Example

Solve the problem

$$\begin{aligned} \textit{minimize} \quad & f(x) \\ \textit{subject to} \quad & x \in \Omega, \end{aligned}$$

i.e., find the global minimum points of $f(x)$ on Ω, if

a. $f(x) = x^2$, $X = (1, 2)$;

b. $f(x) = -x^2 + x + 10$, $X = [-1, 1]$;

c. $f(x) = \frac{x+1}{x^2+3}$, $X = [0, +\infty)$.

Example

Determine whether the matrix

$$A = \begin{pmatrix} 1 & -3 & 0 \\ -3 & 10 & 0 \\ 0 & 0 & -5 \end{pmatrix};$$

is positive definite (semidefinite), negative definite (semidefinite) or indefinite by using only definition.

Example

Determine whether the matrix A is positive definite (semidefinite), negative definite (semidefinite) or indefinite if

$$A = \begin{pmatrix} -1 & 3 & -2 \\ 3 & -9 & -4 \\ -2 & -4 & -5 \end{pmatrix}.$$

Example

Check whether f is convex (strictly convex), concave (strictly concave) on Ω if

$$f(x_1, x_2) = e^{x_1 x_2} + x_3^2, \quad \Omega = \mathbb{R}^3.$$

Example

Find all stationary points of f and check if these points are local maximum, minimum or saddle points for that function:

a. $f(x_1, x_2, x_3) = x_1^2 - 4x_1x_2 + 5x_2^2 + 3x_3^4 - 6x_3^2$;

b. $f(x_1, x_2) = e^{x_1 x_2}$.

Example

Find the global minimizers of f in Ω if

a.

$$f(x_1, x_2) = e^{(x_1 - 1)^4} + x_2^4, \quad \Omega = \mathbb{R}^2.$$

b.

$$f(x_1, x_2) = x_1^6 + x_2^6 + 2x_1^2 + 4x_2^2 - x_1 x_2, \quad \Omega = \mathbb{R}^2;$$

Example

Let $f(x) = e^{(2-x)^2} + 4x$. Our aim is to find the global minimizer x^* of f over $[0, 8]$.

a. Show that $f(x)$ is unimodal in $[0, 8]$.

b. Calculate x_2 approximation of the minimum point using the Golden Section (Ratio) Search Method with $\gamma = 1/4$.

Example

Assume we are solving the minimization problem

$$\begin{aligned} \textit{minimize} \quad & f(x_1, x_2) = x_1 x_2 + x_1^2 + x_2^2 \\ \textit{subject to} \quad & x \in \mathbb{R}^2, \end{aligned}$$

by using line search methods. We start from the initial approximation $x^{(0)} = (1, 0)^T$.

a. Will the vector $\mathbf{d}^{(0)} = (-1, -1)^T$ be a descent direction at $x^{(0)}$?

b. Can we choose $\alpha_0 = 2$ as a step size to calculate the next approximation $x^{(1)}$ by the line search method in the direction of $\mathbf{d}^{(0)}$?

c. Take $\varepsilon = 0.5$, $\alpha_0^{(0)} = 1$ and the backtracking parameter $\tau = 0.5$, and calculate the step size α_0 in the direction of $\mathbf{d}^{(0)}$ using the backtracking algorithm;