Chapter Four
Finite Differences

Introduction:

Consider a function $y = f(x)$ defined on (a, b), x and y are the independent and dependent variables respectively. If the points $x_o, x_1, \ldots, x_n$ are taken at equidistance that is, $x_i = x_o + ih$, for $i = 0, 1, 2, \ldots, n$, then the value of y, when $x = x_i$, is denoted by y_i, where $y_i = f(x_i)$. Here, the values of *x* are called *arguments* and the values of *y* are called as *entries*. The interval *h* is called the *difference interval*. The differences $y_1 - y_o, y_2 - y_1, \ldots, y_n - y_{n-1}$ are called the *first differences* of the function y and are denoted by $\Delta y_o, \Delta y_1, \ldots, \Delta y_{n-1}$ etc. That is

$$\Delta y_o = y_1 - y_o, \ \Delta y_1 = y_2 - y_1, \ldots, \quad \Delta y_{n-1} = y_n - y_{n-1} \tag{4.1}$$

The symbol Δ in these equations is called the *difference operator*.

Finite differences deal with the changes that take place in the value of a function f(x) due to finite changes in x. Finite difference operators include, forward difference operator, backward difference operator, shift operator, central difference operator and mean operator.

4.1. Shift Operators

Actvity4.1: Let $f(x) = x^2 + 2x - 4$. Find $f(x + h), f(x + 2h)$ and $f(x + 3h)$.

Shift operator, E:

Definition 4.1. The shift operator E of a function f denoted by $Ef(x)$ is defined as

$$Ef(x) = f(x + h) \quad \text{or} \quad Ey_i = y_{i+1}$$

Hence, shift operator shifts the function value y_i to the next higher value y_{i+1}. The second shift operator is given as follow.

$$E^2 f(x) = E[Ef(x)] = E[f(x + h)] = f(x + 2h)$$

E is linear and obeys the law of indices. And hence the generalized shift operator is given by

$$\boldsymbol{E^n f(x) = f(x + nh) \quad or \quad E^n y_i = y_{i+nh}}$$

The inverse shift operator denoted by E^{-1} is defined as

$$E^{-1} f(x) = f(x - h)$$

In a similar manner, second and higher inverse operators are given by

$$E^{-2} f(x) = f(x - 2h) \text{ and } E^{-n} f(x) = f(x - nh)$$

The more general form of shift operator E is given by

$$\boldsymbol{E^r f(x) = f(x + rh)}$$

Where ra nonzero rational number (that is, r is positive as well as negative rationals).

Example4.1: Let $f(x) = x^2 + 3x$. Find the first three shift operators.

Solution: the first three shift operators are $Ef(x), E^2 f(x) and\ E^3 f(x)$. Thus

$$Ef(x) = f(x + h) = (x + h)^2 + 3(x + h)$$
$$= x^2 + 2xh + h^2 + 3x + 3h = x^2 + 3x + 2xh + h^2 + 3h$$
$$E^2 f(x) = Ef(x + h) = f(x + 2h) = (x + 2h)^2 + 3(x + 2h)$$
$$= x^2 + (4h + 3)x + 4h^2 + 6h$$

And $E^3 f(x) = E^2 f(x + h) = Ef(x + 2h) = f(x + 3h) = (x + 3h)^2 + 3(x + 3h)$
$$= x^2 + (6h + 3)x + 9h(h - 1)$$

Average operator, μ: The average operator μ is defined as

$$\mu f(x) = \frac{1}{2}[f(x + h/2) + f(x - h/2)]$$

That is $$\mu y_i = [y_{i+1/2} + y_{i-1/2}]$$

4.2. Forward difference operators

Definition 4.2.: The *forward difference* or simply *difference* operator is denoted by Δ and is defined as $$\Delta f(x) = f(x + h) - f(x)$$

or writing in terms of y, at $x = xi$, this above equation becomes
$\Delta f(x_i) = f(x_i + h) - f(x_i)$ or $\Delta y_i = y_{i+1} - y_i$, $i = 0, 1, 2, \ldots, n-1$
The differences of the first differences are called the *second differences* and they are denoted by $\Delta^2 y_o, \Delta^2 y_1, \ldots, \Delta^2 y_{n-1}$.
Hence $\Delta y_o = y_1 - y_o, \quad \Delta y_1 = y_2 - y_1, \ldots, \quad \Delta y_{n-1} = y_n - y_{n-1}$

