Example

Solve the following minimization problem:

$$\begin{aligned} \text{minimize} \quad & x_1 x_2 \\ \text{subject to} \quad & x_1^2 + x_2^2 = 8. \end{aligned}$$

Example

Solve the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & (x_1 - 1)^2 x_2 + x_3^2 x_2 \\ \text{subject to} \quad & x_2 - x_1 = 1, \\ & x_1 + x_2 \geq 2. \end{aligned}$$

Example

Solve the following LPP geometrically

$$\begin{aligned} \text{maximize} \quad & 2x_1 - x_2 \\ \text{subject to} \quad & 3x_1 + 2x_2 \geq 12 \\ & -3x_1 + x_2 \geq -3 \\ & -3x_1 + 2x_2 \leq 14 \\ & x_1 \geq 0,\ x_2 \geq 0 \end{aligned} \quad .$$

Example

Solve the following LPP by solving first its Dual Problem geometrically:

$$
\begin{aligned}
\textit{minimize} \quad & 4x_1 - x_2 + 4x_3 \\
\textit{subject to} \quad & x_1 - x_2 + 2x_3 \geq 10; \\
& -x_1 - 2x_2 + x_3 \leq -2; \\
& x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.
\end{aligned}
$$

Example

Consider the following function

$$f(x_1, x_2, x_3) = x_1^2 + x_1^2 x_2^2 + x_2^2 + e^{x_3^2}.$$

a. Check whether f is a convex (strictly convex), concave (strictly concave) on $\mathbb{R}^3$.

b. Find all stationary points of f and check if these points are local maximum, minimum or saddle points for that function.

Example

Our aim is to find the minimum of the following function

$$f(x_1, x_2) = (x_1 - 1)^2 + x_2^2 + 10\,(x_1 + x_2 - 4)^2\,.$$

a. Is $d = [-1, 1]^T$ a descent direction at $x = [1, 0]$?

b. Find the the steepest descent direction at x.