AP Calculus BC Test 2

H10 Linearization

Linearization is a method used to approximate the value of a function near a given point using the tangent line at that point.

Example: Approximate the cube root of 82 using linearization.

Step 1: Identify the function and the point of tangency.

$$f(x) = \sqrt[3]{x}, \quad a = 81$$

Step 2: Find the derivative of the function.

$$f'(x) = \frac{1}{3x^{2/3}}$$

Step 3: Evaluate the function and its derivative at the point of tangency.

$$f(81) = 3, \quad f'(81) = \frac{1}{27}$$

Step 4: Write the equation of the tangent line.

H11 Optimization

H12 Estimating with Riemann sums

RRAM Right Rectangular Approximation Method - heights of rects = heights of right endpoints

LRAM Left Rectangular Approximation Method - heights of rects = heights of left endpoints

MRAM Midpoint Rectangular Approximation Method - heights of rects = heights of midpoints

Trapezoidal approximation Average of RRAM and LRAM

H13 Writing and interpreting Riemann sums

H14 Riemann sums and definite integrals

H15 Antiderivatives and indefinite integration

H16 The Fundamental Theorem of Calculus

H17 Integration by u-substitution and change of variable

H18 Inverse trig integration

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \arcsin\left(\frac{x}{a}\right) + C$$
$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$
$$\int \frac{1}{x\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\operatorname{arcsec}\left(\frac{|x|}{a}\right) + C$$

(no need to memorize formulas - memorize forms, use u-substitution instead)

Example: Evaluate $\int \frac{1}{\sqrt{16 - 9x^2}}\, dx$.

Step 1: Reduce the constant to 1.

$$\begin{aligned} &\int \frac{1}{\sqrt{16 - 9x^2}}\, dx \\ &= \int \frac{1}{16\sqrt{1 - \frac{9}{16}x^2}}\, dx \\ &= \frac{1}{16}\int \frac{1}{\sqrt{1 - \left(\frac{3}{4}x\right)^2}}\, dx \end{aligned}$$

Step 2: Use u-substitution with $u = \frac{3}{4}x$.

$$
\begin{aligned}
u &= \frac{3}{4}x \\
du &= \frac{3}{4}dx \\
\frac{4}{3}du &= dx \\
\frac{1}{16} \cdot \frac{4}{3} \int &\frac{1}{\sqrt{1-u^2}}\,du \\
= \frac{1}{12} \int &\frac{1}{\sqrt{1-u^2}}\,du
\end{aligned}
$$

Step 3: Use the inverse trig formula.

$$
\begin{aligned}
\frac{1}{12} \int \frac{1}{\sqrt{1-u^2}}\,du &= \frac{1}{12}\arcsin(u) + C \\
&= \boxed{\frac{1}{12}\arcsin\left(\frac{3}{4}x\right) + C}
\end{aligned}
$$

H19 Integration by division

Use long division or synthetic division when the degree of the numerator is greater than or equal to the degree of the denominator.

How to know which integration method to use

- **Basic antiderivatives:** Check if the integral matches a basic antiderivative formula.
- **u-substitution:** If the integral contains a function and its derivative, consider u-substitution.
- **Change of variable:** Use when the
- **Long or synthetic division:** Use when the integrand is a rational function where the degree of the numerator is greater than or equal to the degree of the denominator.
- **u-substitution with trigonometry:** Use when the integrand contains inverse trig derivatives.