```python
import numpy as np
import pdb

"""
This code was originally written for CS 231n at Stanford University
(cs231n.stanford.edu).  It has been modified in various areas for use in the
ECE 239AS class at UCLA.  This includes the descriptions of what code to
implement as well as some slight potential changes in variable names to be
consistent with class nomenclature.  We thank Justin Johnson & Serena Yeung for
permission to use this code.  To see the original version, please visit
cs231n.stanford.edu.
"""


def affine_forward(x, w, b):
    """
    Computes the forward pass for an affine (fully-connected) layer.

    The input x has shape (N, d_1, ..., d_k) and contains a minibatch of N
    examples, where each example x[i] has shape (d_1, ..., d_k). We will
    reshape each input into a vector of dimension D = d_1 * ... * d_k, and
    then transform it to an output vector of dimension M.

    Inputs:
    - x: A numpy array containing input data, of shape (N, d_1, ..., d_k)
    - w: A numpy array of weights, of shape (D, M)
    - b: A numpy array of biases, of shape (M,)

    Returns a tuple of:
    - out: output, of shape (N, M)
    - cache: (x, w, b)
    """

    # ================================================================ #
    # YOUR CODE HERE:
    #   Calculate the output of the forward pass.  Notice the dimensions
    #   of w are D x M, which is the transpose of what we did in earlier
    #   assignments.
    # ================================================================ #

    out = x.reshape(x.shape[0], -1) @ w + b


    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    cache = (x, w, b)
    return out, cache


def affine_backward(dout, cache):
    """
    Computes the backward pass for an affine layer.

    Inputs:
    - dout: Upstream derivative, of shape (N, M)
    - cache: Tuple of:
        - x: Input data, of shape (N, d_1, ... d_k)
        - w: Weights, of shape (D, M)
```

```
    Returns a tuple of:
    - dx: Gradient with respect to x, of shape (N, d1, ..., d_k)
    - dw: Gradient with respect to w, of shape (D, M)
    - db: Gradient with respect to b, of shape (M,)
    """
    x, w, b = cache
    dx, dw, db = None, None, None

    # ================================================================ #
    # YOUR CODE HERE:
    #   Calculate the gradients for the backward pass.
    # ================================================================ #

    # dout is N x M
    # dx should be N x d1 x ... x dk; it relates to dout through multiplication with w, which is D x M
    # dw should be D x M; it relates to dout through multiplication with x, which is N x D after reshaping
    # db should be M; it is just the sum over dout examples

    x_ND = x.reshape(x.shape[0], -1)   #(N, D)
    dx = (dout @ w.T).reshape(x.shape)
    dw = x_ND.T @ dout
    db = np.sum(dout, axis=0)


    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    return dx, dw, db


def relu_forward(x):
    """
    Computes the forward pass for a layer of rectified linear units (ReLUs).

    Input:
    - x: Inputs, of any shape

    Returns a tuple of:
    - out: Output, of the same shape as x
    - cache: x
    """
    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the ReLU forward pass.
    # ================================================================ #

    relu = lambda x: x * (x > 0)
    out = relu(x)
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    cache = x
    return out, cache
```

```python
def relu_backward(dout, cache):
    """
    Computes the backward pass for a layer of rectified linear units (ReLUs).

    Input:
    - dout: Upstream derivatives, of any shape
    - cache: Input x, of same shape as dout

    Returns:
    - dx: Gradient with respect to x
    """
    x = cache

    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the ReLU backward pass
    # ================================================================ #

    # ReLU directs linearly to those > 0
    dx = dout * (x > 0)

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    return dx


def svm_loss(x, y):
    """
    Computes the loss and gradient using for multiclass SVM classification.

    Inputs:
    - x: Input data, of shape (N, C) where x[i, j] is the score for the jth class
        for the ith input.
    - y: Vector of labels, of shape (N,) where y[i] is the label for x[i] and
        0 <= y[i] < C

    Returns a tuple of:
    - loss: Scalar giving the loss
    - dx: Gradient of the loss with respect to x
    """
    N = x.shape[0]
    correct_class_scores = x[np.arange(N), y]
    margins = np.maximum(0, x - correct_class_scores[:, np.newaxis] + 1.0)
    margins[np.arange(N), y] = 0
    loss = np.sum(margins) / N
    num_pos = np.sum(margins > 0, axis=1)
    dx = np.zeros_like(x)
    dx[margins > 0] = 1
    dx[np.arange(N), y] -= num_pos
    dx /= N
    return loss, dx


def softmax_loss(x, y):
    """
    Computes the loss and gradient for softmax classification.

    Inputs:
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    - x: Input data, of shape (N, C) where x[i, j] is the score for the jth class
      for the ith input.
    - y: Vector of labels, of shape (N,) where y[i] is the label for x[i] and
      0 <= y[i] < C

    Returns a tuple of:
    - loss: Scalar giving the loss
    - dx: Gradient of the loss with respect to x
    """

    probs = np.exp(x - np.max(x, axis=1, keepdims=True))
    probs /= np.sum(probs, axis=1, keepdims=True)
    N = x.shape[0]
    loss = -np.sum(np.log(probs[np.arange(N), y])) / N
    dx = probs.copy()
    dx[np.arange(N), y] -= 1
    dx /= N
    return loss, dx
```