Turing Machine 5 States 3 Colors

Basic Information

For every pair of states and colors, the Turing machine rule picks a new state of the Turing machine, and a new color for the active cell, and a direction either left and right for the active cell to move. The active cell is usually called the head of the Turing machine.

For 3 states and 5 colors there are 15 pairs, and for each of those pairs there are 30 possible choices.
So there are 30^15 = 14348907000000000000000 possible Turing machines.

Evaluate a TM

```
$MaxRules = 30^15;
```

```
$Steps = 500;
```

```
$SampleSize = 10 000;
```

```
TM[rn_]:= TuringMachine[{rn,5,3},{1,{{1,0,1},0}},$Steps]
```

```
BigTM[rn_,steps_]:=TuringMachine[{rn,5,3},{1,{{1,0,1},0}},steps]
```

Show a TM

```
ShowTM[rn_]:= ArrayPlot[Last/@ TM[rn],ImageSize→{500,500},PlotLabel→rn,ColorFunction→'
```

```
ShowBigTM[rn_,steps_]:=ArrayPlot[Last/@ BigTM[rn,steps],ImageSize→{Automatic,500},Plot
```

```
ShowTMwithHead[rn_]:=
        ArrayPlot[
            Function[u,MapAt[Red&,u[[2]],u[[1,2]]]] /@
                TM[rn],ImageSize→{Automatic,500},PlotLabel→rn] (* standard size $Step
```

```
ShowHorizontalTM[rn_,steps_,extraopts___]:=ArrayPlot[Reverse[Transpose[Last/@ BigTM[rn
    extraopts,ImageSize→{800,200},ColorFunction→"Rainbow"]
```

```
ShowDivededTM[tm_,rn_,windows_]:=
With[{portion=N[Length[tm]/windows]},
    Do[
        ArrayPlot[
            Function[u,MapAt[Red&,u[[2]],u[[1,2]]]] /@ tm[[Round[i*portion];;Round[(po
            ,
            ImageSize→{Automatic,500},PlotLabel→rn
        ]

    ,{i,0,windows}]
]
```

How to find some interesting TMs

Criteria based on Width

```
MaxWidth[tm_]:=With[
    {relativepos=tm[[All,1,3]]},
    left=Select[relativepos,Positive];
    right=Select[relativepos,Negative];
    If[Length[left]>0,Max[left],0.]-If[Length[right]>0,Min[right],0.]
]
```

```
MaxWidthCriterion[tm_,rn_]:=If[(MaxWidth[tm]<(Length[tm]/3))&&(MaxWidth[tm]>10),rn,"No
```

```
ApplyCriterionAndPrint[criterion_,samplesize_]:=(
    SetDirectory["/Users/Levantina/Documents/WOLFRAM/Homework2013"];
        Do[
            (r=RandomInteger[$MaxRules-1];
            max=criterion[TM[r],r];
            If[max≠"NotInteresting",(max>>> goodWidthTM.txt),""])

            ,
        {i,samplesize}];SetDirectory[];)
```

```
ApplyCriterionAndPrint[MaxWidthCriterion, 1000]
```

Criteria based on Entropy

```
FirstCriterion[tm_,rn_]:= With[
    {heads= tm[[All,1,3]]},
        If[Length[Tally[Partition[Differences[heads],4,1]]]>5,rn,"NotInteresting"]]
```

```
SecondCriterion[tm_,rn_]:= With[
    {heads= tm[[All,1,3]]},
        If[N[Entropy[Partition[Differences[heads],8,1]]]>2,rn,"NotInteresting"]]
```

```
Sample[size_]:= RandomInteger[$MaxRules-1,size]
```

```
ApplyOnSample[criterion_,rules_]:= DeleteCases[criterion[#]& /@ rules,"NotInteresting"
```

```
ApplyOnSample2[criterion_,rules_]:= DeleteCases[criterion[TM[#],#]& /@ rules,"NotInter
```

```
PLogP[x_]:=-Total[x * Log[N[x]]];
TrailEntropy[pos_]:=PLogP @ Normalize[Tally[Partition[Differences[pos],8,1]][[All, 2]]
```

