Analyze noise with time, frequency, and statistics

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Every electronic circuit has some noise and it can affect both analog and digital circuits. Some noise comes from outside interference while some comes from random factors such as thermal effects. Random processes that generate noise are more difficult to characterize than noise from known sources because no individual measurement provides any information about the previous or next measurement. Such processes can only be described by cumulative measurements over many events and then only by the probability of a specific next event. You can use the tools provided in many digital oscilloscopes to characterize noise. Once you have a handle on the characteristics of noise, you can work to mitigate it.

Analyzing a random signal such as electrical noise using a digital oscilloscope requires tools that provide multiple views of the random process. **Figure 1** provides a preview of these multidimensional oscilloscope tools.

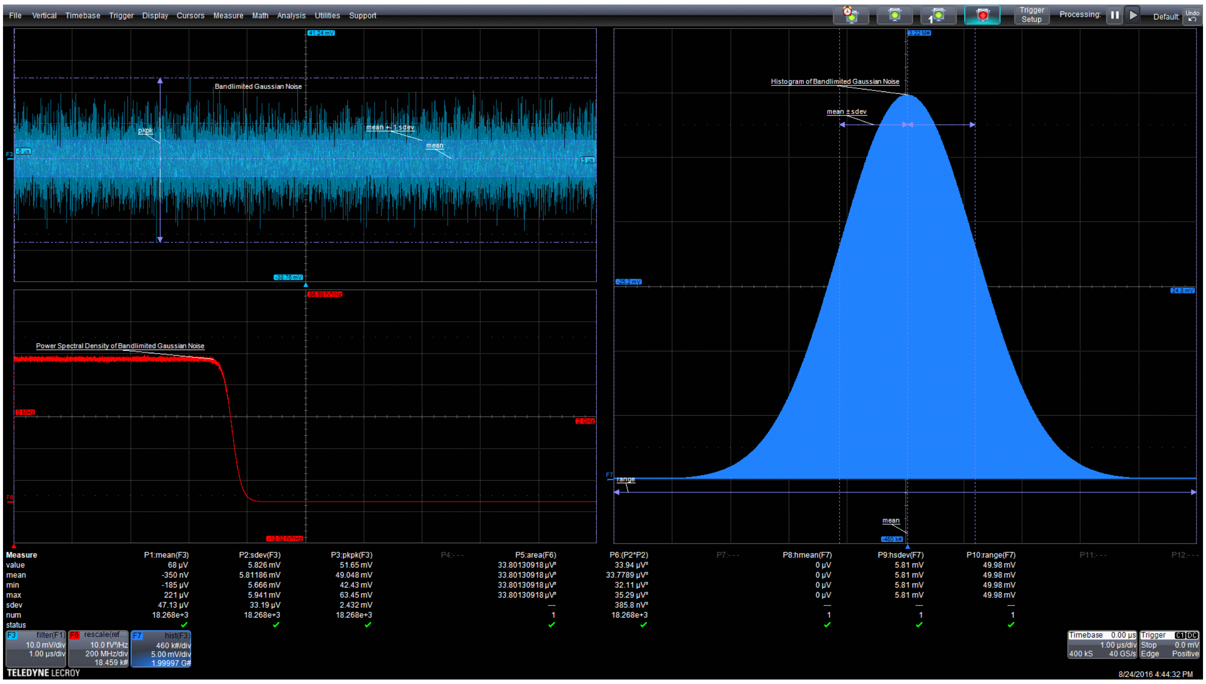

Figure 1. The time-domain view of bandlimited Gaussian noise shown in the top left trace, the next lower trace is the power spectral density, a frequency domain view, of the bandlimited noise. The right grid contains a histogram, a statistical view of bandlimited noise. These three views are augmented by measurement parameters which help quantify the measurements.

The top left trace in Fig. 1 is a time-domain view of band-limited Gaussian noise. We'll refer to this signal throughout this article. . The trace below shows the noise in the frequency domain: the signal's power spectral density (PSD). It displays the noise power per Hertz versus frequency. The right grid contains the histogram of the bandlimited noise. Histograms provide a statistical view by approximating the probability density function (PDF) of the random process. Beneath these traces are a series of measurement parameters that quantify the waveforms derived from the math. Now, we'll look at each of these measurement techniques in detail to see what each method reveals about the bandlimited noise signal.

