svm

January 26, 2021

0.1 This is the svm workbook for ECE C147/C247 Assignment #2

Please follow the notebook linearly to implement a linear support vector machine.

Please print out the workbook entirely when completed.

We thank Serena Yeung & Justin Johnson for permission to use code written for the CS 231n class (cs231n.stanford.edu). These are the functions in the cs231n folders and includes code to preprocess and show the images. The classifiers used are based off of code prepared for CS 231n as well.

The goal of this workbook is to give you experience with training an SVM classifier via gradient descent.

0.2 Importing libraries and data setup

[1]:
```
import numpy as np # for doing most of our calculations
import matplotlib.pyplot as plt# for plotting
from cs231n.data_utils import load_CIFAR10 # function to load the CIFAR-10␣
 ↪dataset.
import pdb

# Load matplotlib images inline
%matplotlib inline

# These are important for reloading any code you write in external .py files.
# see http://stackoverflow.com/questions/1907993/
 ↪autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

[2]:
```
# Set the path to the CIFAR-10 data
cifar10_dir = 'cifar-10-batches-py' # You need to update this line
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

[3]:
```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse',␣
 ↪'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```

[4]:
```
# Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
```

```python
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('Dev data shape: ', X_dev.shape)
print('Dev labels shape: ', y_dev.shape)
```

```
Train data shape:  (49000, 32, 32, 3)
Train labels shape:  (49000,)
Validation data shape:  (1000, 32, 32, 3)
Validation labels shape:  (1000,)
Test data shape:  (1000, 32, 32, 3)
Test labels shape:  (1000,)
Dev data shape:  (500, 32, 32, 3)
Dev labels shape:  (500,)
```

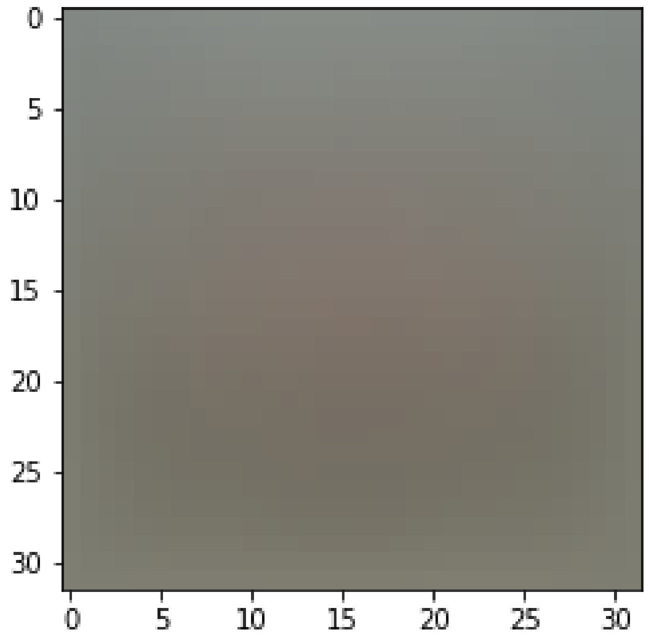
[5]:
```python
# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
print('dev data shape: ', X_dev.shape)
```

```
Training data shape:  (49000, 3072)
Validation data shape:  (1000, 3072)
Test data shape:  (1000, 3072)
dev data shape:  (500, 3072)
```

[6]:
```python
# Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean␣
 ↪image
plt.show()
```

```
[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082
 131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]
```



[7]:
```
# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image
```

[8]:
```
# third: append the bias dimension of ones (i.e. bias trick) so that our SVM
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)
```

```
(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

0.3 Question:

(1) For the SVM, we perform mean-subtraction on the data. However, for the KNN notebook, we did not. Why?

0.4 Answer:

(1) Normalizing the data can help to train the SVM model faster and eliminate bias. KNN's algorithm takes the nearest data points to calculate the distances, so the relative position is important to train the model. Therefore, the mean substraction is not desirable for KNN.

0.5 Training an SVM

The following cells will take you through building an SVM. You will implement its loss function, then subsequently train it with gradient descent. Finally, you will choose the learning rate of gradient descent to optimize its classification performance.