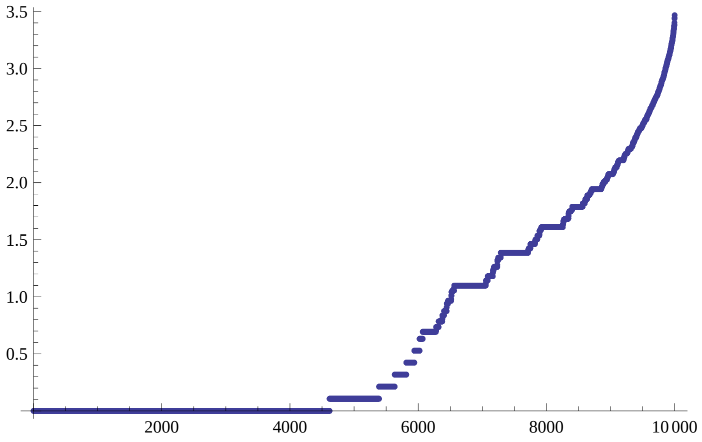
```
RuleEntropy[rn_]:=TrailEntropy[TM[rn][[50;; , 1,2]]];
```

```
FindMaxEntropy[rules_,entropies_]:= With[{max= Max[entropies]},rules[[Flatten[Position
```

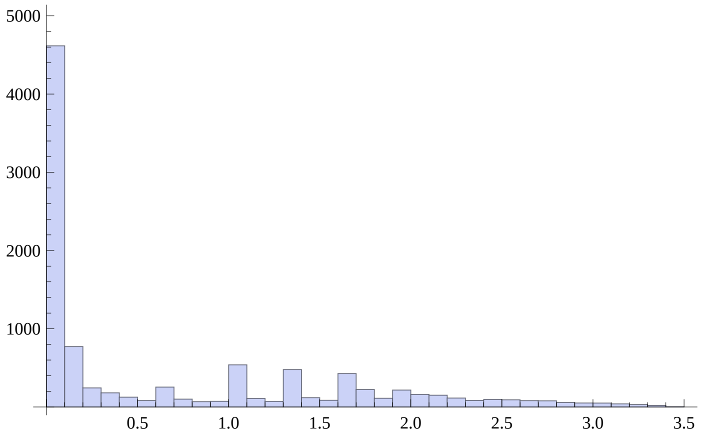
```
rules = RandomInteger[$MaxRules - 1, $SampleSize];
```

```
entropies = Map[RuleEntropy[#] &, rules];
```

```
ListPlot[Sort[entropies]]
```




```
Histogram[entropies]
```



Apply Entropy Criterion to a Set of Samples

```
ExploreEntropies[samplesize_,numberofsamples_]:= Do[
    r=Sample[samplesize];
    Print @ FindMaxEntropy[r,Map[RuleEntropy[#]& ,r]]
    ,
    {i,numberofsamples}]
```

```
ExploreEntropies2[samplesize_,numberofsamples_]:=
    (SetDirectory["/Users/Levantina/Documents/WOLFRAM/Homework2013"];
        Do[
            r=Sample[samplesize];
            FindMaxEntropy[r,Map[RuleEntropy[#]& ,r]] >>> goodTM.txt
            ,
        {i,numberofsamples}];SetDirectory[];)
(* this prints the results step by step in a file *)
```

```
(*On a sample of 10000 rules this
 function is going to find the most entropic rule,
and it repeat that 100 times. The TM are of 500 steps. *)
```

```
ExploreEntropies2[500, 100]
```

How widths are distributed in a random sample of 10^4 TM

```
Width[rules_]:=(SetDirectory["/Users/Levantina/Documents/WOLFRAM/Homework2013"];
        rn=0;
        Do[rn=rules[[i]];
        mytm=TM[rn];
        mov=RelativeAverageHeadMovements[mytm];
            Abs[(mov[[2]]-mov[[1]])/Length[mytm]]>>> widthsTM.txt
            ,
        {i,Length[rules]}];SetDirectory[];) (* I decidet to divide to the number of th
```

```
WidthDistribution[list_,Size_]:= (
    h=Tally[N[Round[1000 #]/1000]& /@ list];
    {#,N[(#2/Size)]}& @@@ h)
```

