ASDS - Applied Statistics with R

Fall 2018, YSU

Homework No. 08

Due time/date: 21:20, 21 December, 2018

Note: Please use R only in the case the statement of the problem contains (R) at the beginning. Otherwise, show your calculations on the paper. Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

Problem 1. Assume we have a random sample

$$X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(0, \sigma^2).$$

The unknown parameter of the model is $\theta = \sigma^2$.

a. Find the MLE and MME of θ.
b. Construct MLE for σ and $(\sigma+1)^3$.

Problem 2. Let we have a random sample

$$X_1, X_2, ..., X_n \overset{IID}{\sim} \mathbb{P}_\theta,$$

where $\mathbb{P}_\theta$ is given by its PFM:

Value of X	-1	1	2
$\mathbb{P}(X = x)$	$\frac{\theta}{4}$	$\frac{\theta}{4}$	$1 - \frac{\theta}{2}$,

with $\theta \in \Theta = (0, 2)$.

a. Find the MLE and MME for θ.
b. Assume we have the following observation from one of the $\mathbb{P}_\theta$, $\theta \in \Theta$:

$$2, 1, 1, 1, 2, -1, 2, -1$$

Estimate θ, using both MLE and MME.

Problem 3. Assume we have a random sample $X_1, ..., X_n$ from the Rayleigh distribution[1] with PDF

$$f(x|\sigma^2) = \begin{cases} \dfrac{x}{\sigma^2} \cdot e^{-\frac{x^2}{2\sigma^2}}, & \text{for } x \geq 0 \\ 0, & \text{for } x < 0. \end{cases}$$

[1]See https://en.wikipedia.org/wiki/Rayleigh_distribution

It is known that if X is a r.v. with Rayleigh distribution with the above PDF, then

$$\mathbb{E}(X) = \sigma \cdot \sqrt{\frac{\pi}{2}}, \qquad \text{and} \qquad Var(X) = \sigma^2 \cdot \frac{4-\pi}{2}$$

a. Find the MLE $\widehat{\sigma^2}$ for the unknown parameter σ^2;
b. Check if the ML estimator is unbiased/consistent.
c. Find the Method of Moments estimator for the parameter σ^2 using the first order theoretical and empirical moments;
d. Find the Method of Moments estimator for the parameter σ^2 using the second order theoretical and empirical moments;
e. Check if the above estimators are consistent;
f. (Supplementary) Check if the above estimators are unbiased;
g. (Supplementary) Prove the above formulas for $\mathbb{E}(X)$ and $Var(X)$.

Problem 4. We consider the parametric model $\{P_\theta : \theta \in (0, +\infty)\}$, where the distribution P_θ is given by its PDF

$$f(x;\theta) = \begin{cases} \theta \cdot x^{\theta-1}, & \text{if } x \in (0,1] \\ 0, & \text{otherwise.} \end{cases}$$

Assume that we have an observation coming from one of the distributions P_θ of that family:

$$x_1 = 0.1; \qquad x_2 = 0.7, \qquad x_3 = 0.61, \qquad x_4 = 0.19$$

and we want to estimate the unknown parameter θ. Find the MLE and Method of Moments estimates for θ.

Problem 5. Assume we want to estimate the mean of the Normal distribution through a random sample: assume we have a random sample

$$X_1, ..., X_{100} \sim \mathcal{N}(\mu, 4),$$

and we want to estimate the mean μ. To that end we choose the following a Confidence Interval (CI):

$$(\overline{X} - 0.1, \overline{X} + 0.1).$$

Can we assert that this is a CI for a confidence level 95% ?

Problem 6. We want to construct a CI for the Uniform Model using the Chebyshev Inequality. Assume we have a random sample

$$X_1, ..., X_n \sim Unif[0, \theta].$$

Construct a CI for θ of the confidence level $1 - \alpha$, $\alpha \in (0,1)$, using the statistics $\hat{\theta} = 2\overline{X}$ and the Chebyshev Inequality.

Problem 7. Assume the results of our Stat Midterm 1 show that we have 16 A grades among 74 students. Construct a 90% confidence interval for the probability of obtaining an A grade.

Problem 8. Assume we have a random sample $X_1, ..., X_n$ from some parametric model with parameter θ. We take any function $L = L(X_1, ..., X_n, \alpha)$, and call an interval

$$(L, +\infty)$$

to be a One-Sided Upper Confidence Interval (CI) for the parameter θ of confidence level $1 - \alpha$, if

$$\mathbb{P}(L < \theta) \geq 1 - \alpha.$$

Similarly, one can define the One-Sided Lower Confidence Interval for the parameter θ of confidence level $1 - \alpha$ to be an interval $(-\infty, L)$ with

$$\mathbb{P}(L > \theta) \geq 1 - \alpha.$$

Construct One-Sided CIs for the following parametric models:

a. $\mathcal{N}(\mu, \sigma^2)$, the parameter is μ, σ^2 is known;

b. $\mathcal{N}(\mu, \sigma^2)$, the parameter is μ, σ^2 is unknown;

c. $\mathcal{N}(\mu, \sigma^2)$, the parameter is σ^2, μ is known;

d. $\mathcal{N}(\mu, \sigma^2)$, the parameter is σ^2, μ is unknown;

Problem 9.

a. Visit the webpage `https://www.aravot.am/2018/08/18/975968/` for some (relatively old) study. Find the 95% CI for proportion of total population of persons in Armenia that think that our political parties are not doing a good job.

b. At the webpage `http://armconsumer.am/news.html` you can download the report about the tobacco use study in Armenia (the report is at `http://armconsumer.am/pdf/report_smoking_%20free_armenia.pdf`). At the pages 4-6 you can read about the sampling methodology (this is for an example, for your knowledge). In this report you will find a lot of information about the smokers in Armenia. Now, the question: based on the very last table, page 53, Q12, estimate the percentage of people in Armenia who has a job, and construct the 99% confidence interval for that percentage.

Problem 10. We want to estimate the average price of a 3-room flat at the Davitashen district. To that end, we want to construct a 95% confidence interval for that average price. Go to `https://www.list.am/en/`, navigate to Real Estate -> For Sale -> Apartments, choose on the left panel the number of rooms and the location, get a sample of 10 prices, and construct the required CI. You can assume that the prices are normally distributed.