Depth-First Search

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Federal University of ABC (UFABC)

Problem 1: Topological Sorting

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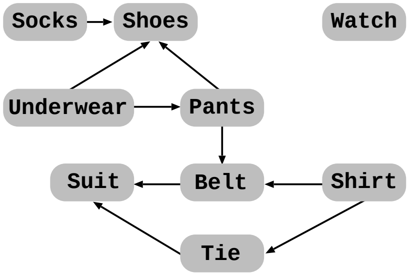

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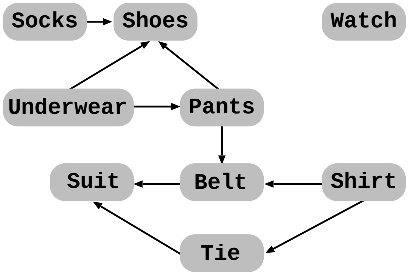

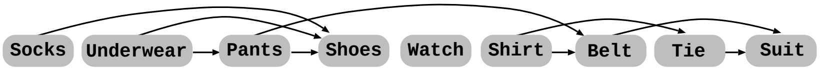

Depth-first search (DFS)

How to find a topological sorting in an acyclic directed graph?

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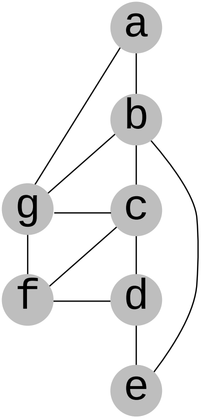
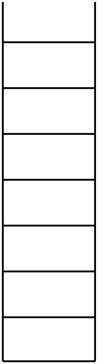
How to find a topological sorting in an acyclic directed graph?

We can use a Depth-first search!

Depth-first search (DFS)





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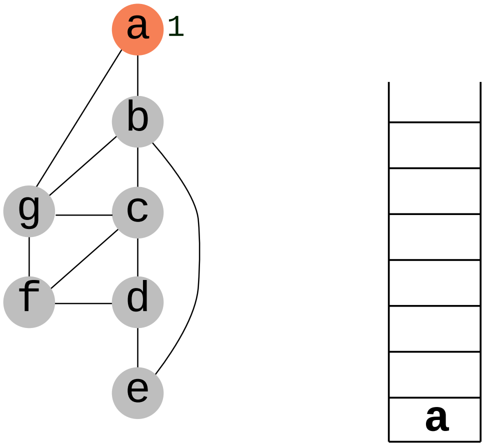

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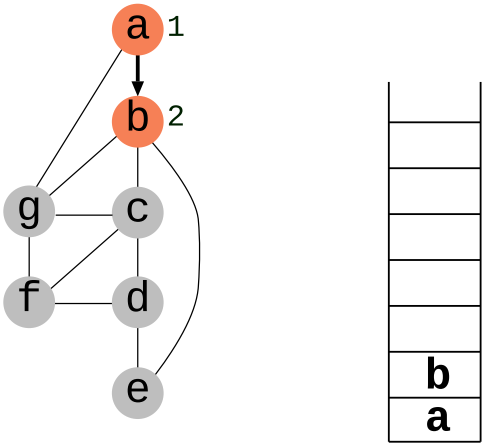

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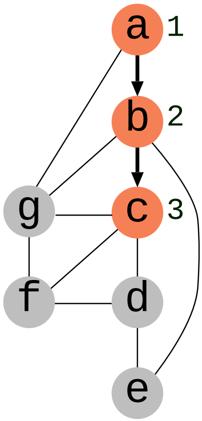

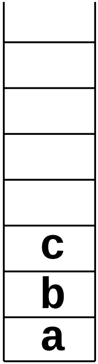

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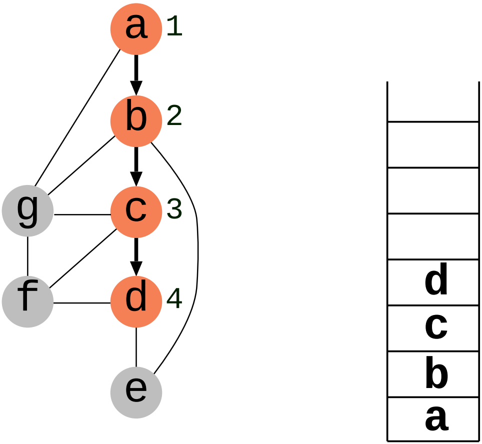