Noise or jitter

Noise and Jitter are related phenomena. Noise is an unwanted vertical signal component added to the desired signal. Jitter is an undesired variation in a signal's timing. Noise becomes jitter when a noisy signal is applied to a threshold comparator such as a logic gate. Amplitude variations due to vertical noise results in the output being early or late relative to the ideal timing of the threshold crossing. The tools and procedures used can equally be applied to measuring jitter.

Noise measurements are made on signals directly as applied to the oscilloscope’s input channels. Jitter measurements are based upon timing measurements such as time interval error (TIE), period, or duty cycle. These timing measurements are performed on a cycle-by-cycle basis on an input signal. The resultant measurements are plotted versus time using a math function called track or time track. This track function is the input signal for subsequent jitter measurements.

The time domain

Measurement parameters can be applied to the noisy waveform in **Figure 2** to gain some insight into this noise signal. The parameters of choice, shown in the figure, are mean, standard deviation, and peak-to-peak values. The parameter readouts appear below the display grid.

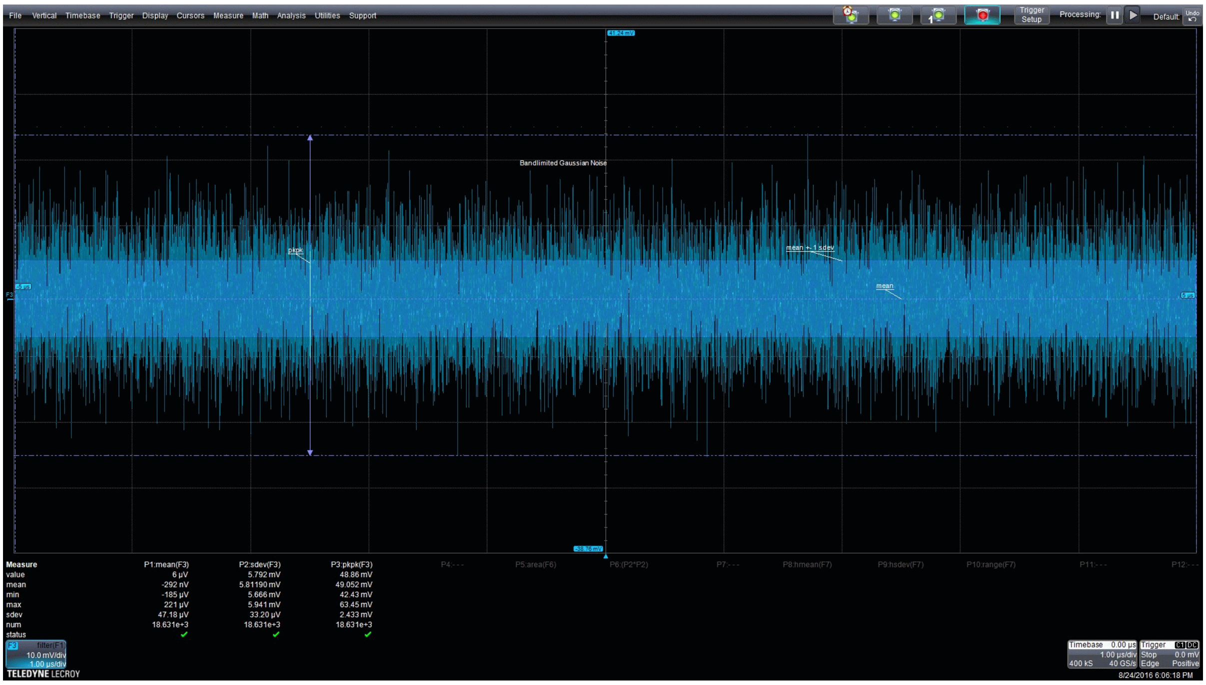

Figure 2. The time domain view of a bandlimited noise signal. Parameter readouts show the basic measurements, mean value, standard deviation or AC rms, and peak-to-peak.

Parameter markers appear as overlays on the random waveform, providing a graphical view of the measurements. The standard deviation, which can also be described as the AC coupled root-mea--square (rms) value, is probably the most useful because it describes the waveform's effective amplitude. The mean reads the average value of the signal while the peak-to-peak values of the difference between the maximum and minimum amplitude values occurring in the acquisition. In addition to reading the selected parameter for a given acquisition, the oscilloscope can compute and display cumulative statistics over multiple acquisitions for each parameter—providing the mean, maximum, minimum, and standard deviation for each parameter.

