Data Compression

Letícia Rodrigues Bueno

Federal University of ABC (UFABC)

Data Compression: Introduction

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  2. minimize the cost for sending files through the network or the time for reading files from disks;
  3. minimize time of searching inside files.

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- **Possible solution:** replace symbols throughout the text by others with smaller number of bits or bytes (**message encoding**);
    1. Given a string of characters (message);
    2. **Problem:** encode message by assigning codes to symbols;
    3. table of codes is stored for decoding;

Run-length encoding (RLE)

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- **main idea:** assign shorter codes (fewer bits) to symbols with high frequencies;
- **traditional implementation:** when considering characters as symbols, it can comprise text in $\approx 25\%$;
- when considering words as symbols, it can comprise text in $\approx 60\%$;

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- Codes for the symbols are binary sequences;
- **Advantage of using prefix codes:** facility for encoding and decoding;

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$$s_4 s_3 s_3 s_1 s_3 s_1 s_4 s_5 s_1 s_3 s_3 s_3 s_3 s_3 s_2 s_3 s_5 s_2 s_2 s_2 s_4$$

Huffman's Algorithm: encoding and decoding

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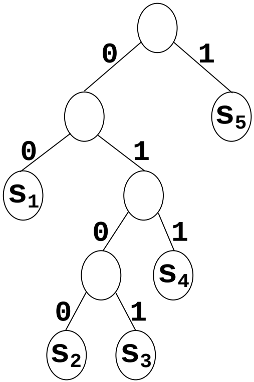
s_1:00
s_2:0100
s_3:0101
s_4:011
s_5:1



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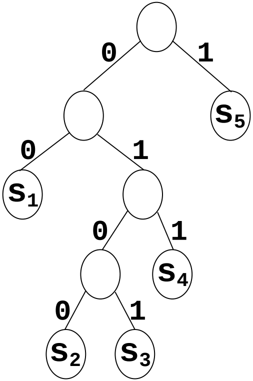

011

Huffman's Algorithm: encoding and decoding

$s_4 s_3 s_3 s_1 s_3 s_1 s_4 s_5 s_1 s_3 s_3 s_3 s_3 s_3 s_2 s_3 s_5 s_2 s_2 s_2 s_4$

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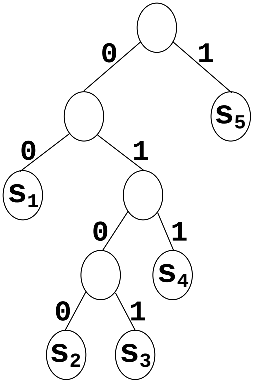

011 0101

Huffman's Algorithm: encoding and decoding

$s_4\, s_3\, s_3\, s_1\, s_3\, s_1\, s_4\, s_5\, s_1\, s_3\, s_3\, s_3\, s_3\, s_3\, s_2\, s_3\, s_5\, s_2\, s_2\, s_2\, s_4$

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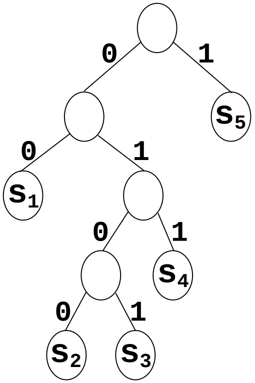

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$s_4 s_3 s_3 s_1 s_3 s_1 s_4 s_5 s_1 s_3 s_3 s_3 s_3 s_3 s_2 s_3 s_5 s_2 s_2 s_2 s_4$

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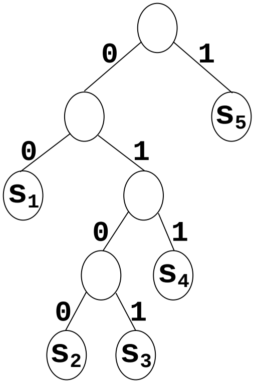

