[FFTs and oscilloscopes: A practical guide]

[Arthur Pini] - September 29, 2016

The FFT (Fast Fourier Transform) first appeared when microprocessors entered commercial design in the 1970s. Today almost every oscilloscope from high-priced laboratory models to the lowest-priced hobby models offer FFT analysis. The FFT is a powerful tool, but using it effectively requires some study. I'll show you how to set up and use the FFT effectively. We’ll skip the technical description of the FFT, because its already implemented in the instruments. Instead I'll focus on the practical aspect of using this great tool.

The FFT is an algorithm that reduces the calculation time of the DFT (Discrete Fourier Transform), an analysis tool that lets you view acquired time domain (amplitude vs. time) data in the frequency domain (amplitude and phase vs. frequency). In essence, the FFT adds spectrum analysis to a digital oscilloscope.

If you look at upper trace in **Figure 1**, you'll see an amplitude-modulated carrier that uses a trapezoidal pulse as the modulation function. If you look at the time-domain view in Fig. 1 and I ask you to tell me the bandwidth of the signal, you'd have a hard time. But take the FFT of this signal and you get another point of view. The signal has a linearly swept frequency and the bandwidth, marked by the cursors, is 4.7 MHz. That’s how the FFT adds to the capability of the oscilloscope, it provides another point of view for the same data.

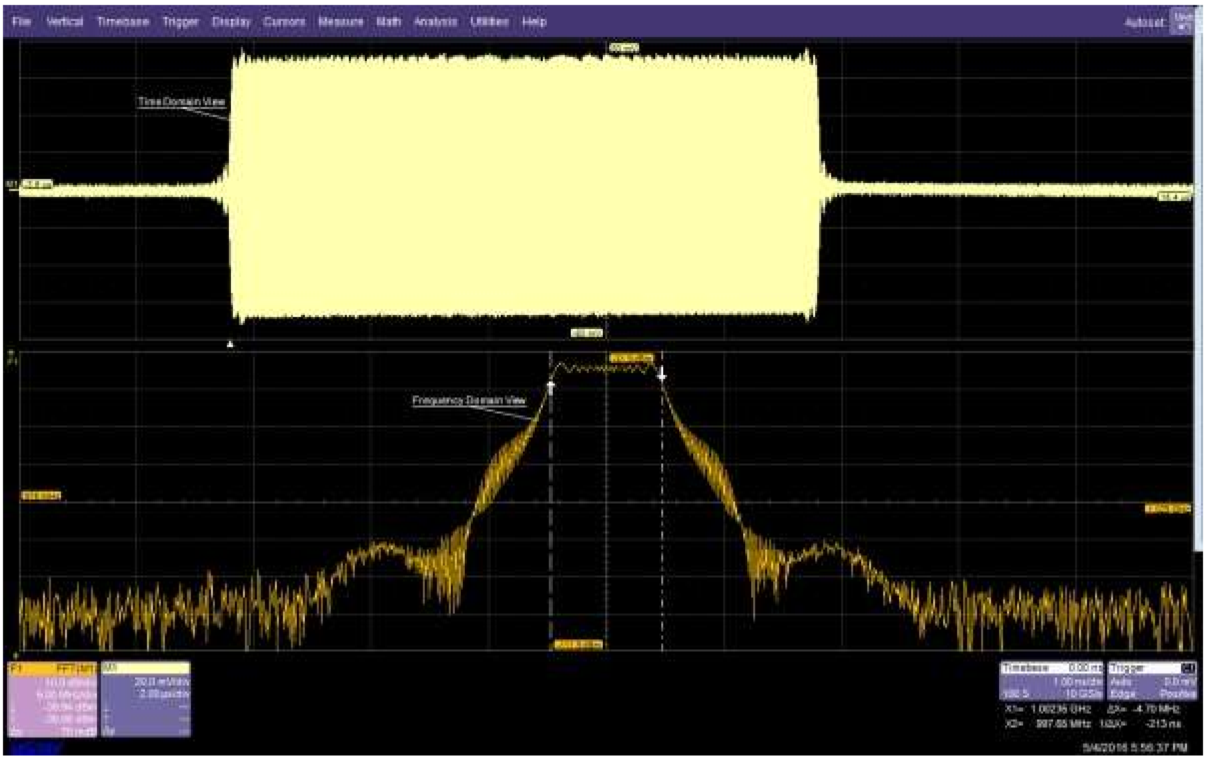

Figure 1. The time domain view in the top grid shows a pulse modulated RF carrier while the frequency domain view in the lower grid shows a uniform distribution of the carrier frequency between 997 MHz and 1002 MHz.

