

## Exercices de calcul littéral

### Exercice 1

1(a) Pour  $a=4$ :  $3a^2 - 2a + 5 = 3 \cdot 4^2 - 2 \cdot 4 + 5 = 3 \cdot 16 - 8 + 5 = 48 - 3 = 45$ .

Pour  $a=-2$ :  $3a^2 - 2a + 5 = 3 \cdot (-2)^2 - 2 \cdot (-2) + 5 = 3 \cdot 4 + 4 + 5 = 12 + 9 = 21$ .

Pour  $a=-1$ :  $3a^2 - 2a + 5 = 3 \cdot (-1)^2 - 2 \cdot (-1) + 5 = 3 \cdot 1 + 2 + 5 = 3 + 7 = 10$ .

(b) Pour  $x=+3$ ,  $3x^2 - 5x + 7 = 3 \cdot 3^2 - 5 \cdot 3 + 7 = 3 \cdot 9 - 15 + 7 = 27 - 8 = 19$ .

Pour  $x=+5$ ,  $3x^2 - 5x + 7 = 3 \cdot 5^2 - 5 \cdot 5 + 7 = 3 \cdot 25 - 25 + 7 = 75 - 18 = 57$ .

Pour  $x=-3$ ,  $3x^2 - 5x + 7 = 3 \cdot (-3)^2 - 5 \cdot (-3) + 7 = 3 \cdot 9 + 15 + 7 = 27 + 22 = 49$ .

(c) Pour  $x=5$ ,  $4x^3 - 12x^2 - 4x + 7 = 4 \cdot 5^3 - 12 \cdot 5^2 - 4 \cdot 5 + 7$   
 $= 4 \cdot 125 - 12 \cdot 25 - 20 + 7$   
 $= 500 - 300 - 13$   
 $= 187$ .

Pour  $x=-3$ ,  $4x^3 - 12x^2 - 4x + 7 = 4 \cdot (-3)^3 - 12 \cdot (-3)^2 - 4 \cdot (-2) + 7$   
 $= 4 \cdot (-27) - 12 \cdot 9 + 8 + 7$   
 $= -108 - 108 + 15$   
 $= -201$ .

Pour  $x=\frac{1}{2}$ ,  $4x^3 - 12x^2 - 4x + 7 = 4 \cdot \left(\frac{1}{2}\right)^3 - 12 \cdot \left(\frac{1}{2}\right)^2 - 4 \cdot \frac{1}{2} + 7$   
 $= 4 \cdot \frac{1}{8} - 12 \cdot \frac{1}{4} - \frac{4}{2} + 7$   
 $= \frac{4}{8} - \frac{12}{4} - 2 + 7$   
 $= \frac{1}{2} - 3 - 2 + 7$   
 $= \frac{1}{2} + 2 = 2,5 \text{ ou } \frac{5}{2}$ .

$$(d): \text{Pour } x=+3: \frac{7x^2}{2} - \frac{8x-6}{3} + \frac{3x+7}{4} = \frac{7 \cdot 3^2}{2} - \frac{8 \cdot 3 - 6}{3} + \frac{3 \cdot 3 + 7}{4}$$

$$= \frac{63}{2} - \frac{24 - 6}{3} + \frac{9 + 7}{4}$$

$$= \frac{63}{2} - \frac{18}{3} + \frac{16}{4} = 31,5 - 6 + 4 = \underline{\underline{29,5}} \left(= \frac{59}{2}\right).$$

$$\text{Pour } x=-2: \frac{7x^2}{2} - \frac{8x-6}{3} + \frac{3x+7}{4} = \frac{7 \cdot (-2)^2}{2} - \frac{8 \cdot (-2) - 6}{3} + \frac{3 \cdot (-2) + 7}{4}$$

$$= \frac{7 \cdot 4}{2} - \frac{-16 - 6}{3} + \frac{-6 + 7}{4}$$

$$= 14 - \frac{-22}{3} + \frac{1}{4}$$

$$= \frac{14 \cdot 12}{12} + \frac{22 \cdot 4}{3 \cdot 4} + \frac{1 \cdot 3}{4 \cdot 3}$$

$$= \frac{168 + 88 + 3}{12} = \underline{\underline{\frac{259}{12}}}$$

$$\text{Pour } x=+5: \frac{7x^2}{2} - \frac{8x-6}{3} + \frac{3x+7}{4} = \frac{7 \cdot 5^2}{2} - \frac{8 \cdot 5 - 6}{3} + \frac{3 \cdot 5 + 7}{4}$$