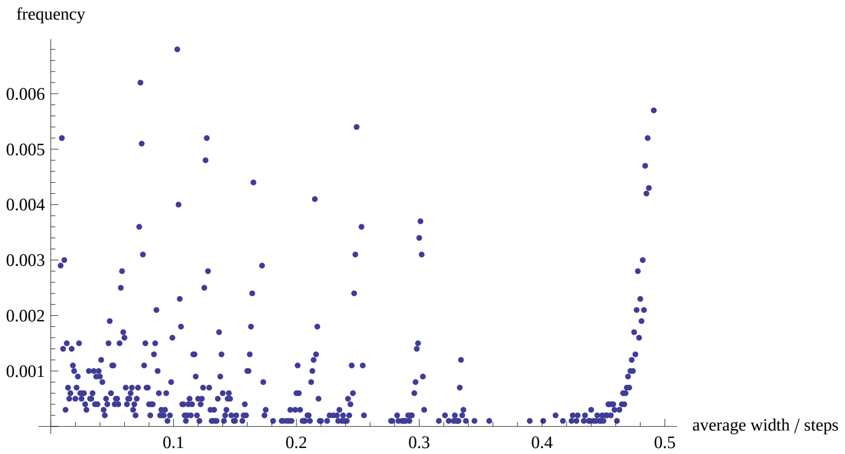
```
Width[Sample[10 000]]
```

```
SetDirectory["/Users/Levantina/Documents/WOLFRAM/Homework2013"];
```

```
l = ReadList["widthsTM.txt", Real];
```

```
SetDirectory[];
```

```
ListPlot[WidthDistribution[l, 10 000],
 AxesLabel → {"average width / steps", "frequency"}]
```

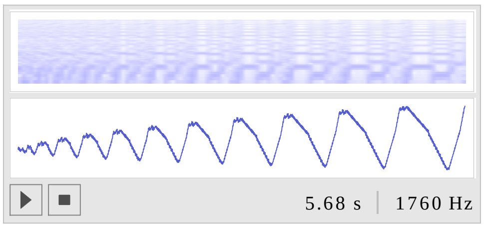


My favorite Turing Machine 5 States 3 Colors

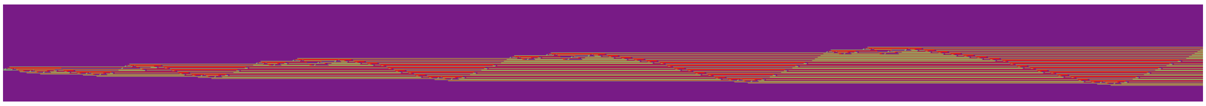
Endurance

The Turing Machine with the rule **484202566025544811003 9** attracted my attention, it has a semi-periodic behaviour, and its period grows in time, and the thing that more interested me was that it can survive without transition at least 10 000 time steps. To study the sample I used primarily a criterion based on Entropy. I wrote also a criterion based on the average width of the TMs, that helps to exclude the trivial Turing Machines; if they have a too large width it means that they evolve almost constantly in the same direction, and with a too small widht they don’t evolve enough to have an interesting behaviour. To find this TM I analyzed a sample of 10 000 TM, evolving for 500 steps, and I selected the one with the largest entropy. I did this selection for 500 times. Plotting the Turing Machines I found some interesting behaviours, but increasing the number of steps often they weren’t interesting anymore. I did a plot of 10 000 steps and I heard the sound due to the movement of the head for 100 000 steps.

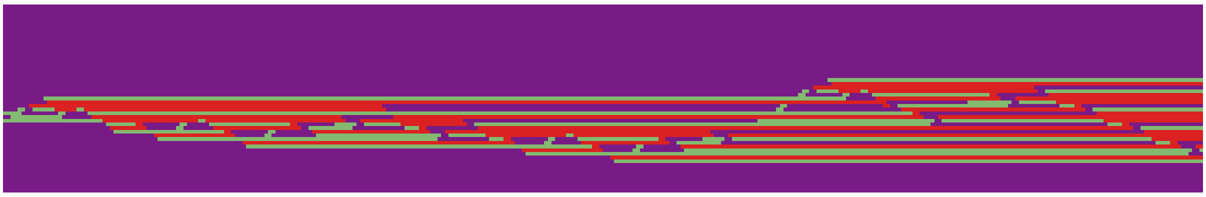
```
ListPlay[BigTM[4 842 025 660 255 448 110 039, 100 000][[All, 1, 2]], SampleRate → 1760]
```




```
ShowHorizontalTM[4 842 025 660 255 448 110 039,
 10 000, PlotLabel -> 4 842 025 660 255 448 110 039,
 ImageSize → 2000, Frame → False, PlotRangePadding → 0]
```



```
ShowHorizontalTM[4 842 025 660 255 448 110 039,
 1000, PlotLabel -> 4 842 025 660 255 448 110 039,
 ImageSize → 1000, Frame → False, PlotRangePadding → 0]
```



```
tmm = BigTM[4 842 025 660 255 448 110 039, 10 000]
```