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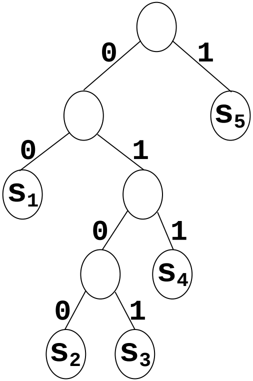

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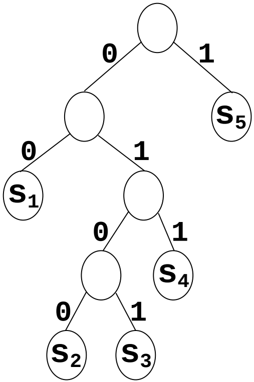

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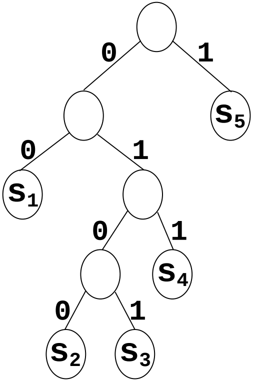

011 0101 0101 00 0101 00 011

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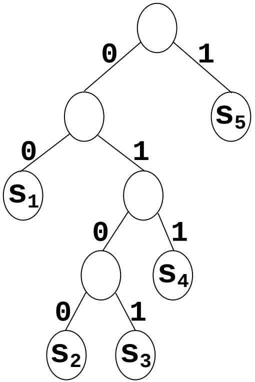

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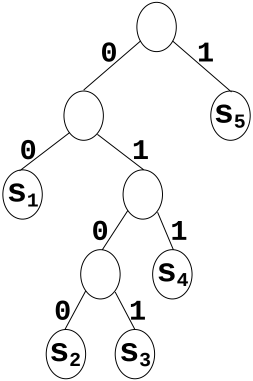

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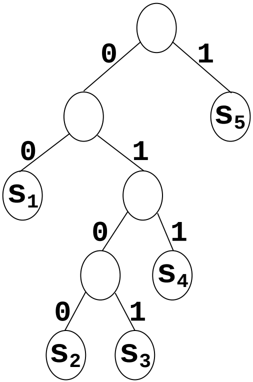

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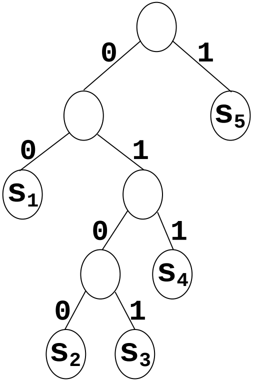

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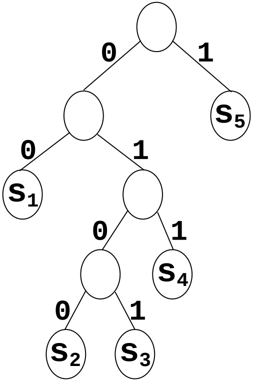

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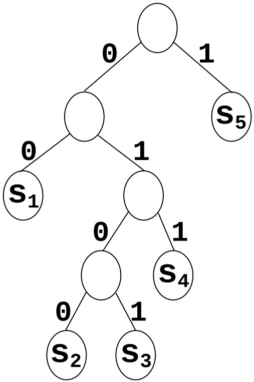

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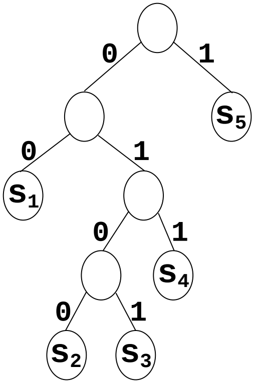

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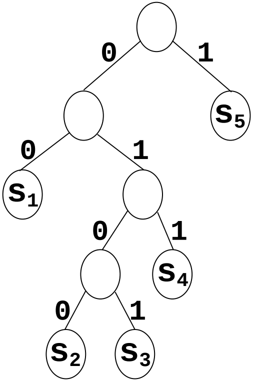