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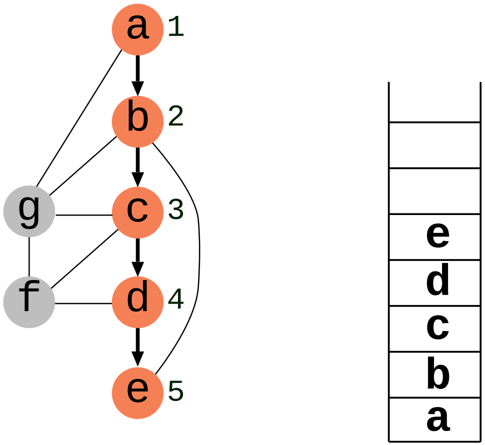

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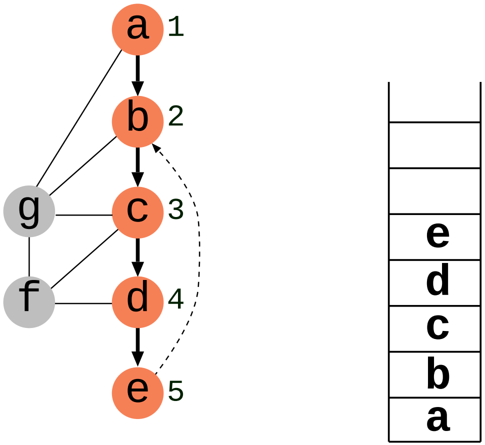

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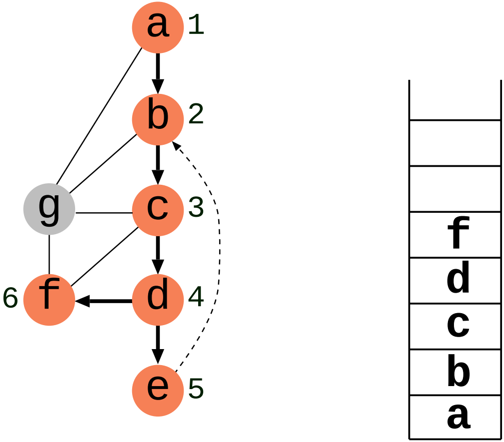

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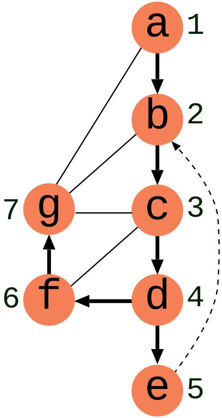

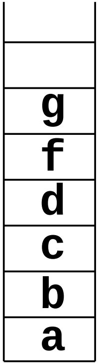

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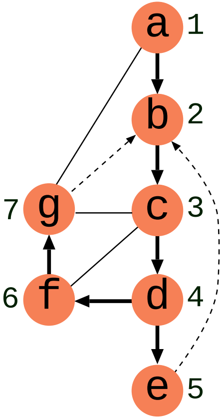

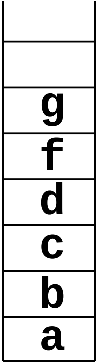

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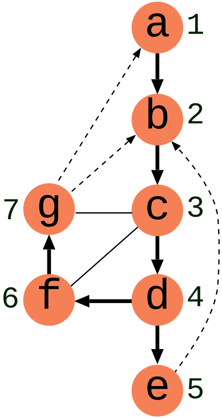

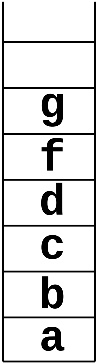

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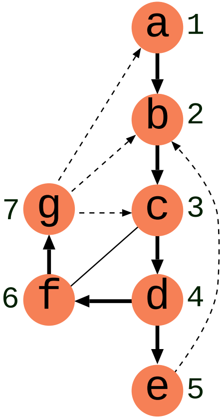