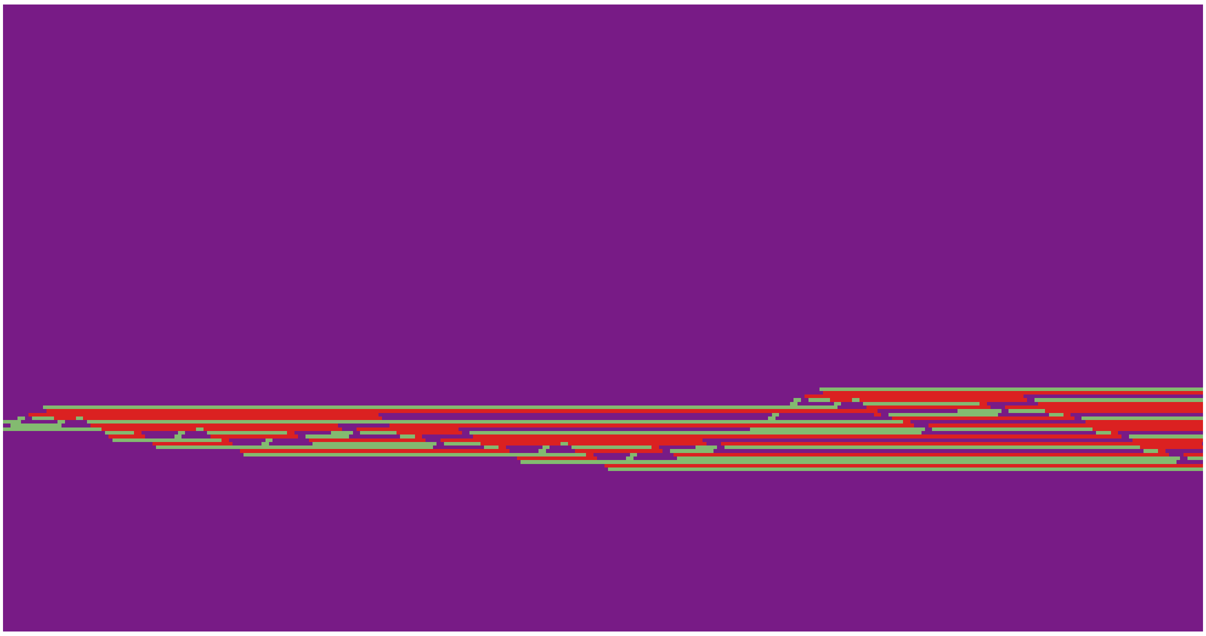
A very large output was generated. Here is a sample of it:

```
{{{1, 56, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}},
 <<9999>>, {{4, 168, 112}, {1, 2, 1, 1, 2, 0, 1, 2, 0, 2, 1,
   1, <<149>>, 1, 2, 1, 1, 2, 1, 0, 2, 2, 2, 1}}}
```