$$= \frac{7 \cdot 25}{2} - \frac{40 - 6}{3} + \frac{15 + 7}{4}$$

$$= \frac{175}{2} - \frac{34}{3} + \frac{22}{4}$$

$$= \frac{175}{2} + \frac{11}{2} - \frac{34}{3}$$

$$= \frac{186}{2} - \frac{34}{3} = \frac{186 \cdot 3}{2 \cdot 3} - \frac{34 \cdot 2}{3 \cdot 2} = \frac{558}{6} - \frac{68}{6} = \underline{\underline{\frac{490}{6}}}$$

$$= \frac{245 \cdot 2}{3 \cdot 2} = \underline{\underline{\frac{245}{3}}}$$

$$\begin{aligned}
 \text{(e): Pour } x=+4: \frac{7-3x}{12} + \frac{2(x-2)}{3} + \frac{5}{4} &= \frac{7-3 \cdot 4}{12} + \frac{2 \cdot (4-2)}{3} + \frac{5}{4} \\
 &= \frac{7-12}{12} + \frac{2 \cdot 2}{3} + \frac{5}{4} = \frac{-5}{12} + \frac{4}{3} + \frac{5}{4} = \frac{-5}{12} + \frac{4 \cdot 4}{3 \cdot 4} + \frac{5 \cdot 3}{4 \cdot 3} \\
 &= \frac{-5}{12} + \frac{16}{12} + \frac{15}{12} = \frac{26}{12} = \underline{\underline{\frac{13}{6}}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Pour } x=+2: \frac{7-3x}{12} + \frac{2(x-2)}{3} + \frac{5}{4} &= \frac{7-3 \cdot 2}{12} + \frac{2 \cdot (2-2)}{3} + \frac{5}{4} \\
 &= \frac{7-6}{12} + \frac{2 \cdot 0}{3} + \frac{5}{4} = \frac{1}{12} + 0 + \frac{5 \cdot 3}{4 \cdot 3} = \frac{1}{12} + \frac{15}{12} = \frac{16}{12} \\
 &= \frac{4 \cdot 4}{4 \cdot 3} = \underline{\underline{\frac{4}{3}}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Pour } x=-3: \frac{7-3x}{12} + \frac{2(x-2)}{3} + \frac{5}{4} &= \frac{7-3 \cdot (-3)}{12} + \frac{2 \cdot (-3-2)}{3} + \frac{5}{4} \\
 &= \frac{7+9}{12} + \frac{2 \cdot (-5)}{3} + \frac{5}{4} = \frac{16}{12} - \frac{10}{3} + \frac{5}{4} \\
 &= \frac{16}{12} - \frac{10 \cdot 4}{3 \cdot 4} + \frac{5 \cdot 3}{4 \cdot 3} = \frac{16}{12} - \frac{40}{12} + \frac{15}{12} = \frac{-9}{12} = \frac{-3 \cdot 3}{4 \cdot 3} = \underline{\underline{-\frac{3}{4}}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(f): Pour } x=-1: (x^2+1)^2 - x^4 - 2x^2 &= (-1+1)^2 - (-1)^4 - 2 \cdot (-1)^2 \\
 &= (1+1)^2 - 1 - 2 \cdot 1 \\
 &= 2^2 - 1 - 2 = 4 - 3 = \underline{\underline{1}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Pour } x=\frac{1}{2}: (x^2+1)^2 - x^4 - 2x^2 &= \left(\left(\frac{1}{2}\right)^2 + 1\right)^2 - \left(\frac{1}{2}\right)^4 - 2 \cdot \left(\frac{1}{2}\right)^2 \\
 &= \left(\frac{1}{4} + 1\right)^2 - \frac{1}{16} - 2 \cdot \frac{1}{4} \\
 &= \left(\frac{5}{4}\right)^2 - \frac{1}{16} - \frac{2}{4} \\
 &= \frac{25}{16} - \frac{1}{16} - \frac{8}{16} = \frac{16}{16} = \underline{\underline{1}}.
 \end{aligned}$$

$$\text{Rq: } (x^2+1)^2 = (x^2)^2 + 2 \cdot 1 \cdot x^2 + 1^2 \\ = x^4 + 2x^2 + 1$$