FFT frequency span and resolution bandwidth

In your earliest circuits course, you learned the frequency (frequency domain) of a periodic signal is the reciprocal of the period (time domain). That same relationship appears throughout the FFT setup.

The best place to start with setting up the FFT is choosing the RBW (resolution bandwidth) because it's related to a single setting. The RBW (Δf) is the incremental step in displaying the FFT frequency axis. In the time domain, the sampling period determines the time between samples. In the frequency domain, RBW is the frequency difference between adjacent "cells" in the spectral view. The RBW is the reciprocal of the time domain record length, also called the capture time as shown in **Figure 2**. You can control this with the oscilloscope's horizontal scale or time/division setting. The acquisition duration in Fig. 1 is 20 µs. Thus, the spectrum view's RBW is the reciprocal of that number, or 50 kHz.

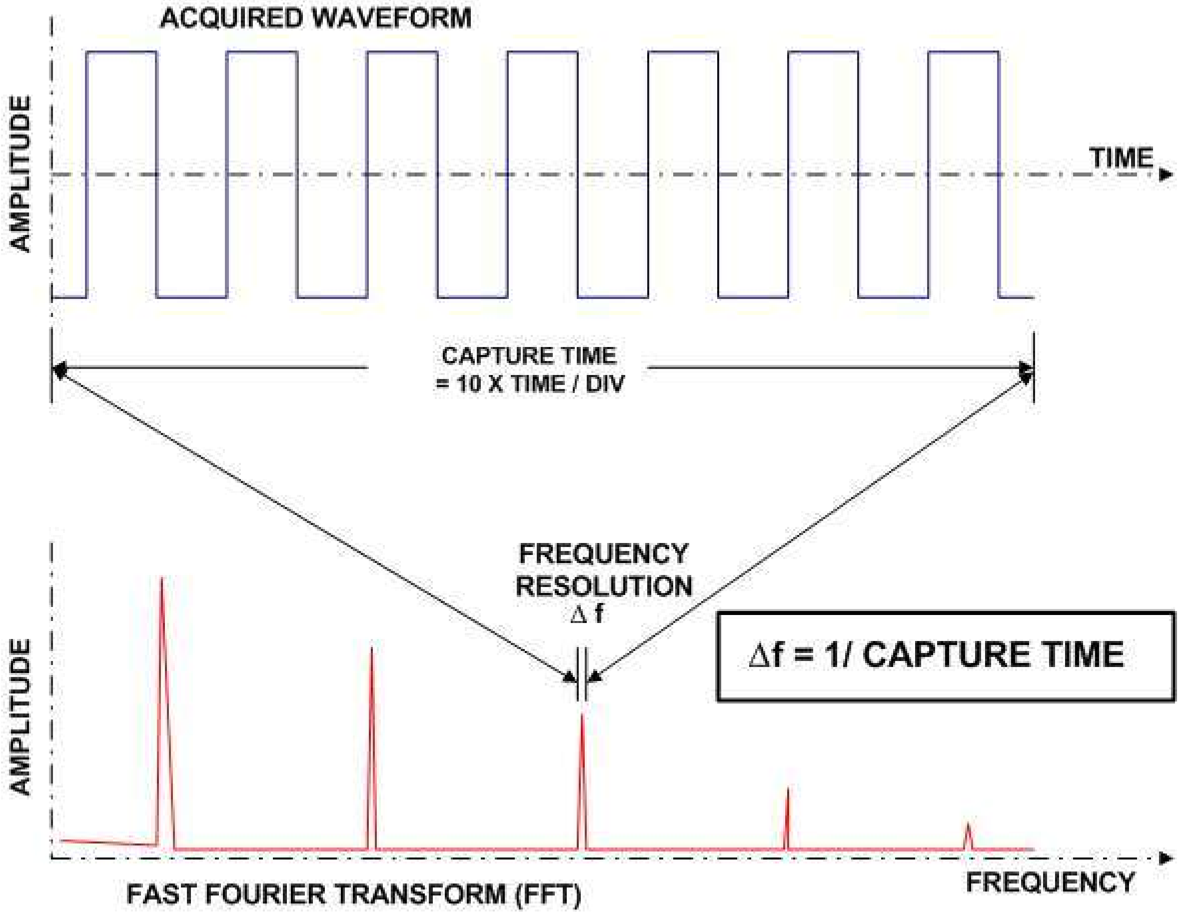

Figure 2. The resolution bandwidth of the frequency spectrum is the reciprocal of the time domain record length, or capture time.

