x + (-1) \cdot 3)(7+4x) \\
 &= (10x^2 + 15x - 2x - 3)(7+4x) \\
 &= (10x^2 + 13x - 3)(7+4x) \\
 &= 10x^3 \cdot 7 + 10x^3 \cdot 4x + 13x \cdot 7 + 13x \cdot 4x + (-3) \cdot 7 + (-3) \cdot 4x \\
 &= 70x^3 + 40x^3 + 91x + 52x^2 - 21 - 12x \\
 &= 40x^3 + (70+52)x^2 + (91-12)x - 21 \\
 &= 40x^3 + 122x^2 + 79x - 21. \\
 &\underline{\underline{40x^3 + 122x^2 + 79x - 21.}}
 \end{aligned}$$

$$(c) 487. (3x^2 - 1)(x+1)(x-1) = (3x^2 - 1)(x^2 - 1) = 3x^2 \cdot x^2 + 3x^2 \cdot (-1) + (-1)x^2 + (-1) \cdot (-1)$$

$$= 3x^4 - 3x^2 - x^2 + 1$$

$$= 3x^4 - 4x^2 + 1.$$

$$(d) 488. \left(x - \frac{3}{5}\right)(5x^2 - 1)(5x + 3) = \left(x \cdot 5x^2 + x \cdot (-1) + \frac{3}{5} \cdot 5x^2 + \frac{3}{5} \cdot (-1)\right)(5x + 3)$$

$$= (5x^3 - x - 3x^2 + \frac{3}{5})(5x + 3)$$

$$= 5x^3 \cdot 5x + 5x^3 \cdot 3 + (-x) \cdot 5x + (-x) \cdot 3 + (-3x^2) \cdot 5x + (-3x^2) \cdot 3 + \frac{3}{5} \cdot 5x + \frac{3}{5} \cdot 3$$

$$= 25x^4 + 15x^3 - 5x^2 - 3x - 15x^3 - 9x^2 + 3x + \frac{9}{5}$$

$$= 25x^4 + 15x^3 - 15x^3 - 5x^2 - 9x^2 - 3x + 3x + \frac{9}{5}$$

$$= 25x^4 - 14x^2 + \frac{9}{5}.$$

$$(e) 489. (2x^2 + 3x - 4)^2 = (2x^2 + 3x - 4)(2x^2 + 3x - 4)$$

$$= 2x^2 \cdot 2x^2 + 2x^2 \cdot 3x + 2x^2 \cdot (-4)$$

$$+ 3x \cdot 2x^2 + 3x \cdot 3x + 3x \cdot (-4)$$

$$+ (-4) \cdot 2x^2 + (-4) \cdot 3x + (-4) \cdot (-4)$$

$$= 4x^4 + 6x^3 - 8x^2 + 6x^3 + 9x^2 - 12x - 8x^2 - 12x + 16$$

$$= 4x^4 + 12x^3 - 7x^2 - 24x + 16.$$

$$(f) 490. (4x^3 - 7x + 2x^2 + 5)^2 = (4x^3 + 2x^2 - 7x + 5)(4x^3 + 2x^2 - 7x + 5)$$

$$= 4x^3 \cdot 4x^3 + 4x^3 \cdot 2x^2 + 4x^3 \cdot (-7x) + 4x^3 \cdot 5$$

$$+ 2x^2 \cdot 4x^3 + 2x^2 \cdot 2x^2 + 2x^2 \cdot (-7x) + 2x^2 \cdot 5$$

$$+ (-7x) \cdot 4x^3 + (-7x) \cdot 2x^2 + (-7x) \cdot (-7x) + (-7x) \cdot 5$$

$$+ 5 \cdot 4x^3 + 5 \cdot 2x^2 + 5 \cdot (-7x) + 5 \cdot 5$$

$$= 16x^6 + 8x^5 - 28x^4 + 20x^3 + 8x^5 + 4x^4 - 14x^3 + 10x^2$$

$$- 28x^4 - 14x^3 + 4x^2 - 35x + 20x^3 + 10x^2 - 35x + 25$$

$$= 16x^6 + 16x^5 - 52x^4 + 12x^3 + 68x^2 - 70x + 25$$

$$\begin{aligned}
 (g) 491. (7x-5)^3 &= (7x-5)(7x-5)(7x-5) \\
 &= (7x \cdot 7x + 7x \cdot (-5) + (-5) \cdot 7x + (-5) \cdot (-5))(7x-5) \\
 &= (49x^2 - 70x + 25)(7x-5) \\
 &= 49x^2 \cdot 7x + 49x^2 \cdot (-5) + (-70x) \cdot 7x + (-70x) \cdot (-5) + 25 \cdot 7x + 25 \cdot (-5) \\
 &= 343x^3 - 245x^2 - 130x^2 + 350x + 175x - 125 \\
 &= \underline{\underline{343x^3 - 735x^2 + 425x - 125}}
 \end{aligned}$$

$$\begin{aligned}
 (h) 492. (x^2-x+2)^3 &= (x^2-x+2)(x^2-x+2)(x^2-x+2) \\
 &= (x^2 \cdot x^2 + x^2 \cdot (-x) + x^2 \cdot 2 + (-x) \cdot x^2 + (-x) \cdot (-x) + (-x) \cdot 2 + 2x^2 \cdot 2 + 2 \cdot (-x) + 2 \cdot 2)(x^2-x+2) \\
 &= (x^4 - x^3 + 2x^2 - x^3 + x^2 - 2x + 2x^2 - 2x + 4)(x^2-x+2) \\
 &= (x^4 - 2x^3 + 5x^2 - 4x + 4)(x^2-x+2) \\
 &= x^4 \cdot x^2 + x^4 \cdot (-x) + x^4 \cdot 2 + (-2x^3) \cdot x^2 + (-2x^3) \cdot (-x) + (-2x^3) \cdot 2 \\
 &\quad + 5x^2 \cdot x^2 + 5x^2 \cdot (-x) + 5x^2 \cdot 2 + (-4x) \cdot x^2 + (-4x) \cdot (-x) + (-4x) \cdot 2 \\
 &\quad + 4x^2 \cdot 1 + 1 \cdot (-x) + 1 \cdot 2 \\
 &= x^6 - x^5 + 2x^4 - 2x^5 + 2x^4 - 4x^3 + 5x^4 - 5x^3 + 10x^2 - 4x^3 + 4x^2 - 8x + 4x^2 - 4x + 8 \\
 &= \underline{\underline{x^6 - 3x^5 + 9x^4 - 13x^3 + 18x^2 - 12x + 8}}
 \end{aligned}$$

$$\begin{aligned}
 8(2) 493. 5 \cdot (3a^2+b^3) - [3(2a^2-b^3) - 2(a^2-5b^3)] &= 5 \cdot 3a^2 - 5 \cdot b^3 - [3 \cdot 2a^2 - 3b^3 - (2a^2 - 2 \cdot 5b^3)] \\
 &= 15a^2 - 20b^3 - [18a^2 - 3b^3 - 2a^2 + 10b^3] \\
 &= 15a^2 - 20b^3 - [16a^2 + b^3] \\
 &= 15a^2 - 20b^3 - 16a^2 - b^3 \\
 &= \underline{\underline{-a^2 - 21b^3}}
 \end{aligned}$$

$$\begin{aligned}
 (b) 494. 3a^2(2b-1) - [2a^2(5b-3) - 2b(3a^2+1)] &= 3a^2 \cdot 2b - 3a^2 - [2a^2 \cdot 5b - 2a^2 \cdot 3 - (2b \cdot 3a^2 + 2b \cdot 1)] \\
 &= 6a^2b - 3a^2 - [10a^2b - 6a^2 - (6a^2b + 2b)] \\
 &= 6a^2b - 3a^2 - [10a^2b - 6a^2 - 6a^2b - 2b] \\
 &= 6a^2b - 3a^2 - [4a^2b - 6a^2 - 2b] \\
 &= 6a^2b - 3a^2 - 4a^2b + 6a^2 + 2b \\
 &= \underline{\underline{2a^2b + 3a^2 + 2b}}
 \end{aligned}$$

$$\begin{aligned}
 (c) 435. & (2a+5b)(3a-2b) - [2a-1(3a+2b)] - [a-2b(5b-1)] \\
 & = [2a \cdot 3a + 2a \cdot (-2b) + 5b \cdot 3a + 5b \cdot (-2b)] \\
 & \quad - [2a \cdot 3a + 2a \cdot 2b + (-1) \cdot 3a + (-1) \cdot 2b] \\
 & \quad - [a \cdot 5b + a \cdot (-1) \cdot (-2b) \cdot 5b + (-2b) \cdot (-1)] \\
 & = [6a^2 - 4ab + 15ab - 10b^2] - [6a^2 + 4ab - 3a \cdot 2b] - [5ab - a - 10b^2 + 2b] \\
 & = 6a^2 + 11ab - 10b^2 - 6a^2 - 4ab + 3a \cdot 2b - 5ab + a + 10b^2 - 2b \\
 & = \cancel{6a^2} - \cancel{6a^2} + 11ab - 4ab - 5ab - \cancel{10b^2} + \cancel{10b^2} + 3a + a + \cancel{2b} - \cancel{2b} \\
 & = 0 + 2ab + 0 + 4a + 0 \\
 & = \underline{\underline{2ab + 4a}}
 \end{aligned}$$

