Quick tips before trying to solve the OLG model. 1- Determine whether the zero steady state is possible:

if f(0) = 0 (or, equivalently, w(0) = 0), then the zero (autarky) steady state exists. It may be stable or unstable, but it exists.

Cobb-Douglas production:

fCobbDouglas[k_] = k^α;

fCobbDouglas[0] = 0

0

CES production with complementary inputs: ρ <0

Important remark: verify the exact functional form of the production function, in particular how the exponents are written.

fCES[k_] = ($\alpha * k$^ρ + (1 − α))^(1/ρ);

Limit[fCES[k], $k \to 0$, Assumptions → {$\rho < 0, \alpha > 0, \alpha < 1$}]

0

If f(0) > 0, then the zero (autarky) steady states does not exist.

Limit[fCES[k], $k \to 0$, Assumptions → {$\rho > 0, \alpha > 0, \alpha < 1$}]

$(1-\alpha)^{\frac{1}{\rho}}$

2- Determine whether the interest rate appears in the savings function:

if log-utility, the interest rate does not appear.

uprimeLogUtility[x_] = 1/x;

Solve[uprimeLogUtility[$w - s$] == $\beta * R *$ uprimeLogUtility[$R * s$], s]

$\left\{\left\{s \to \frac{w\beta}{1+\beta}\right\}\right\}$

Other functional forms: CIES, R appears in the savings function:

uprimeCIES[x_] = x^(−1/σ);

Solve[uprimeCIES[$w - s$] == $\beta * R *$ uprimeCIES[$R * s$], s]

$$\left\{\left\{s \to \frac{R^\sigma w \beta^\sigma}{R+R^\sigma \beta^\sigma}\right\}\right\}$$