The next step in the FFT setup is to determine the span of the frequency-domain view—the difference between the highest and lowest frequency in the FFT. Note that FFTs generally start at 0 Hz and go out to the span. This is very different from the RF spectrum analyzer, which I'll explain shortly.

The span in the FFT is one half of the oscilloscope's effective sample rate (**Figure 3**). The smallest time increment in the time domain—the sample period—determines the largest element in the frequency domain. Similarly, the smallest increment in the frequency domain is a function of the largest duration in the time record. This is that reciprocal relationship between the time and frequency domain.

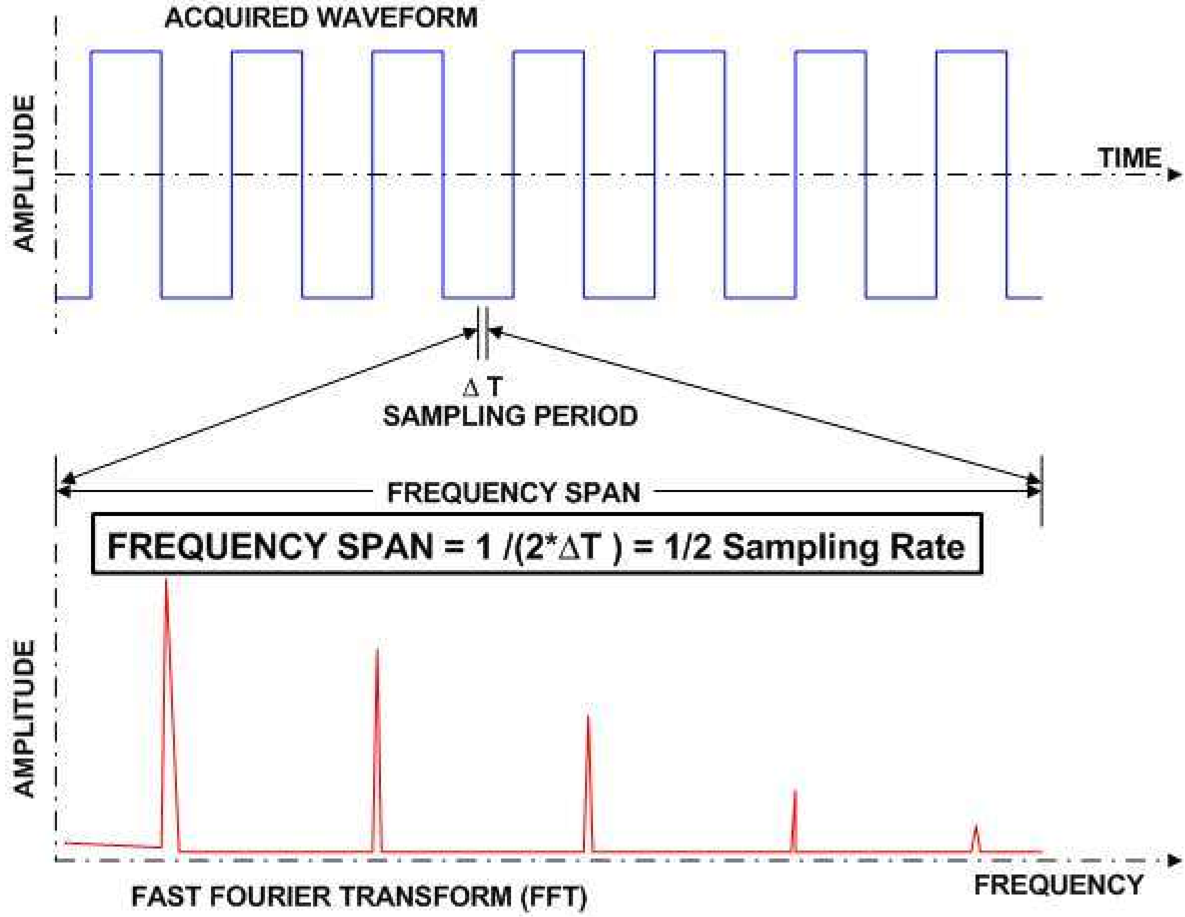

Figure 3. The span of the frequency Spectrum is half of the oscilloscope's effective sample rate.