$$\begin{aligned}
 (d) 436. & (2x-3y)(5x-2y) - [3(x-2y)(2x+1)] - [5(x-y)(3y+1)] \\
 & = [2x \cdot 5x + 2x \cdot (-2y) + (-3y) \cdot 5x + (-3y) \cdot (-2y)] \\
 & \quad - [3x \cdot 2x + 3x \cdot 1 + (-2y) \cdot 2x + (-2y) \cdot 1] \\
 & \quad - [5x \cdot 3y + 5x \cdot (y) \cdot 3y + (-y) \cdot 1] \\
 & = [10x^2 - 4xy - 15xy + 6y^2] - [6x^2 + 3x - 4xy - 2y] - [15xy + 5x - 3y^2 + y] \\
 & = 10x^2 - 19xy + 6y^2 - \cancel{6x^2} - \cancel{3x} + \cancel{4xy} + \cancel{0y} - \cancel{15xy} - \cancel{5x} + \cancel{2y^2} + \cancel{0y} \\
 & = 10x^2 - 6x^2 - 13xy + 4xy - 15xy + 6y^2 + \cancel{5y^2} - 3x - 5x + 2y + y \\
 & = \underline{\underline{4x^2 - 30xy + 9y^2 - 8x + 3y}}
 \end{aligned}$$

$$\begin{aligned}
 (e) 437. & (ax^2-b)(ax^2-2b) + 3b(ax^2-b) + b(b-1) \\
 & = [ax^2 \cdot ax^2 + ax^2 \cdot (-2b) - ba \cdot x^2 + (-b) \cdot (-2b)] + [3ba \cdot x^2 + 3b(-b)] + bb + b(-1) \\
 & = [a^2x^4 - 2abx^2 - abx^2 + 2b^2] + [3ba \cdot x^2 - 3b^2] + b^2 - b \\
 & = a^2x^4 - 3abx^2 + 3abx^2 + 2b^2 - 3b^2 + b^2 - b \\
 & = \underline{\underline{a^2x^4 - b}}
 \end{aligned}$$

$$\begin{aligned}
 (f) 438. & (x-1)(x-2)(x-3) = (x-1)(x^2x - 2x - 3x + 6) \\
 & = (x-1)(x^3 - 5x + 6) \\
 & = x \cdot x^3 - x^2 \cdot 5x + x \cdot 6 - x^2 + 5x - 6 \\
 & = x^3 - 5x^2 + 6x - x^2 + 5x - 6 \\
 & = \underline{\underline{x^3 - 6x^2 + 11x - 6}}
 \end{aligned}$$

$$6(x-1)(x-2) = 6(x^2 - 2x - x + 2) = 6(x^2 - 3x + 2) = 6x^2 - 18x + 12.$$

$$7(x-1) = 7x - 7$$

$$\begin{aligned} \text{Dom: } & (x-1)(6x-2)(x-3) + 6(x-1)(x-2) + 7(x-1) \\ &= x^3 - 6x^2 + 11x - 6 + 6x^2 - 18x + 12 + 7x - 7 \\ &= x^3 - 6x^2 + 6x^2 + 11x - 18x + 7x - 6 + 12 - 7 \\ &= \underline{\underline{x^3 - 1}}. \end{aligned}$$

Another method:

$$\begin{aligned} & (x-1)(x-2)(x-3) + 6(x-1)(x-2) + 7(x-1) \\ &= (x-1)[(x-2)(x-3) + 6(x-2) + 7] \\ &= (x-1)[(x-2)(x-3+6) + 7] \\ &= (x-1)[(x-2)(x+3) + 7] \\ &= (x-1)(x^2 - 2x + 3x - 6 + 7) \\ &= (x-1)(x^2 + x + 1) \\ &= x \cdot x^2 + x \cdot x + x \cdot 1 - x^2 \cdot x - 1 \\ &= x^3 + x^2 + x - x^2 - x - 1 = \underline{\underline{x^3 - 1}}. \end{aligned}$$

$$\begin{aligned} (g) 499. & (x^2 + y^2)(x^2 - y^2)(x - y) + xy(x^3 + y^3) \\ &= (x^2 \cdot x^2 - y^2 \cdot y^2)(x - y) + xy(x^3 + y^3) \\ &= (x^4 - y^4)(x - y) + xyx^3 + xyy^3 \\ &= x^4x - x^4y - y^4x + y^4y + x^4y + x^4y \\ &= \underline{\underline{x^5 + y^5}} \end{aligned}$$

$$\begin{aligned} (h) 500. & \frac{2}{3}x^3y\left(2x^2 - \frac{y}{3}\right) - 2x^2(2x^2 - 1) + \left(2x^2 - \frac{4}{3}\right)\left(1 - \frac{y}{3}\right)(2x^2 - 1) \\ &= \frac{4}{3}x^4y - \frac{2}{3}x^2y^2 - 4x^4 + 2x^2 + \left(2x^2 - \frac{2}{3}x^3y - \frac{4}{3} + \frac{4}{9}\right)(2x^2 - 1) \\ &= \frac{4}{3}x^4y - \frac{2}{3}x^2y^2 - 4x^4 \cdot 2x^2 + \left[4x^4 - 2x^2 - \frac{4}{3}xy + 2\cancel{x^4} - \cancel{2x^4} + \frac{4}{3} + \frac{2}{3}x^3y - \frac{4}{9}\right] \\ &= \left(\frac{4}{3} - 4\right)x^4y + \left(-\frac{2}{3} + \frac{2}{3}\right)x^2y^2 + (-4 + 4)x^4 + (2 - 2)x^2 + \frac{y}{3} - \frac{y^2}{9} = \underline{\underline{\frac{y}{3} - \frac{y^2}{9}}}. \end{aligned}$$

(25)

Exercise 4

$$\begin{aligned}
 1.(a) \frac{1}{2}(a+b)^2 + \frac{1}{2}(a-b)^2 &= \frac{1}{2}(a^2+2ab+b^2) + \frac{1}{2}(a^2-2ab+b^2) \\
 &= \frac{1}{2}(a^2+2ab+b^2 - 2ab + b^2) \\
 &= \frac{1}{2}(2a^2+2b^2) = \frac{1}{2} \times 2(a^2+b^2) = \underline{\underline{a^2+b^2}}.
 \end{aligned}$$

$$\begin{aligned}
 1.(b) \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 &= \frac{(a+b)^2}{2^2} - \frac{(a-b)^2}{2^2} = \frac{a^2+2ab+b^2}{4} - \frac{a^2-2ab+b^2}{4} \\
 &= \frac{a^2+2ab+b^2 - a^2+2ab - b^2}{4} \\
 &= \frac{4ab}{4} = \underline{\underline{ab}}.
 \end{aligned}$$

$$\begin{aligned}
 1.(c) (a-b)(a^3+a^2b+ab^2+b^3) &= aa^3+a.a^2b+a.ab^2+a.b^3 \\
 &\quad - b.a^3 - b.a^2b - b.ab^2 - b.b^3 \\
 &= a^4 + a^3b + a^2b^2 + ab^3 - ab^3 - a^2b^2 - ab^3 - b^4 \\
 &= \underline{\underline{a^4-b^4}}.
 \end{aligned}$$

$$\begin{aligned}
 1.(d) (a+b)(a^3-a^2b+ab^2-b^3) &= a.a^3 - a.a^2b + a.ab^2 - ab^3 + ba^3 - ba^2b + bab^2 - b.b^3 \\
 &= a^4 - a^3b + a^2b^2 - ab^3 + ba^3 - a^2b^2 + ab^3 - b^4 \\
 &= \underline{\underline{a^4-b^4}}.
 \end{aligned}$$

$$\begin{aligned}
 1.(e) (x^2+x+1)(x^2-x+1) &= [(x^2+1)+x][(x^2+1)-x] \\
 &= [x^2+1]^2 - [x]^2 \\
 &= (x^2)^2 + 2 \cdot x^2 \cdot 1 + 1^2 - x^2 \\
 &= x^4 + 2x^2 + 1 - x^2 \\
 &= \underline{\underline{x^4+x^2+1}}.
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & (aa' + bb')^2 + (ab' - a'b)^2 = (aa')^2 + 2aa'bb' + (bb')^2 + (ab')^2 - 2 \cdot ab \cdot a'b + (a'b)^2 \\
 & = a^2 a'^2 + 2 \cancel{aa'bb'} + b^2 b'^2 + a^2 b'^2 - 2 \cancel{aa'bb'} + a'^2 b^2 \\
 & = a^2 a'^2 + a^2 b'^2 + b^2 b'^2 + b^2 a'^2 \\
 & = a^2 (a'^2 + b'^2) + b^2 (b'^2 + a'^2) \\
 & = \underline{\underline{(a^2 + b^2)(a'^2 + b'^2)}}
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & (x-1)(x+1)(x^2+1) = (x-1)(x \cdot x^2 + x \cdot 1 + 1 \cdot x^2 + 1 \cdot 1) \\
 & = \underline{\underline{(x-1)(x^3 + x^2 + x + 1)}}
 \end{aligned}$$