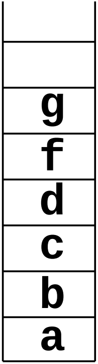

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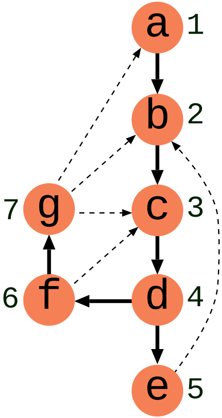

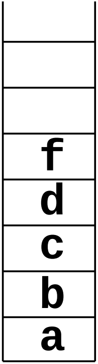

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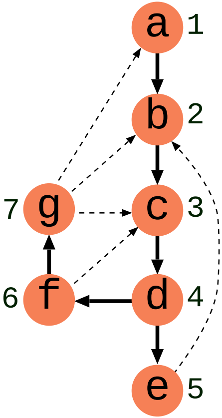

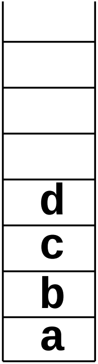

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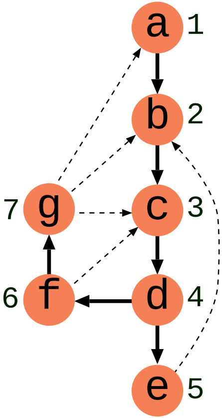

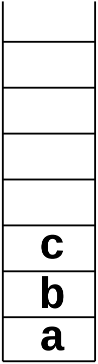

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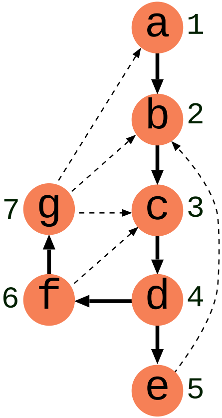

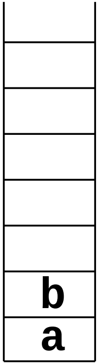

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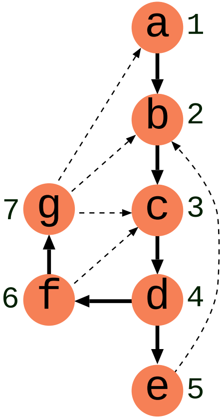

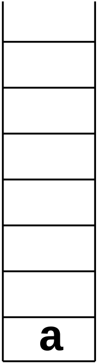

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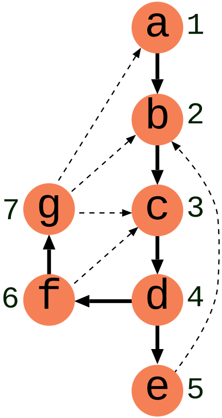

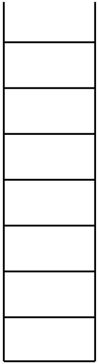

DFS Algorithm

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dfs(G, u, cont):
    u.visited = True
    u.d = cont
    for v in adj(u) do
        if not v.visited then
            v.p = u
            dfs(G, v, cont+1)


for u in V(G) do
     u.visited = False
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- Total complexity: $O(n + m)$.

Topological Sorting

Coming back to the topological sorting problem:

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Topological Sorting

Coming back to the topological sorting problem:

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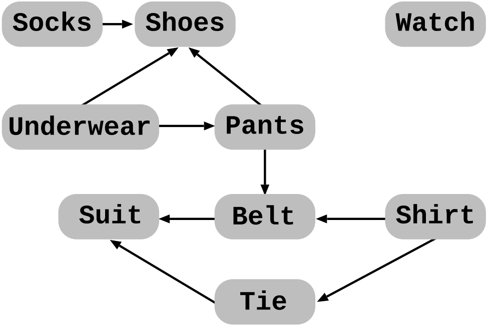
Lemma

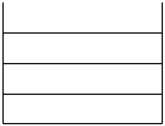
An acyclic directed graph always has a vertex with indegree 0*.*

Proof.

