Multivariate Statistics

HOME WORK # 2

Deadline: 12:00, December 7, 2019

Problem 1. Find the eigenvalues of the matrix

$$\Sigma = \begin{bmatrix} 136 & 104 & 94 \\ 104 & 106 & 71 \\ 94 & 71 & 65 \end{bmatrix}$$

Problem 2. Given the five pairs of poits (x, y) shown below:

$$(4,5); \quad (0,0); \quad (-2,0); \quad (3,6); \quad (1,3).$$

what is the line of the form $Y = X + b$ best fits the data by methods of least squares?

Problem 3. Given the five pairs of poits (x, y) shown below:

$$(4,5); \quad (0,0); \quad (-2,0); \quad (3,6); \quad (1,3).$$

what is the line of the form $Y = aX + b$ best fits the data by methods of least squares?

Problem 4. Find the eigenvalues of the matrix

$$\Sigma = \begin{bmatrix} 7 & 0 & 1 \\ 0 & 7 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Problem 5. Suppose that the following is a variance-covariance matrix:

$$\Sigma = \begin{bmatrix} 8 & 0 & 1 \\ 0 & 8 & 3 \\ 1 & 3 & 5 \end{bmatrix}$$

Determine the eigenvalues λ_1, λ_2, λ_3 and eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ of Σ and verify that:

(a) $\lambda_1 + \lambda_2 + \lambda_3 = \text{tr}(\Sigma)$;

(b) $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = \det\Sigma$;

(c) $\mathbf{v}_1{}'\mathbf{v}_2 = \mathbf{v}_1{}'\mathbf{v}_3 = \mathbf{v}_2{}'\mathbf{v}_3 = 0$.