ASDS - Applied Statistics with R

Fall 2018, YSU

Homework No. 05

Due time/date: 21:20, 9 November, 2018

Note: Please use R only in the case the statement of the problem contains (R) at the beginning. Otherwise, show your calculations on the paper. Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

Problem 1. Assume $X \sim Unif[-4, 4]$.

a. Find and plot the CDF of X;
b. Prove that $Y = \dfrac{X+4}{8}$ is a Standard Uniform r.v., i.e., $Y \sim Unif[0, 1]$;
c. Assume $Z = -X$. Prove that $Z \sim Unif[-4, 4]$;

   Note: This means, particularly, that X and $-X$ are i.d. (identically distributed), although different.
d. Calculate the probability $\mathbb{P}(X > 2)$ and $\mathbb{P}(|X| < 3)$;
e. Describe the distribution of $T = [X]$ (i.e., find the CDF and PDF/PMF of T). Here $[a]$ means the Integer Part of the number a - the largest integer number not exceeding a;

Problem 2. I am modeling the distribution of the final grades for a Probability course. I am assuming that the average grade X will be a Normal r.v., with mean 75, and standard deviation 8, i.e., $X \sim \mathcal{N}(75, 8^2)$.

a. Prove that the r.v. $Y = \dfrac{X-75}{8}$ is a Standard Normal r.v., i.e., $Y \sim \mathcal{N}(0, 1)$;

   Note: This transform is sometimes called a *Standardization* or *Normalization* of a Random Variable.
b. What is the probability that the average grade will be larger than 90?

Problem 3. Assume the time (in hours) X until I will receive next phone call is an Exponential r.v. with a rate $\lambda = 0.5$, i.e. $X \sim Exp(0.5)$ (we will see later that this means that the average waiting time for a call is $\frac{1}{0.5} = 2$ hours. Even at the nights.). Calculate the probability that I will wait for the next phone call more than 3 hours.

Problem 4. Assume $X \sim Bernoulli(0.2)$ and $Y \sim Binom(4, 0.2)$, and they are independent.

a. Construct their Joint PMF in the table form.

b. Calculate the probability that $2X + Y \leq 4$.

Problem 5. Assume that X and Y are discrete r.v.'s, and assume X and Y are independent. Find the Joint PMF of X and Y, if

Y \ X	-2	1	2	PMF of Y
-10				$\frac{1}{4}$
10				$\frac{3}{4}$
PMF of X	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

Problem 6. Assume that the Joint PMF of discrete r.v. X and Y is given by

Y \ X	2	3
-3	0.1	0.3
1	0.2	0.4

Are X and Y independent? Prove your statement.

Problem 7. Assume X and Y are discrete r.v. with the following PMFs:

X	0	2
$\mathbb{P}(X = x)$	0.3	0.7

and

Y	0	2
$\mathbb{P}(Y = y)$	0.3	0.7

Are X and Y dependent? Explain.

Problem 8. Assume $X \sim Binom(4, 0.2)$ and $Y \sim Pois(1)$ and X and Y are independent. Calculate

$$\mathbb{P}(X + Y \leq 2).$$