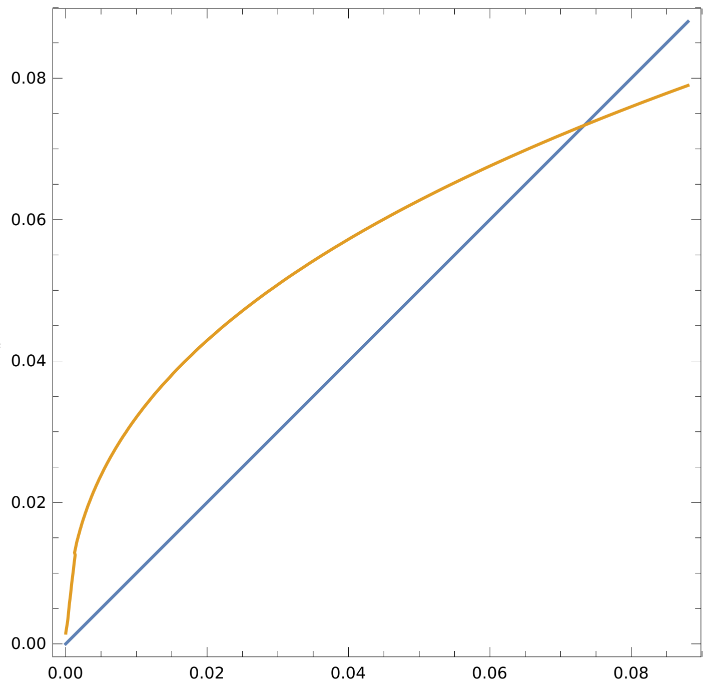
3 - Pick your own model!

```
f[k_, α_, ρ_] = If[ρ == 0, k^α, (α * k^ρ + (1 − α))^(1/ρ)];

u[x_, σ_] = If[σ == 1, Log[x], x^(1 − 1/σ)/(1 − 1/σ)];

wfunction[k_, α_, ρ_] = f[k, α, ρ] − D[f[k, α, ρ], k] * k;

Rfunction[k_, α_, ρ_] = D[f[k, α, ρ], k];

savings[w_, R_, σ_, β_]:=s/.Solve[D[u[w − s, σ], s] + β * D[u[R * s, σ], s] == 0, s][[1]]

derivative[k1_, k_, α_, β_, σ_, ρ_, n_]:= − D[k1 − 1/(1 + n) * (savings[w, R, σ, β]/.{w → wfunction[k, α, ρ], R → Rf
D[k1 − 1/(1 + n) * (savings[w, R, σ, β]/.{w → wfunction[k, α, ρ], R → Rfunction[k1, α, ρ]}), k1]

der[sols_, i_, α_, β_, σ_, ρ_, n_]:=Limit[derivative[k1, k0, α, β, σ, ρ, n], {k0, k1} → {k/.sols[[i]], k1 → k/.sols[[i]]}, Dir

stability[α_, β_, σ_, ρ_, n_]:=
(If[Length[NSolve[k == 1/(1 + n) * savings[w, R, σ, β]/.{w → wfunction[k, α, ρ], R → Rfunction[k, α, ρ]}, k, Rea
sols = NSolve[k == 1/(1 + n) * savings[w, R, σ, β]/.{w → wfunction[k, α, ρ], R → Rfunction[k, α, ρ]}, k, Reals]];
For[i = 1, i ≤ Length[sols], Print[" Steady state ", i, ", Capital level: ", k/.sols[[i]], ": Derivative=", der[sols, i, α
If[−1 < der[sols, i, α, β, σ, ρ, n] < 1, " Stable", " Unstable"]];
i++])

myprogram[α_, β_, σ_, ρ_, n_]:=(Print[
"Production function: f(k)=", f[k, α, ρ], "\n",
"Utility function: u(c)=", u[c, σ], "\n",
"Savings function: s(w(", Subscript[k, t], "),", Subsuperscript[k, "t+1", ""], "))= ", savings[w, R, σ, β], "\n",
"Substituting: s(w(", Subscript[k, t], "),", Subsuperscript[k, "t+1", ""], "))= ",
savings[w, R, σ, β]/.{w → wfunction[k, α, ρ], R → Rfunction[k, α, ρ]}, "\n",
"Steady States: Solve ", Subscript[k, "t+1"], " == 1/(1+n)s(w(", Subscript[k, t], "),", Subsuperscript[k, "t+1",
" With the parameters: ",
Quiet[If[Length[NSolve[k == 1/(1 + n) * savings[w, R, σ, β]/.{w → wfunction[k, α, ρ], R → Rfunction[k, α, ρ]},
```

```
NSolve[k == 1/(1 + n) * savings[w, R, σ, β]/.{w → wfunction[k, α, ρ], R → Rfunction[k, α, ρ]}, k, Reals]]], "\n",
"Stability: Compute the derivative of ", Subscript[k, "t+1"], " with respect to ", Subscript[k, t], " and evaluate
" With the parameters: "];
Print[Quiet[stability[α, β, σ, ρ, n]]];
solsSimple = k/.sols;
If[Max[solsSimple] == 0, maxk = 0.5, maxk = 1.2 * Max[solsSimple]];
Quiet[ContourPlot[{k1==k, k1 == 1/(1 + n) * (savings[w, R, σ, β]/.{w → wfunction[k, α, ρ], R → Rfunction[k1
```

```
myprogram[0.5, 0.8, 2, 0, 0]
```

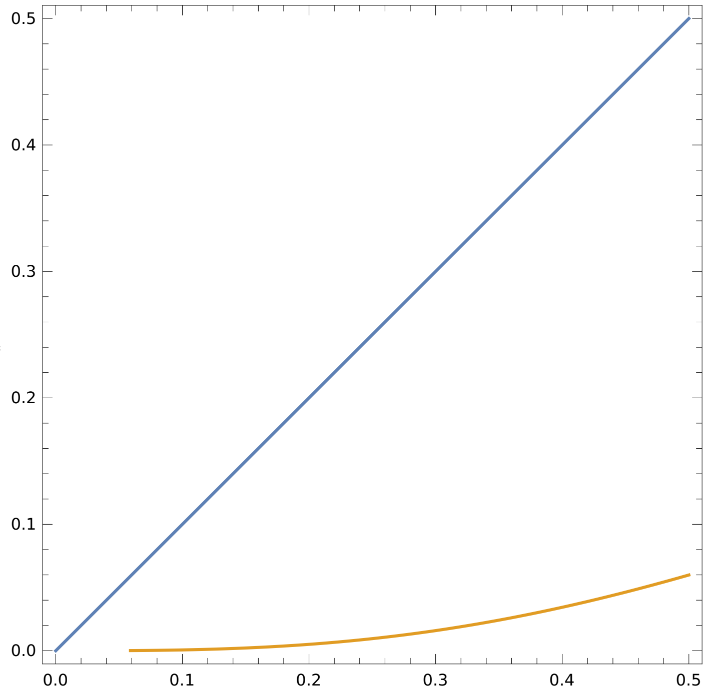
Production function: f(k)=$k^{0.5}$\nUtility function: u(c)=2$\sqrt{c}$\nSavings function: s(w(k_t),Subsuperscript[k, t+1
0.}, {$k \to 0.0733398$}}\nStability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each stea
Steady state 1, Capital level: 0.: Derivative=∞ Unstable
Steady state 2, Capital level: 0.0733398: Derivative=0.406773 Stable
Null