$$\Delta^2 y_o = \Delta y_1 - \Delta y_o = (y_2 - y_1) - (y_1 - y_o) = y_2 - 2y_1 + y_o$$
$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1 = (y_3 - y_2) - (y_2 - y_1) = y_3 - 2y_2 + y_1$$
$$\Delta^3 y_o = \Delta^2 y_1 - \Delta^2 y_o = (y_3 - 2y_2 + y_1) - (y_2 - 2y_1 + y_o) = y_3 - 3y_2 + 3y_1 - y_o$$
$$\Delta^3 y_1 = y_4 - 3y_3 + 3\,y_2 - y_1, \text{ etc.}$$

In general, we have
$$\Delta^{n+1} f(x) = \Delta[\Delta^n f(x)], \quad i.e., \Delta^{n+1} y_i = \Delta[\Delta^n y_i], \quad n = 0, 1, 2, \ldots.$$
Also, $\boldsymbol{\Delta^{n+1} f(x) = \Delta^n[f(x+h) - f(x)] = \Delta^n f(x+h) - \Delta^n f(x)}$
and $\boldsymbol{\Delta^{n+1} y_i = \Delta^n y_{i+1} - \Delta^n y_i, \ n = 0, 1, 2, \ldots.}$
Where Δ^0 is call an identity operator. That is $\Delta^0 f(x) = f(x)$ *and* $\Delta^1 = \Delta$
The tabular representation for forward difference is put as follow.
Table 4.1. Forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_o	y_o					
		Δy_o				
x_1	y_1		$\Delta^2 y_o$			
		Δy_1		$\Delta^3 y_o$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_o$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_o$
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		
x_4	y_4		$\Delta^2 y_3$			
		Δy_4				
x_5	y_5					

The forward differences for the arguments $x_0, x_1, \ldots, x_5$ are shown in Table 4.1. Table 4.1 is called a *diagonal difference table* or *forward difference table*. The first term in Table 4.1 is y_o and is called the *leading term*. The differences $\Delta y_o, \Delta^2 y_o, \Delta^3 y_o, \ldots$, are called the *leading differences*. Similarly, the differences with fixed subscript are called *forward differences*.

Example 4.2.

1. Construct a forward difference table for the following data

x	0	10	20	30
y	0	0.174	0.347	0.518

Solution: The forward difference table for the given data is shown below.

Table4.2. Forward difference table for the above data

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	0			
		0.174		
10	0.174		-0.001	
		0.173		-0.001
20	0.347		-0.002	
		0.171		

30	0.518			

2. Draw the forward difference table for $y = f(x) = x^3 + 2x + 1, for\ x = 1,2,3,4,5.$
   Solution: The forward difference table is given below.
   Table4.3. Forward difference table for (2)

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	4				
		2			
2	6		1		
		3		-1	
3	9		0		3
		3		2	
4	12		2		
		5			
5	17				

4.3. **Backward difference operators**

Definition4.3: The *backward difference operator* denoted by ∇ is defined as

$$\nabla f(x) = f(x) - f(x - h).$$

This equation can be written as

$$\nabla y_i = y_i - y_{i-1},\ i = n, n-1, \dots, 1.$$

or $\nabla y_1 = y_1 - y_0, \quad \nabla y_2 = y_2 - y_1, \dots, \quad \nabla y_n = y_n - y_{n-1}$ (4.2)

The differences in equation (4.2) are called *first differences*. The *second differences* are denoted by: $\nabla^2 y_2, \nabla^2 y_3, \dots, \nabla^2 y_n$.

Hence

$\nabla^2 y_2 = \nabla(\nabla y_2) = \nabla(y_2 - y_1) = \nabla y_2 - \nabla y_1 = (y_2 - y_1) - (y_1 - y_o) = y_2 - 2y_1 + y_0.$

Similarly, $\nabla^2 y_3 = y_3 - 2y_2 + y_1, \nabla^2 y_4 = y_4 - 2y_3 + y_2$, and so on.