[9]:
```
from nndl.svm import SVM
```

[10]:
```
# Declare an instance of the SVM class.
# Weights are initialized to a random value.
# Note, to keep people's initial solutions consistent, we are going to use a␣
↪random seed.

np.random.seed(1)

num_classes = len(np.unique(y_train))
num_features = X_train.shape[1]
```

```
svm = SVM(dims=[num_classes, num_features])
```

SVM loss

```
[11]: ## Implement the loss function for in the SVM class(nndl/svm.py), svm.loss()

loss = svm.loss(X_train, y_train)
print('The training set loss is {}.'.format(loss))

# If you implemented the loss correctly, it should be 15569.98
```

```
The training set loss is 15569.97791541019.
```

SVM gradient

```
[12]: ## Calculate the gradient of the SVM class.
# For convenience, we'll write one function that computes the loss
#   and gradient together. Please modify svm.loss_and_grad(X, y).
# You may copy and paste your loss code from svm.loss() here, and then
#   use the appropriate intermediate values to calculate the gradient.

loss, grad = svm.loss_and_grad(X_dev,y_dev)

# Compare your gradient to a numerical gradient check.
# You should see relative gradient errors on the order of 1e-07 or less if you␣
 ↪implemented the gradient correctly.
svm.grad_check_sparse(X_dev, y_dev, grad)
```

```
numerical: -5.838162 analytic: -5.838162, relative error: 2.466441e-08
numerical: 4.911044 analytic: 4.911043, relative error: 9.019541e-08
numerical: 5.846962 analytic: 5.846962, relative error: 3.093405e-08
numerical: 18.773666 analytic: 18.773666, relative error: 4.042397e-09
numerical: 0.499990 analytic: 0.499991, relative error: 8.427325e-07
numerical: 7.760000 analytic: 7.760000, relative error: 9.871900e-09
numerical: 6.850339 analytic: 6.850339, relative error: 1.834455e-09
numerical: -9.828000 analytic: -9.828000, relative error: 1.953625e-08
numerical: 0.731279 analytic: 0.731280, relative error: 5.622385e-07
numerical: -19.438760 analytic: -19.438760, relative error: 2.125497e-08
```

0.6 A vectorized version of SVM

To speed things up, we will vectorize the loss and gradient calculations. This will be helpful for stochastic gradient descent.

```
[13]: import time
```

```
[14]: ## Implement svm.fast_loss_and_grad which calculates the loss and gradient
#    WITHOUT using any for loops.
from nndl.svm import SVM
# Standard loss and gradient
```

```
tic = time.time()
loss, grad = svm.loss_and_grad(X_dev, y_dev)
toc = time.time()
print('Normal loss / grad_norm: {} / {} computed in {}s'.format(loss, np.linalg.
 ↪norm(grad, 'fro'), toc - tic))

tic = time.time()
loss_vectorized, grad_vectorized = svm.fast_loss_and_grad(X_dev, y_dev)
toc = time.time()
print('Vectorized loss / grad: {} / {} computed in {}s'.format(loss_vectorized,␣
 ↪np.linalg.norm(grad_vectorized, 'fro'), toc - tic))

# The losses should match but your vectorized implementation should be much␣
 ↪faster.
print('difference in loss / grad: {} / {}'.format(loss - loss_vectorized, np.
 ↪linalg.norm(grad - grad_vectorized)))

# You should notice a speedup with the same output, i.e., differences on the␣
 ↪order of 1e-12
```

```
Normal loss / grad_norm: 15472.378211387851 / 2169.1161114350766 computed in
0.08200883865356445s
Vectorized loss / grad: 15472.378211387882 / 2169.1161114350766 computed in
0.0040454864501953125s
difference in loss / grad: -3.092281986027956e-11 / 7.919265564327152e-12
```