If every vertex has indegree greater than 0, we can go backwards through the edges without stopping. Since there is a finite number of vertices, this is only possible in a cycle, but acyclic graphs do not have cycles. □

Depth-first search (DFS)





pt → λ

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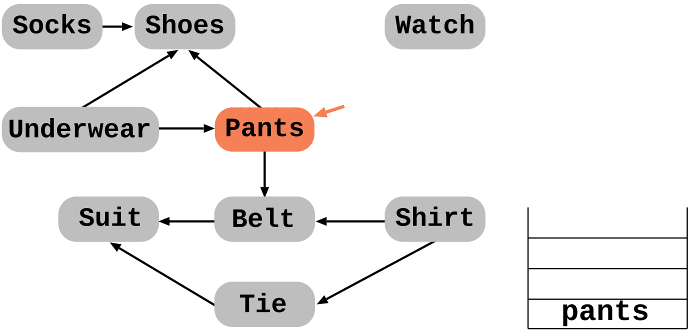

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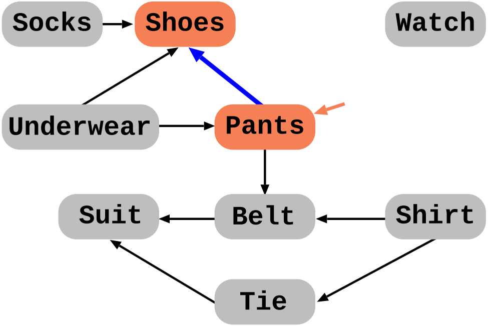

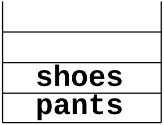

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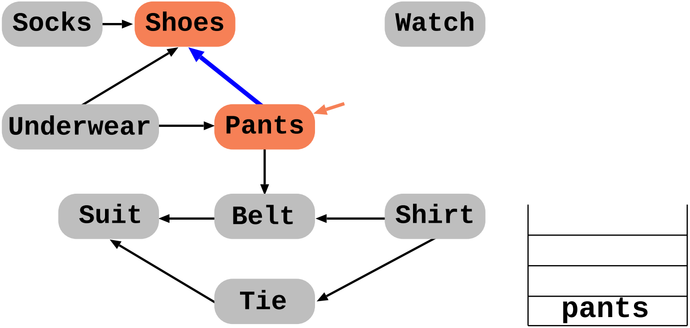

pt → shoes → λ

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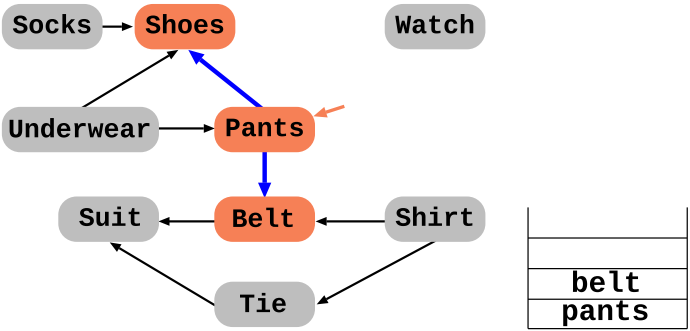

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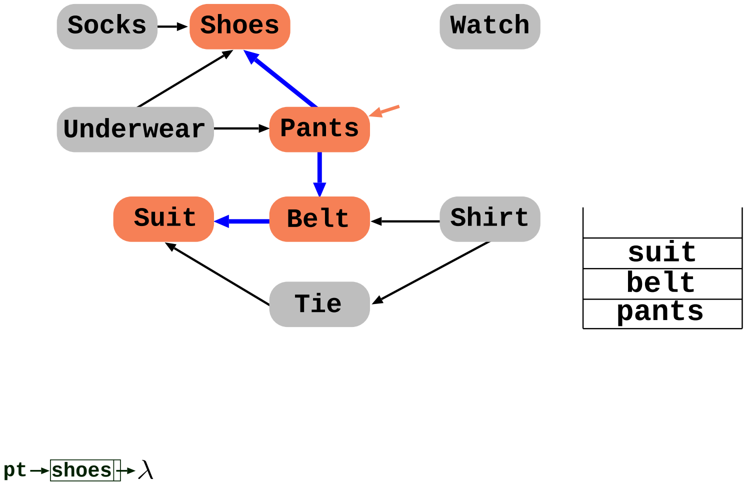