$$\text{donc } (x^2+1)^2 - x^4 - 2x^2 = 1.$$

$$\text{Pour } x = -\frac{1}{2}, (x^2+1)^2 - x^4 - 2x^2 = \left(\left(-\frac{1}{2}\right)^2 + 1\right)^2 - \left(-\frac{1}{2}\right)^4 - 2 \times \left(-\frac{1}{2}\right)^2 \\ = \left(\frac{1}{4} + 1\right)^2 + \frac{1}{16} - 2 \times \frac{1}{4}$$

$$= 1. \quad (\text{reprendre le calcul précédent}).$$

$$(2): \text{Pour } a = -3, (a^2+1)(a^2-1) + 4a^2 = (-3^2+1)(-3^2-1) + 4 \times (-3)^2 \\ = (9+1)(9-1) + 4 \times 9 \\ = 10 \times 8 + 36 \\ = 80 + 36 = 116.$$

$$\text{Pour } a = +3, (a^2+1)(a^2-1) + 4a^2 = (3^2+1)(3^2-1) + 4 \times 3^2 \\ = (9+1)(9-1) + 36 = 116. \quad (\text{voir ci-dessous}).$$

$$\text{Pour } a = \frac{1}{3}, (a^2+1)(a^2-1) + 4a^2 = \left(\left(\frac{1}{3}\right)^2 + 1\right)\left(\left(\frac{1}{3}\right)^2 - 1\right) + 4 \times \left(\frac{1}{3}\right)^2 \\ = \left(\frac{1}{9} + 1\right)\left(\frac{1}{9} - 1\right) + 4 \times \frac{1}{9} \\ = \frac{10}{9} \times \frac{-8}{9} + \frac{4}{9} = \frac{-80}{81} + \frac{4 \times 9}{9 \times 9} \\ = \frac{-80}{81} + \frac{36}{81} = \frac{44}{81}.$$

$$\text{Rq: } (x+y)^2 - x^2 - y^2 \quad (1): \text{Pour } x = -5 \text{ et } y = 2: \frac{(x+y)^2 - (x^2+y^2)}{xy} = \frac{(-5+2)^2 - ((-5)^2+2^2)}{-5 \times 2} \\ = x^2 + 2xy + y^2 - x^2 - y^2 = 2xy$$

$$\text{donc } \frac{(x+y)^2 - (x^2+y^2)}{xy} = \frac{2xy}{xy} = 2.$$

$$\text{Rq: } \frac{a^2-ab}{a^2-2ab+b^2} \quad (2): \text{Pour } a = +7 \text{ et } b = -2: \frac{a^2-ab}{a^2-2ab+b^2} = \frac{7^2 - 7 \times (-2)}{7^2 - 2 \times 7 \times (-2) + (-2)^2} = \frac{49 + 14}{49 + 28 + 4} = \frac{63}{81} = \frac{9 \times 7}{9 \times 9} = \frac{7}{9}.$$

$$\frac{a(a-b)}{(a-b)^2} = \frac{a \times (a-b)}{(a-b) \times (a-b)}$$

$$= \frac{a}{a-b} \rightarrow \frac{7}{7+2} = \frac{7}{9}.$$

$$2. \text{ Pour } a = -5 \text{ et } b = 3: \quad a^3 - ab + b^3 = (-5)^3 - (-5) \cdot 3 + 3^3 \\ = 25 + 15 + 9 \\ = \underline{\underline{49}}.$$

$$\frac{a^3 + b^3}{a+b} = \frac{(-5)^3 + 3^3}{-5+3} = \frac{-125 + 27}{-2} = \frac{-98}{-2} = \underline{\underline{49}}.$$

$$(a+b)^3 - 3ab = (-5+3)^3 - 3 \cdot (-5) \cdot 3 = (-2)^3 + 45 = 4 + 45 = \underline{\underline{49}}.$$

Remarque: ce n'est pas une coïncidence:  $(a+b)^3 - 3ab = a^3 + 3a^2b + 3ab^2 - 3ab = a^3 - ab + b^3$

$$\begin{aligned} \text{Et } (a+b)(a^2 - ab + b^2) &= a \cdot a^2 - a \cdot ab + a \cdot b^2 + ba^2 - b \cdot ab + b^3 \\ &= a^3 - a^2b + ab^2 + ba^2 - ab^2 + b^3 \\ &= a^3 + b^3. \end{aligned}$$

$$2) \text{ où: } \frac{a^3 + b^3}{a+b} = \frac{(a+b)(a^2 - ab + b^2)}{a+b} = a^2 - ab + b^2.$$