D'autre part: $x^4 - 1 = (x^2)^2 - 1^2 = (x^2 + 1)(x^2 - 1)$

$$\begin{aligned}
 & = (x^2 + 1)(x^2 - 1) \\
 & = (x^2 + 1)(x + 1)(x - 1) \\
 & = \underline{\underline{(x-1)(x+1)(x^2+1)}}
 \end{aligned}$$

$$\begin{aligned}
 (1) \quad & a(b-c) + b(c-a) + c(a-b) = ab - ac + bc - ba + ca - cb \\
 & = ab - ba - ac + ca + bc - cb \\
 & = \underline{\underline{0}}
 \end{aligned}$$

$$\begin{aligned}
 (i) \quad & a(bz - cy) + b(cx - az) + c(ay - bx) = abz - acy + bcx - baz + cay - cbx \\
 & = abz - baz + bcx - cbx + cay - acy \\
 & = \underline{\underline{0}}
 \end{aligned}$$

$$\begin{aligned}
 (j) \quad & (x+y)^3 - 3xy(x+y) = (x+y)(x+y)^2 - (3xy(x+y)) \\
 & = (x+y)(x^2 + 2xy + y^2) - (3x^2y + 3xy^2) \\
 & = x \cdot x^2 + x \cdot 2xy + x \cdot y^2 + y \cdot 2xy + y \cdot y^2 - 3x^2y - 3xy^2 \\
 & = x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 - 3x^2y - 3xy^2 \\
 & = x^3 + \cancel{2x^2y} + \cancel{x^2y} - \cancel{3x^2y} + \cancel{xy^2} + \cancel{2xy^2} - \cancel{3xy^2} + y^3 \\
 & = x^3 + 0x^2y + 0xy^2 + y^3 \\
 & = \underline{\underline{x^3 + y^3}}
 \end{aligned}$$

$$\begin{aligned}
 (k) \text{ D'une part: } (x+y)^3 + 2(x^3+y^3) &= (x+y)(x+y)^2 + 2x^3+2y^3 \\
 &= 2x^3+2y^3 + (x+y)(x^2+2xy+y^2) \\
 &= 2x^3+2y^3 + x \cdot x^2 + x \cdot 2xy + x \cdot y^2 + y \cdot x^2 + y \cdot 2xy + y \cdot y^2 \\
 &= 2x^3+2y^3 + x^3+2x^2y+xy^2+x^2y+2xy^2+y^3 \\
 &= 2x^3+x^3+2x^2y+xy^2+xy^2+2xy^2+2y^3+y^3 \\
 &= 3x^3+3x^2y+3xy^2+3y^3
 \end{aligned}$$

$$\begin{aligned}
 \text{D'autre part: } 3(x+y)(x^2+y^2) &= 3(xx^2+xy^2+yx^2+yy^2) \\
 &= 3(x^3+xy^2+xy^2+y^3) \\
 &= 3x^3+3xy^2+3xy^2+3y^3.
 \end{aligned}$$

D'où: $(x+y)^3 + 2(x^3+y^3) = 3(x+y)(x^2+y^2)$.

$$\begin{aligned}
 2.(a) \left(\frac{3}{2}x^3 - \frac{2}{5}y^2\right)^2 &= \left(\frac{3}{2}x^3\right)^2 + 2 \cdot \left(\frac{3}{2}x^3\right) \cdot \left(-\frac{2}{5}y^2\right) + \left(-\frac{2}{5}y^2\right)^2 \\
 &= \frac{3^2}{2^2} (x^3)^2 + 2 \cdot \frac{3}{2} x^3 \cdot -\frac{2}{5} y^2 + \left(-\frac{2}{5}\right)^2 (y^2)^2 \\
 &= \frac{9}{4} x^{3 \times 2} + -\frac{6}{5} x^3 y^2 + \frac{4}{25} y^{2 \times 2} \\
 &= \frac{9}{4} x^6 - \frac{6}{5} x^3 y^2 + \frac{4}{25} y^4
 \end{aligned}$$

$$\begin{aligned}
 (b) \left(\frac{4}{3}x^5 + \frac{2}{5}y^3\right)^2 &= \left(\frac{4}{3}x^5\right)^2 + 2 \cdot \frac{4}{3}x^5 \cdot \frac{2}{5}y^3 + \left(\frac{2}{5}y^3\right)^2 \\
 &= \frac{4^2}{3^2} (x^5)^2 + \frac{2 \times 4 \times 2}{3 \times 5} x^5 y^3 + \frac{2^2}{5^2} (y^3)^2 \\
 &= \frac{16}{9} x^{5 \times 2} + \frac{16}{15} x^5 y^3 + \frac{4}{25} y^{3 \times 2} \\
 &= \frac{16}{9} x^{10} + \frac{16}{15} x^5 y^3 + \frac{4}{25} y^6
 \end{aligned}$$

$$(c) \left(\frac{2}{5}x^2 - \frac{3}{4}y\right)\left(\frac{2}{5}x^2 + \frac{3}{4}y\right) = \left(\frac{2}{5}x^2\right)^2 - \left(\frac{3}{4}y\right)^2$$

$$= \frac{2^2}{5^2}(x^2)^2 - \frac{3^2}{4^2}y^2$$

$$= \frac{4}{25}x^4 - \frac{9}{16}y^2$$

$$(d) \left(\frac{2}{3}a^2x^3 - \frac{1}{2}by^4\right)\left(\frac{2}{3}a^2x^3 + \frac{1}{2}by^4\right) = \left(\frac{2}{3}a^2x^3\right)^2 - \left(\frac{1}{2}by^4\right)^2$$

$$= \frac{2^2}{3^2}(a^2)^2(x^3)^2 - \frac{1^2}{2^2}b^2(y^4)^2$$

$$= \frac{4}{9}a^4x^6 - \frac{1}{4}b^2y^8$$

$$(e) (\overbrace{3x+4y}^{a-b} - 5)(\overbrace{3x+4y}^{a+b} + 5) = (3x+4y)^2 - 5^2$$

$$= (3x)^2 + 2 \cdot 3x \cdot 4y + (4y)^2 - 25$$

$$= 9x^2 + 24xy + 16y^2 - 25.$$

$$(f) \left(\frac{2}{3}x - \frac{4}{5}y - 1\right)\left(\frac{2}{3}x + \frac{4}{5}y + 1\right) = \left[\left(\frac{2}{3}x\right) - \left(\frac{4}{5}y + 1\right)\right] \left[\left(\frac{2}{3}x\right) + \left(\frac{4}{5}y + 1\right)\right]$$

$$= \left(\frac{2}{3}x\right)^2 - \left(\frac{4}{5}y + 1\right)^2$$

$$= \frac{2^2}{3^2}x^2 - \left[\left(\frac{4}{5}y\right)^2 + 2 \cdot \frac{4}{5}y \cdot 1 + 1^2\right]$$

$$= \frac{4}{9}x^2 - \left[\frac{16}{25}y^2 + \frac{8}{5}y + 1\right]$$

$$= \frac{4}{9}x^2 - \frac{16}{25}y^2 - \frac{8}{5}y - 1.$$

$$(g) (3x+4y - 2z)^2 = (3x+4y)^2 + 2(3x+4y)(-2z) + (-2z)^2$$

$$= [(3x)^2 + 2 \cdot 3x \cdot 4y + (4y)^2] + (6x+8y)(-2z) + (-2z)^2$$

$$= [9x^2 + 24xy + 16y^2] - 12xz - 16yz + 4z^2$$

$$= 9x^2 + 16y^2 + 4z^2 + 24xy - 12xz - 16yz.$$

$$\begin{aligned}
 \text{(a)} \quad & \left(\frac{5}{2}x - \frac{3}{4}y + z\right)^2 = \left(\frac{5}{2}x - \frac{3}{4}y\right)^2 + 2\left(\frac{5}{2}x - \frac{3}{4}y\right)z + z^2 \\
 &= \left[\left(\frac{5}{2}x\right)^2 + 2 \cdot \frac{5}{2}x \cdot \frac{3}{4}y + \left(\frac{3}{4}y\right)^2\right] + \left(5x - \frac{3}{2}y\right)z + z^2 \\
 &= \frac{25}{4}x^2 - \frac{15}{4}xy + \frac{9}{16}y^2 + 5xz - \frac{3}{2}yz + z^2
 \end{aligned}$$

$$\begin{aligned}
 \text{3. (a)} \quad & (a+b)(a+x)(b+x) - a(b+x)^2 - b(a+x)^2 \\
 &= (a^2 + ax + ab + bx)(b+x) - a(b^2 + 2bx + x^2) - b(a^2 + 2ax + x^2) \\
 &= (a^2b + a^2x + axb + ax^2 + bab + bax + bxb + bx^2) - ab^2 - 2abx - ax^2 - ba^2 - 2abx - bx^2 \\
 &= a^2b + a^2x + abx + ax^2 + ab^2 + abx + b^2x + bx^2 - ab^2 - abx - ax^2 - a^2b - bx^2 \\
 &= a^2b - a^2b + a^2x + ax^2 - ax^2 + abx + bax - rabx + ab^2 + b^2x + bx^2 - bx^2 \\
 &= a^2x - 2abx + b^2x \\
 &= (a^2 - 2ab + b^2)x \\
 &= (a-b)^2 x
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & bc(b-c) + c(a(c-a) + ab(a-b)) + (b-c)(c-a)(a-b) \\
 &= b^2c - bc^2 + ca^2 - ca^2 + ab - ab^2 + (bc - ba - c^2 + ac)(a-b) \\
 &= b^2c + c^2a + a^2b - bc^2 - ca^2 - ab^2 + (abc - b^2c - ba^2 + ab - ac + bc + a^2c - abc) \\
 &= b^2c - b^2c + c^2a - ac^2 + a^2b - ba^2 - bc^2 + bc^2 - ea^2 + ca^2 - ab + ab^2 \\
 &= 0.
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & (a+b+c) \left[(a-b)^2 + (b-c)^2 + (c-a)^2 \right] = (a+b+c) \left[a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ac + a^2 \right] \\
 &= (a+b+c) (a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac) \\
 &= 2 \left[a^3 + ab^2 + ac^2 - a^2b - abc - a^2c + ba^2 + b^3 + bc^2 + ab^2 - bc^2 - abc \right. \\
 &\quad \left. + ca^2 + cb^2 + c^3 - abc - bc^2 - ac^2 \right] \\
 &= 2 [a^3 + b^3 + c^3 - 3abc]
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & (b-c)(x-a)^2 = (b-c)(x^2 - 2ax + a^2) = b^2x^2 - 2abx + ba^2 - cx^2 + 2acx - ca^2 \\
 & \text{Demi } (c-a)(x-b)^2 = cx^2 - 2bcx + cb^2 - ax^2 + 2abx - ab^2 \\
 & (a-b)(x-c)^2 = ax^2 - 2acx + ac^2 - bx^2 + 2bcx - bc^2
 \end{aligned}$$