Histograms: The statistical domain view

Random processes are best described in the statistical domain using the histogram. **Figure 3** displays the histogram of our bandlimited noise signal along with the source waveform. This histogram breaks the full-scale voltage range into 5000 bins and counts the number of sample values falling into each bin. The vertical axis is the number of samples in each bin, which is proportional to the probability of that value occurring, the horizontal axis is amplitude value, volts in this instance.

The histogram of the band limited noise signal exhibits a classic bell curve, characteristic of the Gaussian or normal PDF. You can completely describe PDF if you know the variance (square of the standard deviation) and mean of the waveform. Also, note that the distribution is symmetric about the mean value.

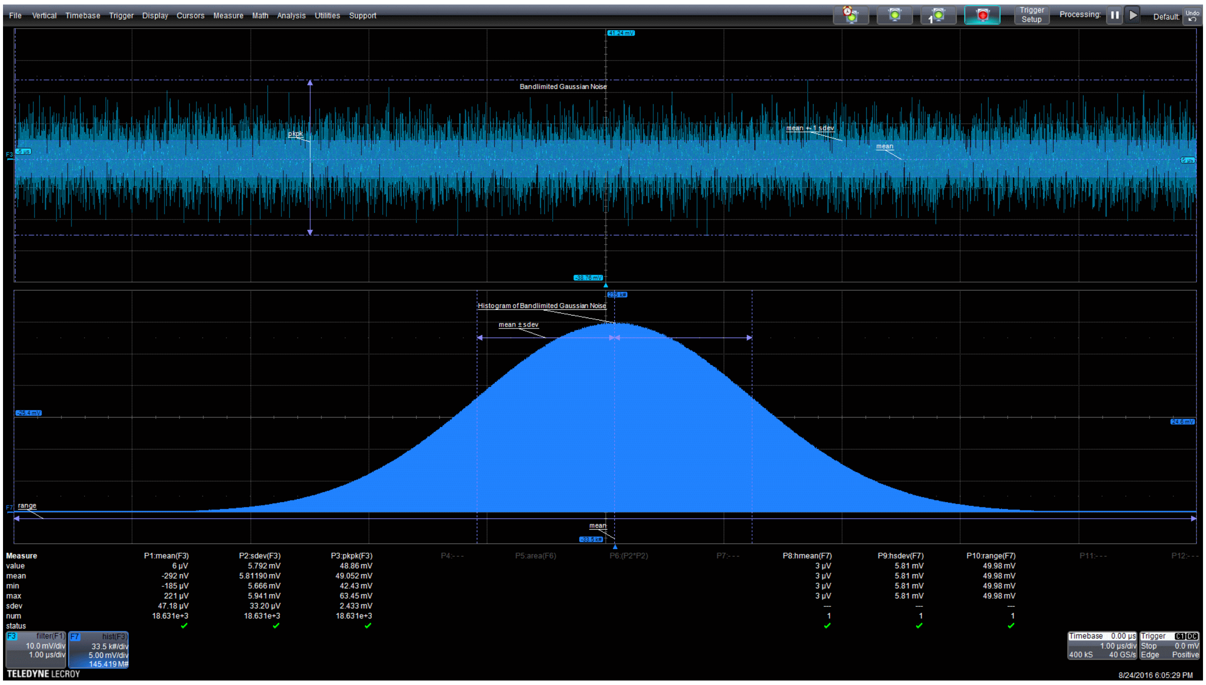

Figure 3. The histogram of the bandlimited noise signal shows a typical Gaussian bell shaped response. Histogram parameters read the histogram mean, standard deviation and range.

Measurement parameters can also be applied to the histogram. In this example the histogram mean (hmean), standard deviation (hstdev), and range (hrange). Note that these readouts are very close to the previously measured mean, standard deviation, and peak-to-peak values for the time waveform. These small differences are due to the "binning" of the histogram samples.

The Gaussian distribution is symmetrical about the mean value with the probability of an amplitude value falling off as the amplitude moves away from the mean. The extreme amplitudes (called tails) have very low but nonzero probability of occurrence. The fact that the tails don't go to zero probability means that the Gaussian distribution is unbounded. Given enough samples, very large amplitude samples may occur. Some typical PDFs are shown in **Figure 4**. The Gaussian distribution is in the top grid.