To achieve finer resolution in the frequency domain, you must increase the amount of data captured by increasing the time/division setting. That's the opposite of what you would do to increase the time resolution in the oscilloscope's time-domain view.

From a practical point of view, the time domain record length is set by the oscilloscope's time/division control. Once you have selected that to achieve the desired resolution bandwidth, the only way to control the sample rate to achieve the required span is to change the oscilloscope's acquisition memory length. Now that may seem complicated, and it can be.

In the recent past, most high-end oscilloscope manufacturers have changed the FFT user interface to more closely resemble a standard RF spectrum analyzer, setting center frequency and span with resolution bandwidth as a parameter. While this type of interface makes the FFT easier to use, it does hide the underlying functionality of the FFT resulting in having to accept the combination of time/division, sample rate, and memory length set by the oscilloscope. With the rules discussed in this section, you can manually set up the FFT and achieve a great deal more freedom in your settings.

Vertical scaling

Vertical Scaling
Depending on your oscilloscope, the FFT may be available with a choice of vertical scales or may be a single fixed vertical format. The most common vertical format is the power spectrum, which displays the vertical amplitude in units of power. Most commonly, this is expressed in decibels relative to a milliWatt (dBm) and is displayed on a logarithmic vertical scale. This selection is also a holdover from RF spectrum analyzers. Laboratory-grade oscilloscopes offer more possibilities including PSD (power spectral density), linear magnitude, magnitude squared, phase, or real/imaginary.

PSD is the power spectrum value normalized to the FFT's resolution bandwidth. It's unit of measure is dBm/Hz and it represents the power per unit bandwidth. PSD is useful for measuring broadband phenomena such as noise. The magnitude format shows the spectral magnitude in linear units which the oscilloscope is measuring like Volts or Amperes.

The magnitude squared display, as its name implies, shows the spectral amplitude as the square of the magnitude. Units can be Volts^2, Amperes^2, etc. and provides a linear scaling for power measurements when normalized to the instrument's input impedance, normally 50 Ω. Normalization is accomplished using the oscilloscope's rescale function, which allows multiplication by a constant. For 50 Ω, the magnitude squared spectrum is multiplied by 0.02 (that's 1/50) and changing the units to Watts ($V^2/50$) for a 50 Ω input impedance.

An FFT spectrum is a complex function (in a mathematical sense) and the magnitude display is only half of the picture. The FFT output consists of real and imaginary parts. Some oscilloscopes can display the both. As an alternative to the real and imaginary components, many oscilloscopes display FFT phase along with magnitude. These two paired output formats (real/imaginary and magnitude/phase) represent the FFT's complete picture. The real/imaginary components are required to compute the inverse FFT. They are also more commonly used in mechanical applications such as vibration measurement. The magnitude/phase format is more commonly seen in electrical measurement. **Figure 4** shows examples of power spectrum magnitude/phase and real/imaginary components for a square wave.