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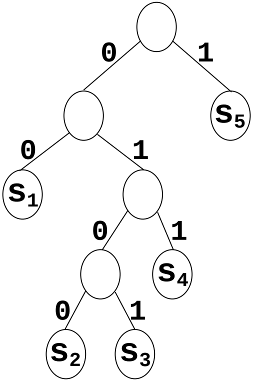

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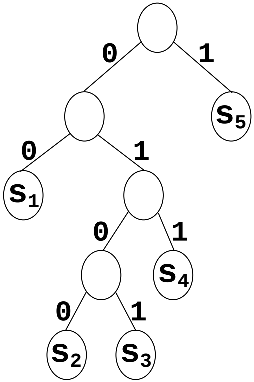

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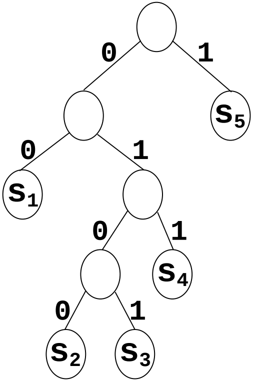

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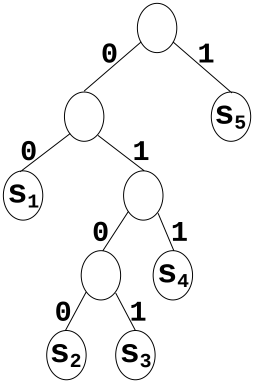

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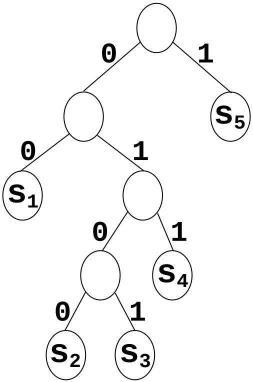

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Huffman's Algorithm: encoding and decoding

$s_4 s_3 s_3 s_1 s_3 s_1 s_4 s_5 s_1 s_3 s_3 s_3 s_3 s_3 s_2 s_3 s_5 s_2 s_2 s_2 s_4$

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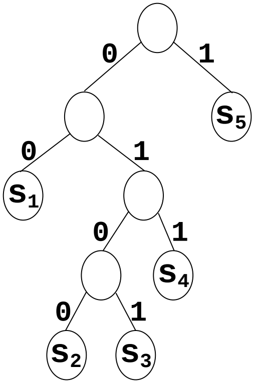

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- If prefix tree T is known, encoding/decoding is made in time $O(m)$ where m is the size of the encoded binary sequence;

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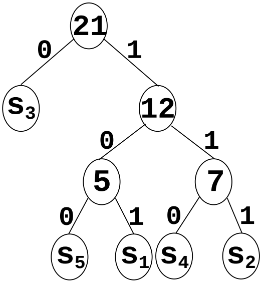
- For 01101010101000101000111000101010101 01010101010001011010001000100011, cost $c(T) = 69$;
- It is necessary to minimize $c(T)$;
- **Minimum Tree** or **Huffman tree**: prefix tree T with the lowest $c(T)$;

Huffman's Algorithm

Huffman's Algorithm

s_1:101
s_2:111
s_3:0
s_4:110
s_5:100



f_1:3
f_2:4
f_3:9
f_4:3
f_5:2

Huffman's Algorithm

s_1:101
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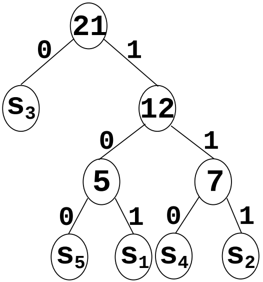
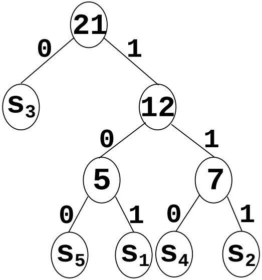


f_1:3
f_2:4
f_3:9
f_4:3
f_5:2

$s_4 s_3 s_3 s_1 s_3 s_1 s_4 s_5 s_1 s_3 s_3 s_3 s_3 s_3 s_2 s_3 s_5 s_2 s_2 s_2 s_4$ gives

