

- Similarly to a DFA, an NFA is a 5-tuple

$$N = (Q, \Sigma, \delta, q_0, F)$$

where, $\delta: Q \times \Sigma \rightarrow 2^Q$

$$F \subseteq Q$$

recall that 2^Q refers to the power set of Q - that is, the set of all possible subsets that may be formed from the elements of Q

- an input string $w \in \Sigma^*$ is accepted by an NFA N if there exists a sequence of states $r_0, r_1, \dots, r_n \in Q$ w/ the following properties

$$\Sigma_\epsilon := \Sigma \cup \{\epsilon\}$$

$\sim r_0 = q_0$ i.e. should be initial

$$- r_n \in F$$

~~there exists a final state~~ machine accepts input if last state is accept st.

$$- r_{i+1} \in \delta(r_i, w_i) \quad (\text{the } \text{allowable} \text{ next state, rel. to current, exists})$$

- $L(N)$ refers to the language accepted by NFA N .

Equivalence of NFA and DFA

Let $L \subseteq \Sigma^*$ be an arbitrary language.

• Notice how the statement is $p \Leftrightarrow q$

$L = L(M)$ for some DFA M iff $L(N)$ for some NFA N

- for \Rightarrow , every DFA is an NFA
 - there exists no ϵ -transitions
 - every set of next states is a singleton set
- for \Leftarrow , construct a DFA $M = (Q', \Sigma, \delta', q_0', F')$ for a given NFA $N = (Q, \Sigma, \delta, q_0, F)$

• In general, corresponding / equiv. DFA is constructed as follows:

- $Q' \subseteq 2^Q$
- $q_0' = \{q_0\}$
- $F' = \{s \in Q' \mid s \cap F \neq \emptyset\}$ refers to dead state / trap state
- $\delta'(s, a) = \bigcup_{q \in s} \delta(q, a)$

1st pt. every st. of M is a set of st. of N ; $P(Q)$ is set of subsets of Q

2nd pt. M starts at the state corresponding to collection containing just the start state of N

3rd pt. M accepts if one of possible states that N could be in is an accept state

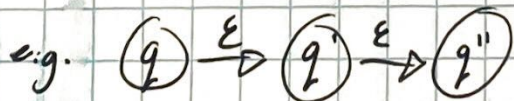
4th pt. if S is a state of M , it is also a ~~subset~~ set of st. of N . When M reads a sym. a in st. ~~from~~ S , it shows where a takes each state in R . ~~B/c~~ B/c each st. may go to a set of st., union of all sets

• if NFA N has n states, the equiv. DFA can have at most 2^n st.s

this is for consideration of ϵ arrows

• Let $S \subseteq Q$, then ϵ -closure would be defined as follows:

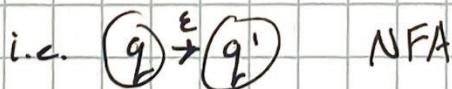
$$E(S) = \{q \in Q \mid q \text{ is reachable from any st. in } S \text{ by } \geq 0 \text{ } \epsilon\text{-transitions}\}$$



$$E(\{q\}) = \{q', q''\}$$

$$q'' = E(\{q_0\})$$

"subset construction" with n



this desc. the collection of st. that can be reached from members of R of M going along only ϵ arrows

$$\delta''(s, a) = \bigcup_{q \in s} E(\delta(q, a))$$

this is in reference to probably