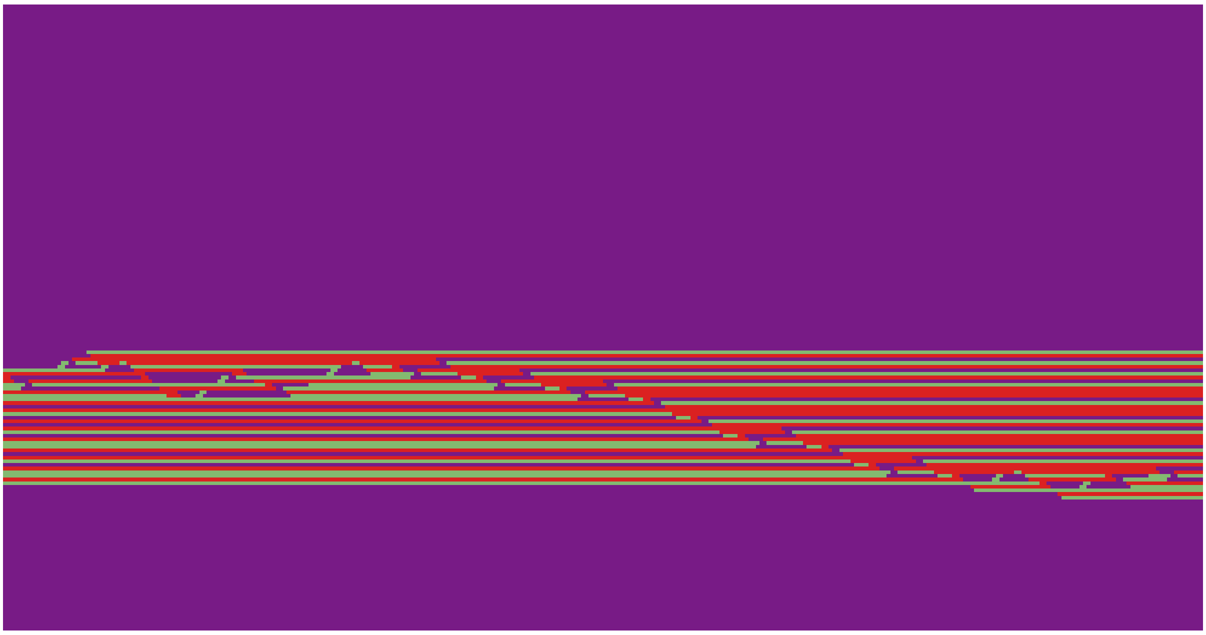
Show Less | Show More | Show Full Output | Set Size Limit...

```
ArrayPlot[Reverse[Transpose[Last /@ tmm[[1 ;; 500]]]],
 PlotLabel → 4 842 025 660 255 448 110 039, Frame → False,
 PlotRangePadding → 0, ColorFunction → "Rainbow"]
ArrayPlot[Reverse[Transpose[Last /@ tmm[[500 ;; 1000]]]], PlotLabel → "500-1000",
 Frame → False, PlotRangePadding → 0, ColorFunction → "Rainbow"]
ArrayPlot[Reverse[Transpose[Last /@ tmm[[1000 ;; 1500]]]],
 PlotLabel → "1000-1500", Frame → False,
 PlotRangePadding → 0, ColorFunction → "Rainbow"]
ArrayPlot[Reverse[Transpose[Last /@ tmm[[2000 ;; 2500]]]],
 PlotLabel → "2000-2500", Frame → False,
 PlotRangePadding → 0, ColorFunction → "Rainbow"]
ArrayPlot[Reverse[Transpose[Last /@ tmm[[9000 ;; 10 000]]]],
 PlotLabel → "9000-10000", Frame → False,
 PlotRangePadding → 0, ColorFunction → "Rainbow"]
```