In general, we have: $\boldsymbol{\nabla^k y_i = \nabla^{k-1} y_i - \nabla^{k-1} y_{i-1},\ i = n, n-1, \dots, k}$

Where $\nabla^0 y_i = y_i$, and hence $\nabla^1 y_i = \nabla y_i$

The backward differences written in a tabular form is shown in Table 4.4 below. In Table 4.4, the differences $\nabla^n y$ with a fixed subscript 'i' lie along the diagonal upward sloping.

Table 4.4: Backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		∇y_1			
x_1	y_1		$\nabla^2 y_2$		
		∇y_2		$\nabla^3 y_3$	
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$
		∇y_3		$\nabla^3 y_4$	
x_3	y_3		$\nabla^2 y_4$		
		∇y_4			
x_4	y_4				

Table 4.4 is called the backward difference or horizontal table.

Example 4.3*:*

1. Construct the backward difference table of the following and find $\nabla^2 y_2, \nabla^2 y_3$ and $\nabla^3 y_3$.

x	0	10	20	30
y	0	0.174	0.347	0.518

Solution: The backward difference of this problem is given as follow.

Table 4.5: backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
0	0			
		0.174		
10	0.174		-0.001	
		0.173		-0.001
20	0.347		-0.002	
		0.171		
30	0.518			

Here $\nabla^2 y_2 = -0.001, \nabla^2 y_3 = -0.002$ and $\nabla^3 y_3 = -0.001$.

2. Obtain the backward differences for the function $f(x) = x^3$ from $x = 1$ to 1.05 to two decimals chopped and find $\nabla y_5,\ \nabla^2 y_5,\ \nabla^3 y_5,\ \nabla^4 y_5$ *and* $\nabla^5 y_5$.

Solution: The backward difference table is computed as below.

Table 4.6: backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1.00	1					
		0.030				
1.01	1.030		0.001			
		0.031		-0.001		
1.02	1.061		0.000		0.002	
		0.031		0.001		-0.003
1.03	1.092		0.001		-0.001	
		0.032		0.000		
1.04	1.124		0.001			
		0.033				
1.05	1.157					

Here, $\nabla y_5 = 0.033, \nabla^2 y_5 = 0.001, \nabla^3 y_5 = 0.000, \nabla^4 y_5 = -0.001, \nabla^5 y_5 = -0.003$

3. Find the missing term in the table below.

x	0	1	2	3	4
y	3	2	3	?	11

Solution: Let the missing term be a, then using backward difference table, we have;

Table 4.7: backward difference

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0	3				
		-1			
1	2		2		
		1		$a-6$	
2	3		$a-4$		
		$a-3$		$18-4a$	
3	a		$14-2a$		
		$11-a$			

4	11				

From the table we see that $a - 6 = 0$, and hence the missing term is 6.

Exercise 4.3:

1. Construct a backward difference table for the following data.

x	45	55	65	75
y	20	60	120	180

2. If m is a positive integer and the interval of differencing is 1, show that $x^m = x(x-1) \ldots [x - (x-1)]$.

3. Find the missing term in the table below.

x	0	1	2	3	4
y	1	3	13	?	81

4.4. Central difference operators

Activity 4.4: Let $f(x) = e^x + 2$. Compute $f(x + h/2) - f(x - h/2)$ for $h = 0, 1, 2, 3$.

Definition 4.4: The central difference operator is denoted by the symbol δ and is defined by

$$\delta f(x) = f(x + h/2) - f(x - h/2)$$

where h is the interval of differencing.

In terms of y, the first central difference is written as

$$\delta y_i = y_{i+1/2} - y_{i-1/2}$$

where $y_{i+1/2} = f(x_i + h/2)$ and $y_{i-1/2} = f(x_i - h/2)$.

Hence $\boldsymbol{\delta y_{1/2} = y_1 - y_o, \delta y_{3/2} = y_2 - y_1, \ldots, \delta y_{n-1/2} = y_n - y_{n-1}}$.

The second central differences are given by

$\delta^2 y_i = \delta y_{i+1/2} - \delta y_{i-1/2} = (y_{i+1} - y_i) - (y_i - y_{i-1}) = y_{i+1} - 2y_i + y_{i-1}$

In general**:** $\boldsymbol{\delta^n y_i = \delta^{n-1} y_{i+1/2} - \delta^{n-1} y_{i-1/2}}$.