Out[ ]=



```
myprogram[0.5, 0.8, 2, −2, 0]
```

Production function: f(k)=$\frac{1}{\sqrt{0.5+\frac{0.5}{k^2}}}$\nUtility function: u(c)=$2\sqrt{c}$\nSavings function: s(w(k_t),Subsuperscript[

0}}\nStability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state\n With t
Steady state 1, Capital level: 0: Derivative=0. Stable
Null

Out[ ]=


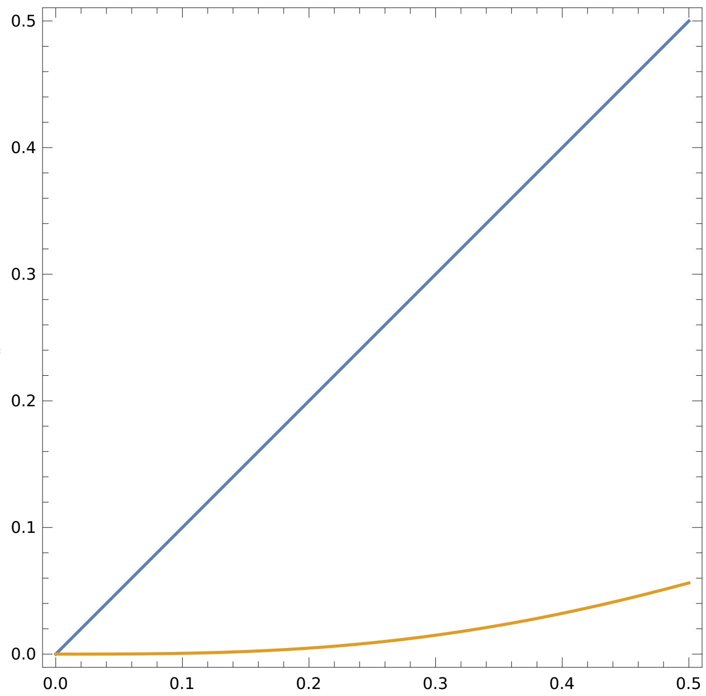
myprogram[0.5, 0.8, 1, −2, 0]

Solve : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a c
Solve : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a c

Production function: f(k)=$\frac{1}{\sqrt{0.5+\frac{0.5}{k^2}}}$\nUtility function: u(c)=Log[c]\nSavings function: s(w(k_t),Subsuperscrip