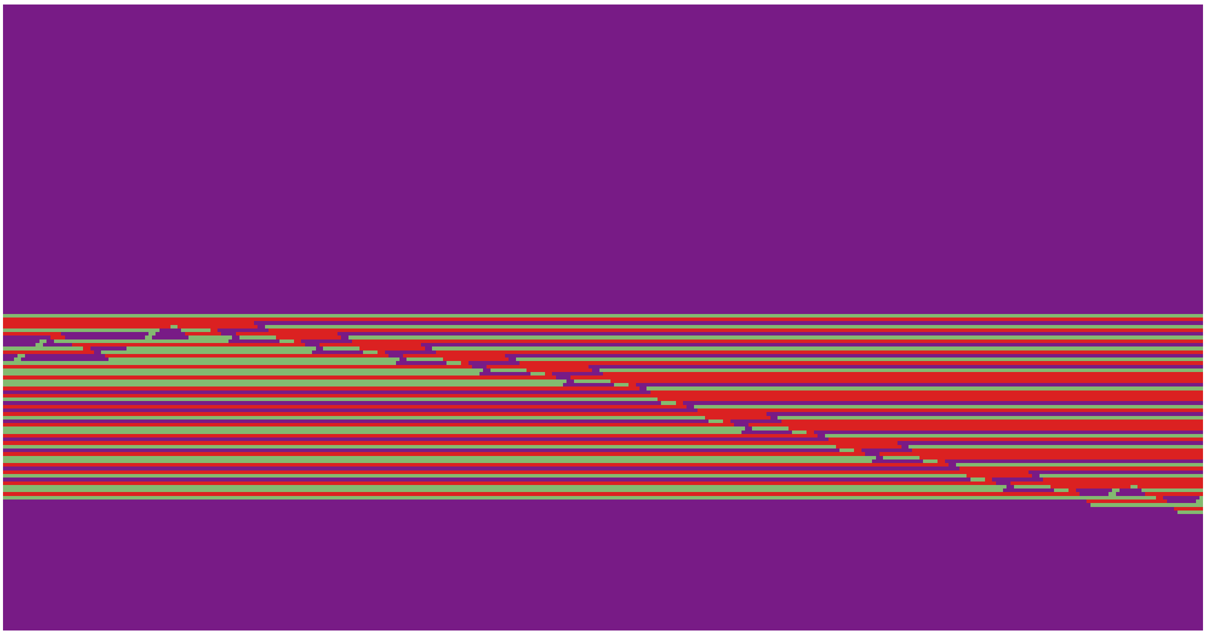
4 842 025 660 255 448 110 039



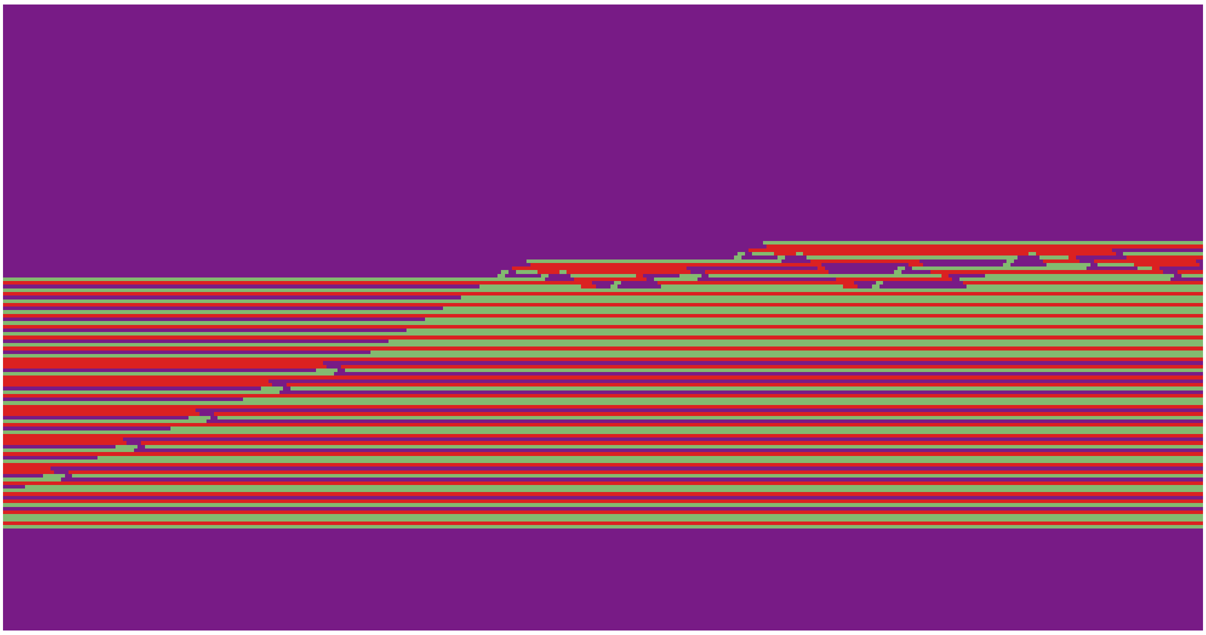
500−1000



1000−1500



2000−2500



9000−10000