0.7 Stochastic gradient descent

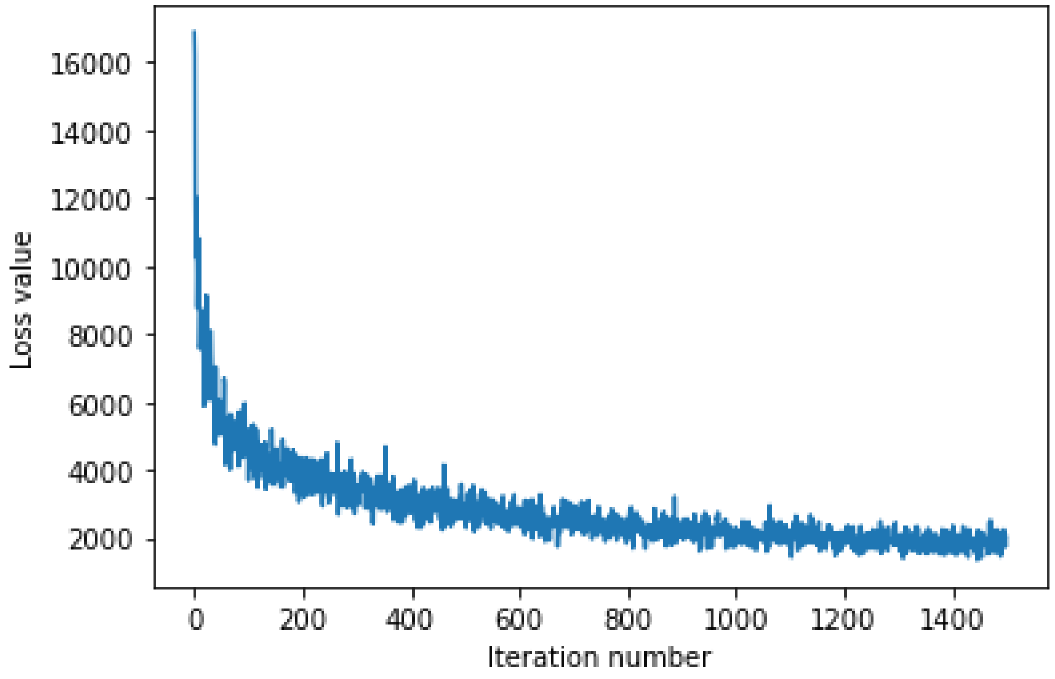
We now implement stochastic gradient descent. This uses the same principles of gradient descent we discussed in class, however, it calculates the gradient by only using examples from a subset of the training set (so each gradient calculation is faster).

[15]:
```
# Implement svm.train() by filling in the code to extract a batch of data
# and perform the gradient step.

tic = time.time()
loss_hist = svm.train(X_train, y_train, learning_rate=5e-4,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took {}s'.format(toc - tic))

plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```

```
iteration 0 / 1500: loss 16878.859625643898
iteration 100 / 1500: loss 3698.37442944583
iteration 200 / 1500: loss 3749.3722795434496
iteration 300 / 1500: loss 3232.2016048804485
iteration 400 / 1500: loss 2786.9285054561765
iteration 500 / 1500: loss 2911.8902158068004
iteration 600 / 1500: loss 2696.1234809338803
iteration 700 / 1500: loss 2959.575637600287
iteration 800 / 1500: loss 2512.8602634753493
iteration 900 / 1500: loss 2105.966992162574
iteration 1000 / 1500: loss 2313.4288377704443
iteration 1100 / 1500: loss 1732.4754464411592
iteration 1200 / 1500: loss 2114.4851353256363
iteration 1300 / 1500: loss 2050.9948002316255
iteration 1400 / 1500: loss 1814.3598110234714
That took 6.833926200866699s
```



0.7.1 Evaluate the performance of the trained SVM on the validation data.

[16]:
```
## Implement svm.predict() and use it to compute the training and testing error.

y_train_pred = svm.predict(X_train)
print('training accuracy: {}'.format(np.mean(np.equal(y_train,y_train_pred), )))
y_val_pred = svm.predict(X_val)
print('validation accuracy: {}'.format(np.mean(np.equal(y_val, y_val_pred)), ))
```