The central difference table for the seven arguments $x_0, x_1, \ldots, x_6$ is given in table 4.7.

Table 4.8 central difference table

x	y	δy	$\delta^2 y$	$\delta^3 y$	$\delta^4 y$	$\delta^5 y$	$\delta^6 y$
x_0	y_0						
		$\delta y_{1/2}$					
x_1	y_1		$\delta^2 y_1$				
		$\delta y_{3/2}$		$\delta^3 y_{3/2}$			
x_2	y_2		$\delta^2 y_2$		$\delta^4 y_2$		
		$\delta y_{5/2}$		$\delta^3 y_{5/2}$		$\delta^5 y_{5/2}$	
x_3	y_3		$\delta^2 y_3$		$\delta^4 y_3$		$\delta^6 y_3$
		$\delta y_{7/2}$		$\delta^3 y_{7/2}$		$\delta^5 y_{7/2}$	
x_4	y_4		$\delta^2 y_4$		$\delta^4 y_4$		
		$\delta y_{9/2}$		$\delta^3 y_{9/2}$			
x_5	y_5		$\delta^2 y_5$				
		$\delta y_{11/2}$					
x_6	y_6						

It is noted in table 4.7 that all the odd differences have fraction suffices and all the even differences are integral suffices.

Exercise 4.4:

1. Construct the central difference table for

x	1	2	3	4	5
y	4	6	9	12	17

2. Let $f(x) = xsinx$. Formulate the central difference table for $x = 0, 2, 4, 6, 8, 10$.

4.5. Properties of the operators

Activity 4.5: ***1.*** Let $f(x) = 6$, find $\Delta f(x)$.

2. Let $f(x) = x^2 + 5x + 3$. Compute $\Delta E f(x)$.

Properties of Δ.

1. If c is a constant then $\Delta c = 0$.
2. $\Delta[f(x) \pm g(x)] = \Delta f(x) \pm \Delta g(x)$
3. $\Delta[cf(x)] = c\Delta f(x)$, for a constant c.
4. If m and n are positive integers, then $\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$
5. $\Delta[f(x)g(x)] = f(x)\Delta g(x) + g(x)\Delta f(x)$
6. $\Delta\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x)g(x+h)}$

Proof (1): Let $f(x) = c$

Hence $f(x+h) = c$, h is the interval of differencing.

Then $\Delta f(x) = f(x+h) - f(x) = c - c = 0$ or

$\Delta c = 0$

Proof (2):

$$\Delta[f(x) + g(x)] = [f(x+h) + g(x+h)] - [f(x) + g(x)]$$
$$= f(x+h) - f(x) + g(x+h) - g(x)$$
$$= \Delta f(x) + \Delta g(x)$$

Similarly: $\Delta[f(x) - g(x)] = \Delta f(x) - \Delta g(x)$

and hence $\Delta[f(x) \pm g(x)] = \Delta f(x) \pm \Delta g(x)$

Proof (3):

$$\Delta[cf(x)] = [cf(x+h) - cf(x)]$$
$$= c[f(x+h) - f(x)]$$
$$= c\Delta f(x)$$

Proof (4)

$$\Delta^m \Delta^n f(x) = \left(\overbrace{\Delta \times \Delta \times \ldots \times \Delta}^{m\ factors}\right)\left(\overbrace{\Delta \times \Delta \times \ldots \times \Delta}^{n\ factors}\right) f(x)$$
$$= \left(\overbrace{\Delta \times \Delta \times \Delta \times \ldots \times \Delta}^{m+n\ factors}\right) f(x)$$
$$= \Delta^{m+n} f(x)$$

Similarly we can prove (5) and (6).

