· Let Σ be an alphabet. The regular expressions over Σ are def. recursively

Lecture 7

Basis $\quad \emptyset$ is a reg. exp.
λ is a reg. exp
$\forall a \in \Sigma$ is a reg. exp.

Recursive Step $\quad$ if α_1 and α_2 are reg. exp., then so are $\alpha_1 + \alpha_2$, $\alpha_1 \cdot \alpha_2$, and α_1^*

union $\quad$ concatenation $\quad$ star operation

$\emptyset$: $\quad L(\emptyset) = \emptyset$
λ : $\quad L(\lambda) = \{\lambda\}$
$a \in \Sigma$: $\quad L(a) = \{a\}$

i.e. reg. exp. = strings as a result o potentially use o op.s (union, concat, star)

$\alpha_1 + \alpha_2$: $\quad L(\alpha_1 + \alpha_2) = L(\alpha_1) \cup L(\alpha_2)$
$\alpha_1 \cdot \alpha_2$: $\quad L(\alpha_1 \cdot \alpha_2) = L(\alpha_1) \cup L(\alpha_2)$
α_1^* : $\quad L(\alpha_1^*) = L(\alpha_1)^* = (L(\alpha_1))^*$

R^+ is the same as $\alpha_1 \alpha_1^*$
~ R^* is all str. that are 0 / > concat. o str. fm R,
R^+ has all str. 1 / > concat. o str. fm R

R^k is shorthand for concat. o k R's w/ each other

· Understand $\varepsilon \neq \emptyset$ b/c ε rep. 1 str. - empty str. - while $\emptyset$ is empty set.

given t. R is a regular exp.:

$R \cup \emptyset = R \quad$ b/c add. empty lang. does nothing
$R \circ \varepsilon = R \quad$ b/c joining empty str. to any string w/ not change it
$R \cup \varepsilon \neq R \quad$ b/c if $R = 0 \Rightarrow L(R) = \{0\}$ but. $L(R \cup \varepsilon) = \{0, \varepsilon\}$
$R \circ \emptyset \neq R \quad$ b/c if $R = 0 \Rightarrow L(R) = \{0\}$ b. $L(R \circ \emptyset) = \emptyset$