```
training accuracy: 0.29612244897959183
```

```
validation accuracy: 0.303
```

0.8 Optimize the SVM

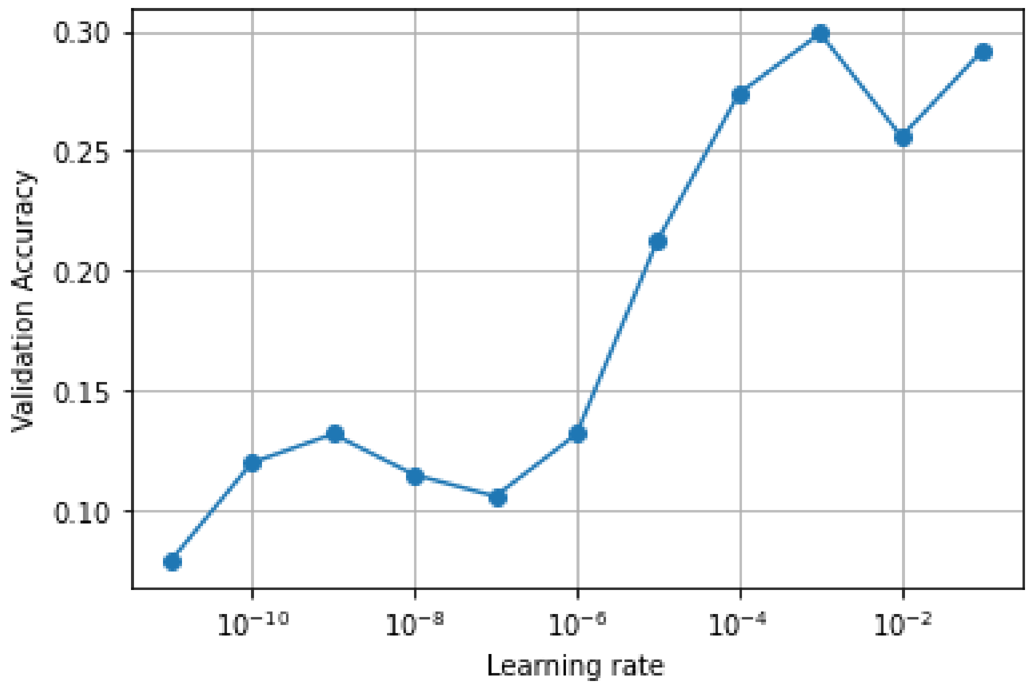
Note, to make things faster and simpler, we won't do k-fold cross-validation, but will only optimize the hyperparameters on the validation dataset (X_val, y_val).

[21]:
```
# ================================================================ #
# YOUR CODE HERE:
#   Train the SVM with different learning rates and evaluate on the
#     validation data.
#   Report:
#     - The best learning rate of the ones you tested.
#     - The best VALIDATION accuracy corresponding to the best VALIDATION error.
#
#   Select the SVM that achieved the best validation error and report
#     its error rate on the test set.
#   Note: You do not need to modify SVM class for this section
# ================================================================ #
lrs = []
y_val_accs = []

for lr in range(11):
    lr = 0.1*10**(-lr)
    lrs.append(lr)
    svm = SVM()
    losses = svm.train(X_train, y_train, learning_rate=lr, num_iters=1500,␣
 ↪verbose=False)
    y_pred = svm.predict(X_val)
    accuracy = 1 - np.count_nonzero(y_val - y_pred) / y_val.shape[0]
    y_val_accs.append(accuracy)
    print("learning rate = ", lr, "; Accuracy = ", accuracy)

plt.plot(lrs, y_val_accs, marker='o')
plt.xscale("log")
plt.grid()
plt.xlabel("Learning rate")
plt.ylabel("Validation Accuracy")
plt.show()
# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```

```
learning rate =  0.1 ; Accuracy =  0.29200000000000004
learning rate =  0.010000000000000002 ; Accuracy =  0.256
learning rate =  0.001 ; Accuracy =  0.29900000000000004
learning rate =  0.0001 ; Accuracy =  0.274
learning rate =  1e-05 ; Accuracy =  0.21299999999999997
learning rate =  1.0000000000000002e-06 ; Accuracy =  0.132
learning rate =  1e-07 ; Accuracy =  0.10599999999999998
learning rate =  1e-08 ; Accuracy =  0.11499999999999999
learning rate =  1e-09 ; Accuracy =  0.132
learning rate =  1.0000000000000002e-10 ; Accuracy =  0.12
learning rate =  1.0000000000000001e-11 ; Accuracy =  0.07899999999999996
```



[24]:
```python
lr = 1e-3
losses = svm.train(X_train, y_train, learning_rate=lr, num_iters=1500,␣
 ↪verbose=False)
y_test_pred = svm.predict(X_test)
accuracy = 1 - np.count_nonzero(y_test - y_test_pred) / y_test.shape[0]
print("learning rate = ", lr, "; Accuracy = ", accuracy)
```

```
learning rate =  0.001 ; Accuracy =  0.263
```

The best learning rate reported is 0.001 with testing accruacy being 0.263