

Oscilloscope rise time and noise explained

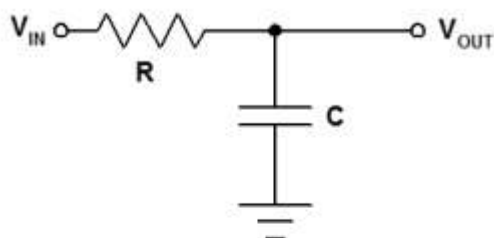
Arthur Pini - November 07, 2016

A reader of the article [Digital oscilloscopes: When things go wrong](#) posed a number of questions regarding two "rules of thumb" that are common in the industry. The first is to ask where the relationship between rise time and bandwidth ($BW=0.35/Tr$) comes from. The second was where the relationship between the rms values of a noise signal to its peak to peak value. I'd like to respond to both.

The rise time/bandwidth issue

The input of early oscilloscopes, and even today's digital oscilloscopes with bandwidths of 1 GHz or lower, can typically be modeled as a single pole RC low pass filter. Figure 1 shows the model and a calculation of the low pass filters bandwidth. The bandwidth of this simple low pass filter is calculated to be $1/(2\pi RC)$

AC FREQUENCY RESPONSE OF THE RC LOWPASS FILTER

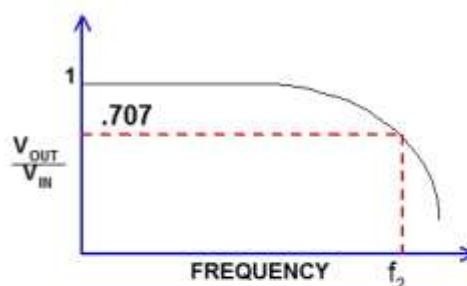


$$\left| \frac{V_{OUT}}{V_{IN}} \right| = \frac{1}{\sqrt{1+(2\pi RCf)^2}}$$

$$= \frac{1}{\sqrt{1+(f/f_2)^2}}$$

$$\text{Where: } f_2 = 1/(2\pi RC)$$

Bode Plot



Bandwidth (f_2) is the frequency where the output falls to 70.7% of the input.

Figure 1. An input model for oscilloscopes with bandwidth 1 GHz or less is a single pole RC low-pass filter with a Gaussian frequency response.

If we now look at the response of this filter to a step function, we see that the filter rounds the edge of the step, which increases its rise time. **Figure 2** shows the calculation of the rise time of the filter output. The filter response to the step function is a rising exponential with a time constant of RC .

Solving for the times of the 10 and 90 percent crossings allows us to determine the rise time as a function of the filter bandwidth. The result for the bandwidth/rise time product is the traditional value of 0.35.

