YEREVAN STATE UNIVERSITY

Department of Mathematics and Mechanics

Fall Semester, 2019   *Instructor Victor K. Ohanyan*

Multivariate Statistics

Mid–Term Examination-1

November 9, 13:30 − 14:50

Student Name   _____

Problem 1 (20%)   _____

Problem 2 (20%)   _____

Problem 3 (20%)   _____

Problem 4 (20%)   _____

Problem 5 (20%)   _____

Total   _____

Please write clearly and state any assumptions you make. You can use only ordinary calculators for computation. The use of mobile phones or tablets is strongly prohibited. Please turn off your cell phones.

PROBLEM 1. The following are five measurements on the variables x_1, x_2 and x_3:

$$\begin{bmatrix} x_1 & 9 & 2 & 6 & 5 & 8 \\ x_2 & 12 & 8 & 6 & 4 & 10 \\ x_3 & 3 & 4 & 0 & 2 & 1 \end{bmatrix}$$

Find the matrices $\bar{\mathbf{x}}, \mathbf{S_n}$, and $\mathbf{R}$.

PROBLEM 2. Let $\mathbf{X}$ have covariance matrix

$$\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

(**a**) Find ρ_{13}.

(**b**) Find the correlation between X_1 and $\frac{1}{2}X_2 + \frac{1}{2}X_3$

PROBLEM 3. Let $\mathbf{X}$ have covariance matrix

$$\Sigma = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find

(a) Σ^{-1}

(b) The eigenvalues and eigenvectors of Σ.

(c) The eigenvalues and eigenvectors of Σ^{-1}.

PROBLEM 4. Given the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$$

find the eigenvalues λ_1 and λ_2 and the associated normalized eigenvectors $\mathbf{e}_1$ and $\mathbf{e}_2$. Determine the spectral decomposition of $\mathbf{A}$.

PROBLEM 5.

(a) Evaluate T^2, for testing $H_0 : \mu_0 = [7, 11]$, using the data

$$\mathbf{X} = \begin{bmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{bmatrix}$$

(b) Specify the distribution of T^2 for the situation in (a).

(c) Using (a) and (b), test H_0 at the $\alpha = .05$ level. What conclusion do you reach?