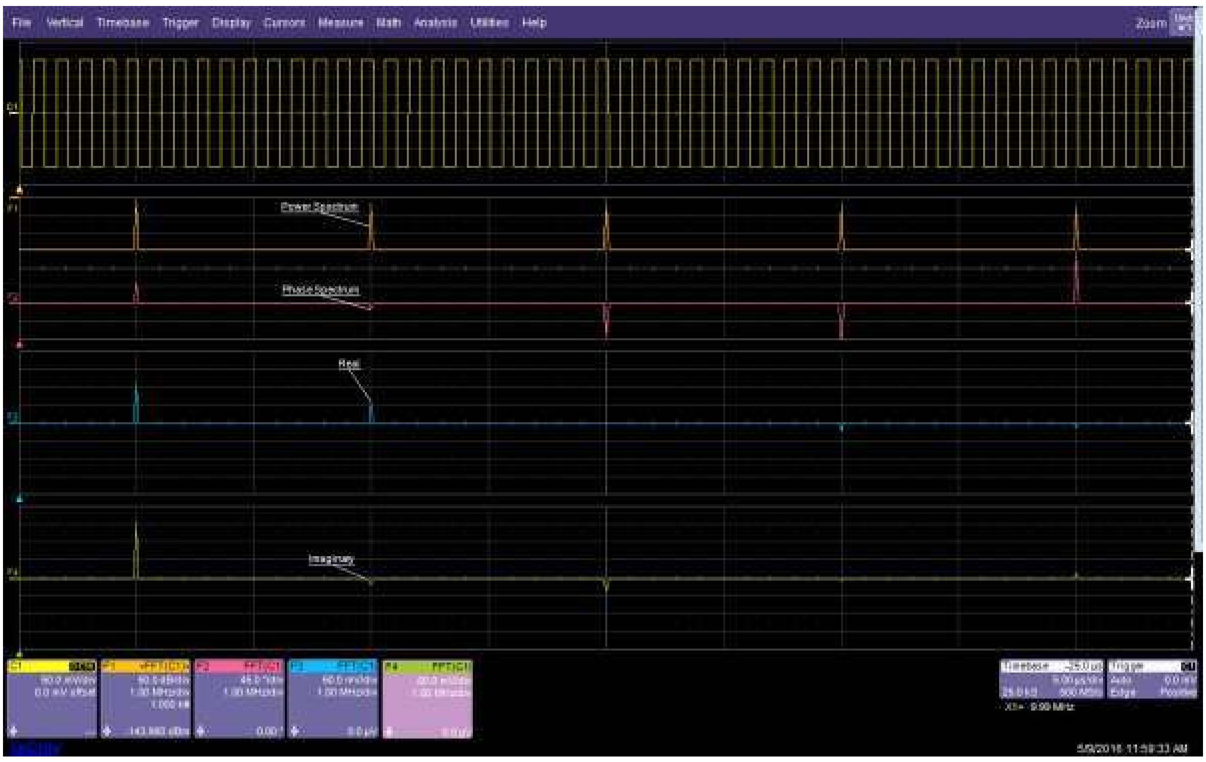

Figure 4. The power spectrum magnitude, phase, real and imaginary components of the FFT of a square wave.

The phase spectrum has vertical units of degrees, the real and imaginary formats have vertical units identical to the source channel, in this case mV. For periodic waveforms such as this square wave the phase, real and imaginary formats have significant values only at the fundamental and harmonic frequencies.

Weighting Functions

The FFT implemented in an oscilloscope has limited record length. This can cause issues in the spectrum display due to continuity issues at the start and end points of the acquired waveforms. **Figure 5** shows how the start and end points affect the spectral shape.

Figure 5. Boundary conditions at the start and end points affect the spectral shape of a signal after undergoing an FFT.

In the upper two grids, the frequency of the acquired signal is a sub-multiple of the sample rate and there are an integral number of cycles in the acquired waveform. The start and end points are at the same amplitude. The resultant spectrum is very narrow. In the lower two grids, the frequency of the acquired signal is not a sub-multiple of the sampling rate. The start and end points are at different levels.

This results in a discontinuity in the time record. The resulting spectrum is broader and the peak level is lower due to the spectral spreading, called leakage, as energy from the acquired signal is spread over adjacent frequency cells. The lower, frequency dependent, peak response is called "picket fence" effect or scallop loss. Weighting (windowing) can help minimize these effects.

Weighting multiplies the acquired waveform by a window function modulating it to zero the end points. The shape of the window function determines the spectral response including the shape of the spectral line and the amplitude of any sidebands. The characteristics of commonly used weighting functions are shown in **Table 1**.

Table 1. The characteristics of common FFT weighting (window) functions.

Window Type	Highest Sidelobe (dB)	Scallop Loss (dB)	Effective Noise Bandwidth (cells)
Rectangular (None)	-13	3.92	1.00
Von Hann (Hanning)	-32	1.42	1.50
Hamming	-43	1.78	1.37
Flat Top	-44	0.01	3.43
Blackman Harris	-67	1.13	1.71

The table summarizes the ability of each window to minimize sidelobes and scallop loss. **Figure 6** shows the effect that the window functions produce on the spectral lines for the same input signal.