$$3. \text{ Pour } a = +4 \text{ et } b = -1: \quad (a+b)^3(a-b) = (4 + (-1))^3(4 - (-1)) \\ = 3^2 \cdot 5 = 9 \cdot 5 = \underline{\underline{45}}.$$

$$(a^3 - b^3)(a+b) = (4^3 - (-1)^3)(4 + (-1)) = (16 - 1)(3) = 15 \cdot 3 = \underline{\underline{45}}.$$

$$\frac{a^4 + b^4 - 2a^2b^2}{a-b} = \frac{4^4 + (-1)^4 - 2 \cdot 4^2 \cdot (-1)^2}{4 - (-1)} = \frac{256 + 1 - 32}{5} = \frac{225}{5} = \underline{\underline{45}}.$$

Remarque: ce n'est toujours pas un hasard:  $(a+b)^3(a-b) = (a+b)(a+b)(a-b) \\ = (a+b)(a^2 - b^2) \\ = (a^2 - b^2)(a+b)$

$$\begin{aligned} \text{et } (a^3 - b^3)(a+b) &= (a^3 - b^3)(a+b) \times \frac{a-b}{a-b} = \frac{(a^3 - b^3)(a+b)(a-b)}{a-b} = \frac{(a^2 - b^2)(a^2 + b^2)}{a-b} \\ &= \frac{(a^2 - b^2)^2}{a-b} = \frac{(a^2)^2 + (b^2)^2 - 2 \cdot a^2 \cdot b^2}{a-b} = \frac{a^4 + b^4 - 2a^2b^2}{a-b} \end{aligned}$$

$$4. \text{ Le dénominateur vaut, pour } a=+5 \text{ et } b=-2 : a(a-1)-bb^2 = 5(5-1)-5(-2)^2 \\ = 5 \times 4 - 5 \times 4 = 0.$$

On ne peut donc pas calculer cette fraction qui aurait un dénominateur nul.

$$5. \text{ Pour } a=+1 \text{ et } b=+2, \text{ le dénominateur vaut : } 4a^2-b^2 = 4 \times 1^2 - 2^2 = 4 - 4 = 0, \text{ donc on ne peut pas évaluer l'expression.}$$

$$6.(a) \left(-\frac{2}{3}\right)a^2x \times (-3y) \times \left(\frac{1}{5}\right) = \left(-\frac{2}{3} \times 3 \times \frac{1}{5}\right)a^2xy = +\frac{12}{15}a^2xy = \frac{4}{5}a^2xy.$$

$$\text{Pour } a=-3, x=2 \text{ et } y=-1, \text{ on trouve : } \frac{4}{5}(-3)^2 \times 2 \times 1 = \frac{-4 \times 9 \times 2}{5} = \frac{-72}{5} (= -3,6)$$

$$(b) xy \times \left(-\frac{2}{3}\right)x^2 \times \frac{3}{4}a^2 = \left(-\frac{2}{3} \times \frac{3}{4}\right)xyx^2a^2 = -\frac{2 \times 3}{3 \times 4}a^2x^3y = -\frac{1}{2}a^2x^3y.$$

$$\text{Pour } a=5, x=-2 \text{ et } y=3, \text{ on trouve : } -\frac{1}{2} \times 5^2 \times (-2)^3 \times 3 = \frac{-25 \times (-8) \times 3}{2} = \frac{600}{2} = 300.$$

$$(c) \frac{2}{7}a^2 \times \left(-\frac{3}{4}\right)xy^3 \times \left(-\frac{2}{5}\right)a^2x = \left(\frac{2}{7} \times -\frac{3}{4} \times -\frac{2}{5}\right)a^2xy^3a^2x \\ = +\frac{2 \times 2 \times 3}{4 \times 5 \times 7}a^4x^2y^3 = \frac{3}{35}a^4x^2y^3.$$

$$\text{Si } a=3,5=\frac{7}{2}, x=3, y=-2 \text{ on trouve : } \frac{3}{35} \times \left(\frac{7}{2}\right)^4 \times 3^2 \times (-2)^3 = \frac{3}{35} \times \frac{7^4}{2^4} \times 9 \times (-2^3) \\ = \frac{-3 \times 9 \times 2^3 \times 7^4}{5 \times 7 \times 2^4} = \frac{-27 \times 7^3}{5 \times 2} \\ = -\frac{27 \times 343}{10} = -\frac{9261}{10} = -926,1.$$

$$(d) \left(-\frac{3}{5}\right)a^2 \times \frac{2}{3}b^3x \times (-x^4) = \left(-\frac{3}{5} \times \frac{2}{3} \times -1\right) \times a^2b^3x^5 \\ = \frac{-3 \times 2 \times -1}{5 \times 3}a^2b^3x^5 \\ = \frac{2}{5}a^2b^3x^5.$$

