ASDS - Applied Statistics with R

Fall 2018, YSU

Homework No. 04

Due time/date: 21:20, 26 October, 2018

Note: Please use R only in the case the statement of the problem contains (R) at the beginning. Otherwise, show your calculations on the paper. Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

Problem 1. For each of the following Random Experiments, construct two different random variables (one discrete and one non-discrete, if that is possible) defined on the Sample Space of that Experiment. Indicate for each r.v. if it is discrete or not.

a. The experiment is tossing 3 fair coins;
b. The experiment is choosing at random a participant of our Probability course;
c. The experiment is choosing at random two participants of our Probability course;
d. The experiment is to consider the weather tomorrow;
e. The experiment is to consider the next football game of our national team.

Problem 2. We are rolling two dice. Let X be the r.v. showing the maximum of the two numbers shown on the top faces. Construct the PMF of X.

Problem 3. Assume $F(x)$ is the CDF of the r.v. X.

a. We have that $F(0) = 0.3$ and $F(3) = 0.4$. Is this possible? What information gives this about X?
b. We have that $F(0) = 0.4$ and $F(3) = 0.3$. Is this possible? What information gives this about X?
c. We have that $F(0) = 0.4$ and $F(3) = 0.4$. Is this possible? What information gives this about X?

Problem 4. Assume X is a discrete random variable given by its PMF:

Values of X	-3	2	7
$\mathbb{P}(X = x)$	0.2	0.5	0.3

Find the CDF $F(x)$ of X (i.e., write $F(x)$ in the analytic form) and give the graph of F.

Problem 5. Assume X is a random variable given by its CDF $F(x)$ (see Fig. 1). Find the PMF of X.

Problem 6. We are given a function $F(x)$ by its graph, see Fig. 2.

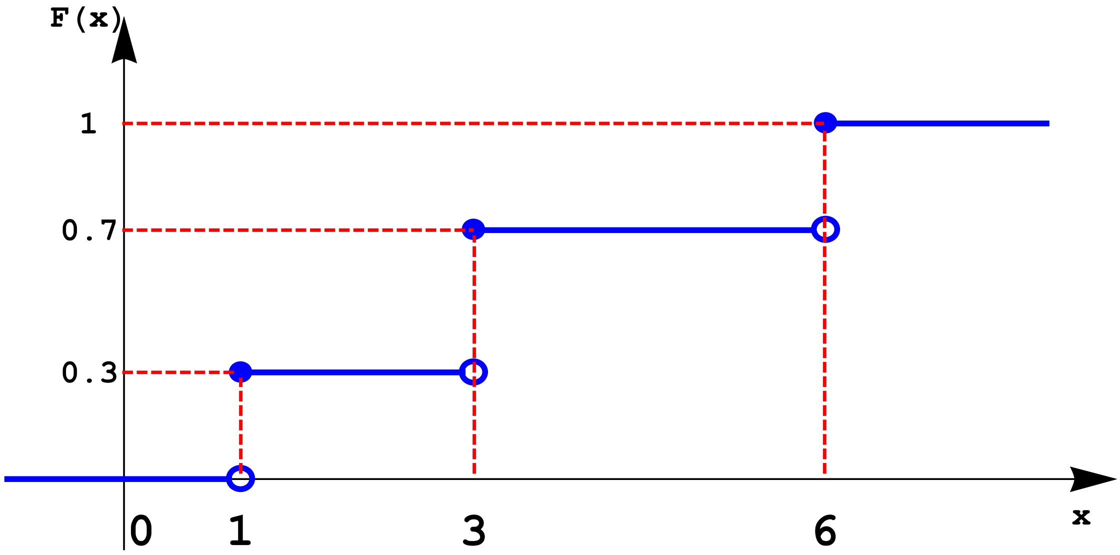

Figure 1: The CDF of X

a. Is this a legal CDF for some r.v. X? Explain. If yes, continue to next tasks, otherwise go to the next problem ☺

b. Is X a discrete r.v.?

c. Calculate the probability $\mathbb{P}(X \leq 2)$.

d. Calculate the probability $\mathbb{P}(1.2 \leq X \leq 3)$.

e. What is the range of X (here by the range of X I mean the smallest closed set A such that $\mathbb{P}(X \in A) = 1$, that is, $\mathbb{P}(X \in \overline{A}) = 0$)?