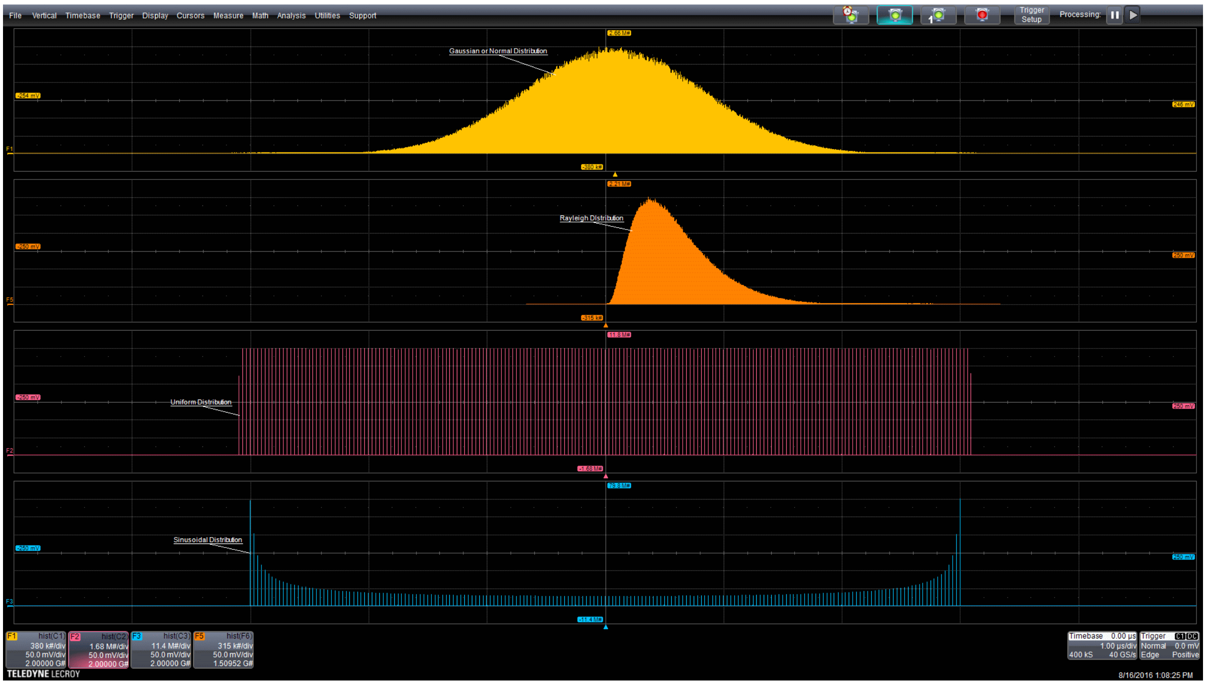

Figure 4. A selection of PDF functions that includes Gaussian, Rayleigh, uniform, and sinusoidal.

The distribution in the second grid from the top is a Rayleigh distribution. This is an asymmetrical distribution that results from applying Gaussian distributed noise to a peak detector. It is included to show that PDFs need not be symmetrical.

The third grid from the top contains a uniform distribution. This distribution arises in timing measurements such as the time between a trigger event and the first sample in an oscilloscope's acquisition. In the uniform distribution, all sample values are equally probable. This distribution is bounded.

A similar bounded distribution is the sinusoidal distribution shown in the bottom grid. This distribution is saddle shaped, with maximum probability occurring at the extreme amplitude values (maxima and minima).

In many applications, two or more random processes may interact. When that happens, the probability densities of the processes are mathematically convolved. A common example is timing jitter where random and deterministic jitter component combine. **Figure 5** shows a Gaussian and a sinusoidal component being combined. The source distributions are in the top two grids. The resultant distribution, third grid from the top, is the convolution of the two sources. Many advanced oscilloscope offer optional analysis packages for jitter or noise that can separate these combined distributions and measure the components separately.