Huffman's Algorithm

s_1:101
s_2:111
s_3:0
s_4:110
s_5:100



f_1:3
f_2:4
f_3:9
f_4:3
f_5:2

$s_4 s_3 s_3 s_1 s_3 s_1 s_4 s_5 s_1 s_3 s_3 s_3 s_3 s_3 s_2 s_3 s_5 s_2 s_2 s_2 s_4$ gives
110001010101110100101000001110100111111111110

Huffman's Algorithm

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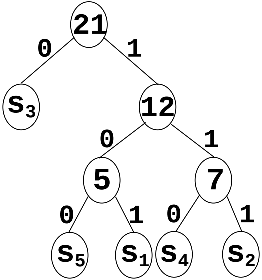

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which has $c(T) = 45$ and it is the minimum.

Huffman's Algorithm Idea

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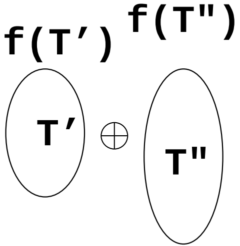

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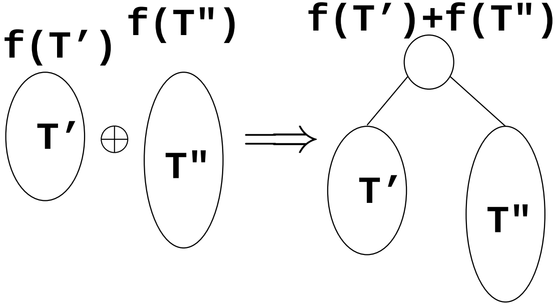

Construction of Huffman Tree

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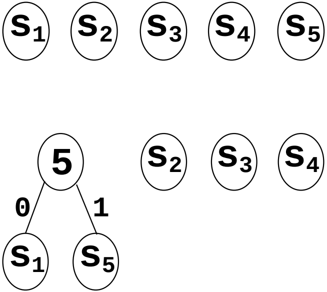

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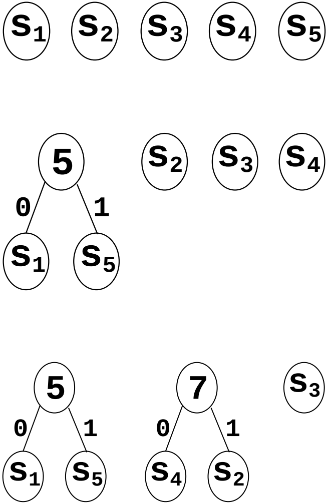

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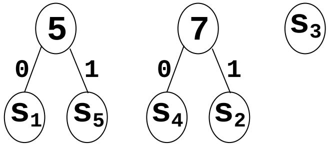

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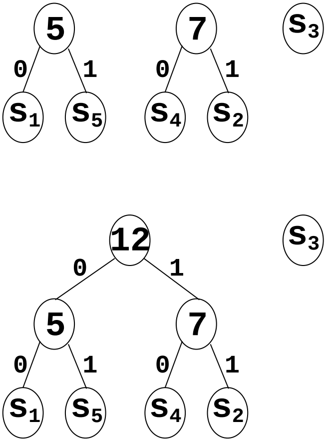