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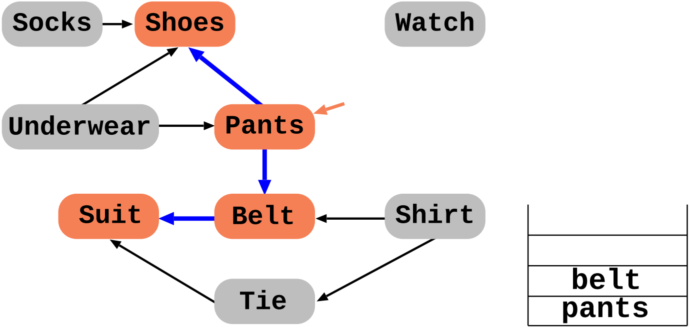

pt → suit → shoes → λ

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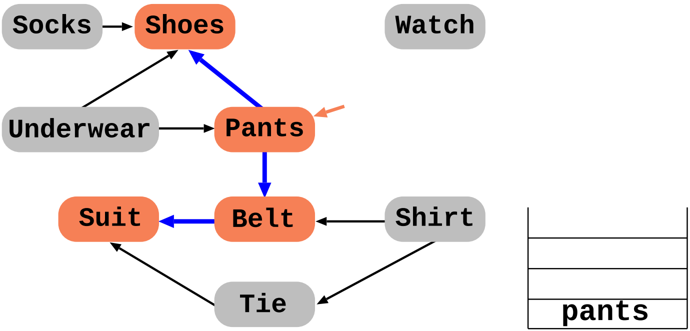

pt → belt → suit → shoes → λ

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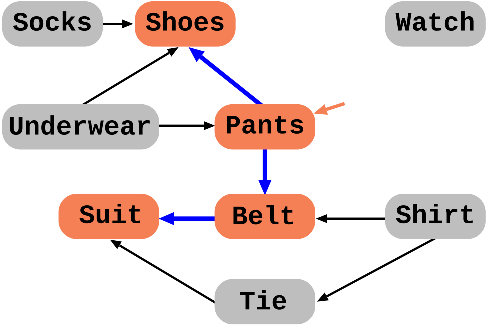

pt → **pants** → **belt** → **suit** → **shoes** → λ

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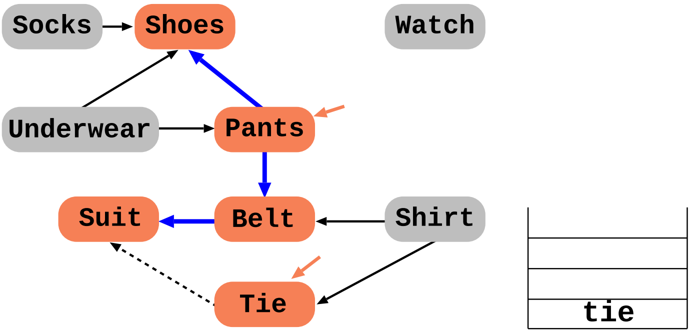

pt→pants→belt→suit→shoes→λ

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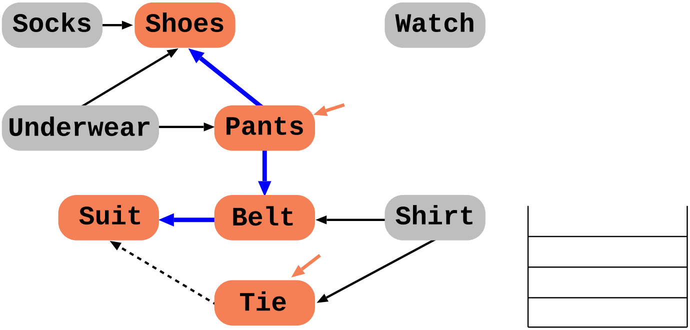

pt → tie → pants → belt → suit → shoes → λ

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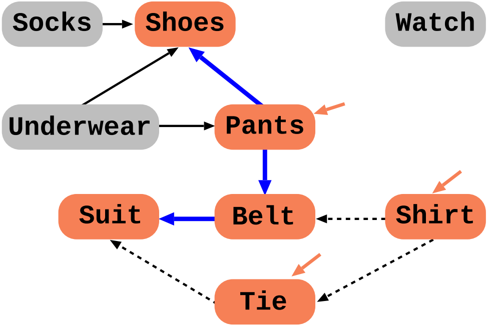