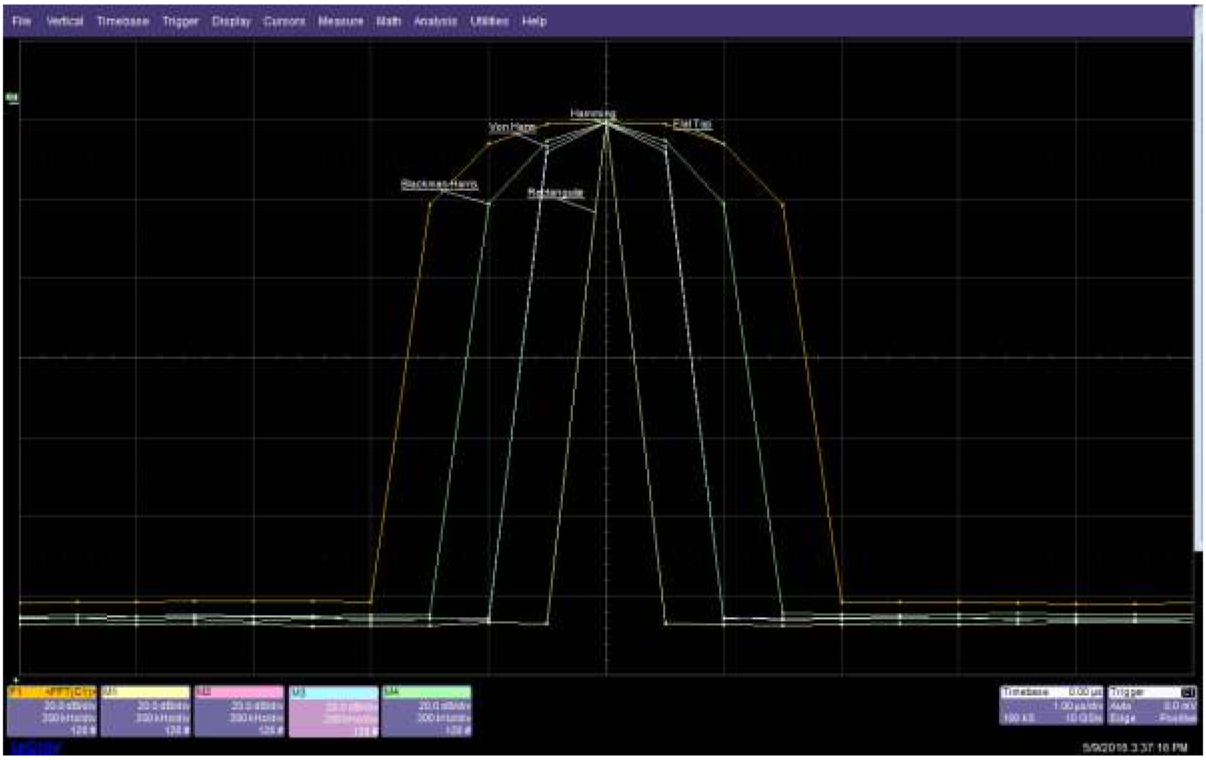

Figure 6. This screen image compares the effects of weighting functions on the spectral response for the same input signal.

The spectral lines broaden for decreasing scallop loss, which makes sense because signals in adjacent cells will overlap at higher amplitudes for broader responses, thereby minimizing scallop loss.

The selection of a window function depends on your needs. If you're measuring transients that are smaller than the acquisition window, then don't use a window function because the amplitude of the spectrum peak will change based on the transient's location in the acquisition window. In that case, the rectangular window (no weighting) is the best choice. The narrower window responses provide better frequency resolution while the broader responses—Blackman Harris or Flat Top windows—produce more accurate amplitude measurements. If you need both, then a good compromise is Von Hann or Hamming window.

Frequency domain averaging

Frequency domain averaging

Averaging is used to improve the signal-to-noise ratio of acquired signals and generally requires multiple acquisitions. Averaging can be applied in the time domain or in the frequency domain. Signals that are not synchronous with the trigger event, such as noise, are attenuated proportional to the number of averages. **Figure 7** is an example of frequency domain averaging.