En regroupant et réduisant, il vient :

$$(b-c)(x-a)^2 + (c-a)(x-b)^2 + (a-b)(x-c)^2 = \underline{ba^2 + cb^2 + ac^2 - cd^2 - ab^2 - bc^2} \\ (= ab(a-b) + bc(b-c) + ac(c-a))$$

$$(e): (a+b)^2 + (b+c)^2 + (c+a)^2 = a^2 + 2ab + b^2 + b^2 + 2bc + c^2 + c^2 + 2ac + a^2 = 2[a^2 + b^2 + c^2 + ab + bc + ac]$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$\text{Donc } (a+b)^2 + (b+c)^2 + (c+a)^2 - (a+b+c)^2 = 2a^2 + 2b^2 + 2c^2 + 2ab + 2bc + 2ac - a^2 - b^2 - c^2 - 2ab - 2bc - 2ac \\ = \underline{a^2 + b^2 + c^2}$$

$$(f): a^2(a-b)(a-c) + b^2(b-c)(b-a) + c^2(c-a)(c-b)$$

$$= a^2(a^2 - ba - ac + bc) + b^2(b^2 - cb - ab + ac) + c^2(c^2 - ac - cb + ab)$$

$$= a^4 - ba^3 - a^3c + a^2bc + b^4 - cb^3 - ab^3 + ab^2c + c^4 - ac^3 - cb^3 + abc^2$$

$$= \underline{a^4 + b^4 + c^4 + a^2bc + ab^2c + abc^2 - ba^3 - cb^3 - cb^3 - ac^3 - ca^3}$$

$$(\text{Si l'on va plus loin: } a^4 + b^4 + c^4 + abc(a+b+c) - ab(a^2+b^2) - ac(a^2+c^2) - bc(b^2+c^2))$$

$$(\text{voire: } a^4 + b^4 + c^4 + 4abc(a+b+c) - (ab+ac+bc)(a^2+b^2+c^2))$$

Exercice 5

$$1(a) \quad 5(2x-3) - 4(5x-7) = 13 - 2(x+11)$$

$$10x - 15 - 20x + 28 = 13 - 2x - 22$$

$$-10x + 13 = 13 - 2x - 22$$

$$\text{donc } -10x + 13 = -2x - 3$$

$$\text{et } -10x = -2x - 3 - 13 = -2x - 16$$

$$\text{d'où: } 2x - 10x = 2x - 2x - 16$$

$$\text{et } -8x = -16$$

$$\text{donc } x = \frac{-16}{-8} = 2.$$

$$\left(\begin{array}{l} \text{Vérifions: } 5(2x-3) - 4(5x-7) = 5x(4-3) - 4x(10-7) = 5x1 - 4x3 = 5 - 12 = -7. \\ 13 - 2(x+11) = 13 - 2x - 22 = 13 - 26 = -7. \end{array} \right)$$

$$(b) \quad 4(x+3) - 7x + 17 = 8(5x-1) + 166$$

$$4x + 12 - 7x + 17 = 40x - 8 + 166$$

$$4x - 7x + 12 + 17 = 40x + 158$$

$$-3x + 29 = 40x + 158$$

$$-3x + 29 - 29 = 40x + 158 - 29$$

$$-3x = 40x + 129$$

$$-3x - 40x = 40x + 129 - 40x$$

$$-43x = 129$$

$$x = \frac{129}{-43} = -3.$$

$$\text{Vérifions: } 4(3+3) - 7(-3) + 17 = 4 \cdot 0 + 21 + 17 = 38.$$

$$8(5 \cdot (-3) - 1) + 166 = 8 \cdot (-15 - 1) + 166 = 8 \cdot -16 + 166 = -128 + 166 = 38.$$

$$(c) \quad 17 - 14(x+1) = 13 - 4(x+1) - 5(x-3)$$

$$17 - (14x + 14) = 13 - (4x + 4) - (5x - 15)$$

$$17 - 14x - 14 = 13 - 4x - 4 - 5x + 15$$

$$3 - 14x = 24 - 9x$$

$$\begin{aligned} \text{Donc } 3 - 14x - 3 &= 24 - 9x - 3 \\ -14x &= 21 - 9x \\ -14x + 9x &= 21 - 9x + 9x \\ -5x &= 21 \\ x &= -\frac{21}{5} (-4,2) \end{aligned}$$

$$\text{Vérifions: } 17 - 14\left(-\frac{21}{5} + 1\right) = 17 - 14\left(-\frac{16}{5}\right) = 17 + \frac{14 \cdot 16}{5} = \frac{85}{5} + \frac{224}{5} = \frac{309}{5}$$

$$\begin{aligned} 13 - 4(x+1) - 5(x-3) &= 13 - 4\left(\frac{-21}{5} + \frac{5}{5}\right) - 5\left(\frac{-21}{5} - \frac{15}{5}\right) = \frac{65}{5} - 4 \cdot \frac{-16}{5} - 5 \cdot \frac{-36}{5} \\ &= \frac{65}{5} + \frac{64}{5} + \frac{180}{5} = \frac{309}{5} \end{aligned}$$

$$(d). 5x + 3,5 + (3x - 4) = 7x - 3(x - 0,5)$$

$$5x + 3x + 3,5 - 4 = 7x - 3x + 1,5$$

$$8x - 0,5 = 4x + 1,5$$

$$8x - 0,5 + 0,5 = 4x + 1,5 + 0,5$$

$$8x = 4x + 2$$

$$8x - 4x = 4x + 2 - 4x$$

$$4x = 2$$

$$x = 0,5 \quad (\text{ou } x = \frac{1}{2})$$

$$\text{Vérifions: } 5 \cdot 0,5 + 3,5 + (3 \cdot 0,5 - 4) = 2,5 + 3,5 + (1,5 - 4)$$

$$= 6 - 2,5 = 3,5.$$

$$\text{et } 7 \cdot 0,5 - 3 \cdot (0,5 - 0,5) = 3,5 - 3 \cdot 0 = 3,5.$$

$$(e). 7(4x+3) - 4(x-1) = 15(x+0,75) + 7$$

$$28x + 21 - (4x - 4) = 15x + 15 \cdot \frac{3}{4} + 7$$

$$24x + 21 + 4 = 15x + \frac{45}{4} + 7$$

$$24x + 25 = 15x + 11,25 + 7$$

$$24x + 25 - 25 = 15x + 18,25 - 25$$

$$24x = 15x - 6,75 \quad \text{donc } 24x - 15x = 15x - 6,75 - 15x$$

$$\text{et } 24x = -6,75$$

$$\begin{array}{r}
 675 \\
 -63 \\
 \hline
 45 \\
 -45 \\
 \hline
 0
 \end{array}
 \quad | \quad 8$$

Donc $x = \frac{-6,75}{8} = -0,75$.

Vérification: $7x(4x - 0,75 + 3) - 4x(-0,75 - 1) = 7x(-3 + 3) - 4x - 1,75$
 $= 7x0 + 7 = 7$.
 $15x(-0,75 + 0,75) + 7 = 15x0 + 7 = 7$.

(8): $17x + 15(x - 1) = -1 - 14(3x + 1)$

$$17x + 15x - 15 = -1 - (12x + 14)$$

$$32x - 15 = -1 - 14 - 12x$$

$$32x - 15 = -42x - 15$$

$$32x - 15 + 15 = -42x - 15 + 15$$

$$32x = -42x$$

$$32x + 42x = 0$$

$$74x = 0$$

$$\underline{x = 0}$$

Vérification: $17 \cdot 0 + 15(0 - 1) = 0 + 15x - 1 = -15$.

$$-1 - 14(3 \cdot 0 + 1) = -1 - 14x + 1 = -1 - 14 = -15$$
.

2. (a) $(x - 1)^2 + (x + 3)^2 = 2(x - 2)(x + 1) + 38$

$$x^2 - 2x + 1 + x^2 + 6x + 9 = 2(x^2 - 2x + 1 - 2) + 38$$

$$2x^2 + 4x + 10 = 2(x^2 - x - 2) + 38$$

$$2x^2 + 4x + 10 = 2x^2 - 2x - 4 + 38$$

$$2x^2 + 4x + 10 = 2x^2 - 2x + 34$$

$$4x + 10 = -2x + 34$$

$$4x + 2x = 34 - 10$$

$$6x = 24$$

$$x = 4$$

(En soustrayant les deux termes de l'équation, on trouve 58 des deux côtés).

$$\begin{aligned}
 \text{(b)} \quad & 5(x^2 - 2x - 1) + 2(3x - 2) = 5(x+1)^2 \\
 & 5x^2 - 10x - 5 + 6x - 4 = 5(x^2 + 2x + 1) \\
 & 5x^2 - 4x - 9 = 5x^2 + 10x + 5 \\
 & -4x - 9 = 10x + 5 \\
 & -10x - 4x = 5 + 9 \\
 & -14x = 14 \\
 & \underline{x = \frac{14}{-14} = -1}
 \end{aligned}$$

En vérifiant, en $x = -1$, les deux termes valent 0.

$$\begin{aligned}
 \text{(c)} \quad & (8x+1)(x-2) = (3x+4)(3x-5) \\
 & 8x^2 - 18x + x - 2 = 9x^2 - 15x + 12x - 20 \\
 & 8x^2 - 17x - 2 = 9x^2 - 3x - 20 \\
 & -17x - 2 = -3x - 20 \\
 & -17x + 3x = -20 + 2 \\
 & -14x = -18 \\
 & \underline{x = \frac{-18}{-14} = \frac{9}{7}}
 \end{aligned}$$

Vérification faite, on trouve de chaque côté $\frac{-440}{49}$.