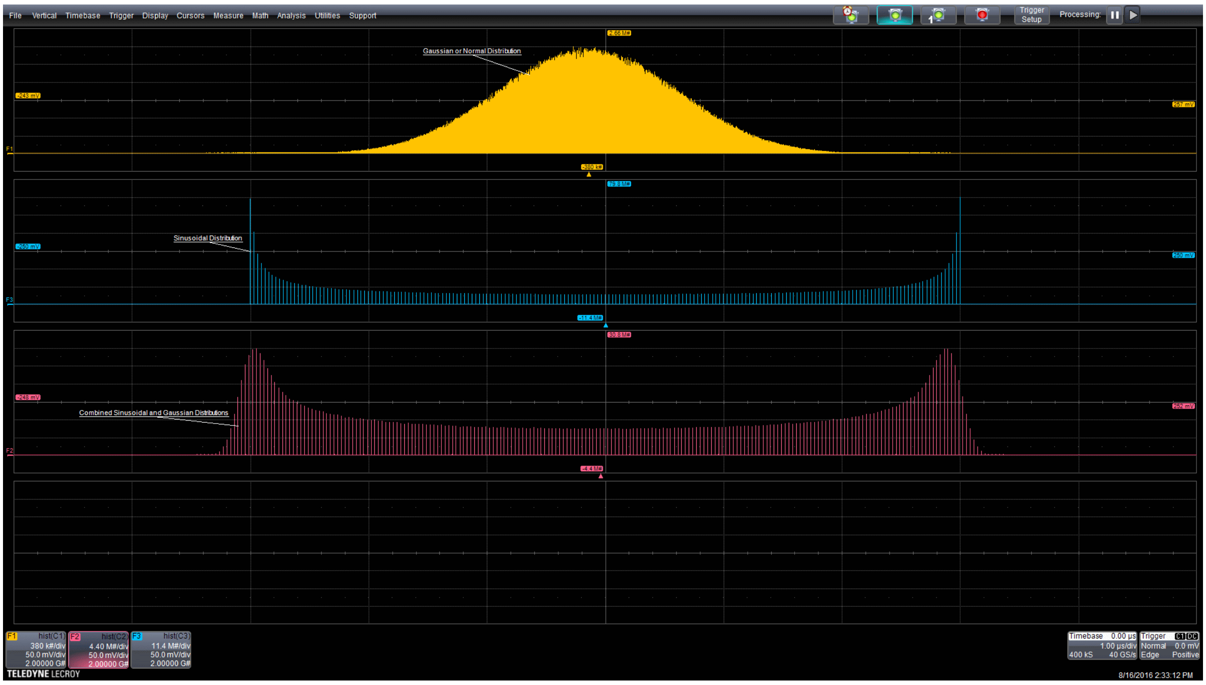

Figure 5. When a Gaussian and sinusoidal distributions combine, the resultant PDF is the convolution of the two source PDFs.

Frequency domain analysis

PSD, power per unit frequency, is the most common frequency domain tool for noise analysis. **Figure 6** shows an example. The upper trace is the time view of the bandlimited Gaussian noise. The lower grid contains the PSD of the band-limited noise.