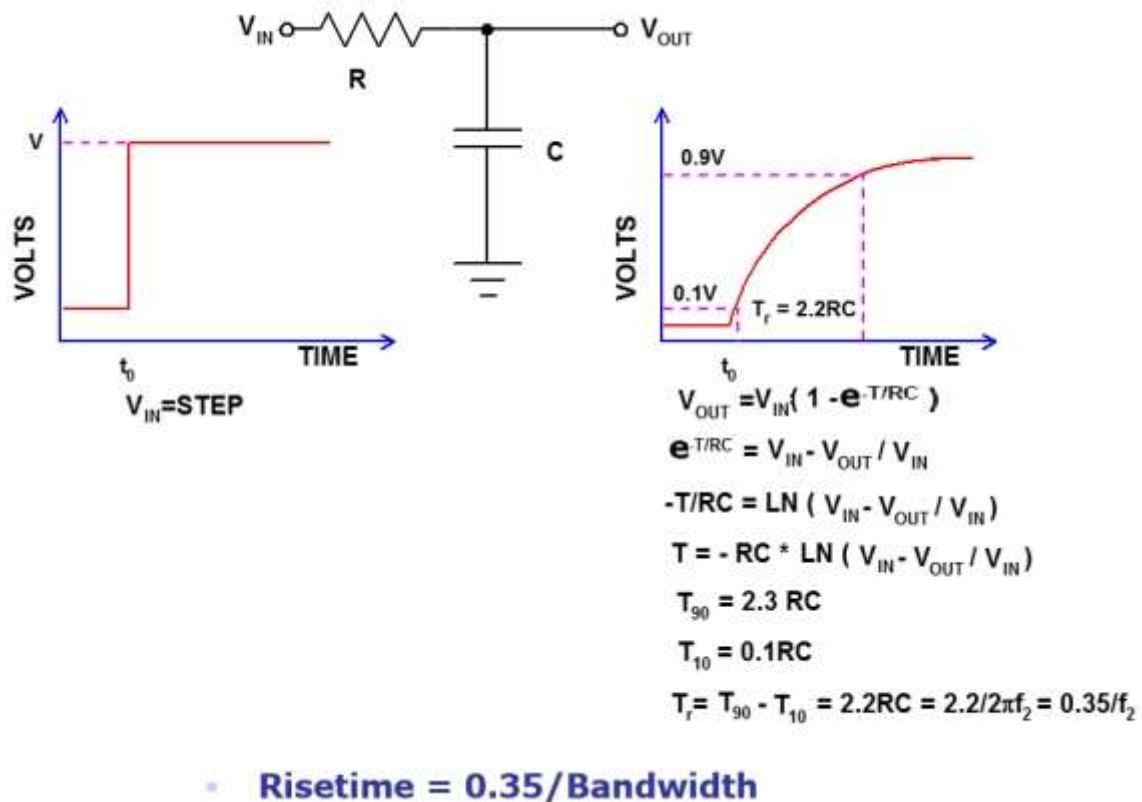


Figure 2. The response of the oscilloscope input model to a step function lets us relate rise time to bandwidth.

Oscilloscopes with higher bandwidths generally use digital signal processing to obtain a frequency response that is very flat out to the scope's bandwidth and then rolls off very rapidly, what is called a "brick wall" response. The bandwidth/rise time products for these scopes are higher, typically in the range of 0.4 to 0.55. Consult the manufacturer to find the value for any particular oscilloscope model.

Peak-to-peak from rms of a noise-like signal

The second question on the relationship of the rms value and peak-to-peak value of a noise signal is rooted in the statistical analysis of the Gaussian or normal random variable.

Random processes like noise or jitter are a statistical phenomenon where the likelihood of a large event increases over time. In order to specify the peak to peak value, the observation time must be specified. **Figure 3** shows histograms for the same jitter measurement over increasing observation time. The range (peak to peak value) for a Gaussian distribution is determined by its standard deviation. The standard deviation is the same as the AC coupled rms value or, if the mean of the process is zero, it is the same as the rms value of the waveform.

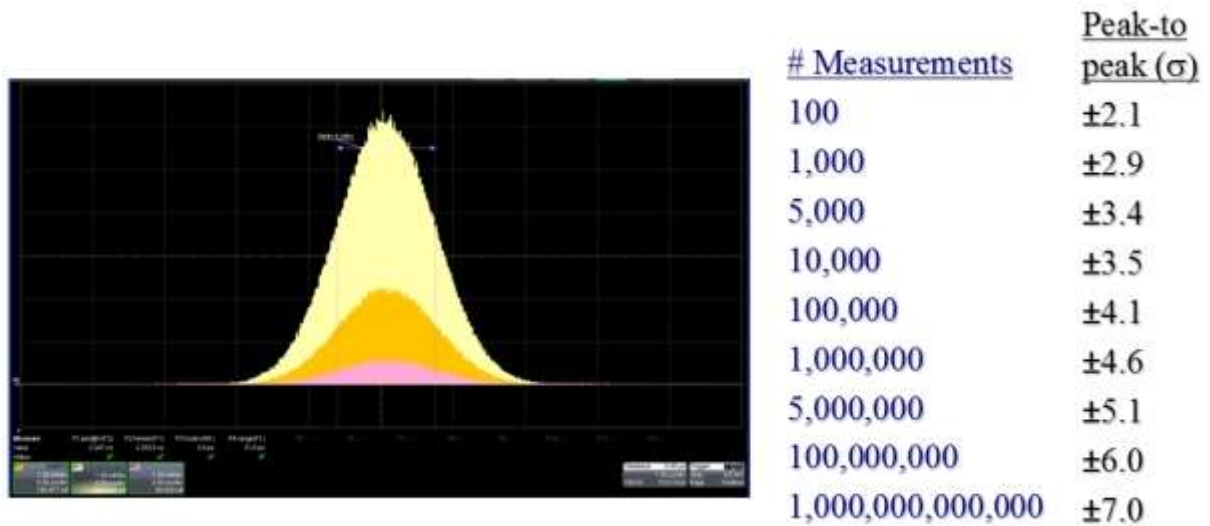


Figure 3. A Gaussian distribution shows how the peak-to-peak value grows as the number of measurements increases. A value of ± 6 standard deviations requires 100 million measurements.

The observation time can be quantified by the number of measurements and the range of the histogram grows with the number of measurements. The number of standard deviations covered by the range for any given number of measurements can be computed from the Gaussian probability density function. The traditional ± 6 standard deviations requires 100 million measurements. In jitter measurements the range for $1e12$ measurements is the standard which requires ± 7 standard deviations.

These common "rules of thumb" are rooted in very classical analysis techniques and you can refer to texts on circuit analysis and random processes, respectively, to get more background.