Duc Viet Le

Use those board as base cases:

Board 3x4:

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 4 | 7 | 10 |
| 8 | 11 | 2 | 5 |
| 3 | 6 | 9 | 12 |

Board 3x7:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 14 | 17 | 20 | 9 | 4 | 7 |
| 16 | 19 | 12 | 3 | 6 | 21 | 18 |
| 13 | 2 | 15 | 18 | 11 | 8 | 5 |

Board 3x8:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 16 | 3 | 22 | 19 | 12 | 7 | 10 |
| 4 | 21 | 18 | 15 | 6 | 9 | 24 | 13 |
| 17 | 2 | 5 | 20 | 23 | 14 | 11 | 8 |

Board 3x13:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 4 | 13 | 16 | 21 | 8 | 23 | 18 | 35 | 38 | 27 | 32 | 29 |
| 12 | 15 | 6 | 3 | 10 | 17 | 20 | 37 | 24 | 33 | 30 | 39 | 26 |
| 5 | 2 | 11 | 14 | 7 | 22 | 9 | 34 | 19 | 36 | 25 | 28 | 31 |

***Lemma 1***: for any, there exists Hamilton path starts at (1, 1) and ends at (2, n-1)

Proof:

We can easily see that by connecting Board 3x4 and Board 3x7, we can construct such path for Board 3x11.

Same for Board 3x12 by connecting Board 3x4 and 3x8

Same for board 3x14 by connecting Board 3x7 and 3x7

Using same strategy: we can construct such path by breaking n down into base cases size:

For example:

Or by picture:

3x(11 or 12 or 13 or 14)

3x4

3x4

3x4

***Lemma 2***: for any, there exists Hamilton path starts at (1, n) and ends at (1, 1)

Proof:

We can easily construct such path by flip the board found in lemma 1 horizontally

Naïve Algorithm to find open tour for any board where

Case 1:

So we can divide the board into stripes of

And these stripes can be easily connected to each others.

But since structure of those stripes are exactly the same, except flipping order and the value of each square in the one stripe is greater than the value of each matched square in previous stripes. I am positive that we can use

Case 2:

Therefore, we can divide the board into (k-2) stripes of. We can use the same strategy used in case 1 for those. Until the very last stripe, now we have to find solution for the stripe.

But remember

Hence, the very last stripe can be divide into and one square board of

Again, we know board is one of our base case, and all we have to do is to solve the board.

Solution for 7 x 7 board can be found by using warnsdorff’s rule

Case 3:

Again, same idea used in case 2, we can reduce any size of board into board.

Below is visualization of case 2 and case 3: position of Orange Square depends on

7 or 8

7 or 8

3

3

3

7 or 8