AP Calculus BC Test 2

H10 Linearization

Linearization is a method used to approximate the value of a function near a given point using the tangent line at that point.

Example: Approximate the cube root of 82 using linearization.

Step 1: Identify the function and the point of tangency.

$$f(x) = \sqrt[3]{x}, \quad a = 81$$

Step 2: Find the derivative of the function.

$$f'(x) = \frac{1}{3x^{2/3}}$$

Step 3: Evaluate the function and its derivative at the point of tangency.

$$f(81) = 3, \quad f'(81) = \frac{1}{27}$$

Step 4: Write the equation of the tangent line.

H11 Optimization

H12 Estimating with Riemann sums

H13 Writing and interpreting Riemann sums

H14 Riemann sums and definite integrals

H15 Antiderivatives and indefinite integration

H16 The Fundamental Theorem of Calculus

H17 Integration by u-substitution and change of variable

H18 Inverse trig integration

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$$
$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$
$$\int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\operatorname{arcsec}\left(\frac{|x|}{a}\right) + C$$

(no need to memorize formulas - memorize forms, use u-substitution instead)

Example: Evaluate $\int \frac{1}{\sqrt{16-9x^2}}\,dx$.

Step 1: Reduce the constant to 1.

$$\begin{aligned}
&\int \frac{1}{\sqrt{16-9x^2}}\,dx \\
&= \int \frac{1}{16\sqrt{1-\frac{9}{16}x^2}}\,dx \\
&= \frac{1}{16}\int \frac{1}{\sqrt{1-\left(\frac{3}{4}x\right)^2}}\,dx
\end{aligned}$$

Step 2: Use u-substitution with $u = \frac{3}{4}x$.

$$\begin{aligned}
u &= \frac{3}{4}x \\
du &= \frac{3}{4}dx \\
\frac{4}{3}du &= dx \\
\frac{1}{16}\cdot\frac{4}{3}&\int \frac{1}{\sqrt{1-u^2}}\,du \\
&= \frac{1}{12}\int \frac{1}{\sqrt{1-u^2}}\,du
\end{aligned}$$

H19 Long division and integration

How to know which integration method to use