   Note: You do not need to prove that the set you are specifying is the smallest one. Just give your intuition behind your choice.

f. Calculate the probability $\mathbb{P}(X \in \{0, 1, 3, 4.5\})$.

g. Calculate the probability $\mathbb{P}(X \in [3, 5])$.

h. Calculate the probability $\mathbb{P}(X \in (0, 1))$.

i. Which is more probable: $X > 4$ or $X < 0$?

j. If X is discrete, calculate its PMF, and if X is continuous, find the PDF of X.

Problem 7. Fig. 3 shows the CDF of the r.v. X.

a. Calculate, as accurate as possible, the probability $\mathbb{P}(X \leq 5)$.

b. Calculate, as accurate as possible, the probability $\mathbb{P}(X \geq 11)$.

c. Which is more probable: $X \in [-5, 0]$ or $X \in [1, 2]$?

d. (Supplementary) Construct, as accurate as possible, the graph of the PDF of X.

Problem 8. Fig. 4 shows some function $f(x)$.

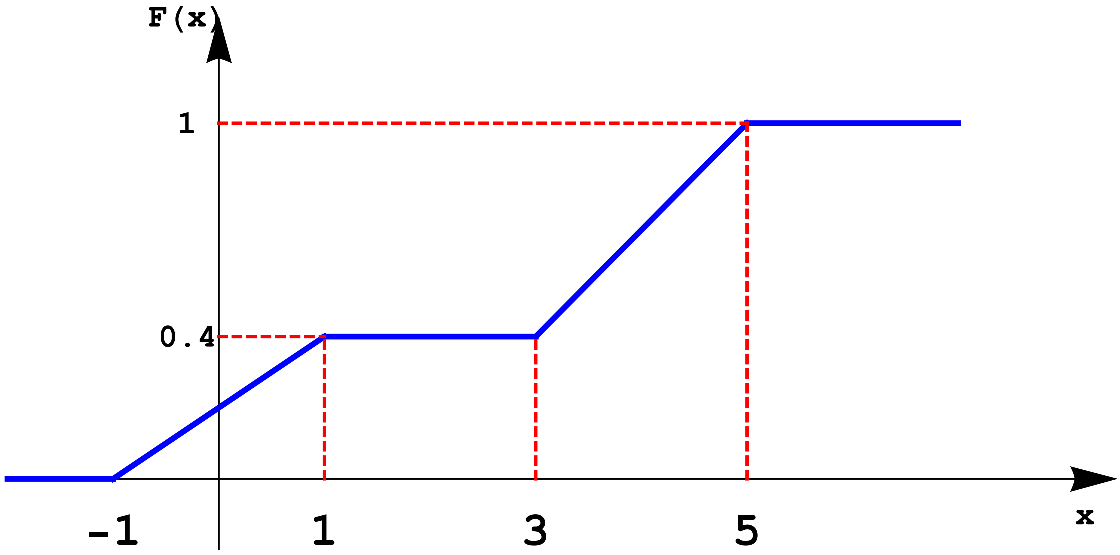

Figure 2: The graph of $F(x)$

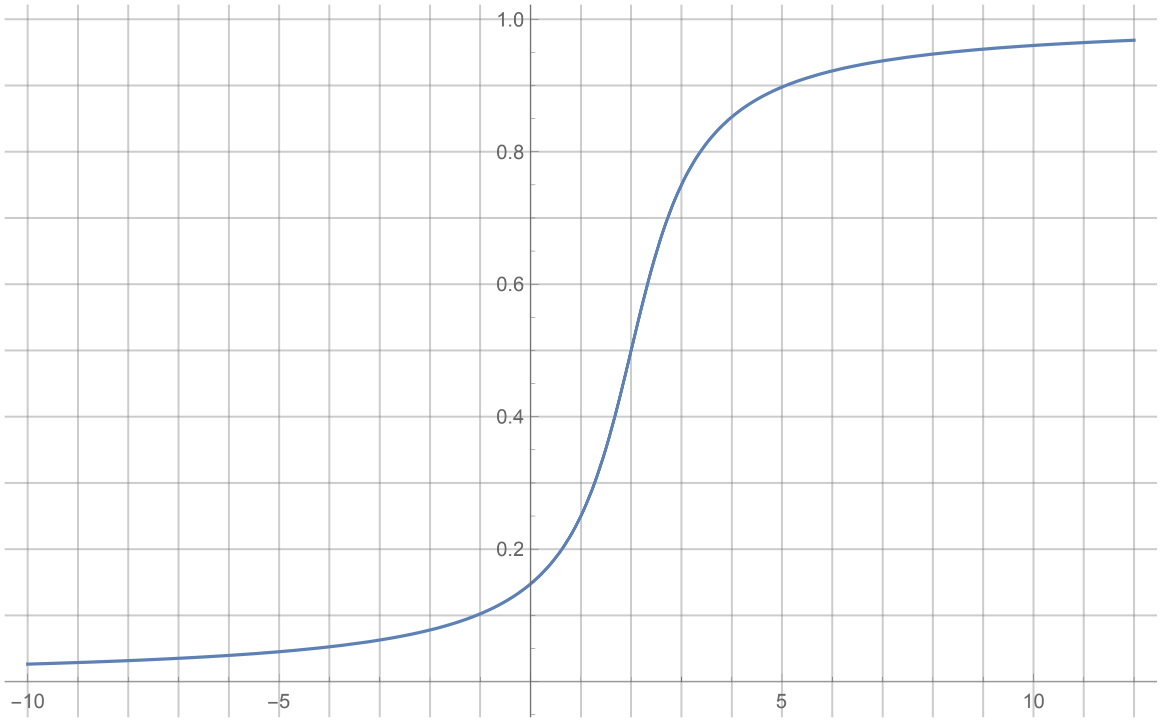

Figure 3: The graph of $F(x)$

a. Is f a PDF for some r.v. X? Explain. If it is, proceed to next tasks, otherwise go to the next problem.