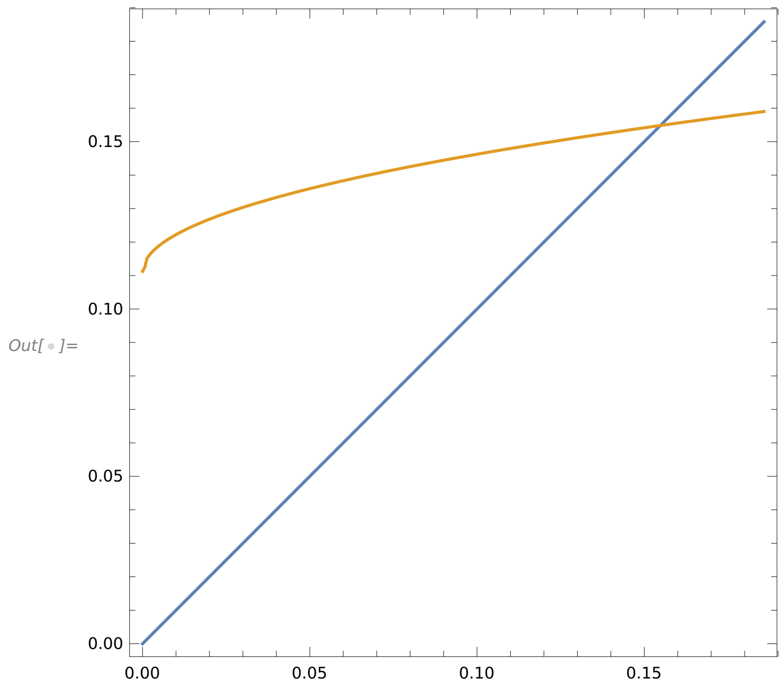
0}}\nStability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state\n With t
Steady state 1, Capital level: 0: Derivative=0. Stable
Null

Out[∘]=



myprogram[0.5, 0.8, 1, 0.5, 0]

Solve : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a c
Solve : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a c
Production function: f(k)= $\left(0.5\ + 0.5k^{0.5}\right)^{2.}$ \nUtility function: u(c)=Log[c]\nSavings function: s(w(k_t),Subsu
0.154832}}\nStability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state\n
Steady state 1, Capital level: 0.154832: Derivative=0.141188 Stable
Null

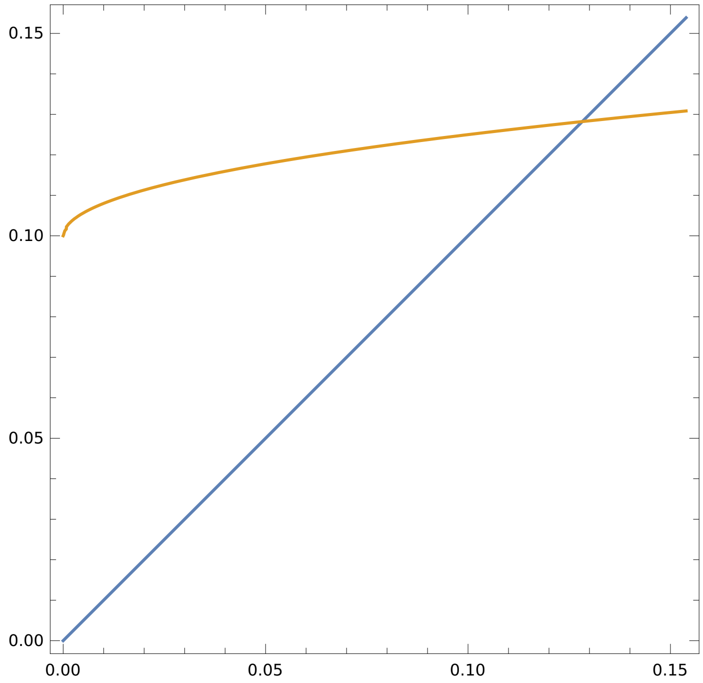


myprogram[0.5, 0.8, 2, 0.5, 0]

Production function: f(k)= $\left(0.5 + 0.5k^{0.5}\right)^{2.}$ \nUtility function: u(c)=$2\sqrt{c}$\nSavings function: s(w(k_t),Subsupe

0.128222}}\nStability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state\n
Steady state 1, Capital level: 0.128222: Derivative=0.107258 Stable
Null

Out[∘]=



myprogram[0.5, 0.8, 1, 0, 0]

Solve : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a c

Solve : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a c

Production function: f(k)=$k^{0.5}$\nUtility function: u(c)=Log[c]\nSavings function: s(w(k_t),Subsuperscript[k, t-

