Yerevan State University
Applied Statistics with R
Midterm 2 Exam Test

27 December, 2018

Exam Time: 18:30 - 20:30

Last Name: ______________________ **First Name:** ______________________

READ THESE INSTRUCTIONS CAREFULLY

- This test consists of 5 Show-Work Problems. The test booklet has 16 pages, including this cover page and empty pages for draft calculations.
- Each Show-Work Problem has its own grade. The overall test grade is 100.
- This is a closed-book test, and no notes, assignments, practice problems, books, formula sheets or other materials are allowed.
- The use of mobile phones or any other electronic devices are strongly prohibited. Please turn off your cell phones and place them out of reach. You can use only simple calculators.
- Sharing of stationery (pens, pencils, erasers, etc.) or calculators is not permitted.
- Talking to another student, looking at another student's paper, or communicating with other students in any way is strictly forbidden.
- Use the scratch pages of the test booklet to do your draft calculations. Please ask the instructor for extra scratch papers if necessary.
- If you run out of the space on the test pages, please use a scratch page to finish your work. Indicate in the test page that you will continue on the scratch page, and mark with the rectangle the portion on the scratch page that contains the solution. Any other work on the scratch page will not be graded.
- Good luck!

DO NOT OPEN THIS BOOKLET
UNTIL YOU HAVE BEEN TOLD TO DO SO

Scratch Paper

Show-Your-Work Problems

1. (10 Points) Assume we have a dataset x:

$$5.7;\quad 8.7;\quad 1.1;\quad 4.4;\quad 0.3;\quad -2.9;\quad 2.2;\quad -1.2;\quad 5.5;\quad 5.4.$$

   We assume that observations are coming from a Normal Distribution $\mathcal{N}(\mu, \sigma^2)$. Construct a Confidence Interval of confidence level 95% for μ;

   Show and explain all your steps.

Scratch Paper

2. (20 Points) Insurance company wants to model the number of car accidents in Yerevan in a day. The company chooses as an appropriate model the Poisson distribution $Poisson(\lambda)$. The company collects information about the number of car accidents in Yerevan for several days, and get the following data:

$$10, 14, 21, 7, 9, 14, 18, 22, 21, 19.$$

The company's statistician is not strong in Statistics (he was not attending my lectures ᵕ̈), and he knows and can use only two estimators for the unknown parameter λ:

$$\hat{\lambda} = \frac{X_1 + 2X_{10}}{3} \qquad \text{and} \qquad \tilde{\lambda} = \frac{X_1 + X_2 + ... + X_{10}}{10}.$$

What is the best **estimate** for λ that the company's statistician can get using only these estimators? Justify your answer.

Show and explain all your steps.

Note: Without justification you will get only 1 point!

Scratch Paper

3. (20 Points) We consider the parametric model $\{\mathbb{P}_\theta : \theta \in [-1,1]\}$, where the PDF of the distribution $\mathbb{P}_\theta$ is given by

$$f(x|\theta) = \begin{cases} \frac{1}{2} \cdot (1 + \theta \cdot x), & x \in [-1,1] \\ 0, & \text{otherwise.} \end{cases}$$

To estimate θ, we use the estimator $\hat{\theta}_n = \overline{X}_n$, where $X_1, ..., X_n \overset{IID}{\sim} \mathbb{P}_\theta$.

a. Calculate $\mathbb{E}(X_k)$;

b. Calculate the bias of the estimator $\hat{\theta}_n$. Is it unbiased?

c. Find a real number α such that the estimator $\alpha \cdot \hat{\theta}_n$ is a consistent estimator for θ.

Show and explain all your steps.

Scratch Paper

4. (25 Points) I have generated in **R** 3 uniformly distributed numbers, 10 times. The result is given in the table below (I made a rounding for the sake of simplicity):

	1	2	3	4	5	6	7	8	9	10
x_1	0.60	-0.073	0.64	0.65	0.32	0.39	0.562	0.704	0.54	0.163
x_2	0.56	0.549	0.60	0.12	0.37	0.59	0.075	0.476	0.90	0.391
x_3	0.15	0.631	0.53	0.71	0.76	0.76	0.651	-0.035	0.45	0.056

I have generated numbers from the distribution $Unif[\theta - 0.5, \theta + 0.5]$, but I will not tell you which value of θ I have used. Your aim will be to estimate that number.

a. First we use the Estimator $\hat{\theta} = X_3$. Show that it is an Unbiased Estimator for θ. Based on the observations we have and the Estimator $\hat{\theta}$, find a good estimate for θ.

b. Now we take another estimator, $\tilde{\theta} = X_{(3)} = \max\{X_1, X_2, X_3\}$. We want to check if it is Unbiased or not. To that end, do the following steps:

b1. Find the CDF $F_{\tilde{\theta}}(x)$ of $\tilde{\theta}$;

b2. Find the PDF $f_{\tilde{\theta}}(x)$ of $\tilde{\theta}$;

b3. Calculate $\mathbb{E}(\tilde{\theta})$;

b4. Calculate the bias $bias(\tilde{\theta}, \theta)$.

Show and explain all your steps.

Scratch Paper

5. (25 Points) Assume we want to estimate the unknown parameter σ^2 in the $\mathcal{N}(1, \sigma^2)$ model, based on a sample of size n. For the sake of simplification, let us denote the variance by θ, i.e., $\theta = \sigma^2$. So we take a Random Sample

$$X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(1, \theta), \qquad \theta \in [0, +\infty).$$

We will use the estimator

$$\hat{\theta}_n = \frac{1}{n} \cdot \sum_{k=1}^{n} (X_k - 1)^2.$$

   a. Is $\hat{\theta}$ unbiased?
   b. Is $\hat{\theta}$ consistent?
   c. Is $\hat{\theta}$ efficient?

Show and explain all your steps.

Note: You can use the fact that for $X \sim \mathcal{N}(1, \sigma^2)$, $\mathbb{E}((X-1)^4) = 3\sigma^4 = 3\theta^2$.

Scratch Paper

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