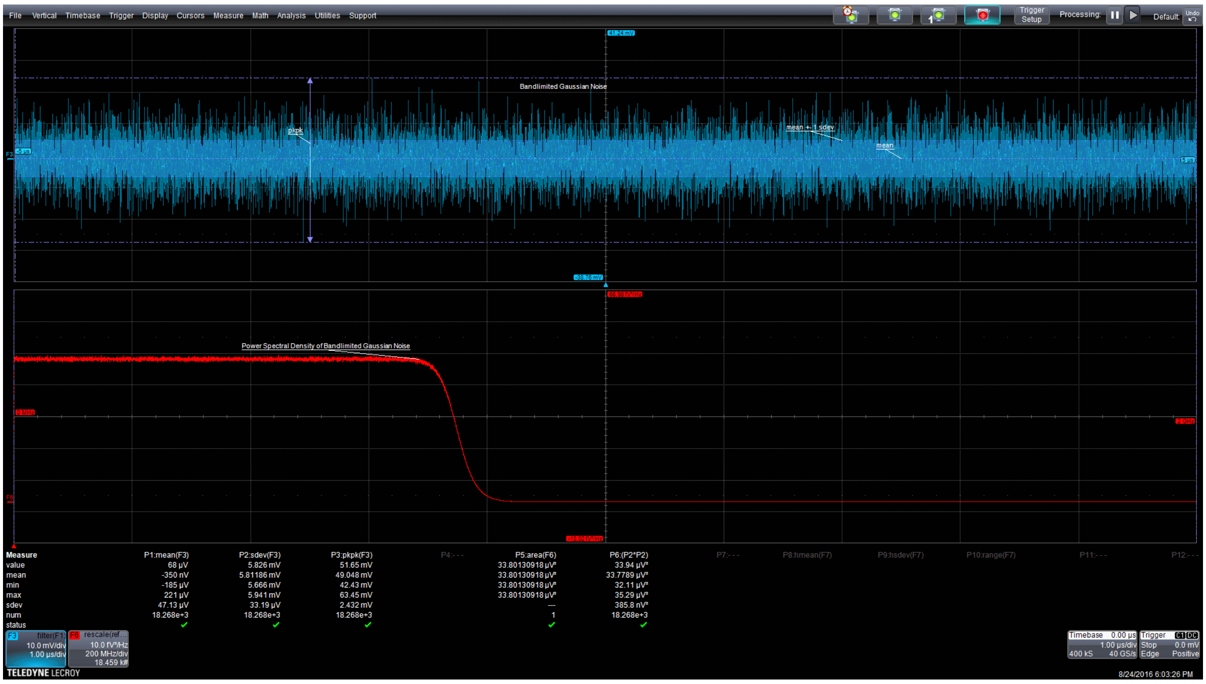

Figure 6. Bandlimited Gaussian noise (upper trace) and its power spectral density (lower trace). The PSD plots power per unit frequency versus frequency. The units of PSD are V2/Hz and the area under the plot is the mean squared value or variance of the signal.

PSD is measured in units of V^2/Hz in this example. This trace is computed using the oscilloscope's Fast Fourier Transform (FFT) selecting the output type magnitude squared instead of the default decibel (dBm) scale. In addition to the output type, we have chosen rectangular weighting and Least

Prime Factor FFT. The FFT setup reports the resolution bandwidth, Δf, which in this case is 100 kHz, as well as the effective noise bandwidth (ENBW) of the weighting function, which is 1.000 for rectangular weighting.

To compute the PSD, the averaged FFT output must be normalized to the effective FFT bandwidth. In addition, this oscilloscope's FFT output is calibrated to read peak values rather than rms values. To convert back to rms values, FFT magnitude values must be multiplied by 0.707 and magnitude-squared values are multiplied by 0.5. You must divide the FFT values by the effective bandwidth of the FFT to normalize the values to a unit bandwidth (1 Hz) using the Rescale math function. The Rescale function lets you rescale by a multiplicative factor and add or subtract offset. In our case, we multiplied by 0.5/100E3 = 5E-6. The factor 0.5 was discussed previously. The other factor is the reciprocal of the effective FFT bandwidth, which is the resolution bandwidth multiplied by equivalent noise bandwidth (ENBW). If a weighting function other than rectangular had been selected, ENBW would be a value higher than 1. The rescale function also has provision to change the units. In this example the units are set to V2/Hz. You may have noted that the reframe math function has also been used to optimize the mapping of the floating-point FFT output into the integer math space used in parameter measurements.

Parameter P2 measures the standard deviation parameter for the time-domain waveform. P6 uses parameter math to square the standard deviation, which is the variance of the noise signal. Parameter P5 takes the area under the PSD plot. This area is also the variance of the noise signal but computed from the PSD.The variance values computed in both methods are essentially equal, the difference being less than 0.1%.

Analyzing a random process in the frequency domain lets you break down contributions from different frequencies. The area measurement in this example is taken over the entire FFT span. You can also use the measurement gates to restrict the measurement to specific frequency bands to determine the noise contribution from specific regions of the frequency spectrum. The oscilloscope's cursor can read PSD at specific spot frequencies with a bandwidth equal to the effective noise bandwidth of the FFT.

Derived parameters
Crest factor—the ratio of the peak value to the RMS value of a waveform—lets you find the dynamic range required to handle the peak variations in a signal. Although the oscilloscope used here has no bipolar "peak" parameter, we can easily create one by taking the absolute value of the signal in channel 1. This will "flip" the negative values into the positive region of the waveform, which lets you use the maximum value parameter (max) to read the maximum positive or negative peak of each acquisition. Note that this works because the signal has a zero mean. We can then use parameter math to compute crest factor as the ratio of peak value to rms value. **Figure 7** shows this measurement.