Relations between the operators

Summary of operators

operators	**Definition**
Forward difference operator Δ	$\Delta f(x) = f(x + h) - f(x)$
Backward difference operator ∇	$\nabla f(x) = f(x + h) - f(x)$
Central difference operator δ	δ f$(x) = f(x + h/2) - f(x - h/2)$
Shift operator E	$Ef(x) = f(x + h)$
Average operator μ	μ f$(x) = 0.5[f(x + h/2) - f(x - h/2)]$

1. For the operators Δ and E, we have the following.

$\Delta = E - 1$ and $E = \Delta + 1$

Proof: $\Delta f(x) = f(x+h) - f(x)$ and $Ef(x) = f(x+h)$

This implies that $\Delta f(x) = Ef(x) - f(x)$

$$= (E-1)f(x)$$
$$\Longrightarrow \qquad \Delta = E - 1$$
And $\qquad E = \Delta + 1$

2. $E\Delta f(x) = E(f(x+h) - f(x)) = Ef(x+h) - Ef(x)$
$$= f(x+2h) - f(x+h)$$
$$= \Delta f(x+h)$$
$$= \Delta E f(x)$$
$$E\Delta = \Delta E$$

Hence $E\Delta = \Delta E$

Example 4.5: Show that $\Delta \log f(x) = \log(1 + \frac{\Delta f(x)}{f(x)})$.

Solution: Let h be the interval of differencing
$$f(x+h) = Ef(x) = (\Delta + 1)f(x) = \Delta f(x) + f(x)$$
$$\Longrightarrow \qquad \frac{f(x+h)}{f(x)} = \frac{\Delta f(x)}{f(x)} + 1$$
Taking logarithms on both sides we get
$$\log\left[\frac{f(x+h)}{f(x)}\right] = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$
$$\Longrightarrow \qquad \log f(x+h) - \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$
$$\Longrightarrow \qquad \Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$

Example 4.5: Evaluate $(\frac{\Delta^2}{E})x^3$

Solution: Let h be the interval of differencing: $\left(\frac{\Delta^2}{E}\right)x^3 = (\Delta^2 E^{-1})x^3$
$$= (E-1)^2 E^{-1} x^3 \quad \text{(Since } \Delta = E - 1\text{)}$$
$$= (E^2 - 2E + 1)E^{-1}x^3$$
$$= (E - 2 + E^{-1})x^3$$
$$= Ex^3 - 2x^3 + E^{-1}x^3$$
$$= (x+h)^3 - 2x^3 + (x-h)^3$$
$$= 6xh$$

Note: If $h = 1$, then $\left(\frac{\Delta^2}{E}\right)x^3 = 6x$

Example 4.6: Prove that $e^x = \frac{\Delta^2}{E}e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$, the interval of differencing being h.

Solution: We know that: $Ef(x) = f(x+h)$, then $Ee^x = e^{x+h}$

Again $\quad \Delta e^x = e^{x+h} - e^x = e^x(e^h - 1)$
$$\Longrightarrow \qquad \Delta^2 e^x = e^x \cdot (e^h - 1)^2$$
Hence $\quad \left(\frac{\Delta^2}{E}\right)e^x = (\Delta^2 E^{-1})e^x = \Delta^2 e^{x-h} = e^{-h}(\Delta^2 e^x)$
$$= e^{-h}e^x(e^h - 1)^2$$
Therefore, the right hand side $= e^{-h}e^x(e^h-1)\frac{e^{x+h}}{e^x(e^h-1)} = e^x$

Relation between *E* and ∇: $\quad \nabla f(x) = f(x) - f(x-h) = f(x) - E^{-1}f(x)$
$$\Longrightarrow \qquad \nabla = 1 - E^{-1}$$
$$\Longrightarrow \qquad \nabla = \frac{E-1}{E}$$

Example 4.7: Prove that (a) $(1+\Delta)(1-\nabla) = 1$ (b) $\Delta\nabla = \Delta - \nabla$

Solution: ***a)*** $(1+\Delta)(1-\nabla)f(x) = EE^{-1}f(x)$
$= Ef(x-h)$
$= f(x) = 1 \cdot f(x)$

Thus $(1+\Delta)(1-\nabla) = 1$.

b) $\Delta\nabla f(x) = (E-1)(1-E^{-1})f(x)$
$= (E-1)[f(x)-f(x-h)]$

And $(\Delta-\nabla)f(x) = ((E-1)-(1-E^{-1}))f(x)$
$= (E-1)[f(x)-f(x-h)]$

Therefore $\Delta\nabla = \Delta - \nabla$.