$$\begin{aligned}
 \text{(d)} \quad & 7(3 - 2x) - 5x(2x - 1) = (5x + 3)(3 - 2x) \\
 & 21 - 14x - (10x^2 - 5x) = 15x - 10x^2 + 9 - 6x \\
 & 21 - 14x - 10x^2 + 5x = 9x - 10x^2 + 9 \\
 & -10x^2 - 8x + 21 = -10x^2 + 9x + 9 \\
 & -8x + 21 = 9x + 9 \\
 & -8x - 9x = 9 - 21 \\
 & -18x = -12 \\
 & \underline{x = \frac{-12}{-18} = \frac{2}{3}}
 \end{aligned}$$

Après calcul, on trouve deux fois $\frac{25}{9}$.

$$\begin{aligned}
 \text{(e)} \quad & (3x-1)^2 - (2x+3)^2 + 7 = (2x+1)(2x-1) + x(2x+7) \\
 & 9x^2 - 6x + 1 - (4x^2 + 12x + 9) + 7 = (2x)^2 - 1^2 + x^2 + 7x \\
 & 9x^2 - 6x + 1 - 4x^2 - 12x - 9 + 7 = 4x^2 - 1 + x^2 + 7x \\
 & 5x^2 - 18x - 1 = 5x^2 + 7x - 1 \\
 & -18x - 1 = 7x - 1 \\
 & -18x = 7x \\
 & -25x = 0 \\
 & \underline{x = 0}.
 \end{aligned}$$

Après calcul en $x=0$, les deux termes valent -1.

$$\begin{aligned}
 \text{(f)} \quad & (x+2)^3 + (x-2)^3 + (x+1)^3 = 3(x+1)(x+2)(x-2) ?
 \end{aligned}$$

Gén: $(x+2)^3 = (x+2)(x+2)^2 = (x+2)(x^2 + 4x + 4) = x^3 + 6x^2 + 12x + 8$.

$$(x-2)^3 = (x-2)(x-2)^2 = (x-2)(x^2 - 4x + 4) = x^3 - 4x^2 + 4x - 8x^2 + 8x - 8 = x^3 - 6x^2 + 12x - 8$$

$$(x+1)^3 = (x+1)(x+1)^2 = (x+1)(x^2 + 2x + 1) = x^3 + 2x^2 + x + x^2 + 2x + 1 = x^3 + 3x^2 + 3x + 1$$

$$\begin{aligned}
 \text{En additionnant: } & (x+2)^3 + (x-2)^3 + (x+1)^3 = x^3 + 6x^2 + 12x + 8 \\
 & + x^3 - 6x^2 + 12x - 8 \\
 & + x^3 + 3x^2 + 3x + 1 \\
 & = 3x^3 + 3x^2 + 27x + 1.
 \end{aligned}$$

$$\begin{aligned}
 \text{D'autre part: } & 3(x+1)(x+2)(x-2) = 3(x+1)(x^2 - 4) \\
 & = 3(x^3 - 4x + x^2 - 4) \\
 & = 3x^3 + 3x^2 - 12x - 12.
 \end{aligned}$$

$$\begin{aligned}
 \text{Donc: } & (x+2)^3 + (x-2)^3 + (x+1)^3 = 3(x+1)(x+2)(x-2) \\
 \Leftrightarrow & 3x^3 + 3x^2 + 27x + 1 = 3x^3 + 3x^2 - 12x - 12 \\
 \Leftrightarrow & 27x + 1 = -12x - 12 \\
 \Leftrightarrow & 27x + 12x = -12 - 1 \\
 \Leftrightarrow & 39x = -13 \\
 \Leftrightarrow & x = \frac{-13}{39} = \frac{-1}{3}.
 \end{aligned}$$

La vérification donne des deux côtés: $\boxed{-\frac{70}{3}}$.

$$3(a) \frac{5}{2}x + 3 - \frac{7}{4}x = x + \frac{9}{4}$$

On multiplie par 4 pour ne plus avoir de dénominateurs:

$$4\left(\frac{5}{2}x - \frac{7}{4}x + 3\right) = 4\left(x + \frac{9}{4}\right)$$

$$4 \cdot \frac{5}{2}x - 4 \cdot \frac{7}{4}x + 4 \cdot 3 = 4x + 4 \cdot \frac{9}{4}$$

$$10x - 7x + 12 = 4x + 9$$

$$3x + 12 = 4x + 9$$

$$12 - 9 = 4x - 3x$$

$$3 = x \text{ donc } x = 3$$

Après calcul en $x = 3$, les deux termes valent $\frac{21}{4}$ (= 5,25)

$$(b) \frac{3x-7}{2} + \frac{x+1}{3} = -16.$$

Pour éliminer les dénominateurs 2 et 3, on multiplie par leur p.p.c.m: $2 \times 3 = 6$.

$$6 \cdot \left(\frac{3x-7}{2} + \frac{x+1}{3}\right) = 6 \cdot -16$$

$$\frac{6(3x-7)}{2} + \frac{6(x+1)}{3} = -96$$

$$3(3x-7) + 2(x+1) = -96$$

$$9x - 21 + 2x + 2 = -96$$

$$11x - 19 = -96$$

$$11x = 19 - 96 = -77$$

$$\text{et } x = -\frac{77}{11} = -7$$

Après calcul, les deux termes valent en $x = -7$: -16 (car $\frac{-28}{2} + \frac{-6}{3} = -14 - 2 = -16$)

$$(c) x + \frac{1}{2} - \frac{x}{6} = 16 - \frac{2x}{3} + \frac{1}{3}$$

Le plus petit multiple commun de 2, 6, 9 et 3 est 18.

$$18\left(x + \frac{1}{2} - \frac{x}{6}\right) = 18\left(16 - \frac{2x}{3} + \frac{1}{3}\right)$$

$$18x + \frac{18}{2} - \frac{18x}{6} = 18 \cdot 16 - \frac{2x \cdot 18}{3} + \frac{18}{3}$$

$$18x + 9 - 3x = 288 - 4x + 6$$

$$15x + 9 = 284 - 4x$$

$$15x + 4x = 284 - 9$$

$$19x = 285$$

$$x = \frac{285}{19} \text{ donc } x = \underline{\underline{15}}$$

$$\begin{array}{r} 285 \\ -13 \downarrow \\ \hline 15 \end{array}$$

En $x = 15$, les deux termes valent 13.

$$(d) \frac{7x}{4} - 2 - \frac{x}{2} = \frac{2x}{13} - \frac{85}{52}$$

Le plus petit multiple commun de 4, 2, 13 et 52 est 52 ($= 13 \times 2 \times 2$).
 $= 13 \times 4 = 26 \times 2$

$$52\left(\frac{7x}{4} - 2 - \frac{x}{2}\right) = \left(\frac{2x}{13} - \frac{85}{52}\right) \times 52$$

$$\frac{52 \cdot 7x}{4} - 52 \cdot 2 - \frac{52x}{2} = \frac{52 \cdot 2x}{13} - \frac{52 \cdot 85}{52}$$

$$13 \cdot 7x - 104 - 26x = 8x - 85$$

$$91x - 26x - 104 = 8x - 85$$

$$65x - 104 = 8x - 85$$

$$65x - 8x = 104 - 85$$

$$57x = 19$$

$$x = \frac{19}{57} \text{ donc } x = \underline{\underline{\frac{1}{3}}}$$

Après calcul en $x = \frac{1}{3}$, les deux termes valent: $-\frac{81}{52}$.

$$(e): \frac{2x}{3} + 4 - \frac{2x}{5} = \frac{x}{2} - \frac{x}{3} + 3,5$$

On multiplie par 30: $\frac{2x \times 30}{3} + 4 \times 30 - \frac{2x \times 30}{5} = \frac{x \times 30}{2} - \frac{x \times 30}{3} + 3,5 \times 30$

$$20x + 120 - 12x = 15x - 10x + 105$$

$$8x + 120 = 5x + 105$$

$$8x - 5x = 105 - 120$$

$$3x = -15$$

$$x = -15 : 3 = -5.$$

Donc $x = -5$, les deux termes de l'équation valent $\frac{8}{3}$.

$$(f) \frac{x}{6} - 1 = \frac{x}{4} - \frac{x}{3} - 1$$

On multiplie par 12, qui est multiple de 3, 4 et 6:

$$12 \left[\frac{x}{6} - 1 \right] = 12 \left[\frac{x}{4} - \frac{x}{3} - 1 \right]$$

$$\frac{12x}{6} - 12 = \frac{12x}{4} - \frac{12x}{3} - 12$$

$$2x - 12 = 3x - 4x - 12$$

$$2x - 12 = -x - 12$$

$$2x + x = -12 + 12$$

$$3x = 0$$

$$\underline{x = 0}$$

Ou plus intelligemment, on écrit directement: $\left(\frac{x}{6} - 1 \right) + 1 = \left(\frac{x}{4} - \frac{x}{3} - 1 \right) + 1$

$$\text{donc } \frac{x}{6} = \frac{x}{4} - \frac{x}{3}$$

$$\text{d'où: } \frac{x}{6} - \frac{x}{4} + \frac{x}{3} = 0$$

$$\text{et } \left(\frac{1}{6} - \frac{1}{4} + \frac{1}{3} \right) x = 0.$$

$$\text{i.e.: } 0 = \left(\frac{1}{12} - \frac{3}{12} + \frac{1}{12} \right) x = \frac{1}{12} x \text{ et } \underline{\frac{x}{12} = 0}.$$

$$4.(2) \quad \frac{x+5}{4} - \frac{x-3}{6} = \frac{x}{3}$$

On multiplie les deux termes par 12 :

$$12 \frac{(x+5)}{4} - 12 \frac{(x-3)}{6} = 12 \frac{x}{3}$$

$$3(x+5) - 2(x-3) = 4x$$

$$3x+15 - 2x+6 = 4x$$

$$3x+15-2x+6 = 4x$$

$$x+21 = 4x$$

$$21 = 4x-x$$

$$21 = 3x$$

$$x = 21 : 3$$

$$\underline{x=7}$$

En effet pour $x=7$, on a des deux côtés $\frac{7}{3}$.