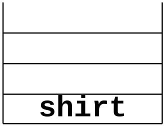

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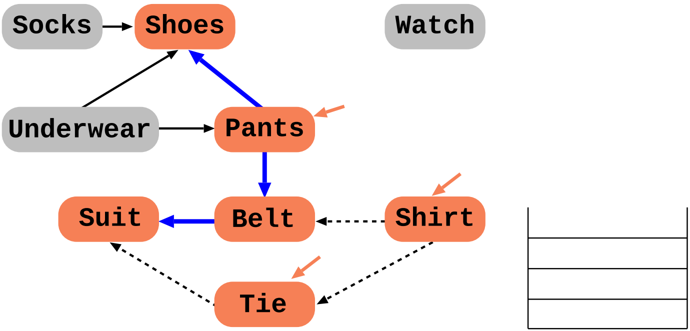

pt → shirt → tie → pants → belt → suit → shoes → λ

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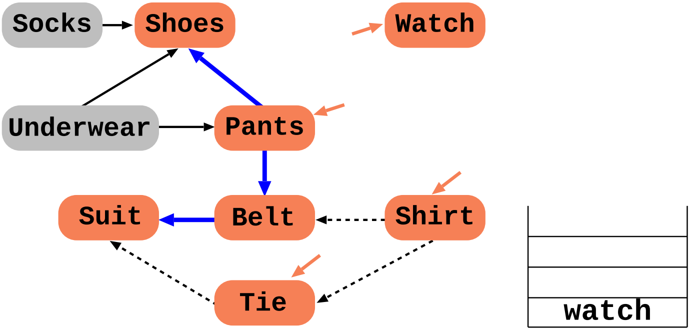

pt → shirt → tie → pants → belt → suit → shoes → λ

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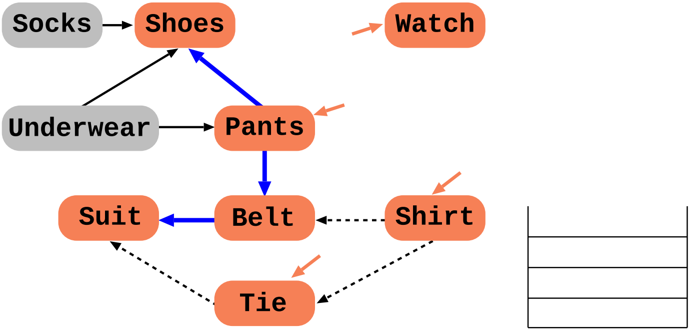

pt → watch → shirt → tie → pants → belt → suit → shoes → λ

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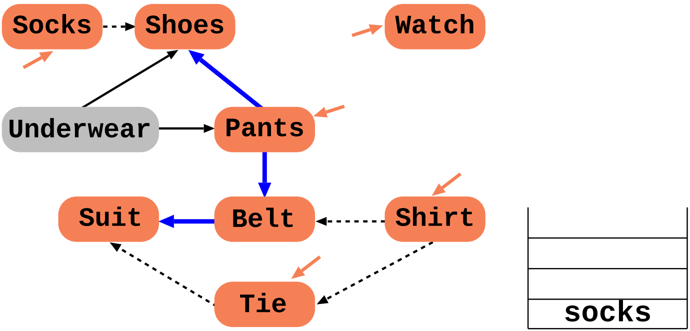

