- similarly to a DFA, an NFA is a 5-tuple

$$N = (Q, \Sigma, \delta, q_0, F)$$

where, $\cdot\ \delta : Q \times \Sigma_\varepsilon \rightarrow 2^Q$

$\cdot\ F \subseteq Q$

recall that 2^Q refers to the power set of Q - that is, the set of all possible subsets that may be formed from the elements of Q

- an input string $w \in \Sigma^*$ is accepted by an NFA N if there exists a sequence of states $r_0, r_1, \ldots, r_n \in Q$ w/ the following properties

$\Sigma_\varepsilon := \Sigma \cup \{\varepsilon\}$

- $r_0 = q_0$ i.e. should be initial
- $r_n \in F$ machine accepts input if last state is accept st.
- $r_{i+1} \in \delta(r_i, w_i)$ (the allowable next state, rel. to current, exists)

- $L(N)$ refers to the language accepted by NFA N.

Equivalence of NFA and DFA

Let $L \subseteq \Sigma^*$ be an arbitrary language.

Notice how the statement is $p \Leftrightarrow q$

$L = L(M)$ for some DFA M iff $L(N)$ for some NFA N

- for $\Rightarrow$, every DFA is an NFA
  - there exists no ε - transition
  - every set o next states is a singleton set
- for $\Leftarrow$, construct a DFA $M = (Q', \Sigma, \delta', q_0', F')$ for a given NFA $N = (Q, \Sigma, \delta, q_0, F)$

- In general, corresponding / equiv. DFA is constructed as follows:
  - $Q' \subseteq 2^Q$
  - $q_0' = \{q_0\}$
  - $F' = \{ s \in Q' \mid s \cap F \neq \emptyset \}$ refers to dead state / trap state
  - $\delta'(s, a) = \bigcup_{q \in s} \delta(q, a)$

1st pt. every st. o M is a set of st. o N; $P(Q)$ is set o subsets of Q
2nd pt. M starts at the state corresponding to collection containing just the start state o N
3rd pt. M accepts if one o possible states that N could be in is an accept state
4th pt. if S is a state o M, it is also a set o st.s o N. When M reads a sym. α in st. S, it shows where α takes each state in R. B/c each st. may go to a set o st., union o all sets

- if NFA N has n states, the equiv. DFA can have at most 2^n st.s

this is for consideration o ε arrows
- Let $S \in Q'$, then ε-closure would be defined as follows:

this is in reference to 2nd pt. probably

$$E(s) = \{ q \in Q \mid q \text{ is reachable fm any st. in } S \text{ by } \geq 0\ \varepsilon\text{-transitions} \}$$

e.g. $q \xrightarrow{\varepsilon} q' \xrightarrow{\varepsilon} q''$ $\quad E(\{q\}) = \{q', q''\}$

- $q'' = E(\{q_0\})$ "subset construction" wdtm

i.e. $q \xrightarrow{\varepsilon} q'$ NFA

$\rightarrow \{q, q'\}$ DFA

this desc. the collection o st. that can be reached fm members o R o M going along only ε arrows

- $\delta''(s, a) = \bigcup_{q \in s} E(\delta(q, a))$