1. $\delta = E^{1/2} - E^{-1/2}$

Proof: We know that $\delta[f(x)] = f(x+h/2) - f(x-h/2)$
$= E^{1/2}f(x) - E^{-1/2}f(x)$
$= \left(E^{1/2} - E^{-1/2}\right)f(x)$

Which implies that $\delta = E^{1/2} - E^{-1/2}$.

2. $\Delta = E\nabla = \nabla E = \delta E^{1/2}$

Proof: $E\nabla f(x) = E[f(x)-f(x-h)]$
$= Ef(x) - Ef(x-h)$
$= f(x+h) - f(x) = \Delta f(x)$

Thus $E\nabla = \Delta$ -- (1)

Again $\nabla Ef(x) = \nabla f(x+h)$
$= f(x+h) - f(x) = \Delta f(x)$

Hence we have $\nabla E = \Delta$ -- (2)

Also $\delta E^{1/2}f(x) = \delta f(x+h/2)$
$= f(x+h) - f(x) = \Delta f(x)$

Which implies $\delta E^{1/2} = \Delta$ -- (3)

From (1), (2) and (3), we have that $\Delta = E\nabla = \nabla E = \delta E^{1/2}$

3. $\Delta\nabla = \nabla\Delta = \delta^2$

Proof: Since $\Delta\nabla f(x) = \Delta[f(x)-f(x-h)]$
$= \Delta f(x) - \Delta f(x\Delta - h)$
$= [f(x+h)-f(x)] - [f(x)-f(x-h)]$
$= \delta f(x+h/2) - \delta f(x-h/2)$
$= \delta^2 f(x)$

$\Longrightarrow$ $\Delta\nabla = \delta^2$ -- (1)

Also $\nabla\Delta f(x) = \nabla[f(x+h)-f(x)]$
$= \nabla f(x+h) - \nabla f(x)$
$= [f(x+h)-f(x)] - [f(x)-f(x-h)]$
$= \delta f(x+h/2) - \delta f(x-h/2)$
$= \delta^2 f(x)$

$\Longrightarrow$ $\nabla\Delta = \delta^2$ -- (2)

Therefore from (1) and (2), we have; $\Delta\nabla = \nabla\Delta = \delta^2$

Example 4.8: Find $\Delta^3(1-3x)(1-2x)1-x)$.

Solution: Let $f(x) = (1-3x)(1-2x)1-x) = -6x^3 + 11x^2 - 6x + 1$

Here, $f(x)$ is a polynomial function of degree three and the coefficient of x^3 is -6.

Therefore $\Delta^3 f(x) = (-6)3! = -36$.

Review exercise

1. Show that: a) $\Delta\nabla = \Delta - \nabla$ b) $\nabla = \Delta E^{-1}$ c) $E^n = (1+\Delta)^n$

2. Find the following a) Δe^{ax} b) $\Delta^2(3e^x)$ c) $\frac{\Delta}{1+x^2}$ d) $\Delta \sin(ax+b)$

3. Let $v_o = 1, v_1 = 5, v_2 = 10, v_3 = 30, v_4 = 30$. Find $\Delta^4 v_o$.

4. Construct table of forward, backward and central differences of the following.

a)

x	40	50	60	70	80	90
y	204	224	246	270	296	324

b) $f(x) = x^2 + 6x - 9$, for $x = 1,2,3,4,6,7,8$

c) $f(x) = e^x + 2x$ for $x = 0,1,2,3,4,5$

d) $f(x) = \ln(x^2)$ for $x = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$

2. Find the missing term of the following.

a)

x	1	2	3	4	5
y	8	17	38	?	140

b)

x	0	1	2	3	4
y	1	-2	-1	?	37

3. Evaluate the following. a) $\Delta(e^{ax} \log bx)$ b) $\Delta tan^{-1}x$ c) $\Delta[\frac{5x+12}{x^2+5x+6}]$ d) $(\frac{\Delta^2}{E})x^3$ (with interval of differencing $h = 1$) e) $x^n[\frac{1}{x}]$

4. Prove the following. a) $\delta = \Delta(1+\Delta)^{-1/2}$ b) $\Delta^3 y_2 = \nabla^3 y_5$ c) $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \Delta + \nabla$

5. Given $u_o = 5, u_1 = 24, u_2 = 81, u_3 = 200, u_4 = 100$ and $u_5 = 8$. Find $\Delta^5 u_o$.