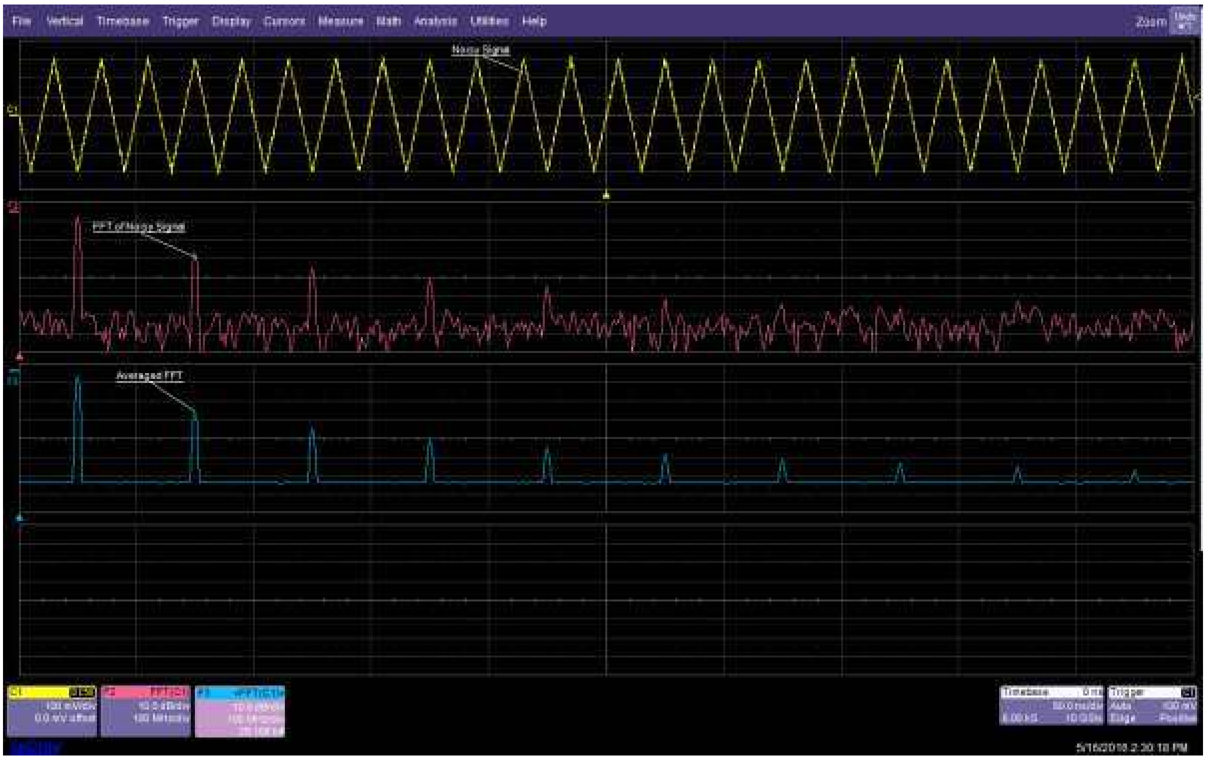

Figure 7. Frequency domain averaging improves signal to noise ratio providing greater dynamic range in measurements. The FFT of a noisy signal contains noise that disappears of the FFT is averaged over many acquisitions, making lower level harmonics visible.

Averaging in the frequency domain is accomplished by summing the contents of each frequency cell over multiple acquisitions and then dividing by the number of acquisitions. Signals that are not synchronous with the acquisition average to zero while synchronous signals add coherently. In Fig. 7, the FFT of the noisy signal contains noise components that are spectrally spread. The noise hides low level harmonics. Averaging improves signal to noise ratio, reducing noise and making harmonics visible. In a similar manner the amplitude any signal not synchronous with the acquisition will be reduced.

Setup example

Consider the need to set up, on a 4 GHz bandwidth oscilloscope, an FFT with a span of 10 MHz, centered at 2.48 GHz with a resolution bandwidth of 10 kHz to analyze a continuous, periodic signal. Based on the earlier discussion, setting the resolution bandwidth can be done with a single setting of the oscilloscope’s time/division setting. The resolution bandwidth requires an acquisition or capture time of 100 µs or time/div setting of 10 µs/division. The oscilloscope’s vertical sensitivity (volts/division) should be set so that the signal occupies at least 90% of the input range to maximize dynamic range.