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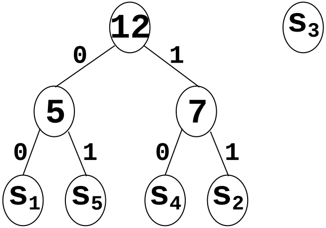
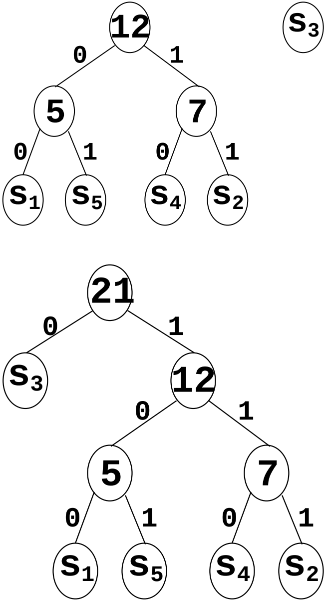


Construction of Huffman Tree



Huffman's Algorithm: inserting into a priority queue

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```
insert(T, f, F):
    F[n + 1] ← T
    n++
    ascend(n)
```

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    F[n + 1] ← T
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```
ascend(i):
    j = ⌊i/2⌋
    if j ≥ 1 then
        if T[i] < T[j] then
            T[i] ⇔ T[j]
            ascend(j)
```

Huffman's Algorithm: inserting into a priority queue

insert(T, f, F):
  $F[n+1] \leftarrow T$
  n++
  ascend(n)

ascend(i):
  $j = \lfloor i/2 \rfloor$
  if $j \geq 1$ **then**
    if $T[i] < T[j]$ **then**
      $T[i] \Leftrightarrow T[j]$
      ascend(j)

- **insert(T, f, F) and ascend(i)**: priority queue methods implemented by a heap data structure;

Huffman's Algorithm: deletion from priority queue

Huffman's Algorithm: deletion from priority queue

minimum(T, f, F):
    if $n \neq 0$ **then**
        remove $T[1]$
        $T[1] \leftarrow T[n]$
        n- -
        descend($1, n$)

Huffman's Algorithm: deletion from priority queue

```
minimum(T, f, F):
    if n ≠ 0 then
        remove T[1]
        T[1] ← T[n]
        n- -
        descend(1, n)
```

```
descend(i, n):
    j = 2 ∗ i
    if j ≤ n then
        if j < n then
            if T[j + 1] < T[j] then
                j = j + 1
        if T[i] > T[j] then
            T[i] ⇔ T[j]
            descend(j, n)
```

Huffman's Algorithm: deletion from priority queue

```
minimum(T, f, F):
    if n ≠ 0 then
        remove T[1]
        T[1] ← T[n]
        n- -
        descend(1, n)
```

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descend(i, n):
    j = 2 ∗ i
    if j ≤ n then
        if j < n then
            if T[j + 1] < T[j] then
                j = j + 1
        if T[i] > T[j] then
            T[i] ⇔ T[j]
            descend(j, n)
```

- **minimum(T, f, F) and descend($1, n$)**: priority queue methods implemented by a heap data structure;

Huffman's Algorithm

huffman():
    for $i \leftarrow 1$ **to** $n-1$ **do**
        minimum(T', f, F);
        minimum(T'', f, F);
        $T \leftarrow T' \oplus T''$;
        $f(T) \leftarrow f(T') \oplus f(T'')$;
        insert(T, f, F);

Huffman's Algorithm

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- Operation of minimization or insertion in the priority queue: $O(\log n)$;

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        $f(T) \leftarrow f(T') \oplus f(T'')$;
        $\mathit{insert}(T, f, F)$;

- Operation of minimization or insertion in the priority queue: $O(\log n)$;
- operation $\oplus$ requires constant number of steps;
- Total: $n - 1$ iterations;
- **Complexity:** $O(n \log n)$;

Exercises

1. Draw a Huffman tree for the following set of keys and frequencies, respectively in the first and second lines of the table:

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
1	6	2	1	1	9	2	3

2. The resulting tree from Huffman's algorithm is unique, i.e., the algorithm always returns the same tree?

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Questions?