$$(1) \quad \frac{3x-7}{2} + \frac{x+1}{3} = -16$$

On multiplie par 6 : $6 \frac{(3x-7)}{2} + 6 \frac{(x+1)}{3} = (-16) \times 6$

$$3(3x-7) + 2(x+1) = -96$$

$$9x-21+2x+2 = -96$$

$$11x-19 = -96$$

$$11x = 19 - 96$$

$$11x = -77$$

$$x = -77 : 11$$

$$\underline{x = -7}$$

Pour $x=-7$, les deux termes valent -16.

$$\text{a)} \quad x - \frac{x+1}{3} = \frac{2x+1}{5}$$

$$\text{On multiplie par 15: } 15x - \frac{15(x+1)}{3} = \frac{15(2x+1)}{5}$$

$$15x - 5(x+1) = 3(2x+1)$$

$$15x - 5x - 5 = 6x + 3$$

$$10x - 5 = 6x + 3$$

$$10x - 6x = 5 + 3$$

$$4x = 8$$

$$\underline{x = 8 : 4 = 2}$$

Pour $x=2$ les deux termes valent 1.

$$\text{d)} \quad \frac{7-3x}{12} + \frac{3}{4} = 2(x-2) + \frac{5(5-2x)}{6}$$

$$\text{On multiplie par 12: } 12 \frac{7-3x}{12} + 12 \cdot \frac{3}{4} = 12 \cdot 2(x-2) + 12 \cdot \frac{5(5-2x)}{6}$$

$$7-3x + 9 = 24(x-2) + 10(5-2x)$$

$$16-3x = 24x - 48 + 50 - 20x$$

$$16-3x = 4x + 2$$

$$16-2 = 4x + 3x$$

$$14 = 7x$$

$$x = 14 : 7$$

$$\underline{x = 2}.$$

Les deux termes valent alors $\frac{5}{6}$.

$$\text{e)} \quad \frac{x}{5} - \frac{3x-1}{6} + \frac{3-x}{4} = 0$$

$$\text{On multiplie par 60: } 60 \frac{x}{5} - 60 \frac{3x-1}{6} + 60 \frac{3-x}{4} = 0$$

$$12x - 10(3x-1) + 15(3-x) = 0$$

$$12x - 30x + 10 + 45 - 15x = 0$$

$$-33x + 55 = 0$$

(41)

$$33x = 55 \quad \text{donc } x = \frac{55}{33} = \frac{5}{3}$$

$$(F) \frac{3(x+3)}{4} + \frac{1}{2} = \frac{5x+9}{3} - \frac{7x-9}{4}$$

$$\text{On multiplie par 12: } 12 \cdot \frac{3}{4}(x+3) + 12 \cdot \frac{1}{2} = 12 \cdot \frac{5x+9}{3} - 12 \cdot \frac{7x-9}{4}$$

$$9(x+3) + 6 = 4(5x+9) - 3(7x-9)$$

$$9x + 27 + 6 = 20x + 36 - 21x + 27$$

$$9x + 33 = -x + 63$$

$$9x + x = 63 - 33$$

$$10x = 30$$

$$x = 30 : 10$$

$$\underline{x = 3}$$

Pour $x = 3$, on a des deux côtés 5.

$$(G) \frac{2x-7}{5} + \frac{x+11}{2} = -4$$

$$\text{On multiplie par 10: } 10 \cdot \frac{2x-7}{5} + 10 \cdot \frac{x+11}{2} = -4 \times 10$$

$$2(2x-7) + 5(x+11) = -40$$

$$4x - 14 + 5x + 55 = -40$$

$$9x + 41 = -40$$

$$9x = -40 - 41$$

$$9x = -81$$

$$x = -81 : 9$$

$$\underline{x = -9}$$

Pour $x = -9$, on trouve bien -4 des deux côtés.

$$(H) \frac{2x-3}{3} - \frac{x-3}{6} = \frac{4x+3}{4} - 17$$

$$\text{On multiplie par 24: } 24 \cdot \frac{2x-3}{3} - 24 \cdot \frac{x-3}{6} = 24 \cdot \frac{4x+3}{4} - 24 \cdot 17$$

$$8(2x-3) - 4(x-3) = 6(4x+3) - 408$$

$$16x - 24 - 4x + 12 = 24x + 18 - 408$$

$$12x - 12 = 24x - 390$$

$$390 - 12 = 24x - 12x$$

$$378 = 12x$$

$$\underline{x = \frac{378}{12} = \frac{63}{2}}$$

Donc $x = \frac{63}{2}$, les deux termes valent $\frac{61}{4}$.

$$(i) \frac{5x-3}{4} - \frac{7x-5}{9} = \frac{x+19}{6}$$

On multiplie par 36: $36 \frac{5x-3}{4} - 36 \frac{7x-5}{9} = 36 \frac{x+19}{6}$

$$9(5x-3) - 4(7x-5) = 6(x+19)$$

$$45x - 27 - 28x + 20 = 6x + 114$$

$$17x - 7 = 6x + 114$$

$$17x - 6x = 114 + 7$$

$$11x = 121$$

$$x = 121 \div 11$$

$$\underline{x = 11}.$$

Pour $x = 11$, les deux termes valent 5.

$$(ii) \frac{5x+1}{8} - \frac{x-1}{3} = \frac{4(2x-3)}{9}$$

On multiplie par 72: $72 \frac{5x+1}{8} - 72 \frac{x-1}{3} = 72 \frac{4(2x-3)}{9}$

$$9(5x+1) - 24(x-1) = 32(2x-3)$$

$$45x + 9 - 24x + 24 = 64x - 96$$

$$21x + 33 = 64x - 96$$

$$21x - 64x = -96 - 33$$

$$-43x = -129$$

$$x = \frac{-129}{-43} \text{ donc } \underline{x = 3}.$$

Pour $x = 3$, on trouve des deux côtés $\frac{4}{3}$.

$$(k) : \frac{2x-1}{3} - \frac{5x+2}{7} = x+13$$

On multiplie par 21: $\frac{21(2x-1)}{3} - \frac{21(5x+2)}{7} = 21(x+13)$

$$7(2x-1) - 3(5x+2) = 21x + 273$$

$$14x - 7 - 15x - 6 = 21x + 273$$

$$-x - 13 = 21x + 273$$

$$-x - 21x = 273 + 13$$

$$-22x = 286$$

$$x = -\frac{286}{22} \text{ donc } x = -26.$$

Pour $x = -26$, on trouve -13 des deux côtés.

$$(l) : \frac{8x+2}{5} - \frac{x-11}{7} = \frac{5x-3}{2} - \frac{3x-1}{4}$$

On multiplie par 140: $\frac{140(8x+2)}{5} - \frac{140(x-11)}{7} = \frac{140(5x-3)}{2} - \frac{140(3x-1)}{4}$

$$28(8x+2) - 20(x-11) = 70(5x-3) - 35(3x-1)$$

$$224x + 56 - 20x + 220 = 350x - 210 - 105x + 35$$

$$204x + 276 = 245x - 175$$

$$245x - 204x = 276 + 175$$

$$41x = 451$$

$$x = 451 \div 41 = 11$$

Pour $x = 11$, on trouve des deux côtés 18.

$$(m) : \frac{2x-7}{3} - \frac{x-5}{6} = \frac{x-3}{8}$$

Pour simplifier, on multiplie par 72: $\frac{72(2x-7)}{3} - \frac{72(x-5)}{6} = \frac{72(x-3)}{8}$

$$8(2x-7) - 12(x-5) = 9(x-3)$$

$$16x - 56 - 12x + 60 = 9x - 27$$

$$4x + 4 = 9x - 27$$

$$9x - 4x = 27 + 4$$

$$5x = 31 \text{ donc } x = \frac{31}{5} = 6.2$$

Pour $x=17$, on trouve des deux côtés 1.

$$(n): \frac{5x+7}{4} - \frac{3x+5}{8} = \frac{4x+3}{5} - \frac{x-9}{3}$$

On multiplie par 120: $120 \cdot \frac{5x+7}{4} - 120 \cdot \frac{3x+5}{8} = 120 \cdot \frac{4x+3}{5} - 120 \cdot \frac{x-9}{3}$

$$30(5x+7) - 15(3x+5) = 24(4x+3) - 40(x-9)$$

$$150x + 210 - 45x - 75 = 96x + 216 - 40x + 360$$

$$105x + 135 = 56x + 576$$

$$105x - 56x = 576 - 135$$

$$49x = 441$$

$$x = 441 \div 49$$

donc $\underline{\underline{x = 9}}$.

Pour $x=9$ les deux termes valent 1.

$$(o): \frac{5x+6}{7} - \frac{3x+1}{4} = \frac{x+16}{5}$$

On multiplie par 140: $140 \cdot \frac{5x+6}{7} - 140 \cdot \frac{3x+1}{4} = 140 \cdot \frac{x+16}{5}$

$$20(5x+6) - 35(3x+1) = 28(x+16)$$

$$100x + 120 - 105x - 35 = 28x + 448$$

$$-5x + 85 = 28x + 448$$

$$-28x - 5x = 448 - 85$$

$$-33x = 363$$

$$x = 363 \div -33$$

$x = -11$.