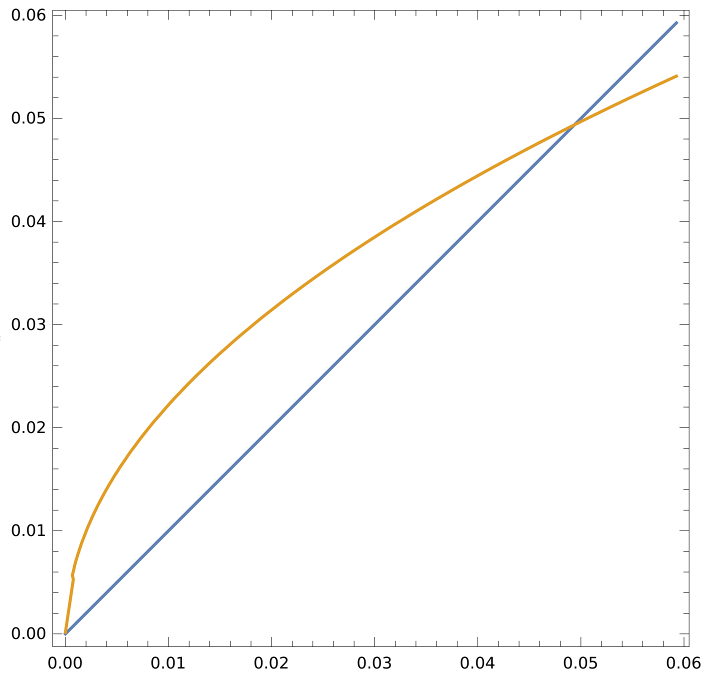
$0.\}, \{k \to 0.0493827\}\}$\nStability: Compute the derivative of $\mathrm{k}_{\mathrm{t}+1}$ with respect to k_{t} and evaluate at each stea

Steady state 1, Capital level: 0.: Derivative=∞ Unstable

Steady state 2, Capital level: 0.0493827: Derivative=0.5 Stable

Null

Out[ ]=



myprogram[0.1, 0.8, 2, −1, 0]

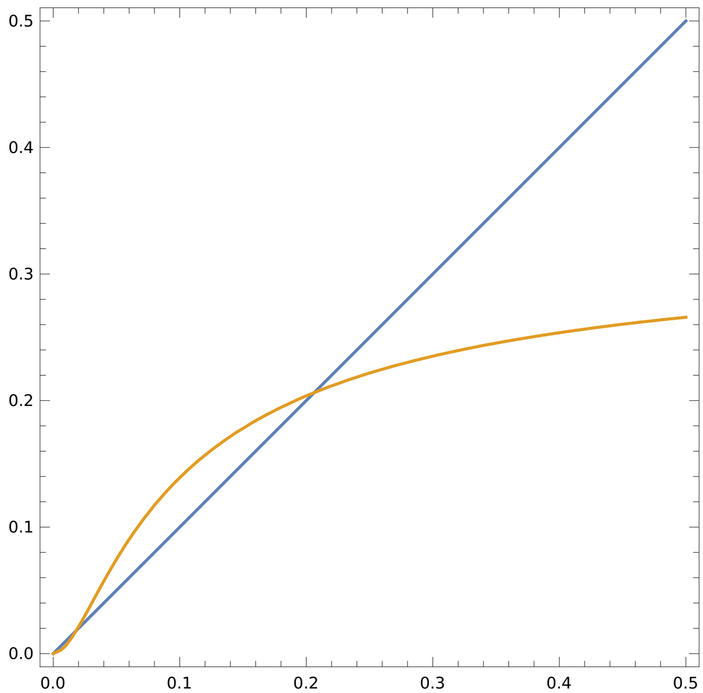
Production function: f(k)=$\frac{1}{0.9+\frac{0.1}{k}}$\nUtility function: u(c)=$2\sqrt{c}$\nSavings function: s(w(k_t),Subsuperscript[k,

0.018251}, {$k \to 0.206304$}}\nStability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at eac

$Aborted

ContourPlot[{k1==k, k1 == 1/(1 + 0) ∗ (savings[w, R, 2, 0.8]/.{w → wfunction[k, 0.1, −1], R → Rfunction[k1, 0

Out[ ]=



Export["xamples.pdf", EvaluationNotebook[]]

xamples.pdf