Optimization

Fall 2018, YSU

PSS

Problem 1. Find the maximum points of $f(x_1, x_2) = 4x_1^2 + x_2^2$ subject to $x_1^2 + x_2^2 = 9$.

Problem 2. Find the maximum and minimum points of $f(x, y, z) = x^2 + 2xy + 3y^2 + 4x + 5y + 6z$ subject to $x + 2y = 3$, $4x + 5z = 6$.

Problem 3. Consider the following constraints on R^2:

$$h(x_1, x_2) = 3x_1 - x_2 = 0, \; g_1(x_1 x_2) = x_2 - x_1^2 - 2 \leq 0, \; g_2(x_1 x_2) = (x_2 - 2)^2 + x_1^2 - 2 \leq 0.$$

Find the set of feasible points. Are all feasible points regular? Justify your answer.

Problem 4. Solve the following problem

$$\begin{aligned} \text{minimize} \quad & (x_1 - 1)^2 + x_2 + e^{x_3^2} \\ \text{subject to} \quad & x_2 - x_1 = 1, \\ & x_1 + x_2 \leq 2, \\ & x_3 \geq 0. \end{aligned}$$

Problem 5. Consider the problem

$$\begin{aligned} \textit{minimize} \quad & x_1^2 + 4x_2^2 \\ \textit{subject to} \quad & x_1^2 + 2x_2^2 \geq 4, \end{aligned} \tag{1}$$

a. Find all the points that satisfy the KKT conditions. Check whether or not each point is regular.

b. Apply the SOSC to determine the nature of the critical points from the previous part.

Problem 6. Solve the following problem

$$\begin{aligned} \textit{minimize} \quad & (1 - x_1)^2 + x_2 - 2 \\ \textit{subject to} \quad & x_2 - x_1 - 1 = 0, \\ & x_1 + x_2 - 2 \leq 0. \end{aligned} \tag{2}$$

Problem 7. Consider the problem

$$\begin{aligned} \textit{minimize} \quad & x_2 - (x_1 - 2)^3 + 3 \\ \textit{subject to} \quad & x_2 \geq 1, \end{aligned} \tag{3}$$

a. Find all the points that satisfy the KKT conditions. Check whether or not each point is regular.

b. Determine whether or not the point(s) in part a satisfy the second-order necessary condition.

c. Determine whether or not the point(s) in part b satisfy the second-order sufficient condition.