Pour $x=-11$, on trouve des deux côtés 1.

$$(p): \frac{4x+7}{5} - \frac{x-5}{6} = \frac{2x+14}{3} - \frac{2x-7}{9}$$

On multiplie par 270: $270 \cdot \frac{4x+7}{5} - 270 \cdot \frac{(x-5)}{6} = 270 \cdot \frac{(2x+14)}{3} - 270 \cdot \frac{(2x-7)}{9}$

$$54(4x+7) - 45(x-5) = 30(2x+14) - 30(2x-7)$$

$$216x + 378 - 45x + 225 = 180x + 1260 - 60x + 210$$

$$171x + 603 = 120x + 1470$$

$$171x - 120x = 1470 - 603$$

$$51x = 867$$

$$x = 867 \div 51$$

$$\underline{x = 17}$$

Pour $x = 17$, on trouve des deux côtés 13.

$$5.6) \frac{(x-1)(x+5)}{3} - \frac{(x+2)(x+5)}{12} = \frac{(x-1)(x+2)}{4}$$

On multiplie par 12: $\frac{12}{3}(x-1)(x+5) - \frac{12}{12}(x+2)(x+5) = \frac{12}{4}(x-1)(x+2)$

$$4(x-1)(x+5) - (x+2)(x+5) = 3(x-1)(x+2)$$

$$4(x^2 - x + 5x - 5) - (x^2 + 2x + 5x + 10) = 3(x^2 + x - 2)$$

$$4(x^2 - x + 5x - 5) - (x^2 + 2x + 5x + 10) = 3(x^2 + x - 2)$$

$$4(x^2 + 4x - 5) - (x^2 + 7x + 10) = 3(x^2 + x - 2)$$

$$4x^2 + 16x - 20 - x^2 - 7x - 10 = 3x^2 + 3x - 6$$

$$3x^2 + 9x - 30 = 3x^2 + 3x - 6$$

$$9x - 30 = 3x - 6$$

$$9x - 3x = 30 - 6$$

$$6x = 24$$

$$x = 24 \div 6$$

$$\underline{x = 4}$$

On peut vérifier que pour $x=4$ les deux termes de l'équation valent $\frac{9}{2}$.

$$(7) \frac{(x+1)^2}{3} + \frac{(x-2)(x-3)}{2} = \frac{(5x-1)(x-4)}{6} + \frac{28}{3}$$

On multiplie par 6: $6 \frac{(x+1)^2}{3} + 6 \frac{(x-2)(x-3)}{2} = 6 \frac{(5x-1)(x-4)}{6} + 6 \cdot \frac{28}{3}$

$$2(x+1)^2 + 3(x-2)(x-3) = (5x-1)(x-4) + 56$$

$$2(x^2 + 2x + 1) + 3(x^2 - 2x - 3x + 6) = (5x^2 - 20x - x + 4) + 56$$

$$2x^2 + 4x + 2 + 3x^2 - 5x + 6 = 5x^2 - 21x + 60$$

$$2x^2 + 4x + 2 + 3x^2 - 15x + 18 = 5x^2 - 21x + 60$$

$$5x^2 - 11x + 20 = 5x^2 - 21x + 60$$

$$-11x + 20 = -21x + 60$$

$$21x - 11x = 60 - 20$$

$$10x = 40$$

$$x = 40 \div 10$$

$$\underline{x = 4}$$

Pour $x = 4$, on trouve des deux côtés $\frac{28}{3}$.

$$(c): \frac{(3x+1)(3x-1)}{9} - \frac{(x-5)(x+1)}{2} = \frac{(3x-1)(x+3)}{18} + \frac{8}{9}$$

$$\text{On multiplie par 18: } 18 \frac{(3x+1)(3x-1)}{9} - 18 \frac{(x-5)(x+1)}{2} = 18 \frac{(3x-1)(x+3)}{18} + 18 \cdot \frac{8}{9}$$

$$2(3x+1)(3x-1) - 9(x-5)(x+1) = (3x-1)(x+3) + 2 \cdot 8$$

$$2(9x^2 - 1) - 9(x^2 - 5x + x - 5) = 9x^2 - x + 27x - 3 + 16$$

$$18x^2 - 2 - 9x^2 + 40x - 5 = 9x^2 + 26x + 13$$

$$18x^2 - 2 - 9x^2 + 36x + 45 = 9x^2 + 26x + 13$$

$$9x^2 + 36x + 43 = 9x^2 + 26x + 13$$

$$36x + 43 = 26x + 13$$

$$36x - 26x = 13 - 43$$

$$10x = -30$$

$$x = -30 \div 10$$

$$\underline{\underline{x = -3}}$$

Pour $x = -3$, on trouve des deux côtés $\frac{8}{3}$.

$$(d): \frac{(4x+7)^2}{4} - \frac{(5x-1)^2}{7} = \frac{(8x-3)(3x+4) - 79x}{56}$$

$$\text{On multiplie par 56: } 56 \frac{(4x+7)^2}{4} - 56 \frac{(5x-1)^2}{7} = 56 \frac{(8x-3)(3x+4) - 79x}{56}$$

$$14(4x+7)^2 - 8(5x-1)^2 = (8x-3)(3x+4) - 79x$$

$$\begin{aligned}
 & 14(16x^2 + \cancel{2x+4x+7} + 4) - 8(25x^2 - 10x + 1) = 24x^2 + 32x - 9x - 12 - 78x \\
 & 224x^2 + 784x + 686 - 200x^2 + 80x - 8 = 24x^2 - 56x - 12 \\
 & 24x^2 + 864x + 678 = 24x^2 - 56x - 12 \\
 & 864x + 678 = -56x - 12 \\
 & 864x + 56x = -12 - 678 \\
 & 920x = -690 \\
 & x = \frac{-690}{920} \text{ donc } x = \underline{\underline{-\frac{3}{4}}}
 \end{aligned}$$

Pour $x = \frac{-3}{4}$, on trouve des deux côtés $\frac{87}{112}$.

$$(e): \left(x - \frac{8}{3}\right)(x + 0,75) = (x + 4,5)(x + 1,5) - \frac{145}{3}$$

$$\left(x - \frac{8}{3}\right)\left(x + \frac{3}{4}\right) = \left(x + \frac{9}{2}\right)\left(x + \frac{3}{2}\right) - \frac{145}{3}$$

$$x^2 - \frac{8}{3}x + \frac{3}{4}x - \frac{8}{3} \cdot \frac{3}{4} = x^2 + \frac{9}{2}x + \frac{3}{2}x + \frac{9}{2} - \frac{145}{3}$$

$$x^2 + \left(\frac{3}{4} - \frac{8}{3}\right)x - 2 = x^2 + \frac{12}{2}x - \frac{145}{3} + \frac{27}{4}$$

$$\left(\frac{3}{4} - \frac{8}{3}\right)x - 2 = 6x - \frac{145}{3} + \frac{27}{4}$$

$$\text{Chmuliplier par 12: } \left[12 \cdot \frac{3}{4} - 12 \cdot \frac{8}{3}\right]x - 12 \cdot 2 = 12 \cdot 6x - 12 \cdot \frac{145}{3} + 12 \cdot \frac{27}{4}$$

$$[9 - 32]x - 24 = 72x - 4 \cdot 145 + 3 \cdot 27$$

$$-23x - 24 = 72x - 580 + 81$$

$$-23x - 72x = 24 - 499$$

$$-95x = -475$$

$$x = \frac{475}{95} = \underline{\underline{5}}$$

Pour $x = 5$, on trouve des deux côtés $\frac{161}{12}$.

$$(8) \frac{(x-5)^2}{5} + \frac{(x+3)^2}{3} = \frac{(3x+1)(3x-1)-x(6x+1)}{15}$$

On multiplie par 15: $\frac{15(x-5)^2}{5} + \frac{15(x+3)^2}{3} = 15 \left[9x^2 - 1 - 6x^2 - x \right]$

$$3(x^2 - 10x + 25) + 5(x^2 + 6x + 9) = 9x^2 - 1 - x^2 - x$$

$$3x^2 - 30x + 75 + 5x^2 + 30x + 45 = 8x^2 - x - 1.$$

$$8x^2 + 120 = 8x^2 - x - 1$$

$$120 = -x - 1$$

$$x = -120 - 1$$

$$\text{donc } x = \underline{-121}.$$

Pour $x = -121$, on trouve des deux termes $\frac{117+48}{15}$ pour

$$(9) \left(3x - \frac{4}{5} \right) \left(5x + \frac{2}{3} \right) = 15(x-1)(x+1) + \frac{7}{15}$$

$$15x^2 + 3x \cdot \frac{2}{3} - \frac{4}{5} \cdot 5x - \frac{4 \cdot 2}{5 \cdot 3} = 15(x^2 - 1) + \frac{7}{15}$$

$$15x^2 + 2x - 4x - \frac{8}{15} = 15x^2 - 15 + \frac{7}{15}$$

$$-2x - \frac{8}{15} = -15 + \frac{7}{15}$$

$$-2x = -15 + \frac{7}{15} + \frac{8}{15}$$

$$-2x = -15 + 1$$

$$-2x = -14$$

$$x = -14 \div -2$$

$$\underline{x=7}$$

Pour $x = 7$, on trouve $\frac{10807}{15}$

$$1.(a) (x-1)(x+2)(x+3)=0 \text{ pour } \begin{cases} x=1 \\ x=-2 \\ x=3 \end{cases}$$

$$(b): (x-3)(x-4)(x-5)=0 \text{ pour } x=3, x=4 \text{ et } x=5.$$

$$(c): (2x+1)(x+1)(4x-3)=0 \text{ si } x=-1$$

ou $2x+1=0$, c'est à dire $x=\frac{-1}{2}$

ou $4x-3=0$, c'est à dire $x=\frac{3}{4}$.