Pour  $a=4$ ,  $b=-1$  et  $x=-2$ , on trouve:  $\frac{2}{5}x^2 \times (-1)^2 \times (-2)^5 = \frac{2}{5} \times 16 \times 1 \times (-32)$   
 $= -\frac{32}{5} \times 32 = -\frac{1024}{5}$   
 $\underline{\underline{(-204,8)}}$

(e)  $4x^3 \times (-3y^2) \times \left(-\frac{5}{6}\right)a^2x^3y^5 = \left(4 \times -3 \times \frac{-5}{6}\right)x^3y^2a^2x^3y^5$   
 $= \frac{+4 \times 3 \times 5}{6} a^2x^3x^2y^2y^5$   
 $= \underline{\underline{10a^2x^5y^7}}.$

Pour  $a=-\frac{1}{2}$ ,  $x=4$  et  $y=\frac{3}{2}$ , on obtient:  $10 \times \left(\frac{-1}{2}\right)^2 \times 4^5 \times \left(\frac{3}{2}\right)^7$   
 $= 10 \times \frac{1}{4} \times \frac{(2^2)^5}{1} \times \frac{3^7}{2^7}$   
 $= 10 \times \frac{1 \times 2^{10} \times 3^7}{2^2 \times 1 \times 2^7}$   
 $= 10 \times 3^7 \times \frac{2^{10}}{2^9} = 10 \times 3^7 \times 2^{10-9}$   
 $= 10 \times 3^7 \times 2 = \underline{\underline{10 \times 3^7 \times 2}}$ .

Enfin  $3^7 = \underbrace{3 \times 3 \times 3}_{= 27} \times \underbrace{3 \times 3 \times 3}_{= (25+2)^2} \times 3$   
 $= (25+2)^2 \times 3$   
 $= (25^2 + 2 \times 2 \times 25 + 2^2) \times 3$   
 $= (625 + 100 + 4) \times 3$   
 $= 729 \times 3 = 2187$

D'après on obtient finalement  $10 \times 2187 \times 2$   
 $= 21870 \times 2$   
 $= \underline{\underline{43740}}$

7. (a)  $\frac{2}{3}ax - \frac{1}{2}ax + \frac{3}{4}ax - \frac{5}{6}ax = \left[\frac{2}{3} - \frac{1}{2} + \frac{3}{4} - \frac{5}{6}\right]ax$   
 $= \left[\frac{8}{12} - \frac{6}{12} + \frac{9}{12} - \frac{10}{12}\right]ax = \underline{\underline{\frac{1}{12}ax}}$

$$\begin{aligned}
 \text{(b)}: -\frac{3}{5}a^2bx + \frac{1}{4}a^2bx - \frac{7}{2}a^2bx + \frac{1}{10}a^2bx &= \left(-\frac{3}{5} + \frac{1}{4} - \frac{7}{2} + \frac{1}{10}\right)a^2bx \\
 &= \left(-\frac{12}{20} + \frac{5}{20} - \frac{70}{20} + \frac{2}{20}\right)a^2bx \\
 &= \frac{-75}{20}a^2bx = \underline{\underline{-\frac{15}{4}a^2bx}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)}: -\frac{4}{7}a^2b^3x + \frac{5}{2}a^2b^3x - \frac{5}{4}a^2b^3x &= \left(-\frac{4}{7} + \frac{5}{2} - \frac{5}{4}\right)a^2b^3x \\
 &= \left[-\frac{16}{28} + \frac{70}{28} - \frac{35}{28}\right]a^2b^3x \\
 &= \underline{\underline{\frac{19}{28}a^2b^3x}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)}: \frac{3}{4}a^2b^3x^4y - \frac{2}{3}a^2b^3x^4y + \frac{1}{4}a^2b^3x^4y &= \left[\frac{3}{4} - \frac{2}{3} + \frac{1}{4}\right]a^2b^3x^4y \\
 &= \left[1 - \frac{2}{3}\right]a^2b^3x^4y = \underline{\underline{\frac{1}{3}a^2b^3x^4y}}.
 \end{aligned}$$