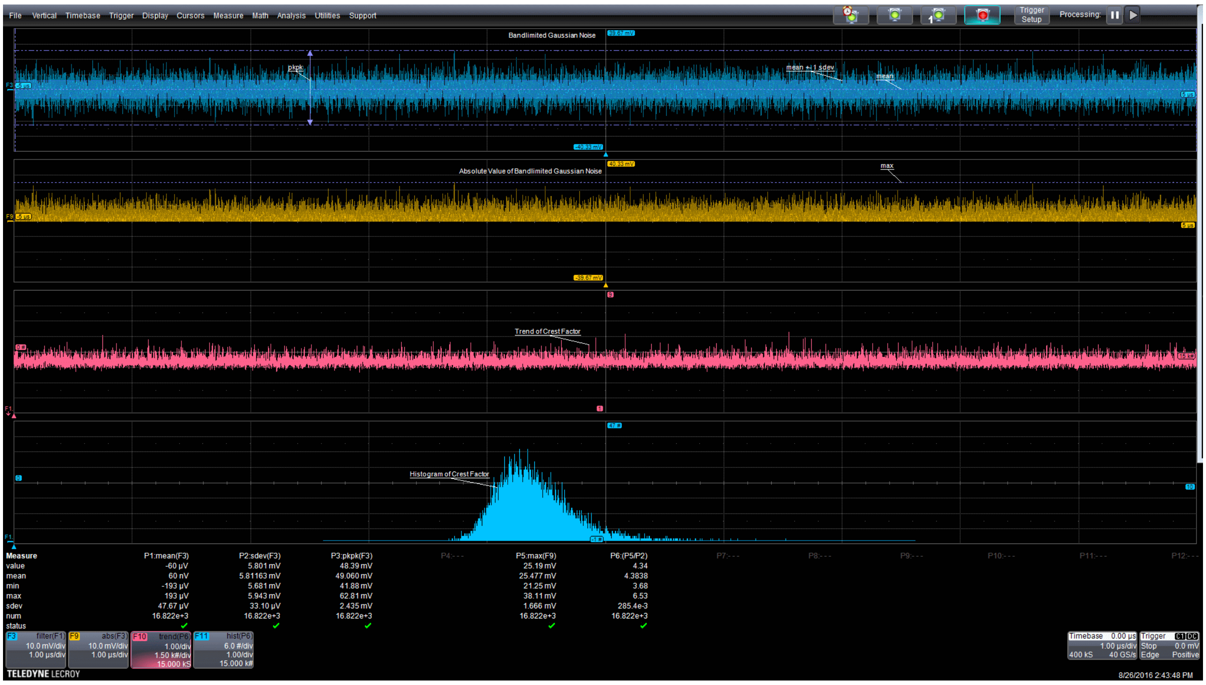

Figure 7. measuring the signal's crest factor, the ratio of peak value to rms. The absolute value of the measured signal makes all peaks unipolar so that the maximum parameter returns the highest peak value per acquisition. Parameter math takes the ratio of the maximum value to the standard deviation (rms) value which is the crest factor.

The top grid contains the bandlimited noise signal. The parameter P2 is the standard deviation (AC coupled rms value) of the noise waveform. The next lower grid shows the absolute values of the noise waveform. This waveform is unipolar. The highest positive or negative peak in the source waveform will be the highest peak in the absolute value. Use the maximum (max) parameter to get this parameter.

Parameter P5 is the maximum value of the absolute waveform trace. Parameter P6 uses parameter math to compute the crest factor as the ratio of P5 (max) to P2 (rms) for each acquisition. Because the noise signal is unbounded, the value of the crest factor will vary with the number of acquisitions. Parameter statistics of P6 show the current, mean, minimum, maximum standard deviation and total number of measurement values of the crest factor. Over the 15,000 acquisitions shown in this example, crest factor varied from 3.68 to 6.53 with an average value of 4.38.

The third grid from the top contains the trend plot of the crest factor. The trend plot shows each crest factor measurement in the order taken. Below the trend plot is the histogram of the crest factor. It shows the bulk of the crest factor measurements clustered about the mean value with a small number of high value measurements in the tail to the far right of the mean.

Conclusion

You can quantify random processes such as noise and jitter using the time, frequency and statistical domain tools augmented by related measurement parameters in a modern digital oscilloscope. Statistical parameters including mean, standard deviation, and range provide insight into the measured process. Parameter math yields derived parameters such as variance and crest factor.

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