$$(d): (2x+1)(x+4)(3x+1)=0 \text{ si } 2x+1=0 \text{ c'est à dire } x=-\frac{1}{2}$$

ou $x+4=0$ c'est à dire $x=-4$

$$(e): x(5x+1)(4x-3)(3x-4)=0$$

si $x=0$

ou $5x+1=0$ donc $x=\frac{-1}{5}$

ou $4x-3=0$ donc $x=\frac{3}{4}$

ou $3x-4=0$ donc $x=\frac{4}{3}$

$$(f): 5x(3x-7)=0 \text{ si } x=0 \text{ ou } 3x-7=0 \text{ donc } x=\frac{7}{3}.$$

$$2.(a) x^2-3x=0$$

$$x(x-3)=0 \text{ donc } x=0 \text{ ou } x=3.$$

$$(b): 5x^2+8x=0$$

$$x(5x+8)=0 \text{ donc } x=0 \text{ ou } 5x+8=0, \text{ c'est à dire } x=-\frac{8}{5}.$$

$$(c): 4x^2-\frac{7x}{3}=0 \Leftrightarrow x\left(4x-\frac{7}{3}\right)=0 \text{ donc } x=0 \text{ ou } 4x-\frac{7}{3}=0$$

$$4x=\frac{7}{3} \\ \text{donc } x=\frac{7}{12}$$

$$(d): \frac{x^2}{5}+x=0 \Leftrightarrow x\left(\frac{x}{5}+1\right)=0 \text{ donc } x=0 \text{ ou } \frac{x}{5}+1=0$$

$$\frac{x}{5}=-1$$

$$\text{donc } x=-5$$

$$(e) : -\frac{3x^2}{5} + x = 0 \Leftrightarrow x \left(-\frac{3x}{5} + 1 \right) = 0 \text{ donc } \underline{x=0}$$

$$\text{ou } -\frac{3x}{5} + 1 = 0 \\ \frac{3x}{5} = 1 \\ \text{donc } x = \frac{5}{3}$$

$$(f) : -\frac{5x^2}{7} - \frac{3x}{4} = 0 \Leftrightarrow -x \left(\frac{5x}{7} + \frac{3}{4} \right) = 0 \text{ donc } \underline{x=0}$$

$$\text{ou } \frac{5x}{7} + \frac{3}{4} = 0$$

$$\frac{5x}{7} = -\frac{3}{4}$$

$$x = -\frac{3}{4} \times \frac{7}{5} \text{ donc } x = \underline{-\frac{21}{20}}$$

$$(g) : x(x+1) = x+1$$

$$\Leftrightarrow x(x+1) - (x+1) = 0$$

$$\Leftrightarrow (x-1)(x+1) = 0 \quad \text{donc } \underline{x=1} \text{ ou } \underline{x=-1}$$

$$(h) : (4x-1)(x-3) = (x-3)(5x+2)$$

$$\Leftrightarrow (4x-1)(x-3) - (x-3)(5x+2) = 0$$

$$(x-3)[(4x-1) - (5x+2)] = 0$$

$$(x-3)(-x-3) = 0$$

$$\text{donc } \underline{x=3} \text{ ou } \underline{x=-3}$$

$$(i) : (x+3)(x-5) + (x+3)(3x-4) = 0$$

$$(x+3)[(x-5) + (3x-4)] = 0$$

$$(x+3)(4x-9) = 0$$

$$\text{donc } \underline{x=-3} \text{ ou } \underline{4x=9} \text{ soit } \underline{x=\frac{9}{4}}$$

$$(j) : 5(x+1)(x+2)(x+3) = 4(x+1)(x+2)(x-4)$$

$$(x+1)(x+2)[5(x+3)] - (x+1)(x+2)[4(x-4)] = 0$$

$$(x+1)(x+2)[5(x+3) - 4(x-4)] = 0$$

$$(x+1)(x+2)[5x-15-4x+16]=0$$

$$(x+1)(x+2)(x+1)=0$$

donc $x = -1$ ou $x = -2$ (au $x = -1$ ceci est redondant)

$$3. (a) (x+5)(4x-1)+x^2-25=0$$

$$(x+5)(4x-1)+x^2-5^2=0$$

$$(x+5)(4x-1)+(x+5)(x-5)=0$$

$$(x+5)[(4x-1)+(x-5)]=0$$

$$(x+5)(5x-6)=0$$

donc $x = -5$

ou $x = 6$ c'est à dire $x = \frac{6}{5}$

$$(b) (x+4)(5x+9)-x^2+16=0$$

$$(x+4)(5x+9)-(x^2-4^2)=0$$

$$(x+4)(5x+9)-(x+4)(x-4)=0$$

$$(x+4)[(5x+9)-(x-4)]=0$$

$$(x+4)(4x+13)=0$$

donc $x = -4$ ou $x = \frac{-13}{4}$

$$(c) x^2-9=0$$

$$(x+3)(x-3)=0 \quad \text{donc } x = -3 \text{ ou } x = 3$$

$$(d) 5x^2-125=0$$

$$5(x^2-25)=0 \quad \text{car } 125 = 5 \times 25$$

$$5(x+5)(x-5)=0$$

donc $x = -5$ ou $x = 5$

$$(e) 4x^2-48=0$$

$$(2x)^2-7^2=0$$

$$(2x+7)(2x-7)=0 \quad \text{donc } x = \frac{-7}{2} \text{ ou } x = \frac{7}{2}$$

$$(1) : x^2 - 100 = 0$$

$$x^2 - 10^2 = 0$$

$$(x+10)(x-10) = 0 \text{ donc } x = -10 \text{ ou } x = 10$$

$$(2) : x^2 = 81$$

$$x^2 - 9^2 = 0$$

$$(x+9)(x-9) = 0 \text{ donc } x = -9 \text{ ou } x = 9$$

$$(3) : 3x^2 = 64$$

$$(3x)^2 - 8^2 = 0$$

$$(3x+8)(3x-8) = 0 \text{ donc } x = -\frac{8}{3} \text{ ou } x = \frac{8}{3}$$

$$(4) : (x+1)^2 - (2x-5)^2 = 0$$

$$[(x+1) + (2x-5)][(x+1) - (2x-5)] = 0$$

$$(3x-4)(-x+6) = 0$$

$$\text{donc } x = \frac{4}{3} \text{ ou } x = 6$$

$$(5) : (2x+7)^2 - (4x-9)^2 = 0$$

$$[(2x+7) + (4x-9)][(2x+7) - (4x-9)] = 0$$

$$(6x-2)(-2x+16) = 0$$

$$\text{donc } x = \frac{2}{6} = \frac{1}{3} \text{ ou } x = \frac{16}{2} = 8$$

$$(6) : (5x+1)^2 = (x-1)^2$$

$$(5x+1)^2 - (x-1)^2 = 0$$

$$[(5x+1) + (x-1)][(5x+1) - (x-1)] = 0$$

$$(6x)(4x+2) = 0$$

$$\text{donc } x = 0 \text{ ou } x = -\frac{2}{4} = -\frac{1}{2}$$

$$(7) : (3x+1)^2 = (x-4)^2$$

$$(3x+1)^2 - (x-4)^2 = 0$$

$$[(3x+1) + (x-4)][(3x+1) - (x-4)] = 0$$

$$(4x-3)(2x+5) = 0$$

$$\text{donc } x = \frac{3}{4} \text{ ou } x = -\frac{5}{2}$$

$$(m) 4(x+1)^2 - 9(x-1)^2 = 0$$

$$[2(x+1)]^2 - [3(x-1)]^2 = 0$$

$$(2x+2)^2 - (3x-3)^2 = 0$$

$$[2x+2+3x-3][2x+2-(3x-3)] = 0$$

$$(5x-1)(-x+5) = 0$$

d'oreille $x = \frac{1}{5}$ ou $\underline{x=5}$.

$$(n) (x+7)^2 - 81(x-5)^2 = 0$$

$$(x+7)^2 - [9(x-5)]^2 = 0$$

$$[(x+7)+9(x-5)][(x+7)-9(x-5)] = 0$$

$$[x+7+9x-45][x+7-9x+45] = 0$$

$$[10x-38][-8x+52] = 0$$

d'oreille $x = \frac{38}{10} = \frac{19}{5}$ ou $x = \frac{52}{8} = \underline{\frac{13}{2}}$.

$$(o) 5x^3 - 5x = 0$$

$$5x(x^2 - 1) = 0$$

$$5x(x+1)(x-1) = 0$$

d'oreille $\underline{x=0}$ ou $\underline{x=-1}$ ou $\underline{x=1}$.

$$(p) (x+1)(x-1)^2 - (x+1)(x-2)^2 = 0$$

$$(x+1)[(x-1)^2 - (x-2)^2] = 0$$

$$(x+1)[(x-1)+(x-2)][(x-1)-(x-2)] = 0$$

$$(x+1)(2x-3)(1) = 0$$

$$(x+1)(2x-3) = 0$$

d'oreille $\underline{x=-1}$ ou $\underline{x=\frac{3}{2}}$.

$$(q) 3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

d'oreille $\underline{x=0}$ ou $\underline{x=4}$

$$(r) (3x+1)(x-3)^2 = (3x+1)(2x-5)^2$$

$$(3x+1)(x-3)^2 - (3x+1)(2x-5)^2 = 0$$

$$(3x+1)[(x-3)^2 - (2x-5)^2] = 0$$

$$(3x+1)[(x-3) + (2x-5)][(x-3) - (2x-5)] = 0$$

$$(3x+1)(3x-8)(-x+2) = 0$$

$$\text{donc } x = \frac{-1}{3} \text{ ou } x = \frac{8}{3} \text{ ou } x = 2$$

$$(8) 7x^3 - 175x = 0$$

$$x(7x^2 - 175) = 0$$

$$7x(x^2 - 25) = 0$$

$$7x(x+5)(x-5) = 0$$

$$\text{donc } x = 0 \text{ ou } x = -5 \text{ ou } x = 5$$

$$(11) (x+5)(3x+2)^2 = x^2(x+5)$$

$$(x+5)(3x+2)^2 - (x+5)x^2 = 0$$

$$(x+5)[(3x+2)^2 - x^2] = 0$$

$$(x+5)[3x+2+x][3x+2-x] = 0$$

$$(x+5)(4x+2)(2x+2) = 0$$

$$(x+5)(4x+2)(x+1) = 0$$

$$\text{donc } x = -5 \text{ ou } x = -\frac{3}{4} = -\frac{1}{2} \text{ ou } x = -1.$$