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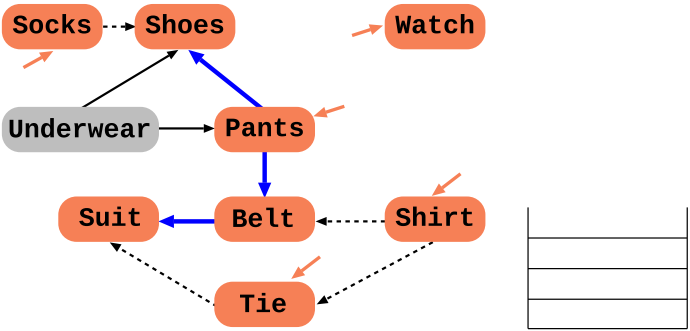

pt→ socks → watch → shirt → tie → pants → belt → suit → shoes → λ

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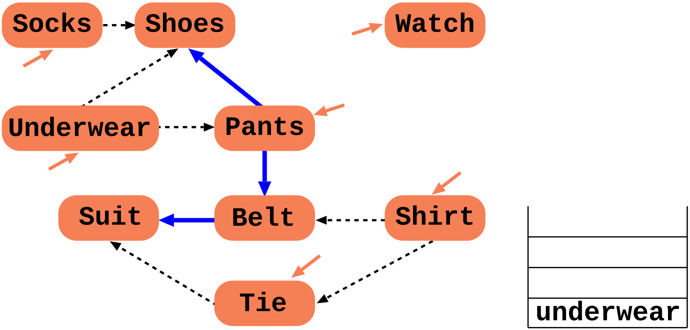

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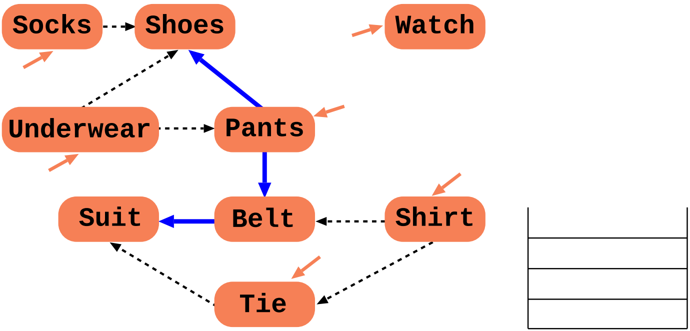

pt→ underwear → socks → watch → shirt → tie → pants → belt → suit → shoes → λ

Topological Sorting Algorithm

```
dfs(G, u):
    u.visited = True
    for v in adj(u) do
        if not v.visited then
            dfs(G, v)
    insertFirst(L, u)


for u in V(G) do
     u.visited = False
L = List();
for u in V(G) do
    if not u.visited then
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Topological Sorting Algorithm

Analysis of complexity:

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Analysis of complexity:

- Each vertex is passed to method dfs once (lines 5 and 12): $O(n)$;

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7  for u in V(G) do
8      u.visited = False
9  L = List();
10 for u in V(G) do
11     if not u.visited then
12         dfs(G, u)
```

Analysis of complexity:

- Each vertex is passed to method dfs once (lines 5 and 12): $O(n)$;
- Adjacency list of each vertex is traversed once (line 3): $O(m)$;

Topological Sorting Algorithm

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1  dfs(G, u):
2      u.visited = True
3      for v in adj(u) do
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- Each vertex is passed to method dfs once (lines 5 and 12): $O(n)$;
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- Total complexity: $O(n + m)$.

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Analysis of Correctness:

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Analysis of Correctness:

- Vertices are added in the beginning of the list only if they do not have more output edges;

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Analysis of Correctness:

- Vertices are added in the beginning of the list only if they do not have more output edges;
- Sinks are added first;

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- Vertices are added in the beginning of the list only if they do not have more output edges;
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- Then “satisfied” vertices are added;

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Analysis of Correctness:

- Vertices are added in the beginning of the list only if they do not have more output edges;
- Sinks are added first;
- Then “satisfied” vertices are added;
- All sources are now added.

Exercises

1. Modify DFS algorithm to verify if a graph is acyclic.
2. Suppose G is connected, how we can use DFS algorithm to get a spanning tree of G?
3. DFS algorithm can be used to verify if a graph is connected?
4. Propose an alternative algorithm to solve the topological sorting problem without using DFS, but with the same complexity of time.

Bibliography

CORMEN, T. H.; LEISERSON, C. E.; RIVEST, R. L. and STEIN, C. *Introduction to Algorithms*, 3rd edition, MIT Press, 2009.