b. Calculate the probability $\mathbb{P}(X = 0.3 \cup X = 7)$.

c. Calculate is the probability $\mathbb{P}(1 \leq X \leq 7)$?

d. What is the range of X (see the note above)?

e. Calculate the probability $\mathbb{P}(X \leq 5)$.

f. Which is more probable: $X \in [-1, 1]$ or $X \in [5, 7]$?

g. (Supplementary) Construct the CDF of X.

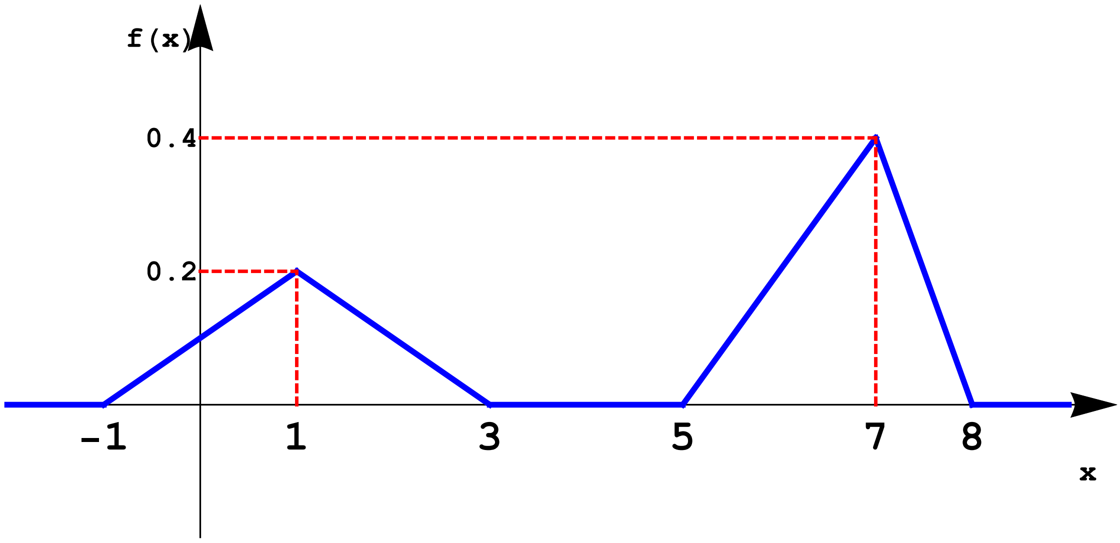

Figure 4: The graph of $f(x)$

Problem 9. Assume the PDF of the r.v. X is given by:

$$f(x) = \begin{cases} c \cdot (x^2 + 1), & \text{if} \quad x \in [0, 1] \\ 0, & \text{otherwise.} \end{cases}$$

a. Find the constant c;

b. Calculate the probability $\mathbb{P}(X \leq 0.2)$;

c. Find the CDF of X.

Problem 10. I have a Bounty Chocolate stick, which is 15cm long, and I will share it with you. I am breaking the chocolate stick at a random place (with uniform probabilities) along the length, and give the right-hand piece to you. My r.v. X is the calories I will get eating my piece, and it is calculated by $X(\omega) = 20 \cdot \omega^2$, where ω is the length of my share.

a. Construct the CDF of X.

b. Find the PDF of X.