The span of the FFT is controlled by the sampling rate. The span has to include the 2.48 GHz signal frequency meaning it must be greater than two times that frequency. A frequency of 5 GHz or greater will work. The oscilloscope has a maximum sampling rate of 20 GS/s. We can set the sample rate by adjusting the acquisition memory length to achieve the sampling rate using the oscilloscope’s timebase settings. In the oscilloscope used for this example setting the memory length to 1 MS achieves the 100 µs acquisition time with a sample rate of 10 GS/s. The FFT setup is shown in **Figure 8**.

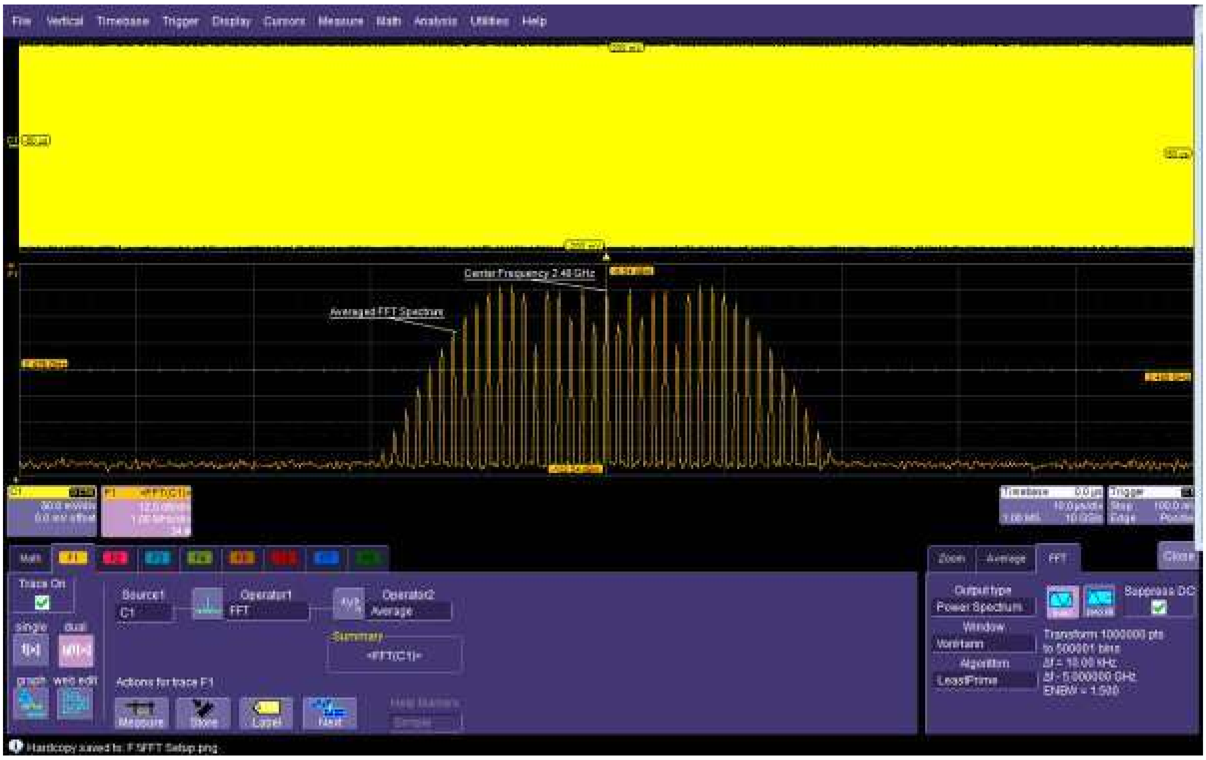

Figure 8. The FFT setup showing the main FFT settings for this example.

The FFT tab of the math function F1 has the main FFT settings and is setup to display the power spectrum. Since the signal was indicated to be continuous the Von Hann window is chosen for weighting function type offering a good compromise for frequency resolution and amplitude flatness.

The FFT tab indicates a resolution bandwidth (Δf) of 10 kHz and a span of 5 GHz. The zoom tab lets you set the center frequency to 2.48 GHz with a horizontal scale to 1 MHz/div as shown in the F1 trace in Fig. 8.

Conclusion

This article has reviewed the key FFT characteristics and explained how they can be used to setup the FFT for effective analysis. Keep it in mind the next time you need to